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BEYOND EQUATIONS AND FORMULAE:
OUR WORLD OF MATHEMATICS

BY

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BEYOND EQUATIONS AND FORMULAE:
OUR WORLD OF MATHEMATICS

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by

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PROLOGUE: HOW IT ALL BEGAN

In the early hours of Sunday, November 10, 1946, a baby boy was born. The usual joy and activities associated with the birth of a new child filled the air, but behind the excitements was a hidden fear; an anxiety over his fate. The reason for this was simple: all previous children of this couple did not live to maturity. This baby was, therefore, not given much chance in their expectationsof surviving beyond a few days or weeks.

When on the fifth day the baby was still breathing, a name had to be given to it as was (and still is) the custom of the village. Not wishing to give another name that might soon agree with a child expected to depart soon, the baby was given a circumstantial name, Sunday. Contrary to all expectations, however, the baby continued to thrive and grow to become a fairly healthy little boy while the parents prayed daily that all might be well. As the child grew up, it was soon noticed that he had an unusually good memory plus a sharp eye for details, two attributes considered remarkable for a boy of his tender age. While some interpreted these things as normal, others saw them as traits often exhibited by children destined not to live long. Although the boy suffered from all imaginable and possible childhood diseases that should ordinarily be enough to confirm this latter view. The child, however weathered the storms and passed the first challenge of staying alive in his village community.

At about the age of four, without any exposure to the four walls of a classroom, the little boy started to scribble and draw with charcoal on every available wall space in the family house, much to the annoyance of everyone who saw no reason for his defacing an otherwise beautiful, brown and black colour of the 'painted' walls. Spanking and rebuking him for that habit did not deter him because the scribblings and drawings appeared as often as they were cleaned by his mother.

Ostensibly, to save the walls and to calm nerves, the boy was sent to live with his maternal grandmother in a nearby village. There, the scribbling continued and was even encouraged by his youthful uncle, while the grandmother cheerfully cleaned after him as often as necessary with little or no complaint from her.

It was the practice that on most Sundays, the boy would be brought by his maternal uncle to visit his parents. On one such Sunday, the headmaster of a nearby village Catholic School came to visit the family with two of his teachers. The oldest member of the family took the opportunity to summon the little boy before the headmaster, with the aim of persuading him to enroll the boy in his school.

The headmaster took a quick look at the small and under-aged boy and concluded that it would be futile asking him to do the test of "put your right hand across your head and touch your left ear". Consequently, the headmaster, having been briefed about the boy's activities in defacing walls, decided to give instead a simpler test of asking the child to state his full names. The test was easy as the boy confidently blurted out in Yoruba language, 'Oruko mi ni Sunday Adepoju'. The test passed and the school master impressed, the boy was asked to report to school on trial the following week, on Monday. He was therefore, enrolled a week later into Infant one class of a Catholic School in 1952. Thus began the boy's journey to a formal education.

The boy's experience at school soon changed the character of his scribblings. Having learnt the alphabet and the first ten counting numbers, and before the teacher could introduce his class to more numbers, the boy had 'invented' what he considered to be the logical sequence of the next set of counting numbers. His scheme looked like this:

1	2	3	4	5	6	7	8	9	10
101	102	103	104						1010

These numbers he pronounced as they appear in the second line:
Ten-one, Ten-two, Ten-three, Ten-four,.... Ten-Ten

After the number "1010", his next sequence was made up as follows:
10101, 10102, 10103, 10104 101010
These numbers were pronounced, just as before, in the way he put them down:
Ten-ten-one, Ten-ten-two, Ten-ten-three, ... Ten-ten-ten.
The numbers were followed again by another new set as indicated here:
1010101, 1010102, 1010103, 1010104 ... 10101010 etc.

Every available wall space was filled with this scheme and the boy would gleefully recite his work before his father, uncles, and friends every evening continuing from where he stopped last night. This was before he learnt from his teacher that his 'Ten-one' Ten-two, ...'Ten-ten', ... 'Ten-ten-ten' had special names of 'Eleven, Twelve, ... 'Twenty',...'Thirty', and so forth.

His father, on noting this strange unending pattern of numbers, once asked the boy when his scheme would come to an end. The boy's reply was to the effect that he could continue the process *ad infinitum* since he was merely using the first ten counting numbers repeatedly. Unknown to the boy, this was a correct observation about the infinite property of the counting numbers, a fact he would learn to prove several years later.

It is of course a fact that the words 'Eleven', 'Twelve' .... Twenty.... etc. as used to this day were coined for the sake of convenience and not because they are a logically consistent scheme for the sequence of numbers. Indeed, it is the 'naïve' scheme of 'Ten-one', 'Ten-ten-two', 'Ten-ten-ten' that is apparently more consistent though a lot more cumbersome. We shall return to the story of the boy later in the lecture.

THE CHALLENGES OF THIS LECTURE

Any mathematician called upon to give a lecture on a topic within his discipline to a general audience, faces such a task with some trepidation. The reason is that mathematicians always have problems in explaining their profession and discipline to non-mathematicians, even though the power and beauty of mathematics abound everywherefrom the mundane issues that are taken for granted to the most sophisticated breath-taking world inventions and discoveries. For other reasons to be discussed later, most professionals do not usually face the same kinds of problem or dilemma at anytime.

The following anecdote typifies the dilemma of the mathematician. A group of four professionals were invited to give a talk on their professions and their impacts on society. The first professional to talk was a medical doctor. He had on his body, a clean white overall on a suit with a bow tie, several fountain pens sticking out of his white overall's breast pocket, and a stethoscope dangling on his neck. He gave a talk on the diagnosis, treatment and management of some diseases. For the diseases chosen, which included cancer, he described the usual symptoms as well as causes and treatment. He showed slides and videos of patients with the diseases, and described how they were treated. For attaining a maximum effect, such slides included those of some patients with carcinoma of the jaw before and after surgery. To add an academic flavour to the presentation, he gave some statistics on the incidence and prevalence of such ailments in the population. He concluded with general advice on early detection and treatment of those diseases. By the time he ended his lecture everyone understood him and many people wished they or their own children were doctors!

The next speaker was a lawyer. He came to his lecture wearing a black suit on a starched white shirt and collar. He spoke about how to win cases on technicalities, when facts alone would not do the trick. For attaining a maximum effect, he interjected high-sounding Latin phrases and words into his presentation and cited famous cases to support his argument. Everyone present, including conmen and women, politicians, land grabbers, etc., listened with rapt attention. He described in detail some specific cases he won just by using his tricks. He ended his talk by discussing the position of the law on human rights abuses and advised listeners on what to do when their rights were being abused, especially by the police. Everyone could relate to and understand him as many admired the profession and wished they were lawyers.

Then came the turn of the mathematician. He wore a simple French suit with thick eye-glasses and holding some sheets of paper full of equations, notations and formulae. Noticing a blackboard not used by any of the previous speakers, he requested for some chalk and a duster. He then walked confidently to the blackboard, writing at the same time as he began his lecture:

> Let $H_n(z)$ be the discrete Heisenberg group. Then the following results are clearly obvious and trivial:
>
> - $H_n(z)$ is a subgroup of $H_n(R)$ which is a smooth manifold of dimension $2n+1$
> - The group operation is smooth
> - When $n = 1$, this manifold has dimension 3, the type made famous by people like Thurston

He went on to state what he would show in the lecture that, contrary to current thinking, an important relationship exists between Heisenberg groups, Fourier analysis and several complex variables.(cf [71], [83])

The whole room was electrified by what he had just said, or not said, and many wondered aloud. "Why can't this man do what the previous speakers did by making it possible for the layman to understand him?" "Who is Heisenberg?" "What is a group?" Meanwhile, others were adjusting their seats and sitting positions for a good snooze through an impending ordeal!

My first and immediate challenge, therefore, is to avoid a similar situation and give a lecture that will not put most people to sleep. But if this one must cause any sleep it should be such that the period would not be long enough for them to snore. Determined to achieve this goal, I will in the course of the lecture keep my audience awake through the Vice Chancellor with a louder than usual call of 'Mr. Vice-Chancellor, Sir'!

OBJECTIVES OF THE LECTURE

Apart from the objective of paying the obligatory debt of an inaugural lecture which every Professor owes, this inaugural lecture is aimed at:

a) a treatise that will be intelligible and accessible to a general audience comprising the specialist, the novice, the curious and the uncommitted alike, without sacrificing rigour and accuracy so that everyone would have something to take home at the end of it all;

b) developing a taste of mathematics in the audience by making a case for better understanding and appreciation of mathematics as a discipline with beauty, power and universality through anecdotes and selected examples from everyday phenomena to which an ordinary person can relate;

c) sharing with the audience the Lecturer's mathematical experience and contribution to the corpus of mathematics literature, and finally;

d) serving as a valediction of a mathematician with sojourn in Administration for over eight years.

MAN AND NUMBERS

Mathematics has been part of mankind's experience since about 3,500 B.C. when man learned to keep a tally of things, counting his sheep or cows, using his fingers, toes or sticks, making marks and symbols on walls or rocks. Since then he has experienced and appreciated the social and practical influence of mathematics and its place in the development of man's way of life. The ability to count has enhanced his skill in calculation, and this is perhaps the reason why the first reaction of most people to the word 'mathematics' is to think of it as meaning numbers and calculations.

There are three main reasons for this reaction. The first is that though we live in a highly mathematical world, the total import and power of mathematics are rarely put on full display as they take the 'back seat' in

order not to make our world too complicated but rather more 'user-friendly' and less frightening. The second is that numbers as the most basic mathematical ideas open to our world need early introduction to our experience, since without them we would be unable to do many mundane things, such as when we count our eggs or conduct census and (successfully) carry out transactions involving simple purchase processes. The third reason for the reaction is that a mathematical knowledge, need and appreciation of most people may rarely go beyond the concept of numbers.

While many of us may not need more than arithmetic processes in our daily lives or professions, our society and culture need more than such limited tools and materials. Indeed, if this had been the highest level of mathematics available, the world would have remained unprogressive or unchallenging and consequently very dull. However, this has not been the situation because the world has continued to change and develop over the centuries; it was, for instance, spurred on in part by the gains and experiences of the industrial revolution in Europe, and by the world wars. These things led to the need for rapid acquisition of new skills and new demands on society, and in turn, have necessitated higher levels of mathematical and scientific training.

Thus, while numbers are considered the 'heart' of mathematics out of which a lot of mathematics developed, they can however form only a relatively small part of the discipline and there are more to mathematics than numbers and calculation. Indeed, there is more to life than drugs, politics, and terrorism, … There is mathematics!

THE SUBJECT MATTER OF MATHEMATICS

What then is mathematics? There is no single and all- embracing answer. Mathematics means many and varied things to different people in view of its universality and diverse applications. We shall, therefore, give here only a summary of classification of the different views and definitions of the subject matter of mathematics as expressed by different people at different times.

(a) ***To those that view mathematics in the context of its role in understanding nature, patterns and in general, the universe, "Mathematics" is and means any of the following: ([57], [79] [12])***

- "The loom upon which God weaves the fabric of the universe"; "the science of patterns"; "a conducting thread connecting scientific ideas with the understanding of our environment"; and "a formal system of thought for recognizing, classifying and exploiting patterns developed by the human mind and culture."

Into this category fits the statement of Galileo (1564-1642) that "the universe is a grand book which cannot be understood unless you understand the language of mathematics in which it is written." It is also in this context that the world is believed to be "built from the threads of mathematics".

(a) ***To those who see mathematics as a practical subject, mathematics is: (cf [21], [79], [65])***

- "a systematic way of digging out the rules and structures that lie behind some observed patterns or regularity, and then using those rules and structures to explain what is going on."

- "a landscape, with an inherent geography that its users and creators employ to navigate through what would otherwise be an impenetrable jungle".

- "the ultimate mental technology transfer in which ways of thinking rather than machines are being transferred."

- "the science that yields the best opportunity to observe the workings of the mind ... and has the advantage that by cultivating it, we may acquire the habit of a method of reasoning which can be applied afterwards to the study of any subject and can guide us in the pursuit of life's object."

- "the language of science and technology."

(b) ***To the logicians, poets, literary scholars, as well as philosophers, "Mathematics" is seen as (cf [12], [30], [26], [54]).***

- "The science of quantity"; "the science of counting"; "a subject concerned with goodness and beauty"; "the Queen and servant of Science"; "Science of self evident things"; "the nearest thing to an international language"; "the most fascinating mental game ever created by man"; "a discipline that includes every subject in which you attempt to reason logically from explicitly recognized underlying assumptions as expected in any subject forming part of the search for truth"; "International activity"; "a necessary evil".

Into this category fits the poetic statement of Prochus (410-485) about mathematics (cf [66]):

"She reminds you of the invisible form of the soul;
She gives life to her own discoveries;
She awakens the mind and purifies the intellect;
She brings light to our intrinsic ideas;
She abolishes oblivion and ignorance which is ours by birth."

From the foregoing views on mathematics, there are some recurring concepts, beliefs and sentiments of varying importance about the discipline. These are that mathematics is a science, a language, an international activity, an art, a system of thought, a landscape and comforter of the 'invisible soul'. No other discipline, to the best of my knowledge, has so much universality measured by many varied views and definitions which are expressed about it as mathematics does.

You may now wish to ask for my own definition of and views on mathematics. While agreeing with all or most of the foregoing, I will take a practical view in the context of Nigerian society. In this regard, therefore, mathematics is to me one of the disciplines that some of our best minds must be encouraged to study and apply to the highest possible level if we are ever to rise above the status of a trading-post nation. For as Chike Obi [18] once remarked: "Any fool can be a diplomat and babble in the United Nations but it requires some innate superiority to invent a themonic value". This 'innate superiority' can only be acquired by harnessing the skills of our best minds in the fields of mathematics, science and technology.

WHO NEEDS MATHEMATICS?

Who needs mathematics and why should anyone be interested in or bothered with a discipline that most people perceive to be unusually difficult, especially when its direct impact on society is often not as felt as in other "easier" fields of study? Surprisingly, this feeling and belief date back to the time of Ptolemy (A.D. 150) and Euclid (300 B.C.). History has it [12] that King Ptolemy, on having problems with mathematics, asked Euclid if there were no easier ways to understand the subject. Euclid replied: "There is no royal road to Geometry" ([Geometry can be taken as the general name given to mathematics at the time]. If, therefore, you are one of those who still consider mathematics too difficult to be worth your while and long for an easier approach to its study and understanding, then you must share that trait with a famous ancient king!.

Perhaps we could consider here some reasons why most people have so much phobia and dislike for mathematics in particular, and for the mathematical sciences generally.

