THE ROLE OF PRACTICAL APPARATUS IN THE TEACHING AND LEARNING OF MATHEMATICS

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ABSTRACT: The use of practical apparatus to help children and adults to learn about Mathematics has a long history, but its utility is currently being questioned by many educators. This article, based on Gestalt theory of learning, reviews the arguments for the value of practical apparatus and examines the practical activities for which the apparatus is used. It tries to explain why some educators seem to find it difficult to translate into proctice the theoretical benefits of practical apparatus in facilitating learning of Mathematics. A number of possible reasons for the divergence and unhelpful use of practical apparatus found are faulty design, faulty selection and faulty utilization of the teaching and learning aid. Aids to appropriate designing, selection and utilization of practical apparatus are recommemded rather than its rejection.

INTRODUCTION

An increasing number of educators worldwide are challenging one of the orthodoxies of a 'good technique' of facilitating learning by rejecting the use of practical apparatus in teaching mathematics (Cockcroft, 1996). Is this a panic reaction to the current economic depression internationally which makes budgeting for practical apparatus more difficult in terms of funding, and the teacher's time to designing, or is it the overdue explosion of another 'progressive myth' about teaching and learning?

Apparatus to help in facilitating the learning of mathematics has been in use for a long time, although in the last 20 years counters, sticks, beards, string and matchsticks or any such practical apparatus is absent in teaching and learning centres or, at times, supplemented by specially designed 'structural' apparatus such as Unifix, Cuisenaire, and Stern Apparatus (Solomon, 1997). In the past, one kind of apparatus or another might have been rejected by educators in harmony with the principle of individual differences, but now educators both in the formal and non-formal educational sub-systems are expressly denying the value of using any such materials. Such opinion is commonly accompanied by anecdotal, even apocryphal, stories about how this learner or that spent years using the available materials without making any progress, and these reports are usually imbued with a sense of wasted time and funds, of having spent so much on a fruitless activity when such time and fund could have been used otherwise in covering more areas in the Mathematics Curriculum.

This article will review the case for practical apparatus in facilitating the learning of mathematics. The discussion will not rest on evidence, because, as Orton (1997) says, it is impossible to control for variations in experience in educational research about the value of materials. After reviewing the specific teaching experiments in the use of practical apparatus by Dienes (1963), Davis (1984) and Hart (1994), he concluded that "the measuring instruments at our disposal are not adequate to prove convincingly to what extent practical apparatus can promote learning of mathematics" (p.93). However, the consensus about materials has been that they are a good thing because they contribute towards the better understanding of useful mathematics.

Dickon et al (1990) report many studies to support the view that the rules of mathematics are forgotten if not supported by understanding and that understanding is facilitated by the use of concrete materials. However, the means by which concrete materials facilitate understanding of mathematical rules have been described in general rather than specific ways such as to constitute a convincing evidence.

Studying critically research reports on the use of practical apparatus in the teaching and learning of mathematics reveals that it is the ineffectiveness of the actual teaching and learning activities with practical materials that lead educators to reject the apparatus in the face of theories that claim better. The argument runs thus: "I have found that it does not work, therefore I will stop using it". This is not a valid inference as it stands, as there is a hidden premise concerning which or how the material was being used. The alternative inference would be "I have found that this apparatus does not work, therefore I should try another one", or "I have found that it does not work in the way I have been using it, therefore I should use it differently". It is therefore necessary in the face of the assault on the practical value of materials to become more specific about what is being claimed for materials in the different activities where they are used. Becoming clearer about the links between practices and purposes may show up the fund and time wasting aspects and reveal those that have real value, which ought to be retained.

