Symbolic Nonlinear Analysis of Viscoelastic Pipes

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ABSTRACT

In this study, a novel application domain of symbolic computation of nonlinear finite element method using a viscoelastic pipes has been presented. The problem of geometric nonlinearity due to the effect of high temperature, large displacement, small strain and moderate rotation of viscoelastic pipes was modeled and validated with numerical, symbolic and graphical computation in a unified manner called symbolic analysis. The current study has shown that the symbolic computation is effective and efficient, it saves computation time which can be seen in the symbolically integrated element-stiffness matrices in the nonlinear finite element method which by-passes time-consuming numerical quadrature operation, especially as the number of Gauss points increases. The symbolic computation also gives the analyst more visibility with respect to the solution method and the engineer can then more easily grasp the inter-relationship of the problem variables, recognize the simplification to be made and do a better and more accurate job. The computer codes of the finite element formulation used in this work was generated through the symbolic programming of the finite element computer code in AceGen and AceFEM computer program.

***Keywords**: Viscoelastic Pipe, Nonlinear Finite Element Method, Symbolic Computation, AceGen & AceFEM*

1.0 INTRODUCTION

In continuum mechanics, “non-linearity” is divided into two phenomena. One is “material non-linearity” the other is “geometric non-linearity”. The former is popular and its characteristic has been expressed by using constitutive model like an elasto-plastic, visco-elastic or hyperelastic model etc. On the other hand, the latter is often neglected because of its complexity. And even when considered in some analyses, at the solution stage the model is linearized in order to simplify it. Instead of considering geometric non-linearity, the infinitesimal deformation theory is used. This theory supposes that deformation during loading is very small and neglected, as if the body does not deform after loading. So, even if an elasto-plastic model is used, deformation obtained from analyses using this theory is assumed to be geometrically linear. However, when the actual body deformed during loading, the geometric nonlinearity appeared, hence necessitates finite deformation theory. In their analysis of long span cabled-stayed bridges, Freire et al (2006) concluded that “Linear analysis” of these modern bridges which have a high flexibility does not provide satisfactory results as their geometrically nonlinear behavior is not modeled in the analysis. Hence, finite deformation theory is necessary in the analysis, and with the help of numerical methods, solutions are achievable.

Numerical methods rule supreme in modern structural analysis, thanks to the truly remarkable advances in computing power over the last decades. This has caused a steady decline in classical analytical techniques, which is to be expected since modern numerical tools such as, the finite-element method which not only permit the analysis of the most complex problems, but are also seen as allowing these problems to be undertaken even by engineers whose background in mechanics is relatively modest. The Finite Element Analysis (FEA) is perhaps the most successful approach to numerical computation of approximate solutions to problems that preclude close-form solution. This coupled with post-processing analysis for simulation and sensitivity analysis make it the most powerful computer oriented method ever devised to analyze practical engineering problems.

Commercial computational tools are widely available to implement several FEA schemes. Such canned programs often create a disconnection between the analyst and the problem as the whole process is rather mechanical. Computer Algebra Systems (CAS) are "expert systems" designed to perform symbolic and numerical manipulation following the rules of mathematics. [Pavlovic M. N. (2003)]. Incorporating these with traditional FEA creates a middle ground where the development of the FEA schemes follows the same modeling approach as the symbolic representation of the underlying problems are directly accommodated. They possess the remarkable capability of manipulating not only numbers, but also abstract symbols which represent numerical quantities. Thus, they are more versatile than traditional computer codes, such as FORTRAN and BASIC, which perform only numerical computations.

Symbolic computations have found broad applications in many areas of science and engineering. It has led to new approaches for problems solving and provide tools that enable an automatic and computerized solution of problems in ways that are not possible with conventional computing systems. Of importance in the study of symbolic computation in structural mechanics is the work of Mattern and Schweizerhof (2010) who used the symbolic programming tool AceGen , a plug-in for the computer algebra software MATHEMATICA to implement a formulated "Solid-shell"- element. The formulation of the element was done with linear and quadratic interpolation of the in-plane geometry and displacement in the thickness as well as in shell surface direction, with "assumed natural strain" and "enhanced assumed strain" in order to reduce artificial stiffness effect on the element. They showed some numerical examples to prove the superiority of AceGen generated element routines over the manually performed implementation and concluded that symbolic computation is clearly advantageous in many applications in structural mechanics.

