HYDRODYNAMICS OF SAND-OIL-GAS MULTIPHASE FLOW IN A DEVIATED PETROLEUM WELL

O. A. Adeyanju
Petroleum and Gas Engineering Dept., University of Lagos, Nigeria

ABSTRACT

A calculation method for predicting pressure profiles along the wellbore based on easily obtainable wellhead parameter has been the preferred method. But, the predictive capability of the existing correlations is a thing of concern. This is due to the inability of the existing models and correlations to account for the presence of the sand particles in the flow stream; also the requirement for the well to be shut-in in order to acquire the needed parameters is counter productive. These inadequacies were corrected in the proposed model. Results showed that the average pressure drop in the multiphase fluid flow using the proposed model is higher than the pressure drops determined using existing models and correlations. The effects of the fluid density, viscosity and velocity on the sand particles lifting were also investigated and results showed that the sand particles suspension and lifting were improved by higher fluid velocity and density and lower fluid viscosity (i.e. higher Reynolds number). Validation of the proposed model with field data showed that the model predicts the field BHP data better than any of the existing models and correlations, based on the Average Absolute Deviation (AAD) value of 6.53 returned by the proposed model compared to AAD value of between 13.56 and 24.67 returned by the existing models and correlations.

NOMENCLATURE

A = Area of pipe flow
B = Oil Formation Volume Factor.
D = pipe diameter
f = Moody friction factor (dimensionless)
F_B = Buoyancy force
F_D = Drag force
F_G = Gravity force
F_M = Momentum transferred force

E-mail: oadeyanju@unilag.edu.ng
INTRODUCTION

Movement and deposition of sand particles inside the well-bore have adverse effect on the productivity as the movement causes increased erosion of the installed wellbore equipment and additional pressure drop along the wellbore while the deposition causes sand to be collect in the low spot along the wellbore and thus constrict the flow of oil through the well at these points. Though sand removals techniques has been successful in fluidizing the settled sand particles and removing them from the wellbore, these operations are time consuming, expensive and require a knowledge of where the sand deposits are.

Accurate flowing bottom-hole pressure (BHP) is important in inflow performance determination, pressure transient analysis, well production efficiency and gas lift performance determination. At present the oil producers depend on correlations for the conversion of surface pressure measurement to bottom-hole pressure, the accuracy of most of these correlation is a thing of concern due to the failure of most of these to take into consideration some of the fluid and particles properties that affects the hydrodynamics of the multiphase fluid flow.

An accurate multiphase flow model is needed for the accurate prediction of flowing bottom-hole pressure as accurate pressure gradient model is a requirement for the determination of the flowing pressure along the wellbore. Given the flowing bottom-hole pressure at the depth of interest, the pressure gradient model can compute the flowing pressure along the tubing string until it reaches the well head pressure, or given the wellhead pressure and the depth of interest it can calculate the flowing pressure until it reaches the predicted flowing bottom hole pressure. Solid-liquid multiphase flow encompasses many different areas of science and technology. As more wells are drilled to recover oil and gas from oil reservoir problems associated with sand production into and their transport inside the wellbore become more prominent. Since large proportion of the worldwide petroleum production comes from regions with unconsolidated sand formations and potential high sand
production zone such as Gulf of Mexico, Gulf of Guinea, Middle East, and North Sea. Hence, sand management is largely gaining attention as old production practice of producing no sand are being superseded by maximum acceptable sand rate (Geilikman and Dusseault, 1997; Tronvoil et al., 2001)

The economic benefits are especially pronounced in offshore environment where the sand management technology can effectively increase the recovery factor. Area of high sand production practices that involves installation of sand exclusion system down-hole are highly capital intensive and this discouraged the installation of such equipment well completion operation (Dusseault et al., 1998, Dusseault and El-Sayed, 2001).

