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Weighted locally convex spaces of measurable functions

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Abstract. In this paper, we make a study of weighted locally convex spaces of measurable functions parallel to the studies of weighted locally convex spaces of continuous functions which has been a subject of intense research for decades. With $L^p, 1 \leq p < \infty$, spaces as our motivation, the completeness and inductive limits of those spaces are studied including their relationship with the weighted spaces of continuous functions leading to new results and generalizations of results true for L^p spaces.

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1. Preliminaries

L^p spaces are some of the most important spaces studied in Mathematics because of its abundant usefulness and applications that run across all the branches of Mathematics. It is a ready source of examples and counter-examples for many mathematical theories. The study of Orlicz spaces, for example, is borne out of an attempt to generalize the results of L^p spaces. This study is also an attempt to generalize the study of L^p spaces with the tool of weighted spaces parallel to that of locally convex spaces of continuous functions (see [6] and [9]), leading us to new results and new proofs of known results.

2. Notation and definitions

Throughout this paper (except otherwise stated), X would denote:

(i) a locally compact Hausdorff space and

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(ii) a measure space, with positive Radon measure μ, on a σ-algebra M such that M contains all Borel sets in X.

We adopt the notations of [6] and [9] for weighted spaces of continuous functions on X. A real-valued non-negative upper semicontinuous function (u.s.c.) v on X is called a *weight* on X. Let V be a non-empty system of weights such that given v_1,v_2 in V and $a > 0$, there is a $v \in V$ such that $av_i \leq v$, $i = 1, 2$ (pointwise on X); if in addition, for each $t \in X$ there is $v \in V$ with $v(t) > 0$, then V is called a Nachbin family on X.

An N_p family V^p on X, $1 \leq p < \infty$, is defined as a set of non-negative measurable functions $v : X \to [0, \infty)$ on X satisfying the following condition: if u and $v \in V^p$ and $\lambda > 0$, there is a $w \in V^p$ such that $\lambda u, \lambda v \leq w$ (pointwise on X).

Members of V^p are also called weights. It should be noted that upper-semicontinuous (u.s.c.) functions on X are measurable. So the Nachbin family V on X and the N_p family V^p on X are comparable. It should be observed that p appears redundant in the notation of N_p family V^p. However, its relevance will be clear in the next section.

Let E be a real (resp.complex) locally convex Hausdorff space, $M(X, E)$ is the space of all measurable functions from X into E and $C(X, E)$ is the vector subspace of $M(X, E)$ consisting of the continuous functions f from X into E. Also $B(X, E)$ is the space of all bounded functions f from X into E. $B_o(X, E)$ is the subspace of $B(X, E)$ consisting of all bounded functions from X into E that vanish at infinity, i.e., those bounded functions f from X into E, such that, given any continuous seminorm (cs(E)) q on E and any $\epsilon > 0$, there is a compact subset K of X such that $q(f(x)) < \epsilon$ for every $x \in X$ outside of K. $M(X, E) \cap B(X, E)$ is denoted by $M_b(X, E)$; $C(X, E) \cap B(X, E)$ is denoted by $C_b(X, E)$ and $C_o(X, E)$ denotes $C(X, E) \cap B_o(X, E)$. $M_m(X, E)$ will denote the subspace of $M(X, E)$ consisting of those functions on X that are identically zero outside some set of finite measure. For example, constant non zero functions from X into E are measurable but are not in $M_m(X, E)$ if $\mu(X) = \infty$. $C_c(X, E)$ shall denote the subspace of $C(X, E)$ consisting of those functions that are identically zero outside some compact subset of X. It is clear that $C_c(X, E) \subseteq M_m(X, E)$. When $E = \mathbf{R}$ or $\mathbf{C}$, the corresponding function spaces on X are written omitting E. Thus $B^+(X)$ is the cone of $B(X)$ consisting of bounded positive valued functions on X, while $B_o^+(X)$ is the cone of $B_o(X)$ consisting of positive valued functions on X that vanish at infinity. We can now introduce the following two spaces:

$$CV_o(X, E) = \{f \in C(X, E) : v.q(f) \text{ vanishes at infinity on } X \text{ for all } v \in V,\ q \in cs(E)\},$$

$$MV^p(X, E) = \{f \in M(X, E) : v.q(f) \in L^p \text{ for all } v \in V^p,\ q \in cs(E)\}.$$