Jiang and Wang (2006) called the unified system of numeric and symbolic manipulation of numbers and abstract symbols "a semi-symbolic program". They concluded that, the semi-symbolic program written for the implementation of finite element method in plasticity is a good compromise between the computational efficiency and human effort in developing non-linear finite element method program. In the paper, while developing the weak form of the governing differential equation, the shape function (or weight function), its derivatives, Jacobian and the strain-displacement matrix for each element are computed symbolically and stored in closed form. However, in order to maintain the equilibrium condition in Newton-Raphson iteration scheme, the evaluation of stiffness matrix and equivalent force of stress were performed numerically by using the Gauss-Legendre quadrature. This resulted in a semi-symbolic program of nonlinear finite element method.

There are many other problems amenable to the present technique based on symbolic matrix inversion; this include two-dimensional elasticity problem. For example, there is the non-collinearly loaded lamina problem (especially useful for purposes of investigating the departure of the shear stress distribution from that of beam theory when rapidly changing shear-force gradients are present). This problem was studied for both isotropic and orthotropic materials: instances were discovered where isotropy leads to the classical parabolic shear-stress distribution, whereas the introduction of orthotropy causes a serious departure from this standard situation [Pavloric M.N. et al (2002)]. A number of additional problems, related to the ones described above but with curvilinear (rather than rectangular) boundaries, have also been tackled in the work of Tahan N. (1991).

A few other people have worked on the use of symbolic computation in the context of finite element method. Korelc (2004) developed a hybrid system in which MATHEMATICA was used for the automatic derivation of material model and the generation of symbolic nonlinear finite element codes. Papusha et al (2008) also developed a symbolic solution to boundary value problems and applied it to solve problems in offshore design technology. Adeleye and Fakinlede (2010) also developed a symbolic finite element solution for the problem of heat transfer in radial fin of triangular profile. The result of their symbolic computation was used for optimization of fin material usage.

Even though much work has been done in symbolic computation of structural mechanics, very little has been done in the area of symbolic computation of geometric nonlinearity of pipes. Other papers have been published on geometric nonlinearity, which include Wataru and Atluri (1995) who developed assumed stress hybrid finite element for nonlinear problems. The elements passed the basic requirements such as coordinate invariance, patch test and eigenvalue test. Areias and Matrous (2008) developed three-dimensional computational framework for the simulation of highly nonlinear viscoelastic reinforced elastomers. The framework can incorporate particle-matrix decohesion and matrix tearing which are requirements in many materials such as solid propellant. Banerjee et al (2008) developed new variation of nonlinear shooting and Adomain decomposition methods in solving the problem of large deflection of a cantilever beam under arbitrary end loading conditions. The procedure used was envisaged to be useful for modeling the actuation of compliant mechanism by discretely distributed smart actuators. Vaz and Caire (2010) derived a mathematical formulation for the deflection of linear viscoelastic beam. The system of equations derived was solved using Forth-Order Runge-Kutta method with an iteration scheme. The results were in agreement with results obtained from finite element method.

The objective of this study is to develop a symbolic finite element solution for the geometric nonlinear viscoelastic pipes. Since temperature effect contributes significantly to the geometric nonlinearity, Fouad E. (2005) concluded that, modeling of small strain viscoelastic material at high temperature may be done using hyperelastic constitutive law since at that temperature, time-dependence behavior of the viscoelastic material is not critical and its finite deformation is irreversible, whereas hyperelastic materials can undergo finite deformation under external load without irreversibility.

2.0 PROBLEM STATEMENT

A uniformly distributed transverse load $q(x)$ is applied to the beam of length L and symmetrical cross-section. The distributed load may include the weight of the beam. For this model the following assumptions were made (a) large transverse displacement (b) strains remain small, (c) rotation is moderate, (d) axial extensibility is neglected, (e) effect of shear deformation is negligible.

2.1 MATHEMATICAL FORMULATION

The bending of beam with large displacement, small strain and moderate rotation can be derived using the displacement field which is written as

$$u_1 = u_0(x) - z\frac{\partial w_0}{\partial x}, \qquad u_2 = 0, \qquad u_3 = w_o(x) \tag{2.1}$$

Where (u, w) are the total displacements along the coordinate direction (x, z) and u_0 and w_0 denote the axial and transverse displacements of a point on the neutral axis at time t.

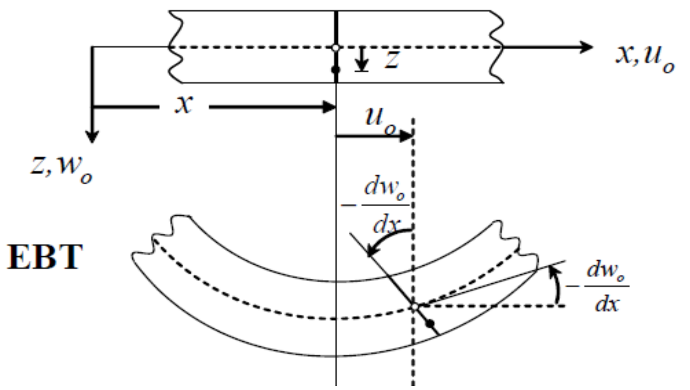

Figure 2.1 Deformation of a transverse normal line in a beam.