Inefficient sand transport in gas-oil-sand multiphase flow through wellbore and flow-line if not addressed may erode the merit of sand management technology by creating numerous problems some of which are pressure loss, enhanced pipeline erosion and corrosion, frequent and expensive cleaning operation and increase downtime (Quedeman, 1993, Appah and Ichara, 1994, Appah et al., 1997). The gas, liquid and solid phases can be categorized into different flow regimes depending on the flow rates, pipe size and geometry of fluid and solid properties in wellbore systems. The main parameters determining the distribution of solid in the liquid are gas-liquid flow patterns, fluid (gas and liquid) velocity, solid loading, solid properties, physical properties of gas and liquid (viscosity and density), solid flow pattern (Bello, 2008). Studies showed that the sand particle input volumetric fraction during oil production may be up to 5% by volume of crude oil in a severe cases (Gillies et al., 1997; Almedeiji and Algharaib, 2005). However, the sand particle input volumetric fraction for most conventional wellbores and flow-lines can be as low as 0.014-0.11 kg sand per cubic meter of crude oil produced (Stevenson et al., 2001).

Meyer, (1986) developed a generalized drag coefficient correlation applicable for all flow regimes for non-Newtonian, power law fluid. It was compared with the experimental data and other empirical correlations to predict the drag coefficient and terminal settling velocity for fracture proppants of various mesh sizes. This correlation fitted experimental data much closer than stokes law which was found to underestimate the drag coefficient and overestimate the terminal settling velocity at large Reynolds number. In addition, the settling velocity was deemed to be independent of power fluid rheological properties such as the fluid viscosity and liquid fraction at large flow Reynolds number.

Gillies et al., (1997) determined a threshold axial pressure gradient for effective sand transport under laminar flow regime. Presence of gas in the mixture was observed to have no impact on transport performance. Salama (2000) developed a model for estimating the minimum mixture velocity to avoid sand deposition in multiphase pipeline. The model was validated using experimental data generated from a multiphase flow loop. The proposed model produced the measured settling (mixture) velocity fairly well in horizontal and near horizontal flow-lines. King et al., (2001) presented experiment and modeling results regarding solid transport in multiphase flow experiment under high and low viscosity fluid system. The pressure gradient predicted by their model was very close to the experimental result generated. Danielson, (2007) developed a theoretical model for predicting critical or minimum transport velocity that will result in sand bed formation in multiphase pipeline based on drift-shift model. By assuming a linear function between the gas velocity and mixture velocity over a wide range of conditions, they proposed a model for predicting sand particles hold up. The model for the sand particle hold up and critical solid carrying velocity gave a good fit to the experimental data. Yang et al., (2007) modeled and simulated sand
transport in a stratified gas-liquid two phase pipe flow system based on one-dimensional multi-fluid model and mixture layer concept. The model predicted the pressure gradient and mean velocity for threshold of particle entrainment into suspension with reasonable accuracy when compared with experimental data.

Bello, (2008) presented Mechanistic model for predicting optimal transport velocity, system performance, critical velocity, particle velocity and hold up in three-phase near horizontal well and pipelines the model showed some appreciable agreement with his generated experimental data.

**MODEL DEVELOPMENT**

The cumulative pressure gradient of the sand-oil gas flow is equal to the addition of fluid flow pressure gradient and the pressure gradient due to transportation of sand in the fluid flow. This is given as

\[
\left( \frac{dP}{dl} \right)_{\text{tot}} = \left( \frac{dP}{dl} \right)_{f} + \left( \frac{dP}{dl} \right)_{s} 
\]

(1)

The Euler’s equation generally accepted for the compressible upward fluid flow in production tubing. The Giles et al (2009) equation is:

\[
\frac{dp}{\gamma} + \frac{vdv}{g} + dx \sin \theta + dh = 0
\]

(2)

The \( h \) is given by Darcy-Weisbach equation as:

\[
h = \frac{f_i v^2}{2gd}
\]

(3)

The wall friction \( f \), is taken as,

\[
f = \frac{2\tau_f}{V^2 \rho_f}
\]

(4)

\( \rho \) is the fluid (oil and gas) density

The continuity equation for compressible fluid flow in pipe is given by:

\[
m = A \rho v
\]

(5)

Also,
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\[ \gamma = \frac{1}{\rho_f} \]  

(6)

Putting equ.(2), and (3) into derivatives of equ. (5), gives:

\[ \frac{dp}{dl} = \frac{\frac{fm^2}{2A^2\gamma dg} + \gamma \sin \theta}{1 - \frac{m^2}{\gamma^2 A^2 g \ dp}} \]  

