ANALYSIS OF DYNAMIC STRESS PROPAGATION OF A VIBRATING SUBSEA STRUCTURE IN A PRESSURIZED ENVIRONMENT

by

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B.Sc. (Hons), M.Sc. Lagos

Thesis Submitted to the School of Postgraduate Studies, University of Lagos in partial fulfilment of the requirements for the degree of Doctor of philosophy

in

Engineering Mechanics

Department of Mechanical Engineering
Faculty of Engineering
University of Lagos, Nigeria
August, 2011

SCHOOL OF POST-GRADUATE STUDIES

UNIVERSITY OF LAGOS, AKOKA, LAGOS

CERTIFICATION

This is to certify that the thesis

"ANALYSIS OF DYNAMIC STRESS PROPAGATION OF A VIBRATING SUBSEA STRUCTURE IN A PRESSURIZED ENVIRONMENT"

Submitted to the School of Post-Graduate Studies University of Lagos for the award of the degree of

DOCTOR OF PHILOSOPHY

is a record of original research carried out by

OGUNMOLA, Ogunbayo Yemisi

in the Department of Mechanical Engineering

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______________________ **SPGS Representative**	______________ Signature	____________ Date

DEDICATION

in

Loving Memory of my Late Father

(Pa Joseph Ogunmola Oguntobi)

and to my

Mother, Wife, Siblings and Teachers

and to

The Almighty God

ABSTRACT

Offshore pipeline and flow line systems define a variety of subsea architectures associated with Floating Production Storage and Offloading units (FPSOs) or Floating Storage and Offloading units (FSOs) that are usually employed for oil and gas production in deep and ultra deep waters. The design of such transmission facilities, must satisfactorily account for various phenomena such as hydrodynamic wave loading, fluid transport velocity, operating pressure and temperature of the internal fluid as well as limitations imposed by the seabed subsoil layer geotechnical properties. In fact the transverse and longitudinal dynamic responses of these pipeline and flow line systems are strongly modulated by these effects. Subsea pipelines are on the high demand to function at high temperatures and pressures. The natural behavior of a pipeline is to relieve the attendant high axial stress in the pipe-wall by buckling. Such uncontrolled buckling can have serious implication on the integrity of a pipeline. Hence, the usual practice to date, in the industry is to restrain pipelines by trenching and burying, or relieving the stress with inline expansion spools. In this work, the effect of transverse and longitudinal vibrations on the dynamic stresses induced by the fluid flow was studied with special reference to onset of buckling or bursting of such pipes. For this purpose, an offshore pipeline was idealized as a fluid conveying elastic beam on an elastic foundation and the corresponding set of equations governing the transverse and longitudinal motion of the pipe were formulated. Particularly, by employing integral transforms, an analytic solution for the induced stresses was computed and simulated for design applications while comparison with corresponding formulae currently in use in the field was also carried out. Furthermore, the earlier work was extended to capture the effect of deliberate or natural sediment covering of pipe that occurs over a long period of time, by examining the dynamic stress propagation through a partially or fully buried offshore pipeline. For this problem a boundary valued partial differential equation for the fluid- structure- soil interaction mechanics was formulated. In particular, by employing operational methods, the burst and buckling pressure profiles as modulated by the seabed sediment layer history were reported for design analysis and applications. Lastly this research reported an analytic solution for the induced stresses in polar coordinates coupled with von Mises yield criterion in conjunction with the corresponding set of equations governing the transverse and longitudinal motions of an offshore pipeline on an elastic foundation. Interesting results were simulated for practical analysis and applications.

ACKNOWLEDGEMENT

Foremost, I am greatly thankful to God Almighty for His divine intervention and mercy throughout my academic pursuit. My tremendous gratitude goes to my supervisors, Distinguished Professor V.OS. Olunloyo NNOM, Dr. O. Damisa and Dr. C.A. Osheku for their exemplary, moral and intellectual commitment both in the problems formulation and analytical methodology. Besides, their efforts throughout the pursuit of this research are highly appreciated.

I give many thanks to Dr. A. Oyediran of AYT Corporation, USA for putting me through the rudiments of the problems formulation at the initial stage of this research.

My warmest regard to my loving wife, Toyin and our dearest daughter Kikelomo for their patience in my absence in their midst while this research lasted.

Also, my sincere appreciation goes to my mother, brothers and sisters for their prayers and moral supports throughout the period of this work.

The Dean, Professor M.A. Salau, of the faculty of Engineering, University of Lagos, is highly thanked for his invaluable support during the series of my seminars and words of encouragement.

I acknowledge the support of my Acting Head, Dr. J.S. Ajiboye for his moral and physical encouragement. The academic staff of Mechanical and Systems Engineering Departments are well acknowledged for standing by me in the course of this research.

My acknowledgement is not complete without mentioning those that we pursued research together under the same Professor at the same period, i.e. Engr. A.O. Adelaja, Engr. O.A. Alli and Engr. Mrs. S.I. Kuye. The encouragement and contribution of one another is highly commendable and appreciated.

Finally, I express my profound gratitude to the experts and earlier workers in the field of offshore engineering.

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NOTATION

A pipe cross sectional area after deformation

A_o original cross sectional area of pipe

A_p surface area of pipe

A' change in the surface area of the pipe

C_1 damping force per unit velocity in the transverse direction

C_2 damping force per unit velocity in the axial direction

C_D hydrodynamic drag coefficient

E Young modulus of elasticity

$F_1(t)$ external force in the transverse direction

$F_2(t)$ external force in the longitudinal direction

g acceleration due to gravity

h depth of pipe below sea level

I moment of inertia

k_b stiffness of the sea bed

L length of pipe

m sum of the masses of pipe and fluid

m_f mass of flowing fluid in side the pipe

m_w mass of sea water displaced by pipe

M sum of masses of pipe, fluid in pipe and external water displaced by pipe

pA pressurization effect

P_h hydrodynamic effect of the ocean

p_o pressure at entry

T_o tension in pipe

t time

u longitudinal displacement

U velocity of fluid flowing inside pipe

U' differential of velocity with respect to x

$\dot{U}$ differential of fluid velocity with respect to time

$\tilde{u}$ longitudinal response in Laplace plane

u^F longitudinal response in Fourier plane

$\tilde{u}^F$ longitudinal response in Fourier-Laplace plane

w transverse displacement

$\tilde{w}$ transverse response in Laplace plane

w^F transverse response in Fourier plane

$\tilde{w}^F$ transverse response in Fourier-Laplace plane

x axial displacement coordinate

z transverse displacement coordinate

r_i internal radius of pipe

r_o external radius of pipe

B_r = the body force in the r direction

$\bar{a}_r$ = the acceleration in the r direction

Greek letters

α coefficient of thermal expansivity

γ coefficient of area deformation

$\Delta\Theta$ temperature change from inlet to outlet

Δp pressure change from inlet to outlet

Θ temperature of the flowing fluid

Θ' temperature gradient

μ coefficient of sliding friction

∇^2 Laplacian operator

∇ gradient operator

ρ_w density of water

Φ velocity potential

σ_y transverse bending stress

σ_x axial bending stress

σ_r radial bending stress

σ_θ Hoop stress

$\tau_{\theta r}$, $\tau_{r\theta}$ radial shear stresses

$\tau_{(xz)1}$, shear stress on the upper layer

$\tau_{(xz)2}$,   shear stress on the lower layer

δ_s sediment layer