Using the nonlinear strain-displacement relation

$$E_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_m}{\partial x_i}\frac{\partial u_m}{\partial x_j}\right] \tag{2.2}$$

Or

$$E_{11} = \frac{\partial u_1}{\partial x_1} + \frac{1}{2}\left[\left(\frac{\partial u_1}{\partial x_1}\right)^2 + \left(\frac{\partial u_2}{\partial x_1}\right)^2 + \left(\frac{\partial u_3}{\partial x_1}\right)^2\right] \tag{2.3}$$

For small strain and moderate rotation, (large strain terms are dropped and moderate rotation of 10^0-15^0 is used) equation 2.1 becomes

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} + \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1}\right)^2 + z\left(-\frac{\partial u_3}{\partial x_1}\right)^2 = \varepsilon_{11}^0 + z\varepsilon_{11}^1 \tag{2.4}$$

Where E_{ij} is the Lagrangian strain tensor and ε_{ij} is the Eulerian strain tensor. Both are equal for small strain.

Hamilton's principle or the principle of virtual displacement requires

$$0 \equiv \delta W_I^e - \delta W_E^e \qquad 2.5$$

$$\delta W_I^e = \int_v \sigma_{ij}\delta\varepsilon_{ij} dV = \int_0^L \int_A \sigma_{xx}(\delta\varepsilon_{xx}^0 + z\delta\varepsilon_{xx}^1) dA dx \qquad 2.6$$

(The volume integral can be expressed as a product of integrals over the length and area of the element)

$$\delta W_E^e = \int_{x_a}^{x_b} q\delta w_0 dx + \int_{x_a}^{x_b} f\delta u_0 dx + \sum_{i=1}^{6} Q_i^e \delta\Delta_i^e \qquad 2.7$$

δW_I^e is the virtual strain energy stored in the element due to the actual stresses σ_{ij} in moving through the virtual strains $\delta\varepsilon_{ij}$ and δW_E^e is the work done by externally applied loads in moving through their respective virtual displacements, $q(x)$ is the distributed transverse load, $f(x)$ is the distributed axial load, Q_i^e are the generalized nodal forces and $\delta\Delta_i^e$ are the virtual generalized nodal displacement of the element.

Substituting equations 2.5, 2.7 and 2.8 into 2.6, the virtual principle equation becomes

$$0 = \int_{x_a}^{x_b} \left[\left(\frac{d\delta u_0}{dx} + \frac{dw_0}{dx}\frac{d\delta w_0}{dx}\right) N_{xx} - \frac{d^2\delta w_0}{dx^2} M_{xx}\right] dx - \int_{x_a}^{x_b} q(x)\delta w_0(x) dx + \int_{x_a}^{x_b} f(x)\delta u_0(x) dx + \sum_{i=1}^{6} Q_i^e \delta\Delta_i^e \qquad 2.8$$

Where $N_{xx} = \int_A \sigma_{xx}\, dA$ and $M_{xx} = \int_A \sigma_{xx}\, z dA$

The differential equation governing nonlinear bending of straight beams is then obtained from the virtual work statement. Integrating equation 2.9, we obtain

$$0 = \int_{x_a}^{x_b} \left\{\left(-\frac{dN_{xx}}{dx} - f\right)\delta u_0 - \left[\frac{d}{dx}\left(\frac{dw_0}{dx} N_{xx}\right) + \frac{d^2 M_{xx}}{dx^2} + q\right]\delta w_0\right\} dx + \left[N_{xx}\delta u_0 + \left(\frac{dw_0}{dx} N_{xx} + \frac{dM_{xx}}{dx}\right)\delta w_0 - M_{xx}\frac{d\delta w_0}{dx}\right]_{x_a}^{x_b} - \sum_{i=1}^{6} Q_i^e \delta\Delta_i^e \qquad 2.9$$

Separating the coefficients of δu_0 and δw_0 results in the equation of equilibrium known as Euler equation which is the governing differential equation governing nonlinear bending of straight beams.

$$-\frac{dN_{xx}}{dx} - f(x) = 0$$

$$\frac{d}{dx}\left(\frac{dw_0}{dx} N_{xx}\right) + \frac{d^2 M_{xx}}{dx^2} + q(x) = 0 \qquad 2.10$$