The first reason is the vocabulary used by mathematicians. This vocabulary comprises words needed and used to denote concepts. Such words are, unfortunately, considered perplexing and odd by many, even though they are no more strange or more difficult than the vocabulary which a student of a foreign language encounters. To mathematicians, there is no reason for this difficulty. The second reason is the use of

symbols. Again, for the mathematician, this difficulty is more apparent than real. The use of symbols is aimed at precision, generalisation, and brevity of words. The general idea is that whenever one looks for a solution to a problem that requires the determination of an unknown result on the basis of some given facts or information, it is more convenient and precise to represent the unknown by any variable (say x or y or t etc) and then use the given facts to determine the unknown quantity. Since mathematical thinking and activity tell a story, to appreciate the story, all students of mathematics need to learn and understand the symbols and vocabulary which are meant to clarify or simplify, and to make precise some statements and ideas that would otherwise be unnecessarily complicated and long. Beginners need not panic about this as an open mind and some patience are all that are needed to overcome the difficulty.

The third problem is in the use of abstraction by mathematicians. Abstraction enhances the development of logical reasoning since it necessitates the justification of processes and facts by means of proofs. This is, perhaps, one of the most useful and challenging creations of mathematicians. In the formulation of abstractions, which are drawn from experience, mathematicians keep the real and the physical objects in mind. We shall subsequently expatiate more on abstraction as an important tool for mathematicians in the lecture.

The fourth problem is the manner in, and the pace at which mathematical materials are presented to learners by those who try to impart the knowledge and skills of the discipline at various levels. Instilling fear in the learner and skipping steps create a serious gap in the understanding of mathematics which in turn may lead to frustration and hardship for the learner.

The other problems that are physical and psychological in nature can be manifested in conditions usually associated with children. They have serious implications in terms of the ability of children and for the ordinary people with these conditions attempting to learn mathematics. These conditions are known as mathematical dysfunctions and there are four common ones. They are dyslexia, dysgraphia, dyscalculia and mathematical anxiety (cf [73]).

Dyslexia is the inability to translate printed language such as mathematical symbols into meaning. Children with this problem may see certain letters, numbers and symbols backwards or upside down. Such children have problems in carrying out sequential instruction, which is an essential element in the learning of elementary mathematics and in the execution of procedural mathematics.

Dysgraphia involves a faulty control of the muscle system used to encode letters and numbers accurately. Children with this condition are unable to copy or draw simple geometric shapes without any distortion.

Dyscalulia is a disorder which leads to the inability to learn arithmetic processes. It is characterised by retardation in calculation achievement resulting in a child with such condition being several years behind his peers.

Mathematical anxiety is the manifestation of the fear of mathematics. A person or child with this problem avoids contact with mathematics. Its manifestation ranges from a mild displeasure in dealing with quantitative information to a complete avoidance of anything mathematical. A mere mention of the word "mathematics" may lead to pathological signs of perspiration, sweating palms, fidgeting and general sense of discomfort.

Consistent failure in mathematics for a period of time often results in mathematical anxiety.

These conditions have to be identified early among children and young adults. Studies have shown [73]] that one way to assist such young people is for a teacher to teach the content (concepts, skills and procedures) first and then develop appropriate cognition skills later, in order to increase the potential and abilities of such people to learn mathematics. This is because it is known [73] that as a child's cognitive functioning improves, so will his/her potential for learning mathematics.

A long-term solution is for attention to be paid to the growing body of knowledge on how young children learn mathematics. Other nations of the world are making progress in this direction with their children [56], [75], [74], [88].

We therefore need our educational psychologists, mathematicians, and physicians to go into basic research to improve the understanding of our children's learning disability in mathematics.

We have not yet answered the question of who needs mathematics. My submission is that everybody needs mathematics. Indeed, as Hardy once remarked 'mathematics has tremendous effect on the daily avocations of man and on the organization of society' [30]. We do not all, however, need the same type and level of mathematics in our daily lives and professions.

- The cook, housewife or chef needs some mathematics to prepare a pot of good soup. (Mixing various condiments in the right proportions is bound to involve mathematics).

- The butcher or fishmonger needs the concepts of weight, measurement and proportions to sell meat or fish appropriately.

- The tailor needs the ideas of measurement, symmetry and proportion to sew good fitting suits and dresses.

- The carpenter needs basic ideas of geometry, symmetry and measurement to construct your house-roof, and make good furniture.

- Nurses need the knowledge of proportion, basic arithmetic, percentages and ratios to prepare and administer correct dosages of drugs and injections at appropriate intervals.

- Bankers and accountants need to know simple and compound interest and arithmetic processes.

Doctors need probability theory and basic statistics to improve the diagnosis of diseases, and they need to have a knowledge of the prevalence and incidence of diseases in their epidemiological studies.

Lawyers and judges need basic logic, probability and decision theories to enhance their arguments and judgments in courts.

Even the simple barber needs some mathematics! Mathematics, for example, tells the barber that in order to obtain a 10% solution from, say a 25% and a 5% solution of mentholated spirit he/she needs to mix the two in the ratio 1:3.

The mathematical theory of optics tells a stage designer or home decorator why it is false to believe that the lower the lighting source is to the ground, the better the illumination - a fact that assists us to arrange proper lighting in our homes and offices. Mathematics can also help to arrange sound systems in our homes or places of religious worship to obtain maximum loudness. It is this latter knowledge that is lacking in our musicians when in setting up their musical instrument, they spend hours screaming 'Testing, Testing' or 'One-two, One two'!

The ordinary man or woman in the street requires consumer mathematics which entails the ability to do basic arithmetic, calculate percentages, compute discounts and averages, and estimate unit prices of goods and items in the market-place as well as interpret tables and graphs.

One immediate advantage of this skill is the ability to compare prices and buy right by knowing when the small size purchase may or may not be cheaper than the large size.

Indeed, people often believe that a small-size purchase is cheaper than a large-size purchase. If you are one of such people, take a look at the following and you will discover why the poor always pay more than the rich for the same item.

ITEM	SIZE	PRICE (N) (N)	PRICE/ UNIT (N)	REMARKS
Honey in Plastic Container	500ml 120ml	450 110	0.90 0.92	Larger size cheaper
Ovaltine in Jar	400g 200g 1,200g	360 220 1,035	0.90 1.10 0.86	Largest size cheaper
Milo in Jar	450g 900g	325 600	0.72 0.67	Larger size cheaper
Mayonnaise	946ml 473ml	345 210	0.36 0.44	Larger size cheaper
Wheetabix	48 Pieces 24 Pieces 12 Pieces	1,560 785 375	32.50 32.71 31.25	Smallest size cheaper
Tooth Paste (Close Up)	125ml 56ml 25ml	135 70 35	1.08 1.25 1.40	Largest size cheaper
Morgan's Hair Pomade	100g 50g	375 275	3.75 5.50	Larger size cheaper
Shoe Polish (Kiwi)	200ml 50ml	275 90	1.38 1.80	Larger size cheaper
Stout	60cl 30cl	150 100	2.50 3.33	Larger size cheaper

***Figure 1:** Do the poor pay more than the rich?*

A quick calculation of proportions will alert the mathematically literate buyer to have a second look at the prices before taking a decision whether or not to add a little more money and buy at a lower price per unit. One may decide to spend less but buy at a higher price per unit.

LEVELS AND TYPES OF MATHEMATICS

The level of mathematics required for the above mundane purposes may be termed 'trivial,' 'school' or 'social' mathematics. This level is accessible and intelligible to almost anyone and requires no mathematical sophistication beyond the general mathematics at the ordinary level, comprising arithmetic, algebra, geometry, elementary statistics and probability and in some cases, further mathematics and with a proficiency not higher than a credit pass at the Senior Secondary Certificate Examination (SSCE) or its equivalent. The required skills can be acquired within the first twelve years of education. For the majority of people, this level is about the limit of their mathematical sophistication and appreciation.

The needs of society over time necessitated the acquisition of higher levels of relevant skills to meet the challenges. This came in the form of the development of higher levels of mathematics leading to breakthroughs in science and technology. This level requires advanced mathematical knowledge, skills and tools at the Higher School Certificate (HSC) or the Advanced General School Certificate level (comprising basically the same secondary school mathematics courses, but at an advanced level treatment) through a good first degree training up to a good two-year Master of Science (M.Sc.) degree in mathematics and the mathematical sciences. The students here meet with a much higher mathematical concepts and topics with abstractions and the idea of mathematical proofs appropriately emphasized at each level. Topics such as abstract algebra, mathematical structures (groups, fields, rings etc.), real and complex analysis, statics and dynamics, mechanics, Differential equations (ordinary and partial), number theory, advanced calculus, topology, advanced statistics and probability, etc. are taught under the rubric of pure mathematics, applied mathematics and statistics. The skills and training for the level are preparatory to careers in teaching,

industry, government and all jobs requiring some amount of quantitative, as well as logical reasoning skills and abilities.

The basic skills and knowledge can be acquired within twenty years of mathematical education, from the primary school to the Master of Science (M.Sc) level, or within thirteen years from the School Certificate up to the M.Sc level. The class of people with the above training and skills are often referred to as mathematically literate, or pupil mathematicians at best. With fruitful teaching and research experience spanning several more years beyond the above level as measured by publications and contributions to the corpus of mathematical knowledge, this class develops to be recognised as full-fledged mathematicians. Mr. Vice Chancellor, Sir, you can imagine our amazement at the sight of posters on some Lagos streets inviting the naïve pupils to:

"BE A MATHEMATICIAN WITHIN TWO WEEKS"

I wish that such a place and feat of scholarship existed during my formative years in mathematical training. More importantly, I wish it were possible to realise such a dream for the sake of many people in Nigeria who cannot do the course(s) they are most ardently praying about due to their mathematical deficiency.

Apart from the above, there is still a higher level of academic attainment by mathematicians. The mathematics required at this level comprises specialised topics such as mathematical visualization, cryptography, wavelets, propulsion mathematics, parallel computing, modelling, imaging, signal processing, control and robotics to mention a few. This level is for the specialists and even then the type of mathematical work here remains the purview of some of the best minds in the world. The relevant background for this level of work includes mathematical training beyond a Ph.D. plus several years of productive research. As I shall mention in this lecture, most of the outstanding advances in science, engineering and technology owe their origins to such a class of mathematicians. Hardy [30] called this level and type of scholarship 'real' or 'significant' mathematics as opposed to what he also considered 'usable' mathematics at the 'mundane' level. This is also the class of mathematicians that are needed to achieve Chike Obi's "innate superiority to invent a themonic value".

The same class of mathematicians create mathematics and mathematical ideas that enhance scientific and technological advancement of society. They often find unexpected connections, hitherto unknown to exist between diverse areas of human endeavour with profound impact on society. A glimpse at the calibre of mathematicians in this category can be found in a popular advertisement by the American National Security Agency (NSA) which seeks to employ: "A higher talented group of mathematicians who deduce structures where it is not apparent, find patterns in seemingly random sets, create order out of chaos ... and (can) apply mathematics to a world of challenges" [89].

Among the early mathematicians and mathematical scientists in this class are:

- **Evariste Galois** (1811-1832): French, who at the age of 18 solved the problem of whether or not the roots of any given polynomial of degree five or more can be expressed as algebraic expressions in its coefficients. He went on to study Group theory in connection with the solution of equations resulting in the development of a field of study known as Galois theory. He died in a duel over a woman at the age of 21.

- **Niels Henrik Abel** (1802-1829): Norwegian, who proved at the age of 22 that it is impossible to write the roots of the general equation of a degree higher than four as algebraic expressions in terms of the co-efficients, an unsolved problem from the 16th century to the early part of the 19th century. Abel went on to contribute to many areas of mathematics including the theory of groups. A particular type of study known as Abelian group bears his name.

- **Karl Fredrich Gauss** (1777-1855): German, popularly known as 'the prince of mathematicians" who at the age of 19 proved that a regular polygon of 17 sides can be 'constructed' with the aid of "a straight edge and compass" alone, a result considered impossible at the time. He went further to show that if $P = 2^{2^n} + 1$ is prime, then a regular polygon of P sides can be constructed.

(for n = 2, $P = 17$). At 22 years of age he proved the Fundamental Theorem of Algebra which states that every polynomial of degree n > 1 has at least one zero, a far-reaching result in the field.

- **Albert Einstein** (1879-1955): German, born with his famous equation, $E = mc^2$, from his work on relativity and gravitation theory.

- **James Clerk Maxwell** (1831-1879): Scottish, with his breakthrough in electromagnetic field theory.

- **Isaac Newton** (1642-1727): British, who invented the calculus and established the laws of motion and gravity among other achievements.

- **Jules Henri Poincare** (1854-1912): French, who was a philosopher, mathematician, physicist and astronomer. He made great discoveries in the area of mathematical functions, one of which is a class of 'Fuchsian' functions called 'Theta-Fuchsian.'

- **G.H. Hardy** (1877-1847), English, a great pure mathematician who collaborated with two of the greatest mathematicians of all time (Srinivasa Ramanujan (1887-1920), an Indian, and a great Number Theorist, also known as "Mathematician's mathematician" and J.E. Littlewood (1885-1980), English, known for his powerful insight into mathematical problems).

- **Pythagoras** (569-500 B.C.): Greek, who is best known for his "Pythagoras theorem" and formula accessible to students of mathematics as early as the secondary school level among other contributions.

- **Euclid** (365-275 B.C.): Greek, known for his "Euclidean" Geometry and his axiomatic approach to the subject as well as for his work in Number theory.

Undoubtedly, these mathematicians made tremendous impacts on the early development of mathematics through their works which can be seen everywhere today. Some of the mathematicians since the last century, whose works have had a tremendous impact on the advancement of mathematics and mathematical sciences, include all the Fields Medalists to date. The Fields Medal is the world highest honour in mathematics which was instituted in 1924 by J.D. Fields, a Canadian mathematician, "to recognize both existing work and the promise of future achievement". The prize is awarded to mathematicians not above forty years of age in the year of the International Congress of Mathematicians (ICM) held every four years. It was instituted in response to the non-inclusion of Mathematics in the disciplines for the Nobel Prize [32].

WHAT IS MATHEMATICS FOR?

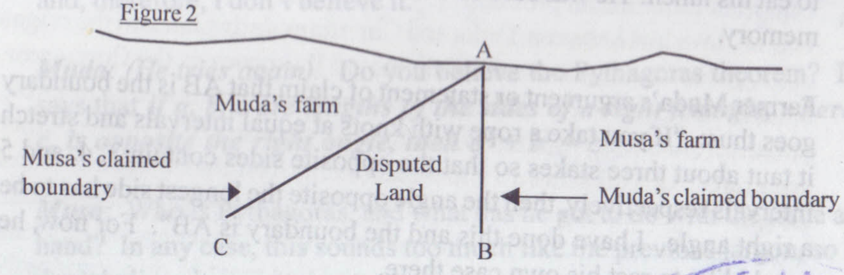
The history of the development and nature of mathematics supports the submission that the purpose of mathematics has continued to change with time in response to the interest and thinking of society, from mathematics being considered as an art for the creation of beauty to the view that it is useful for understanding and mastering nature and with real life applications. At the time of such mathematicians as Euclid, Gauss, Laplace, Newton, Fourier, etc., mathematics was meant to be used "to understand and control natural phenomenon". Fourier (1758-1830), for example, believed that "the principal objective of mathematics is the public utility and the explanation of natural phenomenon", a view shared by applied mathematicians. Carl Jacobi (1804-1851) echoed the views of pure mathematicians that the purpose of mathematics "is the honour of the human spirit and on that basis, a question of the theory of numbers is worth as much as a question about the planetary system". Morris Kline [44], [65] gave a functional summary of what the purpose of mathematics has been to mathematicians over the centuries when he wrote:

> "Apart from the beauty of mathematics and the satisfaction derived by mathematicians from the creative activity of mathematics, their most deep-seated drive has been to aid, through the medium of mathematics, in man's search to understand the universe and his own role in it …, to utilize the forces and phenomena of nature in man's behalf. The meaning and purpose of almost all of mathematics do not lie in the series of logically related collections of symbols but in what these collections have to tell us about our world".