The *weighted topology* w_V on $CV_o(X,E)$ is defined by the family of seminorms

$$p_{v,q}(f) = sup(v(x)q(f(x)) : x \in X) \ for \ v \in V \ and \ q \in cs(E)$$

If $CV_o(X,E)$ is endowed with the weighted topology w_V, it is called a weighted locally convex space of continuous functions. It has a basis of closed absolutely convex neighbourhoods of origin of the form

$$V_{v,q} = \{f \in \ CV_o(X,E) : p_{v,q}(f) \leq 1\}.$$

Much has been done on those spaces. See for example [1], [3], [6] and [9]. Similarly, if $MV^p(X,E)$ is endowed with the *weighted topology* w_{V^p} generated by the family of continuous seminorms

$$p_{v,q}(f) = (\int_X (v.q(f))^p d\mu)^{\frac{1}{p}}$$

as v ranges over V^p and $q \in cs(E)$, then it is called a weighted locally convex space of measurable functions. It has a basis of closed absolutely convex neighbourhoods of the origin of the form

$$V_{v,q} \ = \ \{f \in MV^p(X,E) : p_{v,q}(f) \leq 1\}$$

We shall assume that $MV^p(X)$ is endowed with this topology w_{V^p} henceforth. We shall also assume that $MV^p(X,E)$ is Hausdorff. This is true if there is a $v \in V^p$ such that $v > 0$ a.e. on X. Finally, if $U(resp.U^p)$ and $V(resp.V^p)$ are two $Nachbin(N_p)$ families on X, and for every $u \in U(U^p)$ there is a $v \in V(V^p)$ such that $u \leq v$(pointwise on X), then we write $U(U^p) \leq V(V^p)$. In the case $V(V^p) \leq U(U^p)$ and $U(U^p) \leq V(V^p)$ we write $U(U^p) \sim V(V^p)$.

Examples. Denote $K^+(X)$ as the set of all positive constant functions on X. If $V^p = K^+(X)$, then $MV^p(X,E) = \mathcal{L}^p(X,E)$ both topologically and algebraically. If almost equal functions are identified we have $L^p(X,E)$ spaces. Also if X is the set of natural numbers and μ is the counting measure, then $MV^p(X) = \ell^p$ both topologically and algebraically.

By following the proofs for $0 < p < 1$ in [4], the following result can be easily checked for $1 \leq p < \infty$: If $V^p \leq B(X)$, then

(i) $C_c(X)$ is w_{V^p} dense in $M_m(X)$.
(ii) $M_m(X)$ is w_{V^p} dense in $MV^p(X)$.

For let $f \in MV^p(X)$, $f > 0$, then by [8, Theorem 1.17], there are simple measurable functions s_n on X such that $0 \leq s_1 \leq s_2 \leq \cdots \leq f$ and $s_n(x) \to f(x)$ as $n \to \infty$. Clearly each $s_n \in M_m(X) \subseteq MV^p(X)$ and $|f - s_n|^p \leq f^p$. The dominated convergence Theorem shows that for $v \in V$, $p_v(f - s_n) \to 0$ as $n \to \infty$. $f - s_n \in V_{v,|.|}$ for some n; and since each $s_n \in M_m(X)$ then f is in the w_{V^p} closure of $M_m(X)$. The general case (f complex) follows from this.

Combining (i) and (ii), we have the following:

(iii) $C_c(X)$ is w_{V^p} dense in $MV^p(X)$.

Thus, specifically $C_c(X)$ is $L^p(\mu)$ dense in $\mathcal{L}^p(X)$. This is well known.