2.2 NONLINEAR FINITE ELEMENT FORMULATION

The Weak Form of Equation 2.10 is given as

$$0 = \int_{x_a}^{x_b} \left(\frac{dv_1}{dx} N_{xx} - v_1 f \right) dx - v_1(x_a)[-N_{xx}(x_a)] - v_1(x_b)[N_{xx}(x_b)]$$

$$0 = \int_{x_a}^{x_b} \left[\frac{dv_2}{dx} \left(\frac{dw_0}{dx} N_{xx} \right) - \frac{d^2 v_2}{dx^2} M_{xx} - v_2 q \right] dx - v_2(x_a) \left[- \left(\frac{dw_0}{dx} N_{xx} - \frac{dM_{xx}}{dx} \right) \right]_{x_a} - v_2(x_b) \left[\frac{dw_0}{dx} N_{xx} + \frac{dM_{xx}}{dx} \right]_{x_b} - \left[- \frac{dv_2}{dx} \right]_{x_a} [-M_{xx}(x_a)] - \left[- \frac{dv_2}{dx} \right]_{x_b} [M_{xx}(x_b)]$$

2.11

Where v_1 and v_2 are weight functions

The constitutive relation for linear Viscoelastic material (Kelvin-Voigt model)

$$\sigma_{xx} = k\varepsilon_{xx} + \eta \frac{d\varepsilon_{xx}}{dt}$$

$$\sigma_{xx} = \frac{N_{xx}}{A} = k\varepsilon_{xx} + \eta \frac{d\varepsilon_{xx}}{dt}$$

Therefore $N_{xx} = EA \left[\frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \right] + \eta A \frac{d}{dt} \left[\frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \right]$

Similarly $M_{xx} = -EI \frac{\partial^2 w_0}{\partial x^2} - \eta I \frac{d}{dt} \left(\frac{\partial^2 w_0}{\partial x^2} \right)$ 2.12

Substituting the constitutive relations of viscoelastic material in equation 2.12 into the weak form in equation 2.11, we obtain

$$0 = \int_{x_a}^{x_b} EA \frac{d\delta u_0}{dx} \left\{ \left[\frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \right] + \frac{\eta A}{E} \frac{d}{dt} \left[\frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \right] \right\} dx - \int_{x_a}^{x_b} f(x) \delta u_0 dx - Q_1 \delta u_0(x_a) - Q_4 \delta u_0(x_b)$$

$$0 = \int_{x_a}^{x_b} EA \frac{d\delta w_0}{dx} \frac{\partial w_0}{\partial x} \left\{ \left[\frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \right] + \frac{\eta A}{E} \frac{d}{dt} \left[\frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \right] \right\} dx - \int_{x_a}^{x_b} EI \frac{d\delta^2 w_0}{\partial x^2} \left\{ \frac{\partial^2 w_0}{\partial x^2} + \frac{\eta}{E} \frac{d}{dt} \left(\frac{\partial^2 w_0}{\partial x^2} \right) \right\} dx - \int_{x_a}^{x_b} q \delta w_0 dx - Q_2 \delta w_0(x_a) - Q_3 \delta\theta(x_a) - Q_5 \delta w_0(x_b) - Q_6 \delta\theta(x_b)$$

2.13

2.3 Nonlinear Finite Element Models and Symbolic Solutions

Let the axial and transverse displacements $u_0(x)$ and $w_0(x)$ be interpolated as

$u_0(x) = \sum_{j=1}^{2} u_j \psi_j(x)$, $w_0(x) = \sum_{j=1}^{4} \Delta_j \phi_j(x)$ 2.14

$\Delta_1 = w_0(x_a)$, $\Delta_2 = \theta(x_a)$, $\Delta_3 = w_0(x_b)$, $\Delta_4 = \theta(x_b)$ 2.15

$\psi_j(x)$ are Linear Lagrange interpolation functions and $\phi_j(x)$ are Hermite cubic interpolation functions.

Substituting equation 2.14 for $u_0(x,t)$ and $w_0(x,t)$, 2.15 for $w_0(x,t)$ in equation 2.14, then $\psi_j(x)$ and $\phi_j(x)$ for $\delta u_0(x)$ and $\delta w_0(x)$ all into equations 2.13, we obtain

$$0 = \sum_{j=1}^{2} K_{ij}^{11} u_j + \sum_{J=1}^{4} K_{iJ}^{11} \Delta_J - F_i^1$$

$$0 = \sum_{j=1}^{2} K_{Ij}^{21} u_j + \sum_{J=1}^{4} K_{IJ}^{22} \Delta_J - F_i^2 \qquad 2.16$$