The sum total of these developmental views on the purpose of mathematics leads us to the question of the nature of mathematics and consequently to the thin categorisation of the subject into ***Pure*** mathematics and ***Applied*** mathematics. We shall dwell on this under the subject matter of logic, mathematical proof, and abstract logical system later in the lecture.

We now take some illustrative examples of some mundane uses to which mathematics has been put at various times in history. The first example is how mathematics could be used to resolve a land dispute in a rational and free society. The account is from a historical anecdote which has its origin in the uses to which geometry was put by the early Egyptians and freely adapted from Richardson [65] to illustrate the beauty of a simple mathematical argument. Suppose there is a land boundary dispute between two farmers, say Muda and Musa. The description of the dispute is as diagrammatically shown:

Figure 2



In the diagram, Muda claims AB as the boundary between him and Musa while Musa claims AC. How can the boundary dispute be resolved rationally?

One simple solution would be for one disputant to kill the other and occupy both farms.

This solution though quite fashionable among individuals and to some extent among nations, is considered unfair and uncivilised. Such cases needless to say abound in Nigeria.

Another solution is to appeal to the Oba or village head as an arbiter, for a ruling which may be arbitrary and traditional in nature. Again, another solution is for one of the farmers to abide by the adage that land and whatever is on it belongs to God, and so he would not dare to fight over it. All these solutions have one major defect and that is, none of them may be considered 'satisfactory to the reflexive person' because no common understanding and agreement exists in the approaches enumerated above to the problem.

A more civil and logical approach whereby each man puts his argument and facts across logically is adopted here. For instance, farmer Musa's argument or statement of claim that AC is the boundary was based on memory and the idea that there happened to be an Iroko tree situated at the point C where he used to take a break during the hot afternoons to eat his lunch. He was sure of it because of his confidence in his good memory.

Farmer Muda's argument or statement of claim that AB is the boundary goes thus: "If you take a rope with knots at equal intervals and stretch it taut about three stakes so that the opposite sides contain 3, 4, and 5 intervals respectively, then the angle opposite the longest side has to be a right angle. I have done this and the boundary is AB". For now, he would like to rest his own case there.

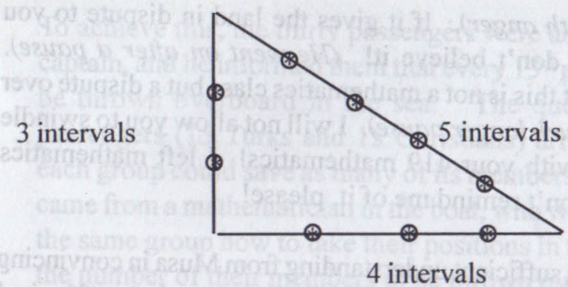


Figure 3: The Argument.

Yet naturally, the following dialogue started in a very comic form of street drama. Mr. Vice-Chancellor, sir, please enjoy this playlet based on mathematics.

Musa: *(Irritated)* I don't believe your 3, 4, 5 jargon.

Muda: OK. Do you believe that ***if the lengths a, b and c of the three sides of a triangle satisfy the equation $a^2 + b^2 = c^2$, then the angle opposite the size BC is right angle?*** (Converse of the Pythagoras theorem)

Musa: What have numbers and triangles got to do with our boundary dispute? In any case, if it were true, then your 3, 4, 5 jargon would be correct and you would win the argument. But your premise is false, and, therefore, I don't believe it.

Muda: *(He tries again)*. Do you believe the Pythagoras theorem? It says that ***if a, b, c are lengths of the sides of a right triangle, where c, is opposite the right angle, then $a^2 + b^2 = c^2$.***

Musa: Who is Pythagoras, and what has he got to do with the issue at hand? In any case, this sounds too much like the previous jargon, so I don't believe it.

Muda: Do you believe that ***if two triangles are similar, then their sides are in proportion?***

Musa: *(Explodes with anger).* If it gives the land in dispute to you (raising his voice), I don't believe it! *(He went on after a pause).* May I remind you that this is not a mathematics class but a dispute over my land. *(Another much longer pause).* I will not allow you to swindle me out of my land with your 419 mathematics! I left mathematics behind years ago. Don't remind me of it, please!

Muda was not getting sufficient understanding from Musa in convincing him that the dispute could be resolved fairly with the help of logical mathematical reasoning based on proposition (or axioms). But he nevertheless went ahead to employ logic and facts from elementary geometry to prove his case that the boundary is AB and, consequently, that the disputed land belonged to him. At the end and in shameful defeat, Musa had to accept the statements and the claims of Farmer Muda, who had a stronger and more logical argument.

This historical anecdote shows that statements can be proved using a few accepted statements that are based on agreement and understanding of issues and problems involved. Of course, more sophisticated boundary cases will require a higher level of mathematical techniques. As we will show, later in this lecture, this logical approach is one of the basic and powerful weapons for creating a sense of beauty or relevance in mathematics.

Our second anecdote is also historical and is called the 'Turks and Christian Decimation problem' [57]. It is recorded in history that there was no love lost between Turks and Christians dating back to the reigns of Emperor Constantine and Mohammed II (known as the Conqueror, a Turk) around 1400. The story shows how mathematical thinking could be used to save a situation in one's favour. The present story goes thus: 15 Turks and 15 Christians were overboard a sailing vessel caught in a hurricane. As was the custom in those days, the captain must appease the gods by throwing half of the passengers overboard whenever there was a hurricane at sea and this must be done randomly so that the selection of those to be thrown overboard would be by chance.

To achieve this, the thirty passengers were arranged in a circle by the captain, and he informed them that every 13th person in the circle would be thrown overboard in the sea. The question is how could the passengers (15 Turks and 15 Christians) arrange themselves so that each group could save as many of its members as possible? A solution came from a mathematician in the boat, who whispered to his fellows in the same group how to take their positions in the circle so as to reduce the number of their members to be thrown overboard.

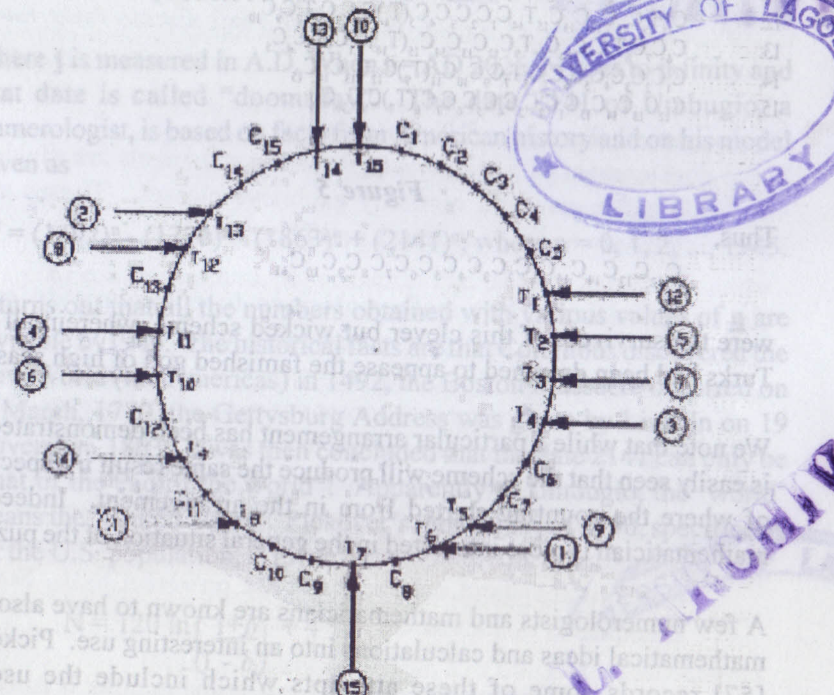


Figure 4: Turks and Christians line up

What was his solution? This appears as the final arrangement of the crowd:

1. $C_1C_2C_3C_4C_5T_1T_2T_3T_4C_6C_7T_5(T_6)C_8T_7C_9C_{10}T_8C_{11}T_9C_{12}T_{10}T_{11}C_{13}T_{12}T_{13}C_{14}C_{15}T_{14}T_{15}$
2. $C_8T_7C_9C_{10}T_8C_{11}T_9C_{12}T_{10}T_{11}C_{13}T_{12}(T_{13})C_{14}C_{15}T_{14}T_{15}C_1C_2C_3C_4C_5T_1T_2T_3T_4C_6C_7T_5$
3. $C_{14}C_{15}T_{14}T_{15}C_1C_2C_3C_4C_5T_1T_2T_3(T_4)C_6C_7T_5C_8T_7C_9C_{10}T_8C_{11}T_9C_{12}T_{10}T_{11}C_{13}T_{12}$
4. $C_6C_7T_5C_8T_7C_9C_{10}T_8C_{11}T_9C_{12}T_{10}(T_{11})C_{13}T_{12}C_{14}C_{15}T_{14}T_{15}C_1C_2C_3C_4C_5T_1T_2T_3$
5. $C_{13}T_{12}C_{14}C_{15}T_{14}T_{15}C_1C_2C_3C_4C_5T_1(T_2)T_3C_6C_7T_5C_8T_7C_9C_{10}T_8C_{11}T_9C_{12}T_{10}$
6. $T_3C_6C_7T_5C_8T_7C_9C_{10}T_8C_{11}T_9C_{12}(T_{10})T_{11}C_{13}T_{12}C_{14}C_{15}T_{14}T_{15}C_1C_2C_3C_4C_5T_1$
7. $C_{13}T_{12}C_{14}C_{15}T_{14}T_{15}C_1C_2C_3C_4C_5T_1(T_3)C_6C_7T_5C_8T_7C_9C_{10}T_8C_{11}T_9C_{12}$
8. $C_6C_7T_5C_8T_7C_9C_{10}T_8C_{11}T_9C_{12}(T_{12})C_{14}C_{15}T_{14}T_{15}C_1C_2C_3C_4C_5T_1$
9. $C_{14}C_{15}T_{14}T_{15}C_1C_2C_3C_4C_5T_1C_6C_7(T_5)C_8T_7C_9C_{10}T_8C_{11}T_9C_{12}C_{13}$
10. $C_8T_7C_9C_{10}T_8C_{11}T_9C_{12}C_{13}C_{14}C_{15}T_{14}(T_{15})C_1C_2C_3C_4C_5T_1C_6C_7$
11. $C_1C_2C_3C_4C_5T_1C_6C_7C_8T_7C_9C_{10}(T_8)C_{11}T_9C_{12}C_{13}C_{14}C_{15}T_{14}$
12. $C_{11}T_9C_{12}C_{13}C_{14}C_{15}T_{14}C_1C_2C_3C_4C_5(T_1)C_6C_7C_8T_7C_9C_{10}$
13. $C_6C_7C_8T_7C_9C_{10}C_{11}T_9C_{12}C_{13}C_{14}C_{15}(T_{14})C_1C_2C_3C_4C_5$
14. $C_1C_2C_3C_4C_5C_6C_7C_8T_7C_9C_{10}C_{11}(T_9)C_{12}C_{13}C_{14}C_{15}$
15. $C_{12}C_{13}C_{14}C_{15}C_1C_2C_3C_4C_5C_6C_7C_8(T_7)C_9C_{10}C_{11}$

Figure 5

Thus,

$$C_{12}C_{13}C_{14}C_{15}C_1C_2C_3C_4C_5C_6C_7C_8C_9C_{10}C_{11}$$

were the survivors of this clever but wicked scheme, wherein all the Turks had been drowned to appease the famished god of high seas.

We note that while a particular arrangement has been demonstrated, it is easily seen that the scheme will produce the same result irrespective of where the counting started from in the arrangement. Indeed, a mathematician is more interested in the general situation of the puzzle.

A few numerologists and mathematicians are known to have also put mathematical ideas and calculations into an interesting use. Pickover [57] records some of these attempts which include the use of mathematical population models, mysticism and calculations as well as passages from the Bible to predict the end of the world. Some of the predicted dates for the end of the world at various times include A.D 2000, (Bible); AD 1999 (Nostradamus); Friday, 13th November, A.D 2026 (Von Foerster); 2141 A.D. (Umbugio), and Pickover [57] whose model predicts 2040 as the "doomsday" for the U.S.

The date A.D. 2000 is from the belief that "the end of the age would come '6000 years' after Adam. Since Adam was created around 4000 B.C. then A.D. 2000 is the end of the world". The 1999 date of Nostradamus, a 16th century prophet, can only be fully explained by himself if he comes back to life.

The date A.D. 2026 of Foerster, a mathematician, is from his equation:

$$N = \frac{1.76 \times 10^{11}}{(2026.87 - t)^{99}}$$

where <u>t</u> is measured in A.D. When t = AD 2026, N goes to infinity and that date is called "doomsday". The date 2141 of Umbugio, a numerologist, is based on facts from American history and on his model given as

$$W = (1492)^n - (1770)^n (1863)^n + (2141)^n, \text{ where } n = 0, 1, 2, \ldots, 1945.$$

It turns out that all the numbers obtained with various values of <u>n</u> are divisible by1946. The historical facts are that Columbus discovered the New World (the Americas) in 1492, the Boston Massacre occurred on 5, March, 1770, the Gettysburg Address was given by Lincoln on 19 November, 1863. It was then concluded that the date 2141 can only be "that of the end of the world"! Apparently to Umbugio, the 'world' means the 'United States'. Pickover's model [57] of 1990, specifically for the U.S. population, is given by

$$N = 120 \ln\left(\frac{1+b}{1-b}\right) + 4$$

When t = 2040, 1-b in the model is zero.

Hence, in the year 2040, the population goes to infinity, N also goes to infinity and we have 'Doomsday', the end of the U.S.!

So far, we have survived some of the predictions, chiefly those of 1999 and 2000. Will the predictions of 2026, 2040 and 2141 come true?

More serious uses to which mathematics has been, and are yet being put will be discussed later.

THE WORKING TOOLS OF MATHEMATICIANS

As in every other profession, there are tools which mathematicians use in doing their work of creating mathematics. The first set of these are terminologies, words and symbols which form the vocabulary of mathematicians. The other indispensable tool of course is logic. Indeed, one major characteristic of mathematics that turns out to be of great importance is the use of logical proof. We shall discuss these tools under this section.

Mathematical Terms and Terminologies

In the development of a mathematical theory, statements are divided into three exhaustive and mutually exclusive classes. Theses are Definitions, Axioms (or Postulates), and Theorems.