(7)

The compressibility of the reservoir fluid is given as:

\[ C_f = \frac{1}{\gamma} \frac{d\gamma}{dp} \]  

(8)

Equ. (7) then becomes,

\[ \left( \frac{dP}{dl} \right)_f = \frac{\frac{fm^2}{2\gamma A^2 dg} + \gamma \sin \theta}{1 - \frac{m^2 C_f}{\gamma A^2 g}} \]  

(9)

**CONSERVATION EQUATIONS**

The conservation equations of mass and momentum which governs the solid phase in the gas-liquid-solid three phase flow in a pipe are given by:

Solid phase conservation equation

\[ \frac{\partial(\rho_s \rho)}{\partial t} + \frac{\partial(\rho_s v_s)}{\partial x} = 0 \]  

(10)

And the solid phase momentum equation

\[ \frac{\partial(\rho_s v_s)}{\partial t} + \frac{\partial(\rho_s v_s^2)}{\partial x} = \sum_{i=1}^{6} F_i \]  

(11)

Assuming a constant solid density and neglecting the acceleration term, equ.(11) becomes
\[ \rho_s \frac{dv_s}{dt} = \sum_{i=1}^{6} F_i \] (12)

\( \rho_s \) = solid density

\( F_i \) = force per unit volume.

Multiplying equ. (12) By the fluid volume, gives

\[ M_p \frac{dv_s}{dt} = F_M + F_B + F_p - F_D - F_G \] (13)

\( M_p \) = mass of solid particle

\[ M_p = \frac{\pi}{6} d_p^3 \rho_s \] (14)

\( d_p \) = solid particle diameter

For a fully developed flow

\[ \frac{dv_s}{dt} = v_s \frac{dv_s}{dx} \] (15)

Substituting equ. (15) into equ. (13)

\[ \left( \frac{dP}{dl} \right)_s = v_s \frac{\pi}{6} d_p^3 \rho_s \frac{dv_s}{dx} = F_M + F_B + F_p - F_D - F_G \] (16)

For an inclined pipe, at angle \( \theta \) to the vertical as shown in Figure 1

\[ \left( \frac{dP}{dl} \right)_s = F_M + F_B \cos \theta + F_p - F_D - F_G \cos \theta \] (17)

The above expression also expressed the force on sand particles in unit volume of the multiphase flow.

Defining the ratio of upward force to downward force on sand particles as

\[ k = \frac{F_M + F_B \cos \theta + F_p}{F_D + F_G \cos \theta} \] (18)
Hence, sand particle with $k$, less than 1 will deposit as sand bed at the lower part of the wellbore. 

The constitutive equations and expressions for the above stated forces are as follows: 

Momentum transferred Force, $(F_M)$ 

Force due to linear momentum transferred between the fluid phase and the sand phase $(F_M)$ is given as:

$$F_M = \frac{q \mu}{2 \rho_s} \left[ 1 + \frac{\rho_f}{2 \rho_s} \right]$$  \hspace{1cm} (19)

$$a = \frac{d_p}{2}, \text{ which is the sand particle radius and } \varepsilon \text{ is the void fraction}$$

where $F^* = \frac{C_d}{24} \left[ \frac{2 \rho_f (v_f - v_s)}{\mu} \right]$ \hspace{1cm} (20)

The Particle-Particle Interaction Force, $(F_P)$ 

The particle-particle interaction force in gas-liquid-solid three phase pipe flow is given by

$$F_p = \frac{\pi}{4} d_p^2 \frac{m_s}{A} (v_f - v_s)$$  \hspace{1cm} (21)

The Viscous Force, $(F_V)$ 

The viscous force of spherical sand particle moving through a flowing fluid is given by the modified Stokes formula

$$F_V = 3 \pi \mu d_p (v_f - v_s)$$  \hspace{1cm} (22)

The Gravity force, $(F_G)$ 

The gravity force is given as

$$F_G = \frac{1}{6} \pi d_p^3 \rho_s g$$  \hspace{1cm} (23)
The Buoyancy force, \( F_B \)
The Buoyancy force is also given as

\[
F_B = \frac{1}{6} \pi d_p^3 \rho_f g
\]  

