

# **BIOMECHANICAL ANALYSIS OF LOWER LIMB SEGMENTS DURING VARIABLE SPEED CYCLING**

by

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***CERTIFICATION***

This is to certify that the thesis

**“BIOMECHANICAL ANALYSIS OF LOWER LIMB SEGMENTS DURING  
VARIABLE SPEED CYCLING”**

Submitted to the School of Postgraduate Studies, University of Lagos for the award of  
the degree of

**Doctor of Philosophy in Mechanical Engineering**  
is a record of original research work carried out

by

**AJAYI, Arinola Bola**

in the Department of Mechanical Engineering

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## DEDICATION

This work is dedicated

To God Almighty, my Justifier and Defender,

**Jehovah Sabaoth, The Lord of Host.**

and

To the evergreen loving memories of my parents,

Chief Joseph Olaitan Eniola AJAYI (Jolly Joe, Omo Olorun)

Omo onimushin ajinon, Omo onimushin Anon 'koru birikiti

Omo Maagushin n'oja omo, Omo 'Laameso Ajagajigi

Omo Oluronbi ..., Omo woye Ikala ti tana osan gan

Omo adasho t'ole ole da, T'ofi 'jade bawon ninu je

Omo olowo alogbabode iyaleta

Omo olowo j'oye meji po, otosi o ri kan je

Omo korokoro boto, ayo bi eni oba nbo

Omo korokoro boto, ayo bi eni oba nbo

Beeke e lo'le oba lo'jeun

Omo Ayonlu' Sonyin

and

Mrs Lucia Olaoluwa Enimowura AJAYI (Mummy Yellow).

Omo Petu airamasa, Omo Petu Ado

Omo abatiwon nda, aba tiwon nda Olorun ko fi tiwon se,

Omo ela ileke, Omo Soroade,

Ela ileke ti gbogbo won nra l'egbefa

Soroade ra ti e l'egbeje,

Omo anigi oro l'egon, Omo abigi owo so bi ewele.

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## ABSTRACT

This work undertakes the biomechanical analysis of the lower limb segments during variable speed cycling using inverse dynamics approach. Cycling interventions have been suggested for cardio-respiratory fitness as well as for patients with spinal cord injuries, stroke, mobility impairments and osteoarthritis. In the course of variable speed cycling, fatigue does set in at the lower limb after a period of usage or when there are abuses. Attempts by cycling athletes to outdo one another can impact negatively on their lower limbs during sports. Similar things do occur in recreation and rehabilitation. Therefore, it is important to understand the dynamics of the biomechanics of the lower limb segments (comprising the ankle, the knee joint and the hip) during variable speed cycling so as to mitigate the effects. The lower limb segments are model as a dynamic five bar linkages with kinematics, kinetics, and joints moments equations developed using inverse dynamics. Parametric studies are carried out using known values in order to evaluate the effects of saddle height; horizontal acceleration and crank speed on the lower limb segments are evaluated. Results show that at crank speeds above 64 rpm, the knee joint moment was the highest, followed by hip and then the ankle. A horizontal acceleration of  $2 \text{ m/s}^2$  was discovered to be appropriate and is therefore recommended. It was also discovered that as the saddle height was reduced below 50% of the total length of the lower limb segments, the joints moments become very high; this can eventually lead to fatigue. A saddle height of 75% of the total length of the lower limb segments minimizes all the three joint moments. It is therefore concluded that in order to minimize knee joint related fatigue which can lead to injuries during recreation and fitness, the saddle height should be 75% of the total length of the lower limb segments and crank speed not exceeding 64 rpm. For competition, the saddle height should be 90 – 100 % of the total length of the lower limb segments and horizontal acceleration of  $2 \text{ m/s}^2$  to be maintained.