Where

$$K_{ij}^{11} = \int_{x_a}^{x_b} \left[E_p A_p \frac{d\psi_i}{dx}\frac{d\psi_j}{dx} + \eta A_p \left(\frac{\partial u_0}{\partial t}\right)\frac{d\psi_i}{dx}\frac{d\psi_j}{dx} \right] dx$$

$$K_{iJ}^{12} = \frac{1}{2}\int_{x_a}^{x_b} \left[\left(E_p A_p \frac{\partial w_0}{\partial x}\right)\frac{d\psi_i}{dx}\frac{d\phi_J}{dx} + \eta A_p \frac{\partial}{\partial t}\left(\frac{\partial w_0}{\partial x}\right)\frac{d\psi_i}{dx}\frac{d\phi_J}{dx} \right] dx$$

$$K_{Ij}^{21} = \int_{x_a}^{x_b} \left[E_p A_p \frac{\partial w_0}{\partial x}\frac{d\phi_I}{dx}\frac{d\psi_j}{dx} + \eta A_p \frac{\partial}{\partial t}\left(\frac{\partial w_0}{\partial x}\right)\frac{d\phi_I}{dx}\frac{d\psi_j}{dx} \right] dx$$

$$K_{IJ}^{22} = \int_{x_a}^{x_b} \left\{ \left[E_p I_p + \eta \hat{I}_p \left(\frac{\partial w_0}{\partial t}\right)\right]\frac{d^2\phi_I}{dx^2}\frac{d^2\phi_J}{dx^2} + \frac{1}{2}\left[E_p A_p \left(\frac{\partial w_0}{\partial x}\right)^2 + \eta A_p \frac{\partial}{\partial t}\left(\frac{\partial w_0}{\partial x}\right)^2 \right]\frac{d\phi_I}{dx}\frac{d\phi_J}{dx} \right\} dx$$

$$F_i^1 = \int_{x_a}^{x_b} f \psi_i dx + Q_i \; F_I^2 = \int_{x_a}^{x_b} q \phi_I dx + Q_I \qquad 2.17$$

$[K_{iJ}^{12}]$, $[K_{Ij}^{21}]$ and $[K_{IJ}^{22}]$ are functions of the unknown $w_0(x)$.

3.0 ITERATIVE SOLUTIONS OF THE NONLINEAR EQUATIONS

Equation 2.16 can be written more compactly as

$$\sum_{p=1}^{2} K_{ip}^{\alpha 1} u_p + \sum_{P=1}^{4} K_{iP}^{\alpha 2} \Delta_P = F_i^{\alpha} \qquad 2.18$$

In Matrix form, we have

$$\begin{bmatrix} [K_{ij}^{11}] & [K_{iJ}^{12}] \\ [K_{Ij}^{21}] & [K_{IJ}^{22}] \end{bmatrix} \begin{Bmatrix} \{\Delta^1\} \\ \{\Delta^2\} \end{Bmatrix} = \begin{Bmatrix} \{F^1\} \\ \{F^2\} \end{Bmatrix} \quad \text{or} \quad [K^e(\{\Delta^e\})]\{\Delta^e\} = \{F^e\} \qquad 2.19$$

The nonlinear behavior occurs as direct stiffness, and it becomes the function of displacement or deformation i.e. in equation 2.19, the direct stiffness matrix K^e is a function of the displacement $\{u\}$.
It is not possible to solve for $\{u\}$ immediately, as K^e and F^e are not known in advance. Therefore an iterative process is needed to obtain $\{u\}$ and the associated K^e and F^e.
Using the Newton-Raphson iterative method

$$[T\{\Delta\}^{(r-1)}]\{\Delta\}^r = -\{R(\{\Delta\}^{(r-1)})\} = \{F\} - ([K^e]\{\Delta^e\})^{(r-1)} \qquad 2.20$$

Where the tangent stiffness matrix $[T^e]$ and residual vector R associated with the Euler-Bernoulli beam element are calculated using the definition

$$[T] \equiv \left(\frac{\partial\{R\}}{\partial\{\Delta\}}\right)^{(r-1)} \qquad \text{or} \qquad T_{ij}^e \equiv \left(\frac{\partial R_i^e}{\partial \Delta_j^e}\right)^{(r-1)} \tag{2.21}$$

$$\left\{R\left(\{\Delta\}^{(r-1)}\right)\right\} = ([K^e]\{\Delta^e\})^{(r-1)} - \{F\} \tag{2.22}$$

The solution at the rth iteration is then given by

$$\{\Delta\}^{(r)} = \{\Delta\}^{(r-1)} + \{\delta\Delta\} \tag{2.33}$$

3.1 MODEL VALIDATION AND DISCUSSION

The validation was done for a simplified case of nonlinear analysis. For the analysis, the model was validated with the case study presented with non-dimensional parameters below; Length $= 100$, thickness $t = 20$, varying load $q = 1,4,8,12$, modulus of elasticity $E = 1000$, Poisson ratio $v = 0.49$.

In this study, the nonlinear analysis of cantilever and simply supported beam are investigated and the results of the case study for model validation have been presented in graphical forms below. The analysis was done with a symbolic software designed for Finite Element analysis; Automatic Code Generation: AceGen and AceFEM. It was done with ease and less vigor as compared with other software used for the same analysis. Figure 3.1 shows the stress distribution of nonlinear deflection of a cantilever beam. The critical region of the beam is shown from the graphical presentation.

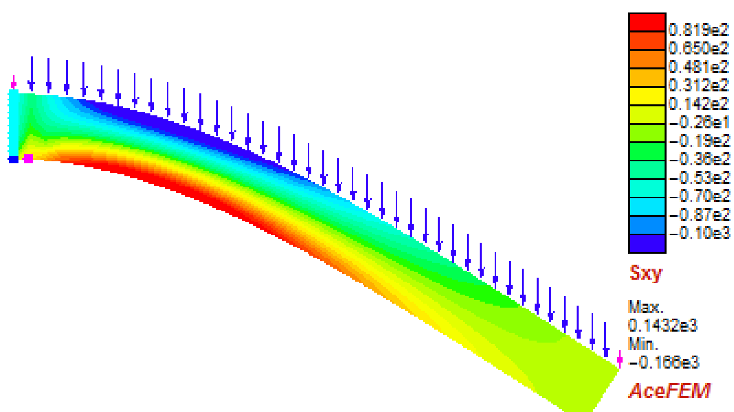

Figure 3.1 Symbolic Result of the stress distribution in the cantilever beam with a uniformly distributed load.

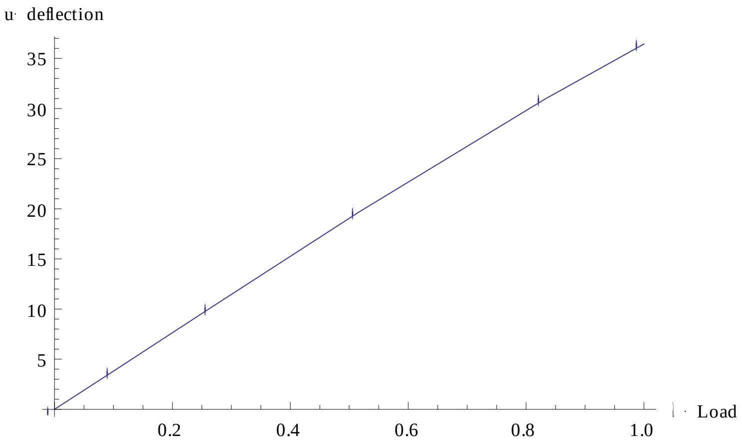

Figure 3.2 Load vs deflection (Linear) curves for a cantilever beam in Figure 3.1 shown above.

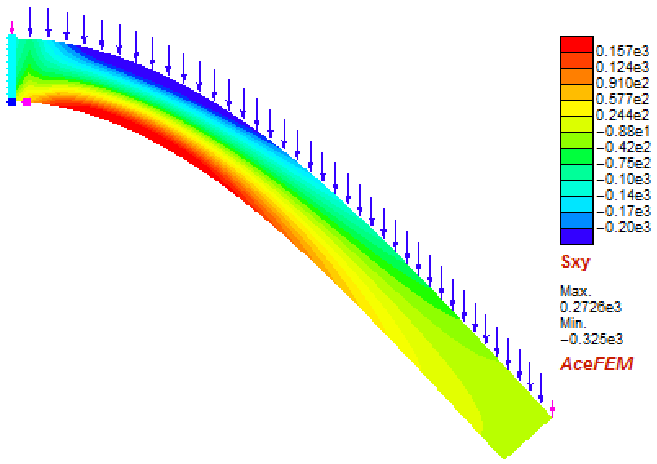

Fig 3.3 Symbolic Result of the stress distribution in the cantilever beam with uniformly distributed load

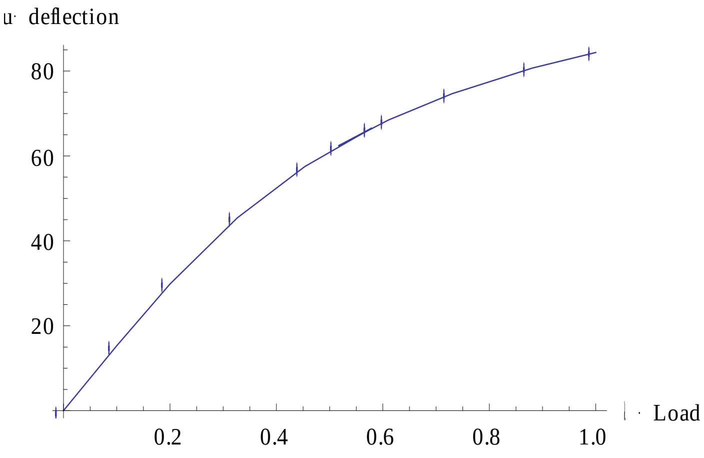

Figure 3.4 Load vs deflection (Nonlinear Linear) curves for a cantilever beam in Figure 3.3