(24)

Frictional Drag Force (\( F_D \))
The drag force transferred from liquid-phase to a single suspended solid particle is calculated by:

\[
F_D = \alpha (v_f - v_s)
\]

(25)

\[
\alpha = C_d \rho_f \frac{v_f - v_s}{2} \left( \frac{\pi d_p^2}{4} \right)
\]

(26)

\( C_d \), is the drag coefficient which depends on the Reynold number (\( R_e \)), for flow around a sphere. The followings are the relationship between the drag coefficient and Reynold number:

For \( R_e \) given as:

\[
R_e = \frac{\rho_f v_f D}{\mu}
\]

(27)

\( \mu = \text{dynamic viscosity} \ (kg/m-s) \)

D = pipe diameter (m)

\[
C_d = \frac{24}{R_e^2}
\]

(28)

Stokes law, \( R_e < 1 \)

Transition region, \((1 < R_e < 1000)\)

\[
C_d = \frac{24}{R_e} \left( 1 - \frac{R_e^2}{6} \right)
\]

(29)

Newton law region, \((1000 < R_e < 2 * 10^5)\)

Procedure for the determination of pressure profiles along the wellbore.

1. The flow rates (fluid and sand particles), fluid properties, temperature, and pressure were measured at the well head and the temperature gradient estimated.

2. Assume a new pressure differential.
3. Assume a new depth differential, and determined the new average depth.
4. From the temperature gradient, determine the average temperature of the increment.
5. Correct the fluid properties for the determined temperature and pressure.
6. Determine the type of flow regimes from the Appendix.
7. Based on step 6, determined the average fluid density, the friction loss gradient from Appendix.
8. Determined the new sand particle velocity.
9. Determined the pressure gradient of the fluid and the sand particle.
10. Calculate the depth differential using equation (1).
11. Iterate, starting from step 3 until the assumed pressure equal to the calculated pressure.
12. Determined the pressure and the depth equivalent for that increment.
13. Repeat the procedure from step 2 until the total sum of the depth differential equal to the total length of the tubing string.

Figure 1. Schematic diagram of forces on sand particle in oil-gas flow.

The proposed model results were validated with field data listed in Orkiszewski (1966), and comparing with empirical correlations for ten wells data using the commonly used Duns and Ross and Hagedorn and Brown correlations in the oil industry as well as the developed models by Yang et al. (2007) and Almedeij and Algarib (2005) model which had been proved to predict the experimental data than the widely used Sukkar and Cornell method, Cullendar and Smith method, Average Temperature and compressibility factor method and the Beggs and Brill method (Yang et al., 2007; Almedeij and Algarib, 2005; Orkiszewski, 1966)

This is through the computation of the average liquid hold up in the tubing. The Average Absolute Deviation (AAD) values of the comparison are listed in Table 1.
Table 1. Average Absolute Deviations (AAD) between measured and predicted pressure drops for different models and correlations

<table>
<thead>
<tr>
<th>Proposed Model</th>
<th>Duns and Ross Correlation</th>
<th>Hagedorn and Brown Correlation</th>
<th>Almedeij and Algharaib Model</th>
<th>Yang et al. Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAD Value</td>
<td>6.35</td>
<td>19.54</td>
<td>24.67</td>
<td>15.23</td>
</tr>
</tbody>
</table>

The AAD is defined as

\[
AAD = \left( \frac{1}{n_d} \right) \sum_{i=1}^{n_d} |\Delta P_E - \Delta P_M |
\]  

(30)

where,

\[\Delta P_E = \text{change in pressure from the field data}\]
\[\Delta P_M = \text{predicted pressure change}\]

The reduction in the error for the oil sample from ten different wells as shown in Table 1 is remarkable in reference to the AAD value of 6.53 from the application of the proposed model compared to AAD value of between 13.56 and 24.67 returned by Hagedorn and Brown’s, Duns and Ros’s correlations, Almedeij and Algharaib’s and Yang et al.’s models. Hence the accuracy of the proposed model over existing models was highlighted in the table 1.