3. Completeness of weighted spaces

Let U^p and V^p be N_p families on X and $\phi : X \to X$ be a continuous mapping such that $U^p \leq V^p \circ \phi$, then the mapping $f \to f \circ \phi$ is a continuous linear mapping from $MV^p(X,E)$ into $MU^p(X,E)$. For if $f \in MV^p(X,E)$ and $u \in U^p$, we can choose $v \in V$ such that $u \leq vo\phi$. Hence, for any continuous seminorm q on E, we have

$$p_{u,q}(f \circ \phi) \leq \left(\int_X ((v \circ \phi).q(f \circ \phi))^p d\mu\right)^{\frac{1}{p}} \leq p_{v,q}(f)$$

Since $v.q(f) \in L^p$ for all $v \in V^p$ and $q \in cs(E)$, it is clear that $u.q(f \circ \phi) \in L^p$. Hence, since u is arbitrary, then $f \circ \phi \in MU^p(X,E)$. We have just shown the following result which is an analogue of [6, Propositions 1 and 2].

Proposition 3.1. *Let U^p and V^p be N_p families on X and $\phi : X \to X$ be a continuous mapping such that $U^p \leq V^p \circ \phi$, then the mapping $f \to f \circ \phi$ is a continuous linear mapping from $MV^p(X,E)$ into $MU^p(X,E)$.*

If ϕ is taken to be the identity map on X, then the first part of the following result follows immediately from Proposition 3.1.

Proposition 3.2. *Let U^p and V^p be N_p families on X with $U^p \leq V^p$, then*

1. *$MV^p(X) \subseteq MU^p(X)$*
2. *the topology induced on $MV^p(X)$ by w_{U^p} is weaker than w_{V^p}.*

Conversely, if (1) and (2) hold and μ is a probability measure such that $V^p \leq B(X)$, then $U^p \leq V^p$.

To prove the converse, we use an argument supplied by the referee which is inspired by Summers' one [9,Theorem 3.3]. It should be observed that the assumptions (1) and (2) imply that for any $u \in U^p$ there is $v \in V^p$ such that $V_v \subseteq U_u \cap MV^p(X)$. We will show that if A $= \{x \in X : (u-v)(x) > 0\}$, then $\mu(A) = 0$. Indeed, suppose $\mu(A) > 0$. For every integer $n \geq 2$, let $B_n = \{x \in X : u(x) > \frac{n+1}{n-1}v(x)\}$; then $B_2 \subseteq B_3 \cdots \subseteq B_n \subseteq B_{n+1} \subseteq \cdots$ and $A = \bigcup_{n=2}^{\infty} B_n$. Then $0 < \mu(A) = \lim_{n\to\infty} \mu(B_n)$ implies that there is $n_o \geq 2$ such that $\mu(B_{n_o}) > 0$. Let

$$f = \frac{1}{(\mu(B_{n_o}))^{\frac{1}{p}}} \frac{2}{u+v} \chi_{B_{n_o}}.$$

Then $f \in V_v$, since

$$\Big(\int_X (v.|f|)^p d\mu\Big)^{\frac{1}{p}} = \frac{1}{(\mu(B_{n_o}))^{\frac{1}{p}}} \Big(\int_{B_{n_o}} (\frac{2v}{u+v})^p d\mu\Big)^{\frac{1}{p}} \leq \frac{(\mu(B_{n_o}))^{\frac{1}{p}}}{(\mu(B_{n_o}))^{\frac{1}{p}}} = 1$$

but

$$\Big(\int_X (u.|f|)^p d\mu\Big)^{\frac{1}{p}} = \frac{1}{(\mu(B_{n_o}))^{\frac{1}{p}}} \Big(\int_{B_{n_o}} (\frac{2u}{u+v})^p d\mu\Big)^{\frac{1}{p}}.$$

Now, for all $x \in B_{n_o}$, $u(x) + v(x) < (1 + \frac{n_o-1}{n_o+1})u(x) = \frac{2n_o}{n_o+1}u(x)$ implies

$$\Big(\int_X (u.|f|)^p d\mu\Big)^{\frac{1}{p}} \geq \frac{1}{(\mu(B_{n_o}))^{\frac{1}{p}}}\big(\frac{n_o+1}{n_o}\big)(\mu(B_{n_o}))^{\frac{1}{p}} = \frac{n_o+1}{n_o} > 1$$

so, $f \notin U_u \cap MV^p(x)$, a contradiction.

