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Abstract. In this study, we introduce the classes of ϕ-strongly pseudocontractive mappings in the intermediate sense and generalized Φ-pseudocontractive mappings in the intermediate sense and prove the existence of fixed points for those maps. The results generalise the results of several authors in literature including Xiang [Chang He Xiang, Fixed point theorem for generalized Φ-pseudocontractive mappings, Nonlinear Analysis 70 (2009) 2277-2279.

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Index to information contained in this paper

1. Introduction

Let E be an arbitrary real normed linear space with dual space E^* and C be a nonempty subset of E. We denote by J the normalized duality mapping from E to 2^{E^*} defined by

$$J(x) = \left\{x^* \in E^* : \langle x, x^* \rangle = \|x\|^2 = \|x^*\|^2 \right\}, \quad \forall x \in E, \tag{1.1}$$

where $\langle ., . \rangle$ denotes the generalized duality pairing.

The following definitions will be needed in this study.

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Definition 1.1. [17]. A mapping $T : C \to E$ is called *strongly pseudocontractive* if there exists a constant $k \in (0, 1)$ such that, for all $x, y \in C$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\langle Tx - Ty, j(x-y)\rangle \leqslant (1-k)\|x-y\|^2. \tag{1.2}$$

T is called *ϕ-strongly pseudocontractive* if there exists a strictly increasing function $\phi : [0,\infty) \to [0,\infty)$ with $\phi(0) = 0$ such that, for all $x, y \in C$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\langle Tx - Ty, j(x-y)\rangle \leqslant \|x-y\|^2 - \phi(\|x-y\|)\|x-y\|. \tag{1.3}$$

T is called *generalized Φ-pseudocontractive* [2] if there exists a strictly increasing function $\Phi : [0,\infty) \to [0,\infty)$ with $\Phi(0) = 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leqslant \|x-y\|^2 - \Phi(\|x-y\|). \tag{1.4}$$

The class of generalized Φ-pseudocontractive mappings is also called uniformly pseudocontractive mappings (see [10]). It is well known that those classes of mappings play crucial roles in nonlinear functional analysis.

T is called *asymptotically generalized Φ-pseudocontractive* [7] with sequence $\{k_n\}$ if for each $n \in \mathbb{N}$ and $x, y \in C$, there exist constant $k_n \geqslant 1$ with $\lim_{n\to\infty} k_n = 1$, strictly increasing function $\Phi : [0,\infty) \to [0,\infty)$ with $\Phi(0) = 0$ and $j(x-y) \in J(x-y)$ satisfying

$$\langle T^n x - T^n y, j(x-y)\rangle \leqslant k_n\|x-y\|^2 - \Phi(\|x-y\|), \tag{1.5}$$

The class of asymptotically generalized Φ-pseudocontractive was introduced by Kim *et al.* [7] as a generalization of the class of generalized Φ-pseudocontractive mappings.

It has been proved (see [14]) that the class of ϕ-strongly pseudocontractive mappings properly contains the class of strongly pseudocontractive mappings. By taking $\Phi(s) = s\phi(s)$, where $\phi : [0,\infty) \to [0,\infty)$ is a strictly increasing function with $\phi(0) = 0$, clearly, the class of generalized Φ-pseudocontractive mappings properly contains the class of ϕ-strongly pseudocontractive mappings.

Bruck *et al.* [1] in 1993 introduced the class of *asymptotically nonexpansive mappings in the intermediate sense* as follows.
The mapping $T : C \to C$ is said to be *asymptotically nonexpansive in the intermediate sense* provided T is uniformly continuous and

$$\limsup_{n\to\infty} \sup_{x,y\in C} (\|T^n x - T^n y\| - \|x-y\|) \leqslant 0. \tag{1.6}$$

Recently, Qin *et al.* [15] introduced the following class of nonlinear mappings.
Definition 1.2. [15]. A mapping $T : C \to C$ is said to be *asymptotically pseudocontractive mapping in the intermediate sense* if

$$\limsup_{n\to\infty} \sup_{x,y\in C} \left(\langle T^n x - T^n y, x-y\rangle - k_n\|x-y\|^2\right) \leqslant 0, \tag{1.7}$$

where $\{k_n\}$ is a sequence in $[1,\infty)$ such that $k_n \to 1$ as $n \to \infty$. This is equivalent to

$$\langle T^n x - T^n y, x - y\rangle \leqslant k_n\|x-y\|^2 + \nu_n, \quad \forall n \geqslant 1,\ x, y \in C, \tag{1.8}$$

where

$$\nu_n = \max\left\{0, \sup_{x,y\in C}\left(\langle T^n x - T^n y, x-y\rangle - k_n\|x-y\|^2\right)\right\}. \tag{1.9}$$

Qin *et al.* [15] proved some weak convergence theorems for the class of asymptotically pseudocontractive mappings in the intermediate sense. They also established some strong convergence results without any compact assumption by considering the hybrid projection methods. Olaleru and Okeke [12] in 2012 proved a strong convergence of Noor type scheme for a uniformly L-Lipschitzian and asymptotically pseudocontractive mappings in the intermediate sense. Olaleru *et al.* [13] established some coupled fixed points results in cone metric spaces.