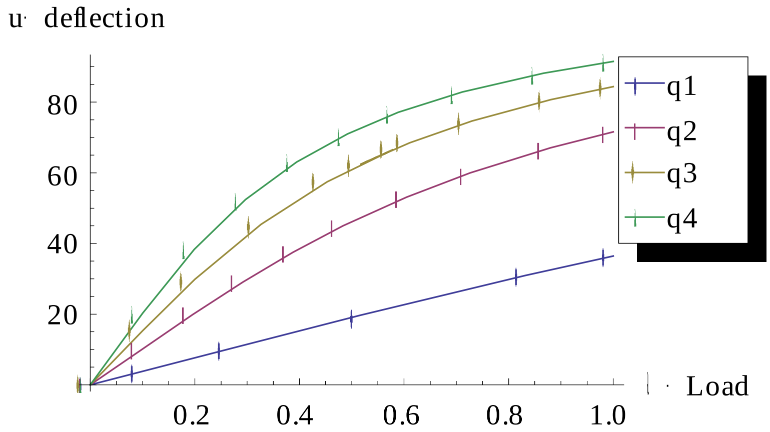

Figure 3.5 Load versus deflection curves for cantilever beam with varying uniformly distributed loads.

The results in Figure 3.2 above shows load-deflection curves for the uniformly distributed loads on cantilever beam in Figure 3.1. For q1, the load is proportional to the deflection as can be observed from the graph, this result agrees with other results from linear theory found in many literatures. At larger loads than q1, that is q2, q3, and q4, the load-deflection curve becomes nonlinear as shown in Figure 3.3 to figure 3.5. The load q is a combination of loads; weight of fluid in beam and weight of beam

Figure 3.6 below shows the stress distribution of nonlinear deflection of a simply supported beam. The beam is simply supported at both ends to allow for axial displacement, since beam would also undergo nonlinear bending. For the linear case, the axial displacement u_0 is uncoupled from bending deflection w_0 and they can be determined independently from the finite element models. But when the beam undergoes nonlinear bending as can be observed from the graph, the coupling between u_0 and w_0 will cause the beam to undergo axial displacement even when there is no axial force. The results would be different if the beam is clamped or fixed at both ends.

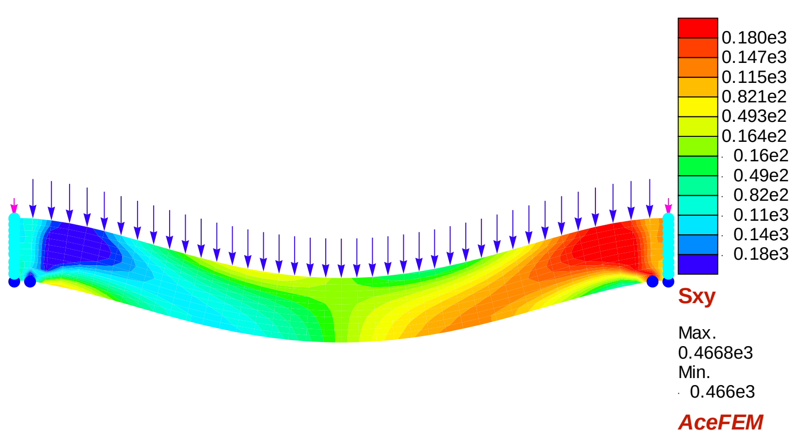

Figure 3.6 Symbolic Result of the stress distribution in the simply supported beam with a uniformly distributed load.

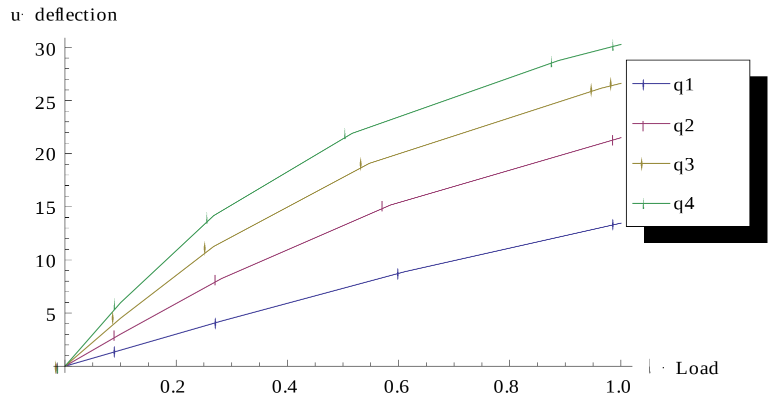

Figure 3.7 Load versus deflection curves for a simply supported beam with a uniformly distributed load.