(a) ***Definitions:*** For mathematicians to communicate and understand one another, there is need to have an agreed working set of meanings of certain terms or words. A definition is, therefore, an agreement to accept one expression (symbol, word or phrase) as being equivalent to another. It is of course not possible to define every term used in order to avoid circular reasoning. The net result is that it would have to be that the presence of undefined terms is inescapable, a fact which makes mathematics stand out to be such a powerful tool. Some examples of mathematical definitions are (cf [40]).

- A positive integer $P > 1$ is a prime number if its only divisors are 1 and itself.
- An equilateral triangle is a triangle having all its three sides equal.
- A circle is the locus of all points equidistant from a given point called its centre.

(b) **Axioms (or Postulates):** Axioms are assumed mathematical statements for which no proof is required. They are statements that are to be accepted without argument. And since we have seen that the presence of undefined terms is unavoidable, all mathematics must, therefore, begin with some axioms for statements cannot be proved about undefined things unless assumptions are made about them. One characteristic of axioms is that they must be consistent to avoid contradictions. A few examples of Axioms are given as follows: (cf. [40], [24]).

- Two points determine a straight line.
- If a, b, x belong to a set of real numbers with the usual operations defined on the set, then the equation $ax = b$ where x is to be determined has a solution, for $a \neq 0$

(c) **Theorems:** A theorem is a mathematical statement for which a proof is required. It consists of two parts: a hypothesis and a conclusion. The hypothesis comes as what is given (or known) and it is usually preceded by the proposition, 'If'. The conclusion is the part to be deduced and it is usually preceded by the preposition 'Then'.

A theorem is thus usually stated in the form: 'If A then B' where A and B are mathematical statements. Other ways of stating a theorem include 'Let A then B'; "Suppose A then B'. Examples of theorems that can be proved are:

- If two tangents are drawn to a circle from an external point, then the tangents are equal angles at the centre.
- Let a, b, c be elements of the set of real numbers with the usual operations defined on it, and if $a + b = a + c$, then $b = c$.

If the hypothesis and the conclusion of a theorem are interchanged, we obtain the <u>converse</u> of the theorem. Where a theorem and its converse are both true, the statement of a theorem can be combined into one statement in this form: 'A if and only if B' where A and B are mathematical statements. The statement 'A if and only if B' means 'condition A implies condition B' (or given A we can prove B). In such formulation we say that 'A is sufficient for B' or that 'B is necessary for A'. Two statements (or conditions) A and B are said to be equivalent if A implies B and B implies A.

To prove the statement 'If A then B' may be difficult to do directly. In that situation, one proves the equivalent statement 'If not B then not A'. The statement 'If not B then not A' is called the **contrapositive** of the original statement 'If A then B'. Thus, since a statement and its contrapositive are equivalent, mathematicians often prove the contrapositive statement in place of the statement. Such a method of proof is called a 'proof by contradiction' or *'reductio ad absurdum'*; a method Hardy [28] considered 'one of a mathematician's finest weapons.'

(d) **Lemma:** A *Lemma* is a 'little theorem' in the sense that it requires a proof. A lemma occurs when a result relating to a major theorem is of sufficient interest and importance to be stated and provided before the main theorem.

(e) **Corollary:** A *corollary* is a consequence of a theorem. It is also a 'little theorem,' but in view of the information from the theorem already established, a proof of a corollary is usually too trivial to be undertaken. Corollaries are 'trivial' results from the main theorem.

(f) **Conjecture:** A *conjecture* is a guess or a 'hunch' at an answer to a problem. That is, something like an argument believed to hold but for which a proof is yet to be provided.

(g) **Mathematical Proof:** A *mathematical proof* is a logical argument that is based on axioms or on known theorems (or results) which are themselves based on the axioms. Thus, to 'prove' a mathematical statement is:

"to devise a well-conceived coherent scheme of operations, of logical mathematical or material operations proceeding from the hypothesis to the conclusion, from the known (or data) to the unknown, from the things we have to the things we want" [60].

ABSTRACT MATHEMATICAL (OR ABSTRACT LOGICAL) SYSTEM

Richardson [65] gave the following broad conception of mathematics:

"An abstract mathematical science is created when we select some undefined terms and a set of consistent assumptions (postulates) about them and then proceed to define new terms in terms of original undefined ones. If meanings are assigned to the undefined terms, then we have a concrete interpretation of the abstract mathematical science. The totality of all abstract mathematical science is called *Pure Mathematics* while the totality of concrete interpretations is called *Applied Mathematics*. Mathematics comprises the two".

In practice, as we pass from an abstract mathematical science to a concrete interpretation, meanings that will make the statements of the assumptions true are substituted for the undefined terms. It follows, therefore, that any system can be converted to an abstract mathematical system by organising such system in such a manner that it begins with undefined terms, some unproved assumptions involving these terms and then defining and proving new statements or theorems.

To mathematise any subject is to put such a subject in the form of an abstract mathematical science. The fact that this abstract logic is independent of a particular subject-matter makes it applicable to any subject matter. If meanings substituted for the undefined terms turn the hypotheses into true statements, then it follows that the conclusion will also be true statements no matter how complicated the arguments are.

Logical Reasoning

Logic as we mentioned earlier plays a crucial role in mathematics, in particular, and in science generally. The achievement in mathematics through the use of logical reasoning in the investigation of mathematical and scientific problems has underscored its indispensable role. Logical reasoning has achieved more than mysticism, authority, or intuition in drawing valid conclusions from available evidence and data.

Valid Arguments

If the conclusion of an argument follows from the hypothesis, the argument is said to be valid. The process of drawing valid conclusions from given hypotheses is called deduction or deductive reasoning. There is a difference between the actual truth of a statement and the correctness (or validity) of the argument. The validity of an argument does not depend on the actual truth of its statements; it depends only on the form of the statements and not on their meaning. This turns out to be exactly what enables us to have an abstract form of argument.

Valid reasoning says that if the hypothesis were true, then the conclusion would have to be true. That is, if it is possible to satisfy the requirements of the hypothesis without satisfying the conclusion then the argument is not valid. In a valid argument, the conclusion must be true if the hypothesis is true. Hence, if the conclusion of a valid argument is false then the hypothesis must be false. However, the conclusion of a valid argument need not be false if the hypothesis is false. The validity of a statement is often tested by means of truth tables.

EXAMPLES

Let us see some illustrative examples of these concepts as given below.

1. Consider the statement: "T was the first Vice Chancellor of the University of Lagos". This is not an assertion until a meaning is given to the undefined term T which will convert the resulting assertion into true or false statement (or assertion). Now if we substitute "Professor J.F. Ade Ajayi" for T, then the statement is clearly false; but if we substitute 'Professor Eni Njoku' for T, then the statement is true. If, however, we

substitute 'Decency' or 'Accountability' for T then the statement will read 'Decency was the first Vice- Chancellor of the University of Lagos'. This is clearly an absurdity.

2. Consider the proposition: "If 5 = 9 then 1 = 1". What can we conclude concerning its validity or otherwise?

We proceed as follows:
If 5 = 9 then it follows that since if equals are divided by equals, the results are equal, $\frac{5}{5} = \frac{9}{9}$ that is 1 = 1.

The reasoning here is valid and the conclusion is true, but the hypothesis is false. Thus, a false hypothesis may give a true conclusion. This is an example where the truth of a conclusion may not imply the truth of a hypothesis, (cf. [3]).

3. Consider the following!

(1) If President Obasanjo is at least 1.62 meters tall, then the United States will not invade North Korea.

(2) President Obasanjo is at least 1.62 meters tall. Therefore, the United States will not invade North Korea.

Even though the height of President Obasanjo has nothing to do with the idea of United States invading North Korea or not, the argument is valid as a simple truth table for $[p \rightarrow q) \wedge p] \rightarrow q$, where *p* is the hypothesis and *q* is the conclusion, will show.

4. *Hypothesis:* (1) All Catholics are communicants
(2) Adepoju is a Catholic

Conclusion: Adepoju is a communicant

This is an invalid argument. Hypothesis (1) is false from experience. Depending on which Adepoju is referred to, hypothesis (2) may be true or false.

5. *Hypothesis:* (1) All residents of Lagos are terrorists
(2) All terrorists live in Nigeria

Conclusion: All residents of Lagos live in Nigeria.

The reasoning is valid, the hypothesis is false but the conclusion is true. This is an example which shows that the conclusion of a valid argument need not be false just because the hypothesis is false.

Abstract Logic

As indicated earlier, the validity of an argument does not depend on the actual truth of the statements, it depends only on the form of such statements and not on their meanings. This enables us to have the following abstract form of an argument (cf. [65]).

Hypothesis: (1) All a's are b's
(2) All b's are c's

Conclusion: All a's are c's

The important point here is that the above argument is valid no matter the meaning given to a, b, c. The statements have no meaning until precise or definite meanings are assigned to the undefined terms a, b, c. Reasoning with the above 'meaningless' but general form is known as abstract logic.

By substituting for a, b and c any meanings which convert the hypothesis into true statements, it follows that the conclusion is also converted into a true statement. For example, if in the above general form we substitute 'Roman Catholic Priests" for *a,* 'celibate people' for *b* and "Christians" for *c,* we obtain the following statements which have meanings:

All Roman Catholic Priests are celibate
All celibate people are Christians
All Roman Catholic Priests are Christians

The fact that this abstract logic is independent of any particular subject matter makes it indispensable in mathematical and scientific work.

THE PROBLEMS OF LOGIC

In spite of the achievements of logical reasoning in mathematical development, there are some inherent problems. The first one is the philosophical question of what truth is and how one can determine whether or not a given statement is true. This problem is partly resolved by assuming that we can determine (by some means) whether given statements are true or false, or that we can accept (without question) that some statements are true while others are false. This is then used to examine the problem of getting new truths from known ones.

The general question is whether the rules of logic are self-evident, in the sense of being free of contradiction, and whether or not the mathematical sciences are consistent or inconsistent. Of course, the problems have resulted in paradoxes in mathematics. A paradox may be defined as "a statement which is self-contradictory and false, though it may seem true or clever" (cf. [31]). Paradoxes arise, for example, in the process of determining the question of what is a cardinal number that indicates the concept of 'how many' (or 'quantity').

Frege (1848-1925) and Russell (1872-1990) independently gave definitions of the "number of a class" in terms of "the class of all those classes that are similar." The riddle is the exact meaning of the phrase "the class of all those classes". To tackle the riddle, Russell in 1902 asked the harmless question, "Is the class of all those classes which are not members of themselves a member of itself"? The conclusion is that either "Yes" or "No" to this question leads to a contradiction. One famous paradox of a similar sort is that of the village barber. It goes thus:

> "In a certain village, there is a barber who must shave all those people and only those people who do not shave themselves" (cf. [12], [54], [44], [62]).

This statement appears simple and suggests nothing unusual, until when you ask the question: "Does the barber shave himself"? Now, if this barber shaves himself then he does not shave himself and if he does not shave himself then he shaves himself. Simply put, 'If he does, he doesn't and if he doesn't, he does. The inevitable conclusion here is that this barber cannot exist.

The lesson from some paradoxes for mathematicians is the need for careful handling of logic and language in mathematical reasoning. The problem of paradoxes in mathematics remain unresolved. As a result of these problems mathematicians have, become more careful and critical in handling mathematical assumptions and logical reasoning. In spite of this, however, logic continues to be used to advance mathematics in a prosperous and progressive manner. The world continues to be spurred on to greater heights in mathematical results and operations through the application of logic to the way we reason and think.

THE WORKING MINDS OF MATHEMATICIANS AND MATHEMATICAL CREATIONS

Mr. Vice-Chancellor, ladies and gentlemen, now we come to the question: what is a mathematical creation or discovery and how do mathematicians use the above tools to create mathematical results? We shall first consider the question of what constitutes a mathematical contribution. Halmos [29] gave us the following way of looking at it:

> "It may be a new proof of an old fact, or it may be a new approach to several facts at the same time. If the new proof establishes some previously unsuspected connections between two ideas, it often leads to a generalization".

Originality as required in a mathematical contribution arises from successfully establishing connections between two or more ideas that are not previously known or shown to exist.

The art of doing this is generally regarded as mathematical creation or discovery. Mathematical creation thus results from persistent productive thinking with ideas generated and organised through a process of creative mental effort. Actually, mathematicians use the tools of logical deduction in creating mathematical results. However, mathematical creation (or

discovery) is more than employing logical deductions from known axioms or theorems. Other processes are involved.

On this matter, Poincare [59] expressed his view when he wrote:

> What, in fact, is mathematical discovery? It does not consist in making new combinations with mathematical entities that are already known. That can be done by anyone, and the combinations that could be so formed would be absolutely devoid of interest. Discovery consists precisely in not constructing useless combinations, but in constructing those which are useful, which are infinitely small minority. Discovery is discernment, selection.

How does the mathematician pick a mathematical problem to work on? Or put in another way, how does he/she know which kinds of problems to work on? This is one of the most difficult issues and challenges faced by every mathematician, especially the prospective mathematician or scientist. In some cases, the problem to be investigated comes from within mathematics, as a result of the desire to improve upon known results, while in other cases problems for investigation come from fields outside mathematics. They may come in response to requests for a solution to a practical problem. Poincare [59] has this to say about the class of mathematical problems worthy of investigation.

> Mathematical facts worthy of being studied are those which, by their analogy with other facts, are capable of leading us to the knowledge of a mathematical law, in the same way that experimental facts lead us to the knowledge of a physical law. They are those which reveal unsuspected relations between other facts, long since known but wrongly believed to be unrelated to each other.

In an attempt to understand fully the processes involved in mathematical creation, certain phenomena and factors that play an important part in mathematical creation by mathematicians have been identified. These include the roles of the subconscious, imagination, curiosity, chance, reason, and intuition. We shall just consider some of these.

Imagination is defined as "the power (of the mind) to form and define new ideas by a synthesis of separate elements of experience. The power to form mental image of objects not perceived or not wholly perceived by the senses," [31]. It is such an important instrument in mathematical creation and any creative work that there is an accepted dictum in research which says "**To know is nothing, to imagine is everything**". Imagination comes to play when seeking answers to such questions as how and why certain patterns (or phenomena) occur resulting in new ways of looking at old things. Imagination serves as a source of new knowledge as it enables the researcher to look deeper than usual and for his/her mind to wander wide resulting in new efforts. When, for example, the mathematician starts with the assumption such as "Suppose A is ..." or "Let X be ...", some element of imagination is involved.

Curiosity serves as an incentive to thinking. It is known that a mathematician (or scientist), who is more likely to succeed in research output or breakthrough, is someone who possesses more curiosity than the usual traits. Curiosity enhances the ability of the researcher to spot, faster, certain possible research problems for deeper investigation.

Intuition is perhaps the most important of the phenomena in mathematical (or scientific) discovery. Though, difficult to define, one recognizes intuition when it comes to play a role in mathematical discovery.