Corollary 3.3. *Let U^p and V^p be N_p families on X such that $U^p \sim V^p \leq B(X)$. If μ is a probability measure, then $MV^p(X) = MU^p(X)$ as topological vector spaces.*

The relationship between $CV_o(X, E)$ and $MV^p(X, E)$ is set forth in the following result, the proof of which can be easily checked.

Proposition 3.4. *Let $V(V^p)$ be a Nachbin(resp.N_p) family on X such that $V^p \leq V \leq B(X)$. If μ is a finite measure, then $CV_o(X) \subseteq MV^p(X)$.*

Remark. Unlike Proposition 3.2(2), when μ is a finite measure the topology induced on $CV_o(X)$ by w_{V^p} is weaker than w_V. If $K^+(X) = V^p$ and $V = B_u^+(X)$, where $B_u^+(X)$ is the set of all upper semicontinuous bounded positive functions on X, w_V is the supremum norm topology $\|.\|$ (see ([3], [9])) and w_{V^p} is the L^p topology. Also, $CV_o(X) = C_o(X)$ algebraically whenever $V = B_u^+(X)$. Since the topology induced on $CV_o(X)$ by w_{V^p} is weaker than w_V when $V^p \leq V$, then in particular, on $C_o(X)$, the L^p topology is weaker than the supremum norm topology. The following example supplied by the referee shows that these two topologies do not coincide. For consider $X = (0, 1)$ with the usual topology, $\mu =$ the Lebesgue measure, and for $n \geq 3$,

$$f_n(x) = \begin{cases} 0 & \text{if } \ x \in (0, \frac{1}{2} - \frac{1}{n}] \cup [\frac{1}{2} + \frac{1}{n}, 1), \\ 1 & \text{if } \ x = \frac{1}{2}, \\ \text{linear} & \text{on } \ [\frac{1}{2} - \frac{1}{n}, \frac{1}{2}] \text{ and on } \ [\frac{1}{2}, \frac{1}{2} + \frac{1}{n}]. \end{cases}$$

Then $f_n \to 0$ in $L^p(\mu)$, $1 \leq p < \infty$, but $\|f_n\| = \sup|f_n| = 1$, $\forall n \geq 3$.

For the remaining part of this section, we define

$$\chi_c(X) = \{\lambda\chi_K; \lambda \geq 0 \text{ and } K \text{ a compact subset of } X\}.$$

Theorem 3.5. *Let $V^p(V)$ be an $N_p(Nachbin)$ family on X and μ be a probability measure. Then w_V and w_{V^p} coincide on the following identities:*

(1) $CV_o(X) = MV^p(X) \cap C(X)$ *if* $V^p \sim V = \chi_c(X)$

(2) $CV_o(X) = MV^p(X) \cap C_b(X)$ *if* $V^p \sim V \sim B_o^+(X)$

(3) $CV_o(X) = MV^p(X) \cap C_o(X)$ *if* $V^p \sim V \sim B_u^+(X)$

(4) $CV_o(X) = MV^p(X) \cap C_c(X)$ *if* $V^p \sim V = C^+(X)$

X is σ compact and w_V is, respectively, the compact open (c-op) topology; strict (β_0) topology; the topology of uniform convergence ($\|.\|$) and ind.lim.top. on $\{C_K = f \in C_c(X) : supp f \subset K\}$ where each C_K is endowed with the topology of uniform convergence on X as K varies over compact subsets of X (e.g. see [2, p50]).

Proof. We first prove the algebraic equalities (1) Let $f \in CV_o(X)$, then fv vanishes at infinity for all $v \in V$ and thus $fv \in L^p \ \forall \ v \in V^p$ since $V \sim V^p$ and μ is a probability measure. Thus $CV_o(X) \subseteq MV^p(X) \cap C(X)$. Also let $f \in MV^p(X) \cap C(X)$. Since $V = \chi_c(X)$, and $f \in C(X)$, then fv vanishes at infinity for all $v \in V$ and so $f \in CV_o(X)$. Thus the algebraic equality of (1) is proved. The remaining three algebraic equalities can similarly be verified. The topological equalities of the four identities follow immediately from Corollary 3.3. The proof is complete since it is well known that w_V is respectively the compact open topology, the strict topology, the topology of uniform convergence and the ind.lim.topology on $CV_o(X)$ whenever V is equivalent ($\sim$) to $\chi_c(X)$, $B_o^+(X)$, $B_u^+(X)$, $C^+(X)$ respectively (see [1], [6], [9]). ☑

We are now in a position to consider the completeness of $MV^p(X, E)$.