**RESULTS DISCUSSION**

The comparison between the pressure profiles of sand-oil-gas flow and that of oil-gas flow is shown in Figure 2. The plot shows that the pressure gradient of sand-oil-gas flow is higher than that of oil-gas flow indicating that inclusion of sand flow into the flow model results in an increase in pressure gradient due to additional pressure loss as a result of frictional forces due to sand movement in fluid flow. Hence the currently used correlations are error prone as most do not include the production of sand in the fluid flow.

Assuming that the hydrodynamic condition at a particular position at time, t, inside the near vertical well is such that the solid particles (sand) settle out of the flowing suspension and deposit at the lower part of the inclined wellbore, it will be possible to correlates the rate of sediment build up at that position along the well length with the hydrodynamics forces acting on the sand particles.

The utility of this model is shown in figures 3, 4, 5, and 6 which plot the defined k-value against the fluid density, viscosity, velocity and Reynolds number respectively. The k-value less than one indicates a tendency of sand particles settling out of the fluid phase (oil) and depositing at the lower part of the inclined wellbore. A k-value greater than one indicates the case where the sand particles are suspended in the oil and transported without settling through the near vertical well interval under consideration.
Figure 3 shows the increase in k-value with the fluid density indicating the higher tendency for the sand particle to be suspended and flow with the fluid (oil) phase as the density increases.

For given oil density and velocity there is higher tendency for the particle to settle out and deposit as the fluid viscosity increases this is shown in fig, 4. this is as a result of higher value of the viscous force resulting from the fluid viscosity value increment. Higher fluid velocity at a given fluid viscosity and density increases the tendency of the solid particles to be suspended in the fluid (oil) and less tendency for the solid particles to be settle out of the fluid flow, this is shown in Figure 5 as the k-value increases with the fluid velocity at constant fluid viscosity and density. Summarizing these trends it could be seen that higher fluid velocity along with lower fluid viscosity and higher fluid density alleviates the tendency of settling and deposition of the sand particles. The combined effect was shown in Figure 6 as the k-value was plotted against the fluid Reynolds number with an almost linear relationship existing between the two properties. Hence the deposition tendency of sand particles in oil flow can be reduced by increasing the fluid Reynolds number.

**CONCLUSION**

In this study, the major hydrodynamic forces relevant to the transportation of sand particle in oil flow in a near vertical well were identified and a model for the sand-oil–gas flow was developed. The main conclusions are:

a. The transport of sand in a near vertical well is characterized by six major forces which include momentum transferred force, particle-particle interaction force, viscous force, gravity force, buoyancy force, and drag force.

b. Analysis of the net forces of solid particles (sand) enables the establishment of a criterion for the deposition of sand particle out of the flowing suspension of sand in oil flow.

c. It was identified that the fluid Reynolds number is the determining factor that control the continue suspension of sand particles in oil flow in a near vertical well.

![Figure 2. Pressure profiles of Gas-Oil and Gas-Oil-Sand Flow at Various Depth.](image)
Figure 3. K-value at different fluid density values.

Figure 4. K-value at different fluid viscosity values.

Figure 5. K-value at different fluid velocity values.
Griffith and Wallis (1961) have defined the boundary between bubble and slug flow, and Duns and Ros, (1963) have defined the boundaries between slug-annular and mist flow. These depend whether the ratio $q_g/q_t$ or $v_{gD}$ or both fall within the limits prescribed.

### Limits Flow Regime

- $\frac{q_g}{q_t} < L_B$  Bubble
- $\frac{q_g}{q_t} > L_B, v_{gD} < L_S$  Slug
- $L_M > v_g > L_S$  Transition
- $v_{gD} > L_M$  Mist

The above variables are defined as

$$v_{gD} = q_g \left( \frac{\sqrt{\rho_g}}{g \sigma} \right) / A_p$$  \text{(A-1)}

$$L_B = 1.071 - \frac{0.2218v_i^2}{d} \text{ with the limit } L \geq 0.13$$  \text{(A-2)}
\[ L_S = 50 + 36 \frac{v_g q_t}{q_g} \]  
(A-3)

\[ L_M = 75 + 84 \left( \frac{v_g q_t}{q_g} \right)^{0.75} \]  
(A-4)

where \( v_{sD} = \) Dimensionless gas velocity

\[ v_t = \text{total fluid velocity} \left( \frac{q_t}{A_p} \right), \text{ ft/sec} \]