Inspired by the above facts, we now introduce the following two classes of nonlinear mappings.

Definition 1.3. A mapping $T : C \to C$ is called *ϕ-strongly pseudocontractive mapping in the intermediate sense* if there exists a strictly increasing function $\phi : [0, \infty) \to [0, \infty)$ with $\phi(0) = 0$ such that, for all $x, y \in C$, there exists $j(x-y) \in J(x-y)$ satisfying

$$\limsup_{n\to\infty}\sup_{x,y\in C}\left(\langle Tx - Ty, j(x-y)\rangle - \|x-y\|^2 + \phi(\|x-y\|)\|x-y\| \leqslant 0\right). \tag{1.10}$$

for all $x, y \in C$ and for some $j(x-y) \in J(x-y)$. Put

$$\tau_n = \max\left\{0, \sup_{x,y\in C}\left(\langle Tx - Ty, j(x-y)\rangle - \|x-y\|^2 + \phi(\|x-y\|)\|x-y\|\right)\right\}. \tag{1.11}$$

Observe that $\tau_n \longrightarrow 0$ as $n \to \infty$. Hence, (1.10) reduces to

$$\langle Tx - Ty, j(x-y)\rangle \leqslant \|x-y\|^2 + \tau_n - \phi(\|x-y\|)\|x-y\|. \tag{1.12}$$

If $\tau_n = 0$ for all $n \in \mathbb{N}$, then (1.12) reduces to the class of ϕ-strongly pseudocontractive mappings. Clearly, the class of ϕ-strongly pseudocontractive mappings in the intermediate sense contains the class of asymptotically ϕ-strongly pseudocontractive mappings and the class of ϕ-strongly pseudocontractive mappings studied by several authors (see, e.g. Deimling [3], Khan *et al.* [6], Ding [4], Liu and Kang [8], Tan and Xu [16], Xu [18]).

Definition 1.4. A mapping $T : C \to C$ is called *generalized Φ-pseudocontractive mapping in the intermediate sense* if there exists a strictly increasing function $\Phi : [0, \infty) \to [0, \infty)$ with $\Phi(0) = 0$ satisfying

$$\limsup_{n\to\infty}\sup_{x,y\in C}\left(\langle Tx - Ty, j(x-y)\rangle - \|x-y\|^2 + \Phi(\|x-y\|)\right) \leqslant 0, \tag{1.13}$$

for all $x, y \in C$ and for some $j(x-y) \in J(x-y)$. Put

$$\xi_n = \max\left\{0, \sup_{x,y\in C}\left(\langle Tx - Ty, j(x-y)\rangle - \|x-y\|^2 + \Phi(\|x-y\|)\right)\right\}, \tag{1.14}$$

we observe that $\xi_n \to 0$ as $n \to \infty$. Hence (1.13) reduces to

$$\langle Tx - Ty, j(x-y)\rangle \leqslant \|x-y\|^2 + \xi_n - \Phi(\|x-y\|). \tag{1.15}$$

Clearly, the class of asymptotically generalized Φ-pseudocontractive mappings in the intermediate sense is a generalization of the class of asymptotically ϕ-strongly pseudocontractive mapping in the intermediate sense, and consequently generalises the classes of asymptotically ϕ-strongly pseudocontractive mappings and asymptotically generalized Φ-pseudocontractive maps studied by several authors (see, e.g Khan *et al.* [6], Ding [4], Kim *et al.* [7], Liu and Kang [8], Mogbademu and Olaleru [9], Osilike [14], Tan and Xu [16], Xu [18]).