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## Notations

### English Alphabets

- $a_x, a_y$  : Magnitude of acceleration vectors in  $x$  and  $y$  directions respectively;
- $b_1, b_2, b_3$  : Centre of gravity of thigh, shank, and foot segments.
- $Col$  : Column matrix
- $F_F$  : Frictional Force
- $F_R$  : Resultant pedal force
- $F_x, F_{fx}, F_{ax}, F_{kx}, F_{hx}$  : Reaction forces at the crank arm spindle, foot/pedal interface, ankle, knee, and hip respectively in the  $x$  direction;
- $F_y, F_{fy}, F_{ay}, F_{ky}, F_{hy}$  : Reaction forces at the crank arm spindle, foot/pedal interface, ankle, knee, and hip respectively in the  $y$  direction;
- $g$  : Acceleration due to gravity;
- $g_r$  : Gear ratio;
- $i, j, k$  : Unit vectors in the  $x, y, and z$  directions respectively;
- $I_{FCG}, I_{SCG}, I_{TCG}$  : Moment of Inertia of the foot, shank, and thigh segments respectively;
- $I_D$  : Moment of Inertia of the crank arm about the crank spindle;
- $J$  : Jacobian Matrix;
- $l_c$  : Crank arm length – from pedal to crank spindle;
- $l_t, l_s, l_f$  : Length of thigh, shank, and ankle to pedal spindle respectively;
- $l_x, l_y$  : Horizontal and vertical distances respectively of the seat from the crank arm spindle;
- $M_a, M_k, M_h$  : Joint moments at the ankle, knee, and hip respectively;
- $M_{BR}$  : Mass of bicycle and the rider;
- $m_f, m_s, m_t$  : Mass of foot, shank, and thigh segments respectively;



$N$	: Normal reaction of the bicycle and the rider;
$q_1, \dot{q}_1, \ddot{q}_1$	: Crank angle, velocity, and acceleration respectively;
$q_2, \dot{q}_2, \ddot{q}_2$	: Position angle, angular velocity, and angular acceleration respectively of the foot segment with the horizontal axis;
$\mathbf{r}$	: Displacement vector of the bicycle wheel radius;
$r_{bw}$	: Radius of the bicycle back wheel;
$\mathbf{s}(t), \dot{\mathbf{s}}(t), \ddot{\mathbf{s}}(t)$	: Position, velocity, and acceleration vectors respectively of the bicycle;
$T_{cog}$	: Number of teeth in the rear cog;
$T_{wheel}$	: Number of teeth in the chain wheel;
$\mathbf{v}(t), \mathbf{a}(t)$	: Velocity and acceleration vectors respectively;
$v_x, v_y$	: Magnitude of velocity vectors in $x$ and $y$ directions respectively;
$W$	: Weight of the bicycle and the rider;
$x, \dot{x}, \ddot{x}$	: Magnitude of position, velocity, and acceleration vectors respectively in $x$ – direction;
$X_{CCG}, \dot{X}_{CCG}, \ddot{X}_{CCG}$	: The position, linear velocity, and linear acceleration of the crank arm in $x$ direction;
$X_{FCG}, \dot{X}_{FCG}, \ddot{X}_{FCG}$	: The position, linear velocity, and linear acceleration of the CG foot in $x$ direction;
$X_{SCG}, \dot{X}_{SCG}, \ddot{X}_{SCG}$	: The position, linear velocity, and linear acceleration of the CG of shank segment in $x$ direction;
$X_{TCG}, \dot{X}_{TCG}, \ddot{X}_{TCG}$	: The position, linear velocity, and linear acceleration of the CG of thigh segment in $x$ direction;
$y, \dot{y}, \ddot{y}$	: Magnitude of position, velocity, and acceleration vectors respectively in $y$ - direction;

$Y_{CCG}, \dot{Y}_{CCG}, \ddot{Y}_{CCG}$  : The position, linear velocity, and linear acceleration of the CG of the crank arm in y direction;

$Y_{FCG}, \dot{Y}_{FCG}, \ddot{Y}_{FCG}$  : The position, linear velocity, and linear acceleration of the CG foot in y direction;

$Y_{SCG}, \dot{Y}_{SCG}, \ddot{Y}_{SCG}$  : The position, linear velocity, and linear acceleration of the CG of shank segment in y direction;

$Y_{TCG}, \dot{Y}_{TCG}, \ddot{Y}_{TCG}$  : The position, linear velocity, and linear acceleration of the CG of thigh segment in x direction;

### **Greek Alphabets**

$\phi, \dot{\phi}, \ddot{\phi}$  : Initial angle, angular velocity, and angular acceleration respectively of the rear bicycle wheel;

$\mu$  : Coefficient of friction;

$\theta_1, \dot{\theta}_1, \ddot{\theta}_1$  : Position angle, angular velocity, and angular acceleration respectively of the shank with the horizontal axis;

$\theta_2, \dot{\theta}_2, \ddot{\theta}_2$  : Position angle, angular velocity, and angular acceleration respectively of the thigh segment with the horizontal axis;

$\sum F_x, \sum F_y$  : Summation of forces in x and y directions respectively.