Intuition has, however, been taken to mean 'a hunch,' 'an inspiration' 'a sudden feeling', etc. Beveridge [13] calls it "a sudden enlightenment or comprehension of a situation, a clarifying idea which springs into the consciousness often, though not necessarily, when one is not consciously thinking of that subject". Whereas there is still no full understanding of how intuition occurs, studies have so far suggested that intuition "arises

from the subconscious activities of the mind which has continued to turn over the problem even though perhaps consciously the mind is no longer giving it attention", (cf. [13]). This explains why ideas can dramatically "spring to the conscious mind" even at a time one is not thinking of such ideas. Though the occurrence of intuition is not easily explained, there are, however, conditions that appear conducive to it.

These are: (cf. [13], [41]):

Preparation of the mind by long conscious contemplation of the problem; avoiding competing interests, worries (both physical and mental fatigue) as well as minimising over concentration on the problem. It also helps to always have a pen (or pencil) handy to record any sudden illumination since ideas often vanish as suddenly as they occur.

EXPERIENCES OF SOME MATHEMATICIANS WITH INTUITION

There are recorded "testimonies" from great mathematicians and scientists on the role of intuition in their research endeavours. We mention just a few here, (cf. [13]).

1. Von Helmholtz, a German physicist, reported that he got his ideas "in the morning or during the slow ascent of wooded hills on a sunny day" but never when his "mind was fatigued or when on his office table".

2. The idea of 'survival of the fittest' as part of the Theory of Evolution came to A.R. Wallace during a bout of malaria fever.

3. Einstein reported that his generalisation equation $E = mc^2$ occurred to him as he was lying in bed unable to sleep.

4. Rene Descartes made his discoveries also while lying in bed in the morning.

5. Intuition occurred to J.R. Baker while lying in the bath, just as Archimedes got his 'displacement' theory when he was taking

a bath and he ran out naked screaming: "Eureka! Eureka!!" In Greek, this is equivalent to the English statement "I have found" or "I have just made a discovery".

6. Henry Poincare found the solution to his long standing problem of establishing a class of the Fuchsian function (called *Theta Fuchsian),* during a night when he drank black coffee and could not sleep.

7. While going along a street (and not at all thinking about the problem in hand), Poincare also got the solution to his other problem on Fuchsian functions.

8 One of the most recent and interesting accounts of how intuition through a dream played a role for a solution is the following: Robert W. Thomason (1952-1995) reported that on January 22, 1988 he had a dream in which his deceased friend, Thomas Trobaugh, told him (in the said dream) how to go about some needed steps to complete the proof of a theorem. On waking up he followed the procedure from the dream and, to his surprise, that suggestion worked and he was able to use the idea to complete his proof. In gratitude, he included his late friend's name as a co-author of the resulting paper which was published in 1990. (cf. [80]).

Many of us must have experienced this phenomenon at one time or another when seriously pondering over a problem and when we have thought of giving up, a sudden illumination or an idea dramatically comes up to the rescue. Not all mathematicians or scientists experience this phenomenon, however, and not all intuitions are correct or fruitful in scientific discovery.

ADEPOJU'S MATHEMATICAL CONTRIBUTIONS

Sometime in 1983, I taught a Number Theory course to final year (Part III) students in the Department of Mathematics. The course was known to have many theorems and results named after their creators. Such popular theorems included Euclid's, Fermat's last, Gaussian Quadratic

Reciprocity law, Diophantine equations, etc. The students observed that each time I came to give a lecture I would discuss one or more of such theorems, go through the proofs and then give illustrative examples on how to use such results to solve appropriate problems in Number Theory.

During one of the lectures, a student got up to ask why they had never come across a "Chike Obi's theorem", an Olubunmo's theorem," "Adepoju's theorem" or any theorem named after a mathematician they knew. He asked whether such theorems existed. That threw the class into a frenzy, followed by loud applause in support of his question.

Mr. Vice-Chancellor, Sir, let me now give an answer and explanation to the question along the lines of the one I gave to the students in 1983. I will then link answer to my own modest contributions to the corpus of mathematical literature, in my general area of Real and Complex Analysis and in my specific area of Basic Sets of functions of Complex Variables in particular. This section may soothe the nerves of the specialists who have so far patiently listened to topics meant for a general audience, and to whom this section will have some appeal.

There is usually some history behind all mathematical results and every mathematician can give a narrative account of how his/her results came about. Indeed, as in each scientific result and breakthrough, the results in mathematics are as important as the history behind their development. My own experience is apparently not different or unique in that respect.

We start with an introduction to my area of research.

A sequence $\{P_n(z)\}$ of polynomials is said to be a basic set if any polynomials, and in particular the polynomials $1, z, z^2, \ldots z^n \ldots$ are uniquely representable by a finite linear combination of the form.

$$z^n = \sum_k \Pi_{n,k} P_k(z), \; n \geq 0 \quad (1)$$

We form the basic series associated with a function $f(z) = \sum_{n=0}^{\infty} a_n (z^n)$ which is analytic about the origin by subsituting for z^n in (1) to obtain

$$\sum_{n=o}^{\infty} a_n z^n = \sum_{k=0} P_k(z)\, \Pi_k(f)$$

where

$$\Pi_k(f) = \sum_{n=0}^{\infty} a_n \Pi_{n,k}, \quad k \geq 0 \qquad (2)$$

and write the associated series as

$$f(z) \;\sum_{k=0}^{\infty} \Pi_k(f) P_k(z) \qquad (3)$$

The basic series in (3) is said to represent $f(z)$ analytic in a circle $|z| \leq r$ if the series converges uniformly to $f(z)$ in $|z| \leq r$ or that the basic set $\{P_n(z)\}$ represents $f(z)$ in $|z| \leq r$. When the basic set $\{P_n(z)\}$ represents in $|z|=r$ every function analytic in $\{|z| \leq R\}$ for $R \geq r$, we say that the basic set $\{P_n(z)\}$ is effective in $|z| \leq r$ for the class of functions $H(R)$ analytic in $|z| \leq R$ When $R=r$ then the basic set is said to be effective in $|z|=r$. If $P_n(z)$ is of degree n, , the set $\{P_n(z)\}$ is necessarily basic and it is called a simple set. The problems of interest in this field are the study of the properties of functions (or polynomials) of one or several complex variables represented by basic sets (series) and their effectiveness properties in suitable regions such as the disk, poly-cylinder, faber region, hypersphere and the annulus as well as their mode of increase (order and type). Contributions in this field can be found under Mathematics Subject Classification (MSC 2000), 30A10, 30B50, 30D05, 30D10, 30D15, 30E10, 32A10, 32A05, 32A15, 32H02, 32E30 etc.

My first serious mathematical research experience was in 1977 when my Ph.D. thesis supervisor, Professor M. Nassif, informed me that he had a mathematical problem for me to work on and that it was a conjecture of his but one which he and others had not been able to resolve since 1952! My first reaction was to accept without question the feeling that my Supervisor did not expect me to obtain a Ph.D., for the simple reason that he was proposing to me a problem he himself could not solve! Although I felt very uncomfortable with this development the man,

however, encouraged me to give it a trial stressing that my being quite young then was a great advantage. My zeal for hardwork and my courage to face challenges soon superseded the initial anxiety.

THE PROBLEM

The problem on my hands involved finding an exact value of the upper bound for the type σ of simple sets of polynomials of order 1 whose zeros lie in the unit disk. Up till 1952, the best known range for the value was $1.3551 < \sigma < 1.3775$. Using previous attempts by researchers on the problem and the properties of the Goncarov Polynomials, Nassif [47] established that the required value satisfies the inequality $\sigma \leq \frac{1}{W}$, where W is the Whittaker constant. The Whittaker constant is defined as the least upper bound of the number **c** with the property that if f(z) is an entire function of exponential type less than **c** and if each of f,f',f''... has a zero in the Unit disk, then $f \equiv 0$ From this Nassif conjectured that the required type would be equal to $\frac{1}{W}$ and stated in the proof that, "the converse inequality does not seem to be obvious." The problem before me was, therefore, to establish the converse inequality, $\sigma \geq \frac{1}{W}$. This would then settle the conjecture and hence the full result on the increase of a simple set of polynomials whose zeros lie in the unit disk.

Related to this is the question of the exact value of the Whittaker constant itself, which so far has been intimately related to the Goncarov polynomials $\{G_n(z, z_0, \ldots z_{n-1})\}$ associated with the sequence $\{z_n\}^o$ of points in the unit disk. It took me months to even understand the problem. Indeed, for over six months working at least four hours a day, I got nowhere with the problem nor on how I might proceed with it.

Just about the time I was to give up, I had a bout of malaria fever! I had a good excuse for the slow pace of work to give to my supervisor, who was eagerly waiting for a progress report. It had always been, however, that whenever I had an attack of malaria I would be unable to sleep well but would instead, be having weird dreams and nightmares in which I would be falling from trees, mountains, and houses. And just when I might fall downwards with a big thud and some injuries, I would instead fall like a feather to the ground, waking up with cold sweats. When that was not happening, I would experience in such dreams that trees, buildings, and other physical objects were turning round and round or upside down or else they were presenting mirror images of their shapes.

But strangely enough and unlike most people that are down with malaria, ideas have often come to me during such bouts which could enhance my creative thinking substantially. It was in this situation that I found myself during the course of my investigation of the problem. On one such troubled nights while certain objects were turning round in my dreams, a number of the equations and formulae arising from the problem appeared but in forms different from the expected type. While some would appear upside down others seemed to be written backwards and they would all disappear as quickly as they came. One pattern kept appearing more frequently than others, and that was the series of equations and formulae written backwards.

What was the significance of the dream in all these contexts? For an answer to this question, we need to mention briefly some of the crucial equations, formulae, and my thinking associated with the problem.

Let,

$$H_n = \max |G_n(0, z_o, z_1 \ldots z_{n-1})|$$

where the maximum is taken over all sequences $\{z_k\}^{n-1}$ whose terms lie in the unit disk $U = \{z: |z| \leq 1\}$ and $\{G_n(z; z_o, z_1, \ldots z_{n-1})\}$ is the Goncarov polynomials associated with the sequence $\{z_n\}^{\infty}$, of points in the unit disk.

Since we know [17] that $W = \dfrac{1}{H}$ where $H = \lim H_n^{1/n}$ then we have that for $H - 1 > \in$ and corresponding to $\in > 0$ there exists an integer m for which

$$m > (2H \log H)/ \in$$

such that

$$H_m^{1/m} > H - {}^{\epsilon}/_{2}$$

Putting

$$Q_n(z_o, z_1, \ldots, z_{n-1}) = G_n(0, z_o, \ldots, z_{n-1}),\ n \geq 1,$$ then we shall have the existence of the point $(a_k)^m_1$ in U such that

$$H_m = |Q_m(a_m, a_{m-1}, \ldots a_1)|$$

Having fixed the integer *m* and the sequence $(a_k)^m_1$ we have the "link" Lemma as follows: Adopting the notation $q_j(z_1, z_2, \ldots z_j) = Q_{(j+I)m+j}(a_m, \ldots, a_1, z_j, a_m, \ldots, a_j; z_{j-1}, \ldots, a_m, \ldots a_1; z_1; a_m, \ldots, a_1)$, $j \geq 1$ then the complex number (t_k) can be chosen such that

$$|t_k| = 1;\ k \geq 1 \text{ and } |q_j(t_1 t_2, \ldots t_j)| > H_m^{j+1},\ j \geq 1$$

The dream was about the order of writing the points $(a_k)^m_1$ This Lemma was then used to prove the theorem which we state as follows:

Theorem (Nassif and Adepoju)
Given a positive number $\in$ a simple set $\{P_n(z)\}$ of Polynomials whose zeros lie in the unit disk can be constructed such that the increase of the set is not less than order 1 type $\frac{1}{W} - \in$. (This is a best possible result, in the sense that the order of $\{Pn(z)\}$ is exactly 1 and type $H = \frac{1}{W}$

Mr. Vice-Chancellor, Sir, the foregoing is a Nassif-Adepoju result (or theorem) and it has been used and so referred to by researchers in the field.

The next result was on the generalisation of results in disks to Faber regions. A Faber curve C in the Z plane is a simple closed regular curve which is the image in the Z plane of the circle $|t| = \gamma$, say, by the transformation. $Z = \phi(t) = t + \sum_{n-0}^{\infty} a_n t^{-n}$ where ϕ is assumed to be conformal in $|t| = T$ for some $T < \gamma$. Such transformations map the exterior domain $\in(\gamma)$of $\not\subset$ in the t-plane conformally onto $\in(\not\subset)$ in the z-plane so that for $T < r < \infty$ the circles $|t| = r$ are mapped onto the simple closed curves $\{C_r$ called Faber curves, such that $\overline{D}(C_{r_1}) \subseteq D(C_{r_2})$ whenever $r_1 < r_2$. The regions $D(C_r)$ and $D(C_r)$ are called Faber regions My investigation was to examine the extent of the generalisation of the effectiveness properties of the transposed inverse of a given basic set of polynomials in Faber regions. The transposed inverse set of a given basic set of polynomials is the set whose matrix of co-efficients is the transposed inverse of that of the given set. On this, I was to be guided by results of Newns [51] who obtained results for inverse sets in disks and of Nassif [50], who obtained results for transposed sets also in disks. With such background works to guide me, I was confident that the task would be easy. But alas! I was proved wrong as the problem turned out to be a surprisingly difficult one for me. After several months without much progress, I began to pray fervently for another bout of malaria fever and the attendant dreams for inspiration! The attack did not come, however, and I was forced to rely on my conscious (instead of subconscious) thought. After much brute force over several months I obtained the appropriate "normalizing substitution" for the given set $\{P_k(z)\}$ that ensured effectiveness properties in Faber regions. This was achieved by the substitution $P_k(z) = P_k(\xi z); k \geq 0$ where ξ is any complex number satisfying $\beta < |\xi| < \infty$

where

$$\beta = \alpha\,[\max\{\alpha, G(P_o^1)\},$$
$$G(r) = \sup_{\theta} |\phi(re^{i\theta})|;\ r > T,$$

$$\rho'_o = \inf\{E(r): r > T'\},$$
$$E(r) = \sup_{\theta} |\varphi(re^{io})|;\ r > T',$$

$$\alpha = G(\gamma)$$

and $t = \varphi(z)$ is the inverse transformation which is conformal for $|z| > T'$. With this substitution and notation the required transposed inverse set $\{\hat{P}_k(z)\}$ of the given set $\{P_k(z)\}$ is the set

$$\hat{P}_k(z) = P(z/_{\xi});\ k \geq 0$$

The outcome of my investigation resulted in three key theorems in an international journal and was reviewed in American Mathematical Review 84f: 30004 by W.F. Newns. These results are also the theorems of Adepoju-Nassif.

This was followed by an investigation on the general conditions for effectiveness of basic sets of polynomials in polycylinders in the space $\not{c}^2$ using a topological approach in Banach spaces as set out by Newns [51]. A polycylinder $\Gamma\, r_1, r_2; r_k > 0; k = 1,2$ is the open connected set $\Gamma r_1; r_2 \{(z_1, z_2): |z_k| < \}\{ r_k \}$

The outcome of the investigation resulted in six key theorems by Adepoju, presented at the III International Conference in Complex Analysis and Applications in Bulgaria (1985) and afterwards published in the refereed proceedings of the conference a year later. The results represent a new approach and a generalisation. It appeared as AMR 89j: 32001 and as Znetral blatt fur Mathematik (ZbL) 621: 32002, the two highbrow Mathematics Reviews in the world.

During the presentation of this paper which was well attended, I was made to know that the reason for the high attendance at my presentation was that one of the world experts in combinatorics, Ya Kazmin (a Russian), visited Bulgaria and Europe so as to present a series of lectures and, in one of his works, he cited my result with Nassif on the Whittaker constant. A few weeks after my return from the conference I received from the AMR the paper of Ya Kazmin which cited our results for me to review. This was for me a rare honour, privilege, and recognition of an achievement.

The other notable contributions to the corpus of mathematics literature was my paper titled, "A Fabry-type Gap theorem for Faber series." This work was on gap theorems which dealt with the question of the relationship between a lacunary (gap) distribution of the exponent of the power series expansion of analytic function f(z) and the impossibility of the analytic continuation of f(z) beyond its circle of regularity. In the extension, power series in disks is replaced by Faber series in Faber regions.

Two major theorems resulted from this work in addition to the deep constructive argument for the required Faber series. It appeared under Adepoju [7] in *Demonstratio Mathematica.* Soon after the appearance of the paper, I received several requests for reprints all over. A Russian sent to me a 99-page booklet (in Russian) comprising his related works in Faber regions, in the hope that we could do some collaborative work in the field. Unfortunately, the language barrier and the political climate of 'Glassnot' and 'Peretroika' stalled our efforts at communication. With the internet now available it is hoped that as we may soon begin to collaborate and share ideas of mutual benefit.

Mr. Vice Chancellor, Sir, permit me to say that many other contributions I have made can be found under effectiveness results in disks, Polychlinders, Faber regions and Hyperspheres, where results are obtained in single complex variables as well as in several complex variables. All of them have been reviewed in either the AMR or Zbl or both and they are all in the area of Basic sets of functions of single and several complex variables. In all, there are at least about **fifty** such theorems and results to date which are my modest contributions either solely or with others including some of my former students.

In mathematical contribution, only ripe and original results are publishable. Views and opinions have no place in mathematical creation. Results that are acceptable as publications in many areas are no more than Lemmas or Corollaries in mathematics. That is why we do not count but weigh and publish only few but ripe results.

I have also contributed to the solution of problems in the teaching and learning of mathematics at the primary and secondary school levels when I served as a Resource person to the Federal Ministry of Education in 1992. The resulting commissioned paper was published by Macmillan Nigeria Limited [5]. Of course, this does not count as a publication in Mathematics.

I still pretend to work in my area but my administrative responsibilities over the years have not given me enough time to do more. But luckily, I will be free of all that handicap in a few days from now! It is a freedom I sincerely look forward to, since I regard myself as an academic and not one of the politicians in the ivory tower.

My goal as a mathematician has always been to be a good scholar of mathematics and effective teacher of the subject, to contribute to the mathematical literature in my field, and to mentor prospective mathematicians who will take over from me. I wanted to enable them make their own contributions either in mathematics or any human endeavour where quantitative reasoning is required in fulfilment of the survival law of continuity, that a system must produce its own kind. Have I achieved any of these, you may ask?

To some extent, I will answer in the affirmative. I can state, however, in the words of G.H. Hardy [30] that "I have added something to knowledge and helped others to add more, and that these somethings have a value". Therefore, Mr. Vice-Chancellor, Sir, please permit me to ask all present here through you – Are there Adepoju's theorems?

The other question is why are these theorems (if any) not known or taught at the level of the students who first asked the question? The reason is simply that they are not as basic or fundamental as the ones taught at the lower levels mentioned earlier. For example, it took me seven years of primary education, six years of secondary schooling, two years of tutelage for a Higher School Certificate, four years of an undergraduate course at the university, another two years for an M.Sc, three years of doctoral research/training, and at least twenty four years of post-doctoral experience till date, all for a total of forty-eight years to obtain and understand some of these results and by God's grace, I am still learning.

Mr. Vice-Chancellor, Sir, when someone with a secondary school level of knowledge in mathematics asks the mathematician like me to explain his research to a layman, you may wonder what exactly the fellow is being asked to do. This is one of the numerous issues that compounds the dilemma of coping with mathematics and the mathematician. The expectation of the honest mathematician in such matters is for our people to have enough knowledge of mathematics to make them ask the 'right' question.

THE MATHEMATICIAN

Many things have been and are still being said about the mathematician. He is as misunderstood as his discipline just as the dislike for his discipline is easily transferred to him/her. In our schools and higher institutions, for example, the mathematician is someone who is often not judged by his competence, intelligence, dedication, or even character but by the students' perception of his subject, especially if he/she is a bad teacher in addition.

Consequently, views about the mathematician is as varied as the sentiments expressed by critics of his discipline. Who is a mathematician then, and what kind of human beings grow up to become mathematicians? To Lord Kelvin [82], the mathematician is the one "to whom (the formula)

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

is as obvious as that two plus two makes four is to you".

Paul Dirac [57], in observing that the world is built from little bits of mathematics and since God created the world, declared that "God is a mathematician." In a similar vein, Plato before then had proclaimed that "God is a geometer". Bell [12] gave another view of mathematicians that they are people who "make meaningless marks on paper and (are) shifting these marks about in accordance with certain tacitly assumed or explicitly formulated rules of play". Still on the misunderstood mathematician, we realise that he/she has been compared with a painter or a poet each of whom makes patterns but "while the painter or poet makes patterns with a combination of shapes, colours and words respectively, the mathematician makes his patterns mainly with ideas" [79].

While mathematicians may or may not be any of these characters, the fact is that mathematicians are essentially not different from any other group of human beings. Indeed, there are all sorts of mathematicians: tall, short, dumb or even handicapped. There are mathematicians who are nice and not so nice. There are those who drink, smoke and socialise,

and there are those who do not. There are sportsmen and women, politicians and some blind ones among them. There are those who do not care about religion, others who grow beards or are clean shaven just as we have those who do not possess mirrors in their homes. Many get married and have children while a few others are celibate. Some are such poor speakers that it is accepted that "An eloquent mathematician is as rare as a talking fish". There are those whom you will never suspect of being mathematicians. The one thing common to all mathematicians, however, is 'a capacity for abstract thought', [12], [30], [54].

We have not really answered the question: Who is a mathematician? Let us examine what mathematicians do in answering this question. A common misconception mentioned earlier is that mathematicians engage mainly in calculations. But as properly put by Condorcet [21], "For the mathematician, not calculation but clear thinking is characteristic, [it is] the ability to disregard inessential things."

The best documents to consult for our answer comprise periodicals devoted to the publication of new mathematical research, which may contain thousands of articles. The two most prominent of such publications are the American Mathematical Reviews (AMR) and the German Zentralblatt fur Mathematik (ZbL). These publications are devoted to short reviews of quality books and research papers circulated by reputable publishers and recognised international journals, respectively. They showcase the activities of today's mathematicians and through them one finds who is working in what areas of mathematics and the mathematical sciences. At the last count, there were 63 broad areas for a total of over 4,367 sub-areas of mathematics appearing under the 2000 Mathematics Subject Classifications (MSC.2000).

Mr. Vice-Chancellor, Sir, not only have all my papers published abroad appeared in these two review publications, I have also been a Reviewer for the AMR and ZbL for about twenty years, reviewing several dozens of papers written by some of the best minds in my specialty of Real and Complex Analysis and Basic Sets. My areas of interest and contributions to date are to be found under major classifications, MSC 30, MSC 32, MSC 41, MSC 26, and MSC 05 with appropriate sub-classifications of about sixteen to reflect the areas of the results obtained.

With an estimated 300,100 mathematical theorems proved each year (or an average of 25,000 a month) and all these appearing in about 60,000 research papers in about 675 Journals worldwide, it is unlikely that any mathematician can lay claim to understanding more than 4% to 5% of the available mathematics. Believing that people can best be described by what they do, we answer our question by defining a **mathematician as anyone who understands and does research in any of the areas listed (or yet to be listed) in the Mathematics Subject Classification (MSC).** Mr. Vice-Chancellor, Sir, it is highly improbable that this feat could be achieved "**within two weeks**".

THE STATE AND CONTRIBUTIONS OF MATHEMATICS TODAY

We have already given some examples of the uses to which mathematics has been put at different times. Our examples, however, have been at a mundane level. There are more serious applications of mathematics in today's world with a profound impact on our scientific and technological development. We shall give and describe a few topical examples.

Cryptology (or Cryptography)

This is the science and art of writing and sending messages in code form. The following is a simple aspect of a more sophisticated cryptography taken from **Why We Struck,** by Major Adewale Ademoyega, formerly of the Nigerian Army. (cf. [1]). As reported in that book, on January 13, 1966 Major Ademoyega sent the following coded message from Lagos to Major Kaduna Nzeogwu in Kaduna:

> "Major Ademoyega will leave Lagos after forty-one days holiday and will arrive in Kaduna after fifty-one days."

What does this mean? Unless you are into the secret of the game you are not likely to be able to decipher the full meaning of that message correctly. In an un coded language it turned out to mean.

> "The coup would take off on the night of the 14th and continue into the morning of the 15th"

The rest is now history as January 15, 1966 remains an important date in Nigeria's political history.

The subject-matter of modern cryptography is mathematical and lies in the field of Number Theory. The objective of cryptography is to be able to "break" or decode any coded message as well as to be able to write (or send) "secure" messages that will be impossible to decipher, without the key, known only to the intended recipient. Mathematicians handle this field by developing a "cryptosystem" considered as "Boolean functions mapping n bits to m bits" with messages sent by means of "cryptographic algorithm (or encryption algorithm) and key" which ensure that decryption of such messages is extremely difficult without the key, (cf. [11], [87]).

Encryption algorithm now has wide applications in military science and other security services. The greatest achievement in the use of cryptography had appeared during World War II, when each side of the war engaged in deciphering the coded messages of the other. The successes of mathematicians such as Rejewski (for Poland), Alan Turing (for the British) and Arne Beurling (for the Swedish) in breaking the German codes, at different times, aided the defeat of Adolf Hitler.

All secure commerce on the internet depend on cryptography. Indeed, without cryptography, what is now called e-commerce, e-learning and all forms of web communication could not exist. The technology behind the security of internet commerce is cryptography mathematics (Public-key cryptography, to be precise). It is pertinent to note in this connection that one of the most widely used encryption method in internet browsers, electronic transactions, value (credit) cards and e-mail services is the "RSA Algorithm" developed by three mathematicians, Rivest, Shamir and Adleman of MIT in 1977 [87]. Therefore, when next you want to access or send e-mails or use your GSM phones, think of mathematics and mathematicians at work!

Wavelet

Closely related to cryptography is wavelet analysis which is another mathematical method for data compression, signal processing and image encoding. The objective is to be able to retrieve signals and image data at different scales with good accuracy and in 'real' time. The mathematics involved include "matrix factorization" and "fractals." (cf [37]). One common application of the wavelet technique is for compressing finger-print images. Next time you are being finger-printed by the Police, SSS, the American Embassy and the American Immigration Officers at the airport, please think of wavelet, mathematics and mathematicians.

Genome

Mathematics plays a key role in mapping and sequencing DNA which is the molecule of life. The genetic material in the chromosomes (DNA molecules) is the *genome*. The problem involved is to identify all the genes and to understand their functions. Some areas of mathematics have come to the rescue in this new research. The areas of mathematics involved include dynamic programming, Fourier series, signal processing techniques (wavelet), combinatorics, optimization, and probabilistic and statistical tools.

New mathematical tools are being developed nowadays to handle the enormous data being collected in projects involving large operations. For example, the Human Genome project is estimated to have "a data base of 3 billion base pairs of human genetic code", while satellites in one remote sensing project can generate "200 billion characters of information in one scan" (cf. [77]). The goal of mathematicians in genome research is to come up with mathematical and computer science tools to build efficient techniques for obtaining and organising such large data for easy analysis and interpretation.

Other Areas Of Applications

Other areas in which mathematics plays major roles in our modern world is its application to studies in robotics, artificial intelligence, dynamics of diseases (e.g. cancer, HIV/AIDS infections), aerodynamics and space science, and to how the brain functions (neural studies) as well as a host of other major problems arising in science and society.

Only recently, it was reported [20] that mathematics and mathematicians have played a major role in the development of the "world's smallest biomolecular computer" which is "capable of identifying changes in the balance of molecules in the body that suggests the presence of certain cancers to diagnose the type and to react by producing a drug molecule to sight the cancer cells". The feat was achieved at the Weizmann Institute in Israel. The leader of that team, Professor Ehud Shapiro, is a mathematician and a computer scientist.

Mr. Vice Chancellor, Sir, I cannot think of any sphere of life that does not have some element of mathematics or to which my subject cannot be applied.

Consider the following poetry:

> Joe, I need a laugh screaming
> To accept facts and jokes laughing
> Havinsham happily confusing Pip
> As her vengeful soul laughs

Is there any chance that there could be something mathematical here? Consider also the following anecdote:

> An Actuarist discussing with a non mathematician on the chance that a certain proportion of some group of people would be alive at the end of a given time, gave a formula involving π and said π represents the ratio of the circumference of a circle to its diameter. The man was alarmed and asked what had the issue of being alive got to do with a circle?

In the secondary school we take the value of π to be $\frac{22}{7}$ which is not exactly correct since π is not a rational number. What is the value of π correct to twenty decimal places? Our poetry gives the answer, where the number of letters in each word stands for the corresponding digits and the comma represents the decimal point.

Thus according to the poetry,

$\pi = 3.14159265358979323846$.

correct to twenty decimal places.

If you need more digits add more stanzas.

New areas that may hold promise for applications to problems and needs of society continue to be created and studied. Some of such areas occasionally appear in a column of the Notices of the American Mathematical Society under the title "WHAT is… ?" (cf. [52]). We take three of such areas.

1. Suppose you are asked the question, "What is a building?". Until recently there is only one definition of a building, - "a permanent construction", or a "structure". However, if you ask the modern mathematician the same question he would tell you that a building is a structure with a topology – like axioms imposed on it. Specifically, he would say that "A building is a simplicial complex that can be expressed as the union of sub-complexes Σ called apartments satisfying certain axioms".

   Buildings arise from connections between geometry and group theory and they provide a geometric framework for understanding certain classes of groups. They also have applications in random walks, potential theory, harmonic maps and manifolds.

2. What is a 'Free Lunch'? Again, until recently, everyone knows what a Free Lunch is since "Free Lunches" abound in our society encouraged in part by governments that have embedded them in their policies and practices of declaring certain things free without anybody responsible for the payment. The modern mathematician, would however, state that the concept of 'Free Lunch' arises from the notion of arbitrage in the theory of finance. An arbitrage opportunity is defined to mean the "possibility to make a profit in financial market without risk and without net investments of capital". A 'Free Lunch' is therefore, a situation of 'no arbitrage'. The concept of a Free Lunch plays an important role in modern theory of finance which involves a lot of mathematics such as 'derivatives'.

3. What is an Amoeba? This again, until recently, is a common animal that every biology student knows – A body with several holes (vacuoles) and straight narrowing tentacles (pseudopods) going to infinity. A mathematical amoeba is, however, "a region in $\Re$ which is the image of the zero locus of a polynomial in two variables under the map Log: $(\mathbb{C}|0)^2 \to \Re^2$: $(z,w) \to (\log|z|, \log|w|)$ Amoebas are used in visualizing complex algebraic varieties in mathematics.

The days when the only thing one could do with a degree in mathematics was to teach is gone for ever. For example, without a degree in Accounting or Business or Banking and Finance you can obtain the professional ICAN qualification or the MBA in record time while a degree in mathematics before or after a Law degree will greatly enhance the quality and practice of the law profession. Lord Denning (1899-1999) proved the latter fact.

Even history is going mathematical with a new field of study known as 'Cliometry' which is the quantitative approach to history.

MATHEMATICS IN THE NIGERIAN ENVIRONMENT

Mathematics like most things in Nigeria is facing daunting problems resulting in too many people avoiding the subject. Even the few that dared to study it often wonder if it was worth all the rigours especially when they could have studied any of the much easier and less challenging but financially more rewarding disciplines. Experience and studies [5] have shown that these problems permeate all levels of our educational system where mathematics remains the most unpopular subject among students.

At the primary school level, mathematics teaching is still dominated, to a large extent, by unmotivated drills and purposeless skills characterised largely by rote learning. The curricula is examination driven while the teachers are unmotivated, ill-equipped, ill-prepared and poorly paid. One easily finds, at this level, people with neither aptitude for nor relevant training in mathematics, teaching mathematics to our children. Knowledge attainment of the pupils could no longer be measured as the primary school leaving certificate examination hitherto used for this purpose has since been jettisoned or de-emphasized by governments. With this situation one could not expect miracles in the performance of the pupils in any subject, especially mathematics, as the pupils transit automatically to the secondary school level.

The situation is similar at the secondary school level. The system is also examination driven and does not allow the student to develop his intuition, imagination and creative ability. This in turn handicaps the student in appreciating the full power, beauty and universality of mathematics in our world of challenges and complexities as the students are more preoccupied with passing examinations than acquiring knowledge. Such a system can only produce passive observers of results and knowledge with little or no chance (or ability) to contribute to or apply knowledge.

The supply of teachers are inadequate while the quality is generally deficient. There are four easily identified categories of teachers in our secondary schools.

1. Those with some knowledge and understanding of mathematics but who do not know how to impart the little they know.
2. Those who have some teaching abilities and skills and can teach but who understand very little mathematics.
3. Those who do not understand any appreciable mathematics at all and who also lack teaching skills and abilities.
4. Those who understand mathematics (reasonably) well and can also impart it (reasonably) well

The largest group are in category (1), followed by category (2), then (3) and lastly, category (4). In other words, those with only a smattering knowledge and understanding of mathematics are the ones teaching mathematics in our schools and many of our teachers cannot therefore, perform effectively, what they have been employed to do. One of the reasons is that, in many secondary schools, graduates of Biology, Physics, Geology, Chemistry, Zoology or even Economics are the Mathematics teachers. The statistics is that only about 19% of all teachers are mathematics graduates with teaching experience and only 48% of mathematics teachers are either degree holders or HND holders [5].

Mr. Vice Chancellor, Sir, mathematics is the only known subject to my knowledge that suffers this anomaly. Unfortunately this trend is creeping into the University system where lecturers whose only qualification in mathematics is the school certificate but are teaching in house Faculty or departmental mathematics courses to undergraduates and postgraduates on the premise that 'their students' are failing mathematics taught by mathematicians. Yet the same apostles of this practice will not hear of someone from the mathematics department teaching any of their courses even though it is quite possible, in many cases, for the mathematician to read up such course materials over a weekend and teach it effectively the following Monday! This practice is one of the reasons for the low level of quantitative reasoning abilities in many of our graduates. The mathematics is lacking. This must be addressed.

The end result of the secondary situation on the students is the production of confused, ill-prepared, discouraged and mathematically traumatised and deficient students who view mathematics as an impossible subject not meant to be taken seriously and understood by anyone. The consequences and evidence of the damage done can be seen from the following statistics of the performance of our children, at the senior secondary certificate examinations (SSCE) from 1992-2002 (cf. [9]).

From the table, the percentage credit passes range from 10% to 36.55% in Mathematics, 9.4% to 47.66% in Physics, 18.70% to 36.70% in Chemistry, 11.40% to 34.45% in Biology; 7.80% to 47.26% in Technical Drawing and 15.20% to 41.30% in Agricultural Science. It is noteworthy that in none of these science and technology subjects was a 50% credit pass rate achieved in the ten year period. Furthermore, Mathematics records the worst result although it has the largest number of entries for the examination, the highest occurring in 2002 with 1,078,901 candidates.

STATISTICS OF PERFORMANCE IN STM SUBJECTS AT THE SSCE WASSCE (1992-2002)

YEAR	AGRIC SCIENCE		BIOLOGY		CHEMISTRY		MATHS		PHYSICS		TECH. DRAWING	
	TOTAL ENTRY	%CREDIT PASS	TOTAL ENTRY	% CREDIT PASS	TOTAL ENTRY	%CREDIT PASS	TOTAL ENTRY	% CREDIT PASS	TOTAL ENTRY	%CREDIT ENTRY	TOTAL ENTRY	TOTAL ENTRY PASS
1992	273,040	29.70	358,961	27.90	142,379	18.70	365,491	21.60	124,351	16.20	9,400	7.80
1993	378,607	38.50	481,034	18.70	170,537	23.00	491,755	10.90	152,275	24.50	8,027	12.70
1994	395,278	33.10	508,384	11.40	161,232	23.70	518,118	16.10	146,000	14.70	7,239	12.50
1995	361,973	41.30	453,353	18.90	133,188	36.70	462,273	16.50	120,768	18.90	5,560	15.00
1996	401,676	22.90	506,628	15.90	144,990	33.50	514,342	10.00	132,768	12.80	4,924	24.10
1997	490,108	15.20	609,026	15.80	172,383	23.60	616,923	7.60	157,700	9.40	5,164	21.90
1998	513,130	23.37	637,021	34.45	185,430	21.40	640,624	11.15	172,223	11.34	5,959	30.96
1999	599,101	31.46	745,102	27.81	223,307	31.08	756,680	18.25	210,271	30.57	5,289	38.32
2000	508,369	19.30	639,020	19.31	201,369	31.89	643,371	32.81	193,052	30.06	5,506	46.24
2001	792,986	36.44	995,345	23.25	301,740	36.25	1,023,102	36.55	287,993	34.46	6,115	47.26
2002	832,949	33.31	1,047,235	31.39	309,112	34.89	1,078,901	34.50	298,059	47.66	6,635	9.81

*Table 5: **Sources:** From WAEC's Performance Statistics records for each of the years. cf. [9]*

Mr. Vice Chancellor, Sir, from this statistics, one does not need a crystal ball to conclude that our scientific and technological future is not in sight.

At the University level, students that were not interested in mathematics or were discouraged from learning mathematics will suddenly turn up to want to read courses for which mathematics is indispensable. The result is usually mass failure, below average performance and frustration, their high score from Jamb or even school certificate not withstanding. Even those who eventually show up in mathematics departments are there as a last resort, having been rejected in their first and second choices which are usually, Engineering, Computer Science, Accounting, etc. Consequently, most of those who cannot go elsewhere are the ones to be found in mathematics departments. Teaching such a group of people to learn some mathematics has been an unpleasant task, - a case of the "unwilling horses" being forced to drink water! The implications are that the best are no longer doing mathematics, only a few of them would qualify for higher degrees in mathematics and in mathematical sciences and only a few would successfully complete the degree and be found appointable to Lectureship position. Mr. Vice Chancellor, Sir, we are already faced with this situation today in the universities including our University.

SOMETHING TO TAKE HOME

Mr. Vice Chancellor, Sir, it would be nice for us to take something home at the end of this lecture. I shall do this in the form of two sets of challenges. The first set is backed by real money while the second set is not but our future depends on correct solutions. How would you like to win seven million dollars for attending my lecture? Here is how:

On May 24, 2000 the Clay Mathematics Institute, (CMI) founded in 1998 by Landon T. Clay, a Boston businessman announced in Paris, that the Institute would award prizes of $1 million each for solutions to seven unsolved mathematical problems of the last century. The purpose is to "celebrate the new Millennium and to increase the visibility of mathematics among the general public." This challenge came to be

known as "The Millennium Prize Problems". Since the popularisation and awareness of mathematics is one of the main objectives of this lecture, I believe this information has a place in this lecture. The problems are for the mathematicians and non mathematicians of today and of tomorrow. I quote them fully (cf. [22], [35]):

1. **The "P versus NP" problem.** A problem is in **P** if it can be solved by an algorithm that runs in polynomial time (i.e., the running time is at most a polynomial function of the size of this input). A problem is in **NP** if a proposed solution can be checked in polynomial time. The question is: Does P = NP?

2. **The Riemann Hypothesis:** Every non-trivial zero of the Riemann zeta function has real part equal to ½.

3. **The Poincare conjecture:** 3-manifold is homeomorphic to the 3-sphere.

4. **The Hodge conjecture:** On a non singular complex projective algebraic variety, any Hodge class is a rational linear combination of classes of algebraic cycles.

5. **The Birch and Swinnerton – Dyer conjecture:** For every elliptic curve over the rationals, the order of vanishing of its L-function at 1 is equal to the rank of the abelian group of rational points on the curve.

6. **The Navier-Stokes Equations:** Prove or disprove the existence and smoothness of solutions to the 3-dimensional Navier-Stokes equations (a set of nonlinear partial differential equations that describe many kinds of fluid flow), under reasonable boundary and initial conditions.

7. **Yang-Mills Theory:** Prove that the quantum Yang-Mills fields exist and have a mass gap.

To qualify (for any of the prizes) your correct solution must have appeared in a recognised journal for two years.

Perhaps this may be considered a waste of time by many, in the sense that they may never have the wherewithal to solve these problems. This should not be so if one recalls that Andrew Wiles first came in contact with the Fermat's last theorem when he was only ten years old, yet he went on to solve the over 350 years old problem in 1995! The same Andrew Wiles in his remarks at the announcement of the problems stated that they are meant to "excite and inspire future generations of mathematicians and non-mathematicians alike". They are therefore, not beyond anyone. Let someone sitting in this lecture today challenge himself (or herself) to solve one or more of these problems someday. A lot of mathematics would be learned and discovered in the process apart from the reward.

Perhaps these problems may not be what anyone would consider solvable in the immediate future. There are, however, problems that are not as difficult and are not even mathematical but which require our immediate attention and solutions. There are no prizes for answers to the questions and problems but correct solutions will guarantee our future and that of generations after us.

Here are some of them:

- Why is it that other nations have been able to harness knowledge and skills from various disciplines to develop their societies and we have not been able to do the same?

- Is the failure due to irrelevant or inadequate training?

- Is it a question of misplaced priorities or no priorities at all?

- Is there a painful possibility that we might just be plainly incapable of achieving any meaningful development due to our genetic make up?

- Why is it that we have no respect for continuity as we do not care to put in place sustainable structures or systems with the result that each time there is a change of government (or personality) we dismantle everything put in place by our predecessors (no matter how laudable or relevant the programme being dismantled) and we start all over again?

  A President initiates a programme of landing a man on the moon, for example, and as soon as such a President leaves office, his successor immediately on assumption of office, scraps the programme to announce his own programme of landing two men on Pluto! Why?

- How much crude oil do we have and how long will it last us? Do you know the answer? Does our government know? Shouldn't we know?

- How many Nigerians have the technical know-how and skills to determine the quantity independent of the foreign partners and countries? Should this be so after over fifty years of Oloibiri?

- Do we have sufficient number of qualified Nigerian as well as equipment to drill, refine and distribute our oil should we decide to ask all the foreign oil companies to go? I can assure you that should we make such an announcement this evening, there would be fuel crises within the next three hours and we would never recover until we call them back and with apologies. Why is this so?

VALEDICTION

Mr. Vice Chancellor, Sir, in a few days, I would have served for four years as Deputy Vice Chancellor. I would, therefore, take this opportunity to say a few valedictory words as I take my final exit on July 31st ,2004.

We have come a long way – as colleagues in the Department of Mathematics, as Deputy Vice Chancellors and now as my Vice-Chancellor. It has, therefore, been a long association throughout which time we shared mutual respect for and understanding of each other. I thank my Vice Chancellor for this and for his confidence in my ability to serve the system.

It is my belief that if this Vice Chancellor has sincere friends on this campus, then I should be counted among them. The word 'friend' is clearly an English word. Mr. Vice Chancellor, Sir, what is the mathematical definition or meaning of the word 'friend' or 'friendship'? Pythagoras [57], [28], defined 'friend' as "someone who is another me" and two people are said to be friends "if each is the sum of everything that measures the other." Mathematically, two numbers are said to be friendly (or amicable) if the sum of the divisors of the first number is equal to the second number and *vice versa*. That is, the sums of their divisors equal each other. It follows therefore, that if I know the number of the Vice Chancellor and I know mine, then to confirm whether indeed we are friends or not, we need only check if our numbers are friendly (or amicable).

Mr. Vice Chancellor, Sir, let me assign you the number 1210 and let me assign myself the number 1184 and let us see if indeed we are friends.

The divisors of 1210 are:

1, 2, 5, 10, 11, 22, 55, 110, 121, 242, 605

The sum of these numbers is

$$1 + 2 + 5 + 10 + 11 + 22 + 55 + 110 + 121 + 242 + 605 = 1184 =$$

My number!

Now the divisors of 1184 are:

1, 2, 4, 8, 74, 16, 32, 37, 148, 296, 592

The sum of these numbers is

$$1 + 2 + 4 + 8 + 74 + 16 + 32 + 37 + 148 + 296 + 592 = 1210 =$$

Your number!

From the above, I have shown that I am the "sum of everything that measures you" and I have proved that we are friends. It follows, as a corollary, that if we could assign friendly numbers to the 5093 members

of staff of the University of Lagos, we would all be 'friends' as it should be. Our homework is now to determine if we have that many friendly numbers and to find them if we have. Other known examples of friendly numbers are, 220 and 284, 2620 and 2924, 6232 and 6368. How many more can we find?

As friends, permit me, therefore, the privilege to say a word or two to my Vice Chancellor. I urge the Vice Chancellor not to be distracted from his mission and vision to move the University to greater heights. I am confident that he has all it takes to achieve this. He certainly has the physique and the energy. In the process of putting things in place and in exercising his powers and in discharging his responsibilities to achieve these goals, the Vice Chancellor must remain guided by the rules as well as the limitations which must never be ignored. As individuals, groups, cliques, unions and those who believed that their "cheese has been moved" jostle for the soul of the University, the Vice Chancellor must be able to sieve through all the 'booby traps' and ensure that the 'buck' stops on his table.