Accretive maps are firmly connected with pseudocontractive maps. In fact, T is accretive (strongly accretive) if and only if $(I-T)$ is pseudocontractive (strongly pseudocontractive) (see, e.g. Deimling [3], Chidume and Chidume [2]). The study of accretive maps is enhanced as a result of its application since many physically significant problems can be modelled in terms of an initial value problem of the form

$$\frac{du}{dt} = -Tu, \qquad u(0) = u_0 \tag{1.16}$$

where T is accretive or strongly accretive in an appropriate Banach space (see, Khan *et al.* [6], Mogbademu and Olaleru [9]). Thus we have the following definitions:

Definition 1.5. A mapping $A : C \to C$ is said to be (Φ, ξ_n)-strongly accretive if there exists a strictly increasing function $\Phi : [0,\infty) \to [0,\infty)$ with $\Phi(0) = 0$ satisfying

$$\langle Ax - Ay, j(x-y)\rangle \geqslant \|x-y\|^2 + \xi_n - \Phi(\|x-y\|), \tag{1.17}$$

where ξ_n is as defined in (1.14).

A mapping $T : C \to C$ is called *generalized Φ-pseudocontractive mapping in the intermediate sense* if and only if $(I-T)$ is (Φ, ξ_n)-strongly accretive.

Xiang [17] in 2009 obtained the following existence results for the class of generalized Φ-pseudocontractive mappings.

Theorem 1.6. (Xiang [17]). Let E be a real Banach space, C be a nonempty closed convex subset of E, and $T : C \to C$ be a continuous generalized Φ-pseudocontractive mapping. Then T has a unique fixed point in C.

It is our purpose in this study to prove the existence of fixed points for our newly introduced class of asymptotically generalized Φ-pseudocontractive mappings in the intermediate sense, thus generalizing the results of Xiang [17] and several other authors in literature.

The following lemmas will be needed in this study.

Lemma 1.7. [2]. Let E be a real normed linear space. Then for all $x, y \in E$, we have

$$\|x+y\|^2 \leqslant \|x\|^2 + 2\langle y, j(x+y)\rangle, \quad \forall\, j(x+y) \in J(x+y).$$

Lemma 1.8. [10]. Let $\psi : [0,\infty) \to [0,\infty)$ be a strictly increasing function with $\psi(0) = 0$ and let $\{\theta_n\}$, $\{\sigma_n\}$ and $\{\nu_n\}$ be nonnegative real sequences such that $\sigma_n = o(\nu_n)$, $\sum_{n\geqslant 0} \nu_n = \infty$, $\lim_{n\to\infty} \nu_n = 0$. Suppose that

$$\theta_{n+1}^2 \leqslant \theta_n^2 - \nu_n \psi(\theta_{n+1}) + \sigma_n, \quad n \geqslant 0.$$

Then $\theta_n \longrightarrow 0$ as $n \to \infty$.

2. Main Results

Theorem 2.1. Let $C \subset E$ be closed and convex, $A : C \to E$ continuous and (Φ, ξ_n)-strongly accretive and the following boundary conditions are satisfied

(i) $\|x - \lambda Ax - C\| = o(\lambda)$ as $\lambda \to 0$ holds for each $x \in C$.

(ii) If $x \in C$, $x^* \in E^*\backslash\{0\}$ and $\|x^*\|\|x\| = \sup_{y\in C} \|x^*\|\|y\|$ then $\|x^*\|\| - Ax\| \leqslant 0$,

If either $\langle Ax, x\rangle \geqslant 0$ for $\|x\| \geqslant R$ or $\|Ax\| \to \infty$ as $\|x\| \to \infty$, then $0 \in A(C)$.

Proof. Since C is translation invariant, except $\langle Ax, x\rangle \geqslant 0$, we assume that $0 \in C$, but we have to change $\langle Ax, x\rangle \geqslant 0$ into $\langle Ax, x + x_0\rangle \geqslant 0$ for $\|x + x_0\| \geqslant R$ (for some $x_0 \in C$ fixed). Let $A_n = A + \frac{1}{n}I$. Suppose $x \in \partial C$, $x^* \in E^*\backslash\{0\}$ and $\|x^*\|\|x\| = \sup_{y\in C} \|x^*\|\|y\|$ then

$$\|x^*\|\| - A_n x\| = \|x^*\|\| - Ax\| - \frac{1}{n}\|x^*\|\|x\| \leqslant 0, \tag{2.1}$$

since $\|x^*\|\| - Ax\| \leqslant 0$ by conditions (i) and (ii) and $\|x^*\|\|x\| \geqslant 0$ (since $0 \in C$). Hence, condition (i) is also true for A_n. In addition, A_n is (Φ, ξ_n)-strongly accretive with $\Phi_n(r) = \frac{1}{n}$. Let $f(u) = -Au$ for $u \in C$. Since C is convex,

$$\langle -(A_n u - A_n v), u - v\rangle = -\langle A_n u - A_n v, u - v\rangle \leqslant 0 \tag{2.2}$$

is sufficient for (1.16) to have a unique global solution. Hence, A_n has a zero $x_n \in C$, i.e. $Ax_n = -\frac{1}{n}x_n$ for every n. Now, suppose that $\|Ax\| \to \infty$ as $\|x\| \to \infty$ holds. Since A is accretive, we have $\|Ax_n\| = \|\frac{1}{n}x_n\| \leqslant \|A(0)\|$. Hence, $\|x_n\|$ must be bounded.

However, if $\langle Ax, x + x_0\rangle \geqslant 0$ for $\|x + x_0\| \geqslant R$, then $\|x_n + x_0\| \geqslant R$ implies $\langle x_n, x_n + x_0\rangle \leqslant 0$. Let $x^* \in F(x_n + x_0)$. Then $\|x^*\| = \|x_n + x_0\|$ and

$$\|x_n + x_0\| = \|x^*\|\|x_n\| + \|x^*\|\|x_0\| \leqslant \|x^*\|\|x_n\| + \|x_n + x_0\|\|x_0\|. \tag{2.3}$$

This implies

$$\|x_n + x_0\|^2 \leqslant \langle x_n, x_n + x_0\rangle + \|x_n + x_0\|\|x_0\|. \tag{2.4}$$

Therefore, $\|x_n\| \leqslant \max\{R + \|x_0\|, 2\|x_0\|\}$ for every n.

Since in both cases, $\|x_n\| \leqslant c$ for some $c > 0$ and every n, we obtain

$$\Phi(\|x_n - x_m\|)\|x_n - x_m\| \leqslant \langle Ax_n - Ax_m, x_n - x_m\rangle \leqslant c(\frac{1}{n} + \frac{1}{m})\|x_n - x_m\|, \tag{2.5}$$

hence $\Phi(\|x_n - x_m\|) \to 0$ as $n, m \to \infty$. Therefore, $\{x_n\}$ is a Cauchy sequence and thus convergent to some $x \in C$. Since A is continuous and $Ax_n = -\frac{1}{n}x_n \to 0$ as $n \to \infty$, $Ax = 0$. □

We now obtain the following corollary as a consequence of Theorem 2.1.

Corollary 2.2. Let $C \subset E$ be closed and convex, $T : C \to E$ continuous and *generalized* Φ*-pseudocontractive mapping in the intermediate sense.* Suppose that the condition $\langle (1-\lambda)x + \lambda Tx, D\rangle = o(\lambda)$ as $\lambda \to 0$, for each $x \in C$. If C is unbounded, assume either $\|x - Tx\| \to \infty$ as $\|x\| \to \infty$ or $\langle Tx, x\rangle \leqslant \|x\|^2$ for $\|x\| \geqslant R$. Then T has exactly one fixed point.

Theorem 2.3. Let E be a real Banach space, C be a nonempty closed convex subset of E, and $T : C \to C$ be a continuous generalized Φ-pseudocontractive mapping in the intermediate sense. Then T has a unique fixed point in C.

Proof. For each $u \in C$, the mapping $S : C \to C$ defined by $Sx = \frac{1}{2}u + \frac{1}{2}Tx$ for each $x \in C$ is a continuous generalized Φ-pseudocontractive mapping in the intermediate sense. By Corollary 2.2, we see that S has a unique fixed point in C. Hence, given $x_0 \in C$, the sequence $\{x_n\}$ defined by $x_{n+1} = \frac{1}{2}x_n + \frac{1}{2}Tx_{n+1}$ $(\forall n \geqslant 0)$ is well defined.

For each $n \geqslant 1$, we have

$$x_{n+1} = x_n - x_{n+1} + Tx_{n+1}, \qquad x_n = x_{n-1} - x_n + Tx_n. \tag{2.6}$$

Using Lemma 1.7 and (1.15), it follows that there exists $j(x_{n+1}-x_n) \in J(x_{n+1}-x_n)$ such that

$$\begin{aligned}
\|x_{n+1} - x_n\|^2 &= \|(x_n - x_{n-1}) - (x_{n+1} - x_n) + (Tx_{n+1} - Tx_n)\|^2 \\
&\leqslant \|x_n - x_{n-1}\|^2 - 2\langle x_{n+1} - x_n, j(x_{n+1} - x_n)\rangle \\
&\quad +2\langle Tx_{n+1} - Tx_n, j(x_{n+1} - x_n)\rangle \\
&\leqslant \|x_n - x_{n-1}\|^2 - 2\|x_{n+1} - x_n\|^2 \\
&\quad +2\left\{\|x_{n+1} - x_n\|^2 + \xi_n - \Phi(\|x_{n+1} - x_n\|)\right\} \\
&= \|x_n - x_{n-1}\|^2 - 2\|x_{n+1} - x_n\|^2 + 2\|x_{n+1} - x_n\|^2 \\
&\quad +2\xi_n - 2\Phi(\|x_{n+1} - x_n\|).
\end{aligned} \tag{2.7}$$