Theorem 3.6. *Let V^p be an N_p family on X such that $0 < V^p \leq B^+(X)$. If E is complete, then $MV^p(X, E)$ is complete.*

Proof. Let ϕ be a Cauchy filter in $MV^p(X, E)$ and U be a closed neighbourhood of the origin in $L^p(X, E)$. Then we can find a set H in ϕ such that $v.(f - g) \in U \ \forall \ f, \ g \in H \ and \ v \in V^p$. Clearly $\phi.V^p = \{vH : H \in \phi, \ v \in V^p\}$, where $vH = \{vf : f \in H\}$, is a Cauchy filter in $L^p(X, E)$. Since each v is bounded, it is clear that ϕ is a Cauchy filter in $L^p(X, E)$ and thus converges to $f_o \in L^p(X, E)$ by the completeness of $L^p(X, E)$. Thus $v.q(f_o) \in L^p$ for all v in V, $q \in cs(E)$, (since each v is bounded). Therefore $f_o \in MV^p(X, E)$ and it is the limit of ϕ in the space $MV^p(X, E)$. ☑

If $V(V^p)$ is a $Nachbin(N_p)$ family on X such that $CV_o(X, E)$ is contained in $MV^p(X, E)$ and $V^p \leq B^+(X)$, then in the light of Theorem 3.6, $CV_o(X, E)$ is complete if and only if $CV_o(X, E)$ is closed in $MV^p(X, E)$. Suppose $\mu(X) < \infty$ and $V^p \leq V$, then $CV_o(X, E)$ is contained in $MV^p(X, E)$. If E is complete and $\chi_c(X) \leq V$, then $CV_o(X, E)$ is complete [6, Theorem 3] and thus from Theorem 3.6, we have the following result.

Proposition 3.7. *Suppose V^p and V be respectively N_p and Nachbin families on X such that $\chi_c(X) \leq V$, $V^p \leq B^+(X)$ and $V^p \leq V$. If $\mu(X) < \infty$ and E is complete, then $CV_o(X, E)$ is w_{V^p} closed in $MV^p(X, E)$.*

Corollary 3.8. *If E is complete and X is such that $\mu(X) < \infty$, then $C_o(X, E)$ is L^p closed in $L^p(X, E)$.*

Proof. Set $V = B_u^+(X)$ and $V^p = K^+(X)$, then the result follows immediately from Proposition 3.7. ☑

4. **Inductive limits**

Let $\{V_n^p,\ n \in N\}$ be a sequence of N_p families on X such that $V_{n+1}^p \leq V_n^p$ for each $n \in N$. We shall denote *ind* $MV_n^p(X)$ by $V^pM(X)$. We want to describe the weighted inductive limit $V^pM(X)$, analogous to the case of weighted spaces of continuous functions, in terms of an associated N_p family on X. Let $v_n \in V_n^p$ and $\alpha_n > 0$ for each n; if we put $\overline{v}(x) = inf\{\alpha_n v_n(x), n \in N\},\ x \in X$, then $\overline{v}(x)$ is clearly a weight on X. Scalar multiples of all those weights on X form an N_p family on X which we will denote $\overline{V}^p$. Clearly $\overline{V}^p$ contains every N_p family V^p on X that satisfies $V^p \leq V_n^p$ for each $n \in N$.