Figure 3.7 above shows the load-deflection curves for varying loads on the beam. For load q1, the relationship is linear. But at larger loads, q2, q3 and q4, the relationship becomes nonlinear. When the loads are large, the linear load–deflection relationship ceases to be valid for obvious reasons.

For simple cases considered, the software used for the computational proved to be very efficient; the load increments were done with ease and less vigor. The changing of beam profile from cantilever to simply-supported was done with ease in the symbolic computation. Doing it conventionally would require a significant increase in the complexity of mathematical manipulation.

3.2 SENSITIVITY ANALYSIS OF THE DEFLECTION OF LINEAR ELASTIC BEAM

The finite element analysis of the deflection of a beam of linear elastic material (steel) simulated with the symbolic program (AceFEM) was compared with its exact analytical solution. The result shows a perfect agreement between the two analyses as presented in Figure 3.8 below. This means that the results obtained from our nonlinear analysis simulated by AceFEM can be trusted as true solution. When the steel material was replaced with a viscoelastic material and a nonlinear analysis was carried out with AceFEM (with same quantity of load), a geometric nonlinear response was obtained as shown in Figure 3.9. This also shows that a linear analysis cannot give a satisfactory result in a nonlinear problem.

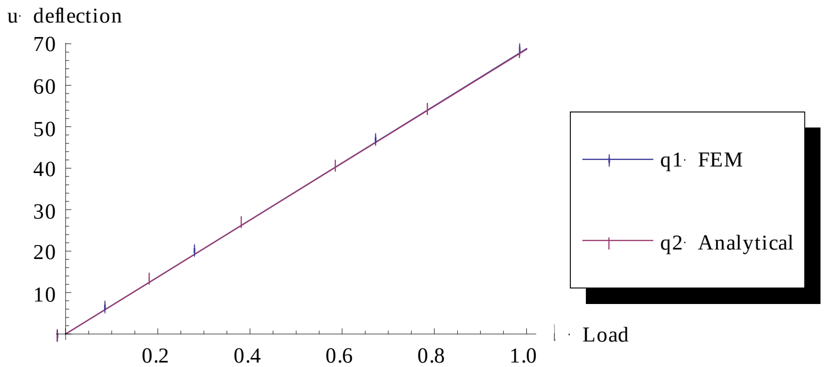

Figure 3.8 The sensitivity analysis of linear deflection of beam; the comparison between analytical and FEM solution

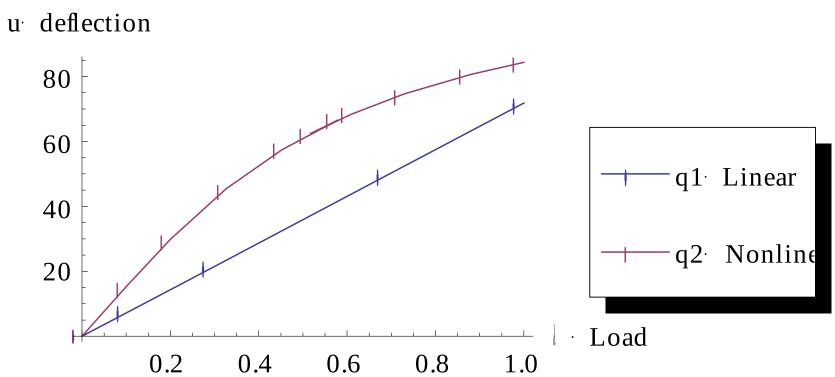

Figure 3.9 Load versus deflection curves for cantilever beams with same quantity of uniformly distributed loads but different Materials (Linear and Nonlinear deformation responses)

4.0 CONCLUSION

In this study, the nonlinear finite element model and symbolic computation for a geometrically nonlinear viscoelastic pipe has been presented. The implementation concept was done using the Automatic Code Generation tool AceGen and AceFEM, based on Computer Algebra program MATHEMATICA and the advantages regarding the programming and computational efficiency was discussed.

The study has shown that the symbolic computation is effective and efficient; it saves computation time which can be seen in the symbolically integrated element-stiffness matrices in the nonlinear finite element method which by-passes time-consuming numerical quadrature operation, especially as the number of Gauss points increases. The symbolic computation also gives more visibility with respect to the solution method and we can then more easily grasp the inter-relationship of the problem variables, recognize the simplification to be made and do a better and more accurate analysis.

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