THEORETICAL BENEFITS

As far back as 1920s, Gestalt theorists were able to prove that suitable environment and suitable practical meaterials would lead to insightful learning (Wertheimer, 1925; Kaffka, 1935; Kohler, 1958). Thus, the belief that practical apparatus might facilitate the teaching and learning of mathematics is not a 'modern' phenomenon. Orton (1997) suggests that in the 19th century Tillich and Froebel "both advocated the use of concrete equipment in teaching of elementary mathematics" (p.91) and McIntosh (1992) finds recommendations for the use of practical activities in educational reports from 1840 onwards till date. Despite that, it would not be true to say that the practical approach to mathematics has had a fair empirical trial in the classroom. McIntosh, after listing the features of the recommendations in the reports from the past, comments that "it is doubtful if a student in a million has received a mathematical education consistently following these principles at every stage (p.11). The justification for the value of practical materials is theoretical, based on Gestalt theories on the promotion of insight and discovery. Orton (1997) suggests that this underpins all use:

> All who use such equipment presummably believe that it provides the learners with insight into number relationships and into the structure, enabling learning to take place. This is the essence of Gestalt theory applied to the provision of learning situations (p.91).

Other theorists, such as Dienes (1963), have elaborated the mechanisms by which this may occur, using words like 'abstraction' and 'discovery'. He suggests that mathematics is best learned by abstraction from artificial environments specifically designed to 'embody' mathematical structures and that the learning of these structures brings an understanding which will enhance performance in mathematics and its application elsewhere. The right way to use these materials, he suggests, is through discovery. He asserts that the use of structured material is designed to enable the learners "to discover for themselves the mathematical structures the educators want them to learn" (p.172).

How is this done? According to Orton (1997) the notion of 'learning by discovery' is that manipulating the apparatus "takes the learners a very long way towards mastery of conceptual and structural relationships" (p.84). However, Dearden (1967) vigorously attacks such approach which Dearden says is based on the assumption that just allowing the learners to freely manipulate apparatus would enable them to see the 'structure' that the apparatus 'embodies'. Dearden says sarcastically:

> In some mysterious way, a special potency is thought to inhere in practical apparatus such that if learners manipulate it, significant experiences must be had, and important concepts must be abstracted (p.146).

More recently, Mason (1989) warns that the notion of an 'embodiment' of characteristics of mathematics in such concrete apparatus is a perception of educators and is not necessarily how learners see the materials when using them. Also, Solomon (1992) points out that "simple presentation of concrete materials cannot, in point of logic, make an idea clear to someone who does not already have that idea" (p.149).

Thus, it is necessary for the educator to be much more than the hovering provider of materials, or even the stucturer of an environment from which new concepts are supposed to be abstracted in the course of undirected activity. The educator needs to be one who questions, discusses, hints, suggests and instructs what to do to find out. In other words, a form of discovery which guides experience, by the subtle use of language, towards something that is regarded as educationally valuable is acceptable, where passive discovery methods and 'abstractionism' is not. The acceptance of the need for the educator to be active may reflect a change in belief about the nature of mathematics, expressed by Solomon (1992) when she says that mathematics is all around us only because it is imposed on experience, it is not there in some mystical way and cannot be abstracted from experience.

When used by a purposeful educator practical apparatus can be helpful for the following reasons.

1. The characteristics of mathematical concepts such as addition, subtraction, angle, line and so on are on the surface and can be seen and talked about, such that the thinking of both educator and learner concerning the meanings of mathematics is more accessible to the other person. Activity with the materials which represent number relationships can be described directly and abstract mathematical words are given concrete reference, which makes it easier for learner and educator to demonstrate meaning and share thinking. The learner's misunderstandings can be revealed through observation and help can be given by example, even one learner to another. For example, to introduce the concept of volume, Lackie (1987) suggests using building blocks to make different sizes of prisms.

2. The materials are picturable (Resmick and Ford, 1981) and can be remembered through mental imagery rather than through a word or symbol, which helps the learner to remember and review experience of mathematics. For example, Rodda (1991) uses sets of sticks to introduce the concepts of number and numeration.

3. The materials can be a context for the learner's own exploration of possibilities in number, a means to try things out in a concrete way, to generate examples which can form the basis of generalisation and pattern recognition, which is how Lehn (1986) suggests mathematics is most easily learned.

4. Activities can involve an element of choice and be self-corrective, which can help to sustain concentration on the activity. For example, the use of counters such as sticks or coins to introduce the concept of addition and subtraction makes the understanding and self-directed practice of the concepts easier.