We first state the following results:

Lemma 4.1. *Let V^p be an N_p family on a σ-compact space X and μ a probability measure, then $M\overline{V}^p(X)$ and $V^pM(X)$ induce the same topology on $M_m(X)$.*

Proof. We follow the proof of the analogous result in the weighted spaces of continuous functions (see [2,p114,Lemma 4]) with some modifications. Since the canonical injection of $V^pM(X)$ into $M\overline{V}^p(X)$ is continuous, we can fix an arbitrary neighbourhood U of zero in $V^pM(X)$ and then have to prove that the intersection of $M_m(X)$ with some zero neighbourhood in $M\overline{V}^p(X)$ is contained in U. By the description of a basis of zero neighbourhoods in an inductive limit, we may assume without loss of generality that U is an absolutely convex hull of the form $\Gamma(\bigcup_n B_n)$, where

$$B_n = \{f \in MV_n^p(X) : p_{v_n}(|f|) \leq \rho_n,\ v_n \in V_n\}$$

and ρ_n is positive for each $n \in N$. Put $\overline{v} = \inf\lim_{n\in N} \frac{2^n}{\rho_n} v_n \in \overline{V}^p$. It remains to show that $\{f \in M_m(X) : p_{\overline{v}}(|f|) < 1\} \subset U$. Fix $f \in M_m(X)$ with $p_{\overline{v}}(|f|) < 1$. For each n, let F_n denote the measurable subset $\{x \in X : \frac{2^n}{\rho_n} v_n(x)|f(x)| \geq 1\}$ of X. We observe that $\bigcap F_n$ is empty because, for any $x \in \bigcap F_n$, $\frac{2^n}{\rho_n} v_n(x)|f(x)| \geq 1$ holds for each n, whereby $p_{\overline{v}}(|f|) \geq 1$ contradicting $p_{\overline{v}}(|f|) < 1$. If $U_n = X \backslash F_n$, then U_n is measurable for each n. Hence by [8, Theorem 2.17a], there is an open set V_n such that $U_n \subset V_n$ for each n. Clearly $(V_n, n \in N)$ is an open covering of X. Let $(\psi_n)_n \subset C_c(X)$ be a continuous partition of unity on *supp f* which is subordinate to $(V_n)_n$. We then take $g_n = 2^n\psi_n f \in M_m(X) \subset MV_n^p(X)$ for each n and estimate $p_{\overline{v}}(|g_n|)$ $= |\psi_n 2^n| p_{v_n}(|f|) = |\rho_n \psi_n \frac{2^n}{\rho_n}| p_{v_n}(|f|) \leq \rho_n$. Thus each $g_n \in B_n$, and hence f = $\sum \psi_n f$ is an element of $\Gamma(\bigcup_n B_n) = U$ and the proof is complete. ☑

The following result will also be needed.

Lemma 4.2. [1, Lemma 1.2] *Given a locally convex space (E_1, ϵ_1), let E_2 denote a linear subspace and ϵ_2 a locally convex topology on E_2 which is finer* *than the topology induced by* ϵ_1. *If* ϵ_1 *and* ϵ_2 *induce the same topology on some dense linear subspace* D *of* (E_2, ϵ_2), *then* $\epsilon_2 = \epsilon_1/E_2$.

We now have the following result which is an analogue of [2, Theorem 1.3].

Theorem 4.3. *Let* X *be a* σ*-compact space and* μ *a probability measure.*

(1) *If* $\{V_n^p, n \in N\}$ *is a sequence of* N_p *families on* X *such that* $V_{n+1}^p \leq V_n^p$ *for each* $n \in N$, *then the canonical injection from* $\mathbb{V}^p M(X)$ *into* $M\overline{V}^p(X)$ *is a topological isomorphism.*

(2) *Suppose* $V_n^p \leq B^+(X)$ *for each* $n \in N$, *then* $M\overline{V}^p(X)$ *is the completion of* $V^p M(X)$.

Proof. (1) If $(E_1, \epsilon_1) = M\overline{V}^p(X)$, $(E_2, \epsilon_2) = V^p M(X)$ and D=$M_m(X)$ in Lemma 4.2, then the proof follows clearly from Lemma 4.1. (2) Since $M\overline{V}^p(X)$ is complete by Theorem 3.6, and the fact that $M_m(X)$ is dense in $V^p M(X)$, an application of (1) completes the proof. ☑

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