From (2.7), we obtain

$$\begin{aligned}
\|x_{n+1} - x_n\|^2 &\leqslant \|x_n - x_{n-1}\|^2 - 2\Phi(\|x_{n+1} - x_n\|) + 2\xi_n \\
&\leqslant \|x_n - x_{n-1}\|^2 - 2\xi_n\Phi(\|x_{n+1} - x_n\|) + 2\xi_n
\end{aligned} \tag{2.8}$$

where $\Phi : [0, \infty) \to [0, \infty)$ is a strictly increasing function with $\Phi(0) = 0$. Let $\theta_n = \|x_n - x_{n-1}\|$ $(\forall\ n \geqslant 1)$, $\sigma_n = 2\xi_n$, $\nu_n = 2\xi_n$ and $\psi(s) = \Phi(\sqrt{s})$. Then $\theta_{n+1}^2 \leqslant \theta_n^2 - \nu_n\psi(\theta_{n+1}) + \sigma_n$ for all $n \geqslant 1$. By Lemma 1.8, we obtain $\lim_{n\to\infty} \|x_n - x_{n-1}\|^2 = \lim_{n\to\infty} \theta_n^2 = 0$. Hence,

$$\lim_{n\to\infty} \|x_n - x_{n-1}\| = 0. \tag{2.9}$$

Observe that $x_n - x_{n-1} = Tx_n - x_n$ for each $n \geqslant 1$. We obtain

$$\lim_{n\to\infty} \|Tx_n - x_n\| = 0. \tag{2.10}$$

For each $\epsilon > 0$, we take $\delta = \frac{\Phi(\epsilon)}{2\epsilon} > 0$, it follows from (2.9) and (2.10) that there exists a natural number N such that $\|x_{n+1} - x_n\| < \epsilon$ for every $n \geqslant N$ and

$\|(Tx_m - x_m) - (Tx_n - x_n)\| < \delta$ for each $m > n$. Next, we prove by induction that

$$\|x_m - x_n\| < \epsilon, \quad \forall\ m > n \geqslant N. \tag{2.11}$$

For each natural number $n \geqslant N$, if we take $m = n+1$, then we observe that (2.11) holds for some $m \geqslant n+1$. Then

$$\|x_{m+1} - x_n\| \leqslant \|x_{m+1} - x_m\| + \|x_m - x_n\| < 2\epsilon. \tag{2.12}$$

Using (1.15), we obtain

$$\langle Tx_{m+1} - Tx_n, j(x_{m+1} - x_n)\rangle \leqslant \|x_{m+1} - x_n\|^2 + \xi_n - \Phi(\|x_{m+1} - x_n\|). \tag{2.13}$$

From (2.13), we obtain

$$\begin{aligned}\Phi(\|x_{m+1} - x_n\|) &\leqslant \|x_{m+1} - x_n\|^2 + \xi_n - \langle Tx_{m+1} - Tx_n, j(x_{m+1} - x_n)\rangle \\ &\leqslant \langle (x_{m+1} - Tx_{m+1}) - (x_n - Tx_n), j(x_{m+1} - x_n)\rangle + \xi_n \\ &\leqslant \|(x_{m+1} - Tx_{m+1}) - (x_n - Tx_n)\|\|x_{m+1} - x_n\| + \xi_n \\ &\leqslant \delta.2\epsilon + \xi_n \\ &\leqslant \delta 2\epsilon \quad \text{as} \quad n \to \infty \\ &= \Phi(\epsilon).\end{aligned} \tag{2.14}$$

Since Φ is a strictly increasing function, we have that $\|x_{m+1} - x_n\| < \epsilon$, meaning that (2.11) holds for $m+1$. By induction, (2.11) holds for all $m > n \geqslant N$, which implies that $\{x_n\} \subset C$ is a Cauchy sequence. But E is a Banach space and C is closed, hence $\{x_n\}$ converges to some $p \in C$. Since $T : C \to C$ is continuous, we conclude that $Tp = p$ using (2.10). From (1.15), we see that the fixed point of T is unique. The proof of Theorem 2.3 is completed. $\square$

Remark 2.4. Theorem 2.3 is a generalization of Theorem 2.1 of Xiang [17] and the references therein since the class of generalized Φ-pseudocontractive mappings in the intermediate sense is more general than those defined by these authors.

3. Conclusion

The existence of fixed points established for the class of generalized Φ-pseudocontractive mappings in the intermediate sense in this study generalizes and extends several known results in literature.

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