A STATISTICAL MODEL FOR COMBINING JAMB AND POSTJAMB SCORES: WITH AN ILLUSTRATION BASED ON UNIVERSITY OF LAGOS' 2006/2007 ADMISSIONS DATA

BY
RAY OKAFOR\textsuperscript{a}, ISMAILA ADELEKE\textsuperscript{b} AND ATHANASIUSS OPARA\textsuperscript{c}

\textsuperscript{a} Department of Mathematics, University of Lagos, Lagos, Nigeria
\textsuperscript{b} Department of Actuarial Science & Insurance, University of Lagos, Nigeria
\textsuperscript{c} Distance Learning Institute, University of Lagos, Nigeria

ABSTRACT

The aim of this research is to find a statistical model for combining JAMB and PostJAMB scores. The problem arises from the ad hoc manner in which most universities combine the two scores for admission purposes. Our model achieves a combined score using a most statistically efficient technique. We recommend it to the universities.
The University of Lagos (from henceforth UNILAG, for short) first proposed a follow-up test or further screening of prospective students who have written and "passed" the university matriculation examination (UME) which is given annually by the Joint Admission and Matriculation Board (JAMB). The latter is a parastatal of the Federal Ministry of Education mandated to conduct annual entrance examinations into institutions of higher learning in Nigeria. UNILAG and other Nigerian universities implemented the said screening in 2005/2006 academic year after it was approved by government. Prior to the advent of screening, JAMB's mandate was exclusive, so it vehemently opposed the idea. But the universities and others advocates of screening seem to have a cast-iron case. In recent years, many universities in Nigeria have complained about a pattern that seems to have emerged; a pattern in which many students admitted into the universities with very high JAMB scores performed well below expectation in their university examinations. This worrisome development got authorities in universities thinking, culminating in the proposition for a further test. Proponents argue that scores from the latter test will serve as an instrument for cross-validation of students' JAMB scores, and the end result of cross-validation cannot but have a salutary effect on students' later performances in their university examinations.

Having secured government approval for screening, the question begging for plausible answer is how best to use the result of screening to advance the admission process. We are persuaded that a uniform, well-thought-out and robust method of using screening scores is required. What happened in the last two admissions processes was that individual universities fashioned out their own methods of use. We believe that the latter
methods, mostly adhoc in nature, can be improved upon on the basis of statistical analysis.

This article proposes a statistical model for combining JAMB scores and the results of PostJAMB screening. In the frame work of the model, we propose a weighting scheme, whereby each prospective student’s JAMB score $X_{JM}$ and PostJAMB score $X_{PJM}$ are combined to yield a single (combined) score $X_w$ defined as

$$X_w = \lambda_1 X_{JM} + \lambda_2 X_{PJM}$$

(1)

where,

$\lambda_1$ and $\lambda_2$ are estimable constants that weight $X_{JM}$ and $X_{PJM}$ respectively. It is instructive that $\lambda_1$ and $\lambda_2$ are estimated entirely from the bivariate $(X_{JM}, X_{PJM})$ data, depending specifically on the variances of $X_{JM}$ and $X_{PJM}$ as well as covariance between $X_{JM}$ and $X_{PJM}$. Also, we show that the distribution of $X_c$ has small variances than that of $X_{JM}$. Consequently this method of combining the scores from JAMB and PostJAMB is plausible.

The remainder of the article is sectionalized as follows: we make an empirical comparison of the JAMB and PostJAMB scores in Section 2; in Section 3 we review strategies for combining scores; the proposed model is shown in Section 4 The article is concluded in Section 5 with a summary and comments

2. **Empirical Comparison of JAMB and POSTJAMB Scores**

A comparison of the distribution of JAMB and PostJAMB scores of five departments namely; Actuarial Science, Insurance, Mathematics, Computer Science, and Education
Admin Departments shows that the distribution of JAMB scores differs quite remarkably from that of PostJAMB scores in all the departments. The descriptive statistics shown in Table 1 below show that PostJAMB scores have greater variability than JAMB scores in all the departments. In Actuarial science for instance, the coefficient of variation (CV) of PostJAMB is 33.1% while that of JAMB is 9.64%.

Table 1
Comparison of JAMB and PostJAMB scores of Various departments using some statistical measures

<table>
<thead>
<tr>
<th>Departments</th>
<th>JAMB</th>
<th>PostJAMB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>10.27</td>
<td>32.21</td>
</tr>
<tr>
<td>Actuarial Science</td>
<td>9.64</td>
<td>33.1</td>
</tr>
<tr>
<td>Computer Science</td>
<td>10.15</td>
<td>29.27</td>
</tr>
<tr>
<td>Insurance</td>
<td>10.13</td>
<td>31.71</td>
</tr>
<tr>
<td>Education Admin</td>
<td>10.25</td>
<td>34.96</td>
</tr>
</tbody>
</table>

EDA has shown that the distribution of JAMB and PostJAMB scores differ from one another in all the departments. But the Scores claim to measure the performance of the same set of students. Greater care should therefore be taken to combine the two scores since they represent different distributions.

3. A Review of Strategies For Combining scores

Combining examination scores of candidates in more than one examination to get a final combined score for each candidate and the consequent final rank order poses a problem. When simply added or averaged together to get the combined score, “the components will not all contribute to the same extent”, (Adams & Wilmut, 1981). Also see (Adams,
R.M and Murphy, R.J.L 1980, Forrest, G.M., 1974, Thyne, J.M., 1966) for more insight. In two component examinations for instance, PAPER I may achieve greater weight in determining the final results than PAPER II even though the original intention of the examiners was that the two papers should contribute equally. One property of the marks on a component that determines its contribution to the aggregate is their dispersion. To illustrate this vividly, Adams and Wilmut have given the example shown in Table 2 where there are two papers. By virtue of their greater dispersion, the PAPER II marks so dominate the aggregate that the final rank order is the same as that resulting from PAPER II marks alone.

<table>
<thead>
<tr>
<th></th>
<th>Paper I</th>
<th>Paper II</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mark</td>
<td>Rank</td>
<td>Mark</td>
<td>Rank</td>
</tr>
<tr>
<td>72</td>
<td>(1)</td>
<td>12</td>
<td>(6)</td>
</tr>
<tr>
<td>71</td>
<td>(2)</td>
<td>16</td>
<td>(5)</td>
</tr>
<tr>
<td>70</td>
<td>(3)</td>
<td>20</td>
<td>(4)</td>
</tr>
<tr>
<td>69</td>
<td>(4)</td>
<td>24</td>
<td>(3)</td>
</tr>
<tr>
<td>68</td>
<td>(5)</td>
<td>28</td>
<td>(2)</td>
</tr>
<tr>
<td>67</td>
<td>(6)</td>
<td>32</td>
<td>(1)</td>
</tr>
</tbody>
</table>

A number of measures were proposed such as scaling to achieve intended weights. A weakness of this approach is that such intended weights are not data determined.

Our approach is quite different. Conceptually, we regard JAMB and PostJAMB as two examinations that measure the same thing which is students’ ability to secure admission and cope with subsequent university work. If measured on the same scale, the two sets of scores should be distributed alike. If they differ, the one with less variation has better information content regarding student’s relative abilities and should have greater weight in the combination.
4. Proposed Strategy for Combining JAMB and POSTJAMB Scores

There are two sets of examination scores for the same set of prospective students. One is from JAMB examination \( X_{JM} \) and the other from PostJAMB examination \( X_{PJM} \). The challenge is to find the best way the two scores can be combined into one score \( X_w \) so that decision on admission (or other decisions) can be made [about the students] on the basis of the single set of scores \( X_w \).

4.1 Definition of “Best”

Best in this regard can be viewed as a combination of \( X_{JM} \) and \( X_{PJM} \) which takes into account the relative precision (or reliability) of the information content of the two sets. If a set of figures has a larger variance, then it is less precise. It has low information content.

4.2 The Task

The above considerations imply that a weighted average of \( X_{JM} \) and \( X_{PJM} \) which minimizes overall variation of \( X_w = \lambda_1 X_{PJM} + \lambda_2 X_{JM} \) would certainly give more weight to the more precise variable.

4.3 Variation and Scale of Measurement

Variation is affected by the scale of measurement. Hence, the two sets of figures XJM(i) and XPJM(i) need to be converted to the same scale before the analysis can be done. JAMB is a single examination with four papers. The English paper is compulsory for all candidates while the remaining three papers are drawn from the candidate’s intended discipline and/or related disciplines in the University. The maximum JAMB score is 400. The University of Lagos’ PostJAMB examination consists of three papers, English and two other papers drawn from disciplines like in the JAMB case. Maximum PostJAMB
score is 100. Transformation to scale of JAMB scores means multiplying students’ PostJAMB scores by 4.

4.4 Result

\[ X_w^{(i)} = \hat{\lambda}_1 X_{PJM}^{(i)} + \hat{\lambda}_2 X_{JM}^{(i)} \]

is the best linear combination where estimates \( \hat{\lambda}_1 \) and \( \hat{\lambda}_2 \) of \( \lambda_1 \) and \( \lambda_2 \), respectively are given as

\[
\hat{\lambda}_1 = \frac{k_3 - k_2}{2k_3 - k_2 - k_1}
\]

\[
\hat{\lambda}_2 = \frac{k_3 - k_1}{2k_3 - k_2 - k_1}
\]

\( k_1 = \text{Var}(X_{PJM}^{(i)}) \)

\( k_2 = \text{Var}(X_{JM}^{(i)}) \)

\( k_3 = \hat{\rho}_{PJM,JM} \sqrt{\text{Var}(X_{PJM}^{(i)}) \text{Var}(X_{JM}^{(i)})} \)

\( \hat{\rho}_{PJM,JM} \) is the correlation coefficient between \( X_{PJM}^{(i)} \) and \( X_{JM}^{(i)} \)

Proof

Let \( X_w^{(i)} = \hat{\lambda}_1 X_{PJM}^{(i)} + \hat{\lambda}_2 X_{JM}^{(i)} \)

\[
E(X_w^{(i)}) = E[\hat{\lambda}_1 X_{PJM}^{(i)} + \hat{\lambda}_2 X_{JM}^{(i)}]
\]

\[
= \hat{\lambda}_1 E(X_{PJM}^{(i)}) + \hat{\lambda}_2 E(X_{JM}^{(i)})
\]

\[
\text{Var}(X_w^{(i)}) = E\left[X_w^{(i)} - E(X_w^{(i)})\right]^2
\]

\[
= \hat{\lambda}_1^2 E\left[X_{PJM}^{(i)} - E(X_{PJM}^{(i)})\right]^2 + \hat{\lambda}_2^2 E\left[X_{JM}^{(i)} - E(X_{JM}^{(i)})\right]^2 + 2\hat{\lambda}_1\hat{\lambda}_2 E\left[(X_{PJM}^{(i)} - E(X_{PJM}^{(i)}))(X_{JM}^{(i)} - E(X_{JM}^{(i)}))\right]
\]
\[ \text{Var}(X_w^{(i)}) = \lambda_1^2 \text{Var}(X_{PJM}^{(i)}) + \lambda_2^2 \text{Var}(X_{JM}^{(i)}) + 2\lambda_1\lambda_2 \text{Cov}(X_{PJM}^{(i)}, X_{JM}^{(i)}) \]

\[ \text{Var}(X_w^{(i)}) = \lambda_1^2 \text{Var}(X_{PJM}^{(i)}) + \lambda_2^2 \text{Var}(X_{JM}^{(i)}) + 2\lambda_1\lambda_2 \hat{\rho}_{PJM,JM} \sqrt{\text{Var}(X_{PJM}^{(i)})\text{Var}(X_{JM}^{(i)})} \]

where \( \hat{\rho}_{PJM,JM} \) is the correlation coefficient between \( X_{PJM}^{(i)} \) and \( X_{JM}^{(i)} \)

Let:

\[ k_1 = \text{Var}(X_{PJM}^{(i)}) \]

\[ k_2 = \text{Var}(X_{JM}^{(i)}) \]

\[ k_3 = \hat{\rho}_{PJM,JM} \sqrt{\text{Var}(X_{PJM}^{(i)})\text{Var}(X_{JM}^{(i)})} \]

\[ \text{Var}(X_w^{(i)}) = k_1\lambda_1^2 + k_2\lambda_2^2 + 2\lambda_1\lambda_2 k_3 = \phi(\lambda_1, \lambda_2) \]

We minimize \( \phi(\lambda_1, \lambda_2) \) subject to \( \lambda_1 + \lambda_2 = 1 \)

We apply the principle of Lagrange multiplier

\[ \phi^*(\lambda_1, \lambda_2) = k_1\lambda_1^2 + k_2\lambda_2^2 + 2\lambda_1\lambda_2 k_3 + m(\lambda_1 + \lambda_2 - 1) \]

\[ \frac{\partial \phi^*}{\partial \lambda_1} = 2\lambda_1 k_1 + 2\lambda_2 k_3 + m = 0 \quad (1) \]

\[ \frac{\partial \phi^*}{\partial \lambda_2} = 2\lambda_2 k_2 + 2\lambda_1 k_3 + m = 0 \quad (2) \]

and

\[ \lambda_1 + \lambda_2 = 1 \quad (3) \]

Solving the above system of linear equations we obtain

\[ \hat{\lambda}_1 = \frac{k_3 - k_2}{2k_3 - k_2 - k_1} \]

\[ \hat{\lambda}_2 = \frac{k_3 - k_1}{2k_3 - k_2 - k_1} \]
4.5 Theorem

The formula for $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are valid only when $-1 < \hat{\rho} < 1$.

Proof

Both $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are undefined if $2k_3 - k_2 - k_1 = 0$

Suppose $2k_3 - k_2 - k_1 = 0$, leads to absurdity, then $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are valid for values of $\hat{\rho}$ for which the absurdity holds.

$$2k_3 - k_2 - k_1 = 0 \Rightarrow 2\hat{\rho}_{\text{PJM,JM}} \sqrt{\text{Var}(X^{(i)}_{\text{PJM}}/\text{Var}(X^{(i)}_{\text{JM}}))} = \text{Var}(X^{(i)}_{\text{PJM}}) + \text{Var}(X^{(i)}_{\text{JM}})$$

$$2\hat{\rho} = \frac{\sqrt{\text{Var}(X^{(i)}_{\text{PJM}})}}{\sqrt{\text{Var}(X^{(i)}_{\text{JM}})}} + \frac{\sqrt{\text{Var}(X^{(i)}_{\text{JM}})}}{\sqrt{\text{Var}(X^{(i)}_{\text{PJM}})}}$$

Let $\frac{\sqrt{\text{Var}(X^{(i)}_{\text{PJM}})}}{\sqrt{\text{Var}(X^{(i)}_{\text{JM}})}} = u$

Then $2\hat{\rho} = u + \frac{1}{u} \Rightarrow u^2 - 2\hat{\rho}u + 1 = 0$

$$u = \frac{2\hat{\rho} \pm 2\sqrt{\hat{\rho}^2 - 1}}{2}$$

$$u = \hat{\rho} \pm \sqrt{\hat{\rho}^2 - 1}$$

Thus, for $-1 < \hat{\rho} < 1$, $u$ is not in $\mathbb{R}$ which is absurd.

Hence for $-1 < \hat{\rho} < 1$; $2k_3 - k_2 - k_1 \neq 0$ which means that

$$\hat{\lambda}_1 = \frac{k_3 - k_2}{2k_3 - k_2 - k_1} \quad \text{and} \quad \hat{\lambda}_2 = \frac{k_3 - k_1}{2k_3 - k_2 - k_1}$$

are valid
4.6 Application

The method was applied to the five departments, and the resultant weights are shown in Table 3. PostJAMB weights are uniformly lower than JAMB weights because PostJAMB scores are more variable.

<table>
<thead>
<tr>
<th>Departments</th>
<th>$\lambda_1$ (JAMB)</th>
<th>$\lambda_2$ (PostJAMB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>0.82</td>
<td>0.19</td>
</tr>
<tr>
<td>Actuarial Science</td>
<td>0.85</td>
<td>0.15</td>
</tr>
<tr>
<td>Computer Science</td>
<td>0.83</td>
<td>0.17</td>
</tr>
<tr>
<td>Insurance</td>
<td>0.61</td>
<td>0.39</td>
</tr>
<tr>
<td>Education Admin</td>
<td>0.78</td>
<td>0.22</td>
</tr>
</tbody>
</table>

The JAMB and PostJAMB scores of each department were combined using $\lambda_1$ and $\lambda_2$ to obtain weighted scores. They were also combined using simple average (average scores). A comparison of the combined scores with JAMB scores was made and the weighted scores have the lowest variation as shown by the respective coefficient of variation in Table 4.

<table>
<thead>
<tr>
<th>Departments</th>
<th>CV(%)</th>
<th>JAMB</th>
<th>Weighted scores</th>
<th>Average scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>10.27</td>
<td>9.96</td>
<td>14.27</td>
<td></td>
</tr>
<tr>
<td>Actuarial Science</td>
<td>9.64</td>
<td>9.43</td>
<td>14.65</td>
<td></td>
</tr>
<tr>
<td>Computer Science</td>
<td>10.15</td>
<td>9.97</td>
<td>13.98</td>
<td></td>
</tr>
<tr>
<td>Insurance</td>
<td>10.13</td>
<td>12.06</td>
<td>14.03</td>
<td></td>
</tr>
<tr>
<td>Education Admin</td>
<td>10.25</td>
<td>9.69</td>
<td>13.98</td>
<td></td>
</tr>
</tbody>
</table>
5. **Summary and Conclusion**

This article has dealt with finding a statistics model for a combined score for JAMB and PostJAMB. Thus said, we wish to emphasize that we were not concerned about the use of the achieved combined score in the admission process. It is left to individual universities to decide on how best to use the combined scores. What motivated this research was our knowledge that the use of PostJAMB scores in most universities in the first two editions of PostJAMB scores was completely ad hoc, not analytical, in nature. What we have achieved in this article is to combine two scores using the most statistically efficient techniques. The weighted score for each prospective students draws from the abilities of the candidate in both exams. Consequently we recommend that universities should use the weighted scores in preference to either JAMB or PostJAMB scores.

**Reference**