\( \rho_L = \text{liquid density, ib/cu ft.} \)

\( \sigma = \text{liquid surface tension, ib/sec}^2 \)

**EVALUATION OF AVERAGE DENSITY AND SHEAR STRESS**

The variable fluid density (\( \rho_f \)) and fluid shear stress (\( \tau_f \)) for different flow regimes are determined as follows:

**Bubble Flow**

The void fraction of gas (\( F_g \)) in bubble flow can be expressed as:

\[ F_g = \frac{1}{2} \left[ 1 + \frac{q_t}{v_s A_p} - \sqrt{\left(1 + \frac{q_t}{v_s A_p} \right)^2 - \frac{4q_g}{v_s A_p}} \right] \]  
(A-5)

where \( v_s = v_g - v_l \) is the slip velocity in ft/sec

Griffith, 1962 suggested that a good approximation of average \( v_s \) is 0.8 ft/sec, hence equ. (A-5) can be evaluated to derive the average fluid flowing density as:

\[ \rho_f = \left(1 - F_g \right) \rho_l + F_g \rho_g \]  
(A-6)

The friction gradient is,
where \( \nu_L = \frac{q_L}{A_p(1 - F_g)} \), and

\( f, \) is the friction factor obtained by using the Moody relative roughness factor. The Reynolds number, \( R_e \) is calculated as

\[
R_e = \frac{1488 \rho_L d_p \nu_L}{\mu_L}
\]

where \( d_p \) is the hydraulic pipe diameter, ft

\( \mu_L \) is the liquid viscosity, cp

**Slug Flow**

The average density term is,

\[
\rho_f = \frac{m_t + \rho_L \nu_b A_p}{q_t + \nu_b A_p} + \Re \rho_L
\]

where \( \Re \) is the coefficient correlated from oilfield data.

Griffith and Wallis (1961) correlated the bubble rise velocity, \( \nu_b \), by the relationship

\[
\nu_b = C_1 C_2 \sqrt{g d_p}
\]

where \( C_1 \) is a function of bubble Reynolds number and \( C_2 \) is a function of both bubble Reynolds number.

The shear stress term independently derived, is expressed as.

\[
\tau_f = \frac{f \rho_L \nu_t^2}{2 g d_p} \left[ \frac{q_L + \nu_b A_p}{q_t + \nu_b A_p} + \Re \right]
\]

\( \nu_t \) = velocity of liquid and gas \( \left( \frac{q_L}{A_t} \right) \) ft/sec.
**Transition Flow**

Duns and Ros, 1963 determined the variable fluid density \( (\rho_f) \) and fluid shear stress \( (\tau_f) \) for transition flow. The method is first to calculate these terms for both slud and mist flow, and then linearly weight each term with respect to \( v gD \) and the limits of the transition zone \( L_M \) and \( L_S \).

\[
\frac{L_M - v_{sd}}{L_M - L_S} [\rho_f]_{slag} + \frac{v_{sd} - L_S}{L_M - L_S} [\rho_f]_{mist}
\]  
(A-12)

The shear stress term would be weighted similarly.

**Mist Flow**

The average flowing density for the mist flow is given by,

\[
\rho_f = (1 - F_g) \rho_l + F_g \rho_g
\]  
(A-13)

Since there a virtually no-slip between the phase, \( F_g \) is given as;

\[
F_g = \frac{1}{\left(1 + q_L/q_g\right)}
\]  
(A-14)

Duns and Ross(1963), expressed the shear stress as,

\[
\tau_f = \frac{f \rho_g v_g^2}{2g d_p}
\]  
(A-15)

Hence the friction factor is given by

\[
f = \frac{2g \tau_f d_p}{\rho_g v_g^2}
\]  
(A-16)

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Hydrodynamics of Sand-Oil-Gas Multiphase Flow in a Deviated Petroleum Well


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