5. Written recording can follow the activity, giving meaning to the symbols as a result of the representation of action.

6. Different materials can be used by the educator to draw attention to different aspects of number. For example, cuisenaire rods, which are unmarked, can be used to emphasise the wholeness of numbers rather than the counting aspects, and base 10 apparatus allows a clear focus on the place value system for representation of quantity.

7. The educator can demonstrate specific teaching points about procedural methods when paper and pencil methods are being learned. For example, Lackie (1987) illustrates how an educator can use cardboard to teach the concept of different types of triangles in trigonomentry and geometry.

PRACTICAL LIMITATIONS

The use of practical apparatus listed above were made with the background assumption that the educator would be involved in the teaching, not a passive observer of 'discovery learning'. The role of the educator is to guide, and to provide the opportunity for making discoveries. However, error can creep in with the extent of such guidance provided. If the path is too rigidly laid down, the discovery can easily become a mockery of what was intended. If on the other hand, the situation is left too open, purposeful learning may not take place at all. Skill in mathematics teaching methods, effort and a great deal of ingenuity are required on the part of the educator to strike a balance. If the balance is not struck, the use of practical apparatus may not show its full value.

A further limitation to the value of the apparatus arises because knowledge and skill are progressive, and if the learner does not have the necessary experience, knowledge and skill, no practical apparatus could be of value in providing meaning and insight. For example, the use of the practical apparatus to generate number combinations would not have any value if the learner's conception of addition, subtraction or multiplication was flawed or absent.

Thus, the practical apparatus will not show its full value if inappropriate one is chosen, or appropriate one is being used inappropriately with regard to the knowledge and awareness of the learner. As far back as 1961, Lovell argued that practical apparatus might not enhance performance if we tried to force a concept on a learner before he was ready for it. If so "there is little hope of the concept becoming more generalised or remaining stable when the apparatus is withdrawn" (p.148).

CONCLUSION

Mathematics which is to be useful has to be meaningful or else we would not know when and how to apply it. It also has to be slick, or else we would be de-skilled at critical moments.

It also has to be familiar and 'easy', to provide the flexibility in thinking needed to cope with the range of situations in which the concepts are embedded. It is important therefore for there to be three strands in learning mathematics: (i) understanding its meaning; (ii) gaining familiarity; and (iii) learning how to get answers efficiently in a range of ways, including by paper and pencil methods. Practical apparatus has a role to play in all of these, by providing contexts in which meanings can be established and extended, in which relationships can be exemplified and explored and in which techniques can be demonstrated.

It is therefore important that the designing, selection and use of practical apparatus should be emphasised at all levels of training mathematics teachers. Also, the education of every mathematics teacher should be continuous by his familiarizing himself with the up-to-date teacher's guides of the texts he is using to teach mathematics at all levels. Many of the activities suggested in most of the teacher's guides available do have the potential to be helpful in teaching mathematics effectively. However, there is a category of suggested activity of which this cannot be said. It is the use of practical apparatus as a calculation aid, for helping the learner to complete calculation exercises. Thyer and Maggs (1992) suggest the use of 'practice word cards' for completion with and without apparatus and in the teacher's guide to New Curriculum Mathematics for Schools (Cockcroft, 1996) the pupils books are referred to in that respect: "Page 39. Some learners will require beards or counters to help them complete this page" (p.70). To use practical apparatus to get answers is to obscure its purpose and prevent it from having its true value, which is to bring meaning into mathematics. Even worse, the use of practical apparatus as an aid to calculation inserts an instrusive and unnecessary stage in the process of learning how to calculate, delaying and confusing the development of paper and pencil skills without the longer term benefits to understanding that would justify such a delay.

Thus, inappropriate design, selection or use of practical apparatus can actually distract attention from the points an educator is trying to make. It is perfectly sensible not to use a practical apparatus than to work with the wrong ones or to use the right ones wrongly. It would be a shame, however, to lose the possibility of tapping into the great potential of practical materials to contribute to meaning in mathematics, most especially when introducing a new concept, if we reject completely the use of practical materials instead of making the necessary efforts to use them well.

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