Throughout my stewardship, I tried to carry out my duties, diligently, courageously, faithfully, loyally and with complete dedication. Some of these duties involved silently alerting the Vice Chancellor of some of these 'booby traps' so as to avoid them. I advised at all times what I would have done if I were faced with the issues at hand and under the same situation. These were not done to seek favours, recognition or praises but because I believe it is the right thing to do. I had no other agenda other than to serve well and I would do the same for whoever I serve. Mr. Vice Chancellor, Sir, I wish you well and I will always wish you well.

RECOMMENDATIONS

We now give some suggestions on the way forward based on the information shared and the situation discussed in this lecture. Mr. Vice Chancellor, Sir, our number one position among the Universities has put a lot of responsibilities and challenges on us. We must put the right things in place to be able to sustain that position. Not only because it would be keenly competed for next time around but because it is more difficult to sustain a position than to achieve it. Some of the things we need to do and put in place include the following:

- A periodic self-assessment of how we do things and to continually find new and better ways of solving old and new problems.

- A periodic and a comprehensive review of our programmes in terms of courses offered, contents and relevance to societal needs.

- We should gradually begin to move towards achieving 'selective expertise' in our programmes. In this context, all polytechnic-like and colleges of education-like programmes should go to the polytechnic and colleges of education.

- Our various ratios and parameters, though due for review, must be adhered to and used to guide our planning, budgeting, resource allocation and future development and growth. These ratios and parameters include: staff/students ratio, staff/facilities ratio, students/facilities ratio, student/teaching staff ratio, student/non teaching staff ratio, teaching staff/non teaching staff ratio, etc.

- We must work toward achieving total quality in the number and qualifications of our teaching staff. All capable and relevant hands must be on deck and retained to sustain our position while efforts should be intensified to identify younger ones at home and abroad for recruitment.

- Our School of Postgraduate Studies should continue to be strengthened from where the 'best of the best' among graduate students should be identified for recruitment. It should be the best of the best because our current problem with non-PhD holders among our staff is partly attributable to past recruitment exercises which were based more on need than on quality.

- Our products must excel in a competitive environment measured by the contributions and achievements of our alumni at home and abroad.

- Quality and adequacy of our teaching and research facilities in the laboratories, workshops, classrooms and in the offices should continue to be improved upon.

- On the problems facing the study of mathematics, we need a change in societal attitude. This attitude is that unless the study of a discipline (or profession) will translate into instant wealth, material things and social recognition, such a discipline is not considered worth studying even if our future depends on it such as the mathematical sciences and basic sciences. Until the same opportunities that are available in other disciplines also exist for the mathematical sciences, problem of good students and good teachers will remain unsolved. We need better conditions of service for teachers and students of mathematics and the mathematical sciences to improve the situation.

- Teachers at all levels must be continually trained through refresher courses which should be well funded. Academic staff in tertiary institutions should also have training opportunities with exposures to seminars, workshops, conferences, exchange programmes both local and international. Access to the internet, journals and library services is indispensable. To whom much is expected, much should be given.

- The return of the Higher School Certificate programme is a welcome development, which will improve the standard of our educational system especially in the mathematical sciences. This will, in the long run improve the quality of our students and staff in the tertiary institutions. Our International School which has since got an approval to run such a programme should be assisted to commence without further delay.

- We need to take a second look at our one-year M.Sc. programmes. It is one of the contributory factors to the inability of some of our staff to obtain a Ph.D. Our current one-year M.Sc. is deficient in producing capable candidates for the PhD in the mathematical sciences. We need to return to our two year M.Sc. with a Thesis. This is because it takes a considerable more effort and skills to get a higher degree in the mathematical sciences than in other disciplines. A second class upper degree should be the minimum entry requirements. We have produced enough of one-year Masters for the Polytechnics, Secondary schools and Colleges of education and even, for some newer universities. Let us now produce those that can competently replace us in a first class university.

Government and Nigerians should continue to support the Nigerian Mathematical Centre (NMC) to enable it fulfill its mission of improving the standard of the mathematical sciences. Its Endowment drive which kicked off on May 8th, 2004 deserves the support of all Nigerians, (Individuals, Corporate bodies Philanthropists, Foundations, etc). If properly funded and managed, the Centre would make a significant difference to our mathematical and technological development. The NMC should be akin to similar centers and institutes such as the International Centre for Theoretical physics (ICTP), Trieste; the Institute for Advanced Study (IAS), Princeton; the Oberwolfach, Germany., Institut des Hautes Etudes Scientifiques (IHES), France; Isaac Newton Institute for Mathematical Sciences, Cambridge England; Steklov Institute of Mathematics, Moscow; Tata Institute of Fundamental Research, India and the Weizmann Institute, Israel, etc. Most of the advances and breakthroughs in the mathematical sciences emanate from such Institutes and Centres.

Government and Nigerians must identify and support their best Universities, Institutes and creative minds whose resources and talents must be harnessed to serve as Nigeria's Future in mathematical, scientific and technological development. We should not be satisfied with the mere production of ill prepared, under utilised and unchallenged graduate manpower just because we can. The USA has her hope and future in Institutions such as the MIT, Harvard etc, Britain has hers in Cambridge, Isaac Newton's Institute, etc. while Israel has hers in the Technion

and the Weizmann Institute. Nigeria is yet to think along this line.

The efforts of Promasidor Nigeria Limited, manufacturers of Cowbell Milk in encouraging the study and popularisation of Mathematics through their "Cowbell Mathematics Competition" must be commended and appreciated. Their mathematics Olympiad is encouraging, our youths to be challenged and excited by a once dreaded subject. Similar efforts of the Mathematical Association of Nigeria (MAN) at the primary and junior secondary school levels and of the NTA Quiz programme are also meant to achieve popularization of and interest in the subject. All deserve our support. Government is called upon to encourage Individuals, Corporate bodies and Foundations through tax relief incentives to support such programmes as well as the study of and research in mathematics and the mathematical sciences.

We should all learn to appreciate mathematics and what mathematics can do for us. This will enhance the employment opportunities for mathematicians. We call on the Federal government to play more significant role in challenging the Universities on the direction of mathematical research considered indispensable to our technological development. This is the situation in all the developed world where National (Federal) governments through appropriate agencies provide the desired leadership and a substantial percentage of the required funding for mathematical research. In the United States, for example, the Federal government through its agencies is one of the largest employers of mathematicians in addition to providing the enabling environment and funding. Notable agencies in this effort are, the National Security Agency, the Air-force, the Department of Energy, the National Science Foundation, and the Army Research Office. In these agencies, research is funded in all areas of mathematics relevant to the mission of each agency.

Mr. Vice-Chancellor, Sir, our future lies in our appreciating and investing in mathematics and in the mathematical sciences. In this effort, no Nigerian should be left out. In the course of this lecture we mentioned that there are all kinds of human beings who are mathematicians and that, indeed, there are some who are blind. How is this possible you might ask? Perhaps the best explanation would be that nature has a way of compensating for any defect in the human body. For the sight impaired mathematicians, what they lack in vision is compensated for in the possession of an imaginative skills, resulting in an extraordinary ability to think in their heads without the blackboard or paper. Some famous blind mathematician include (cf. [34]):

Pontryagin, S.,(1908 – 1988) was blind at the agé of fourteen. He became famous in the field of topology, homotopy theory, and Topological groups. I wrote my M.S. research paper at the University of Akron on Topological groups.

Euler, L., (1707 – 1783), one of the greatest mathematicians of all time, was blind for the last seventeen years of his life. The famous formula, $e^{it} = \cos t + i \sin t$ bears his name.

Morin, B., became blind at the age of six from glaucoma. He got his Ph.D. in 1972.

Saunderson, N., (1682 – 1739), became blind in his first year due to small pox. He never earned a degree having been denied admission to Cambridge. He, however, became the Lucasian Professor of Mathematics at Cambridge, a position formerly held by Isaac Newton!

Vitushicin, A.G., now at the Steklov Institute in Moscow. He works in the area of complex analysis.

Salinas, N., obtained his Ph.D. at the University of Michigan. Until recently, he was at the University of Kansas, where he retired. He became blind at the age of 10.

People with disabilities can also create something of 'thermonic value'. The Nemeth (Braille) code was invented by Abraham Nemeth, a blind mathematician and computer scientist and a Professor at the University of Detroit, U.S.A. The latest code called GS8, which improves the Nemeth code, was invented by a blind physicist named John Gardner. He also developed a programme called TRIANGLE for the sightless. A blind computer scientist at Cornell University, T.V. Raman, developed a programme called AsTeR. (cf. [34]). They are all, therefore, as capable as any of us.

Mr. Vice-Chancellor, Sir, I went into this detail to show that the blind (or the visually impaired) can do mathematics and can contribute to the mathematical sciences if they are empowered and encouraged. Many are in our universities today including the University of Lagos. Their learning conditions must be improved upon. Unfortunately, nothing is currently in place to assist, encourage and to challenge them. As a result of their predicament, Senate has had occasions to address the academic problems of some of these students, which are traceable to inadequate facilities for their needs.

Mr. Vice-Chancellor, Sir, a first choice and the current number one University should be able to provide an enabling environment for these and other categories of our students with disabilities. We therefore, call on the Vice-Chancellor to have in place as soon as practicable, a CYBERCAFE with all the necessary gadgets and equipment (such as the GS8 and the Nemeth codes, the AsTeR and the TRIANGLE software, Hearing aids etc.) for use of students with disabilities to enhance their academic pursuits. All other schemes and policies that would improve their lives on campus should also be put in place. I thank the Vice-Chancellor for accepting this challenge.

ACKNOWLEDGEMENTS

Mr. Vice Chancellor, Sir, I wish to now express some appreciations. My first acknowledgement and appreciation go to the Almighty God who breathed earthly life into me and has sustained me since then to make today's occasion possible. May His blessings, protection and goodness continue to be with us all. Amen.

My profound gratitude goes to my parents, Mr. Job Adepoju Adeniran and Mrs. Rebecca Asabi Adepoju Adeniran who gave me everything they had to raise me and to send me to school. I thank my paternal Uncle, His Royal Highness Oba Joshua Kayode Adeniran, (of blessed memory), the 15th Onisosan of Sosan through whom I started school at an early age and my maternal uncle, His Royal Highness Oba Zacheous Oguntoye Agbi (also of blessed memory) the 16th Onisosan of Sosan who encouraged me with his constant prayers.

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To all members of the University Community both Junior and Senior, Academic and Non-Academic in all Faculties and Units, I say thank you for being part of my working life experience. As a policy I tried to learn from everyone I have ever met. I have learned from all of you and you have enriched my life. My thanks go to Mr. S. O. Oduntan who assisted with the processing of this lecture and also to Senior Evangelist S. O. Ojelabi who recently retired from my office and who shared with me the policy of hardwork and complete dedication to duty. Though he had no vehicle and was living as far as Abule Egba, Evangelist Ojelabi gets to the office before 7 a.m. **EVERY** working day of the eight years I have known him. He word processed this lecture even after retirement.

I thank my friends and well wishers both within and outside the University who stood by me and my family with prayers, advice, encouragement and goodwill. Among them are Sir and Mrs. Oyalana, Justice and Mrs. Ade Alabi of the Lagos High Court, His Royal Highness, Oba Dapo Tejuoso, The Osiele, Karunwi III, and all the Oloris, Mama Chief (Mrs) Bisoye Tejuoso (of blessed memory), His Royal Highness, Oba Charles Adesunloye, Oyolola I, and the Olori, the Olisua of Isua, Prince and Olori Olukoya, Captain and Mrs. Benoni Briggs, Former Minister of Aviation, Deacon and Mrs. Taiwo, Engr. and Mrs. Ron Aborowa, Mr and Mrs. Akin Mateola and Mr. and Mrs. Isichei. I thank Professors, Segun Awonusi, G.O. Williams, Union Edebiri and Dr. E. A. Babalola for translating some of my mathematics into understandable English.

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To my sister, Mrs. Mary Omoare and her family, and my brother, Mr. Kuye Adepoju and his family, I say thank you for your support and

understanding at all times. I pray that at our reincarnation, we would come back again as brothers and sisters.

I thank my nuclear family, first, my wife who, like every Police officer, is taught to be tough, assertive and resolute but soon learnt to leave all that behind at the police station and to come home as a dutiful Mrs. Adepoju, the wife of Professor Adepoju, the Kabiyesi of the family. I thank you for your hard work patience and understanding and for the care of the children while I was engrossed in my teaching, research and administrative duties over the years.

To the children, Seyi (Kosemani Eni Olorun da kose farawe), Biodun (Bibi), Toyin (Smally), Dayo (Amaju Oritse) and Babatunde, my father reincanate, I thank you all for your patience, understanding and for bearing with me for my frequent absence from home. But you all knew where I was at any time. I know that I have been hard on all of you but it is to prepare you for a better life and future. I know you will one day thank me for it. Never depend on anyone for anything, not even on Daddy but accept whatever is given to you gratefully and work hard always to achieve your goals. I love you all.

Mr. Vice-Chancellor, Sir, but why are we seven in this nuclear family? Is it because 'Adepoju' consists of seven letters? Is it because the number seven is the fourth smallest prime number but the third smallest odd prime number? Is it because $7! - 2 = 5038$ and the sum of the digits equals 7?. Is it because the last words of Christ on the Cross comprise seven words? Is it because pilgrims to Mecca must circle Kaaba seven times? (cf [57]). Numerologists often excite themselves with these kinds of questions because of their interest in the magic meaning of numbers. I am not a numerologist, however.

Mr. Vice Chancellor, Sir, the reason is none of these but simply that we lost one of us, Master Olalekan Adepoju who would have been twenty-three years old this year. I had concluded long before the tragedy that he would be a better mathematician than I am. We continue to remember him in our prayers.

Mr. Vice Chancellor, Sir, you will recall the story of the boy in the prologue of this lecture. From that humble beginning, the boy went through stages of educational training at home and abroad and rose to become the first Director of Academic Planning and a Deputy Vice Chancellor of this University. Mr. Vice Chancellor, Sir, that boy has, today July 7, 2004, about 1,710 million seconds after his birth, delivered the 151st Inaugural Lecture of your University as a Professor of Mathematics. Unfortunately, his parents who went through all the anxieties of bringing up a 'Kasimawo' child and who supported him with everything (they had) could not be present today. Not that they would have understood the grammar or the mathematics but they would have been happy and proud that their boy gave the lecture. All these must have been destined by the Almighty God who makes all things possible and to whom we owe everything we do and are. May His will be done.

I thank the Vice Chancellor for the opportunity given me to pay my debt. I know that 'Oshomalos' insist on debts being paid! I thank my good audience for sitting through the trauma of listening to a mathematician's inaugural lecture. I hope the sleep was not deep enough to snore. You have been a wonderful audience and great mathematicians "within two hours"!

My Vice Chancellor, Sir, this is my inaugural lecture.

I thank you all.

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