

Research Paper

Journal of Applied and Computational Mechanics



Dynamic Analysis of the Biomechanical Model of Head Load Impact Using Differential Transform Method

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Received May 02 2019; Revised August 19 2019; Accepted for publication August 21 2019. Corresponding author: O.A. Adeleye (oadeleye@unilag.edu.ng) © 2020 Published by Shahid Chamran University of Ahvaz & International Research Center for Mathematics & Mechanics of Complex Systems (M&MoCS)

Abstract. The dynamic analysis of the biomechanical model of the head load impact using the Differential Transform Method is presented in this paper. In many parts of the world, the problem of traumatic brain injuries (TBI) has led to neurodegenerative dementing disorders and diseases as a result of head load impact from sporting activities, accidents involving the head, etc. have serious effects on humanity. The head load impact and its control have been modeled as a rigid linkage head-neck manipulator. This rigid link manipulator is governed by a system of nonlinear ordinary differential matrix equations with three degrees of freedom which requires special techniques for its solution. The system of equations was solved using Differential Transform Method (DTM) and the results were compared with results obtained in earlier studies and validated with the fourth-order Runge-Kutta numerical method (RK4). Good agreements are reached in all these results. From the model, the effects of head loads, head mass, neck mass, upper and lower linkage lengths, head and neck moments of inertia were investigated. As the head loads increased, there were increases in both axial and angular displacement of the head motion and the neck region. The study provides a theoretical basis for the design and understanding of the effects of head load carriage on vital organs that are susceptible to pains, damages, and even failure.

Keywords: Head Loads; Biomechanical Model; Differential Transform Method; Runge-Kutta Method.

1. Introduction

The problem of dynamics of human head load impact has received considerable attention as a result of its effect on humanity. Various sporting activities (see Figures 1a, b) involve the use of the head as a major contact component and therefore make the head highly exposed to traumatic brain injuries (TBI), which often lead to Psychiatric Disorders, such as Addictive behaviours, Alzheimer's disease (AD), Anxiety disorders, Schizophrenia, Parkinson's syndrome, and Personality disorders [1-3]. In order to understand the dynamical properties and simplify the control problems, analysts model the musculoskeletal systems as rigid-link manipulators [4-5], (see Figures 2a, b) where the head is used as the end-effector. This rigid-linkage end effector phenomenon is also observed in many animals where the head is used to manipulate and interact with their immediate environment. An example is a woodpecker that repeatedly beats the tree bark while searching for food [6]. Similarly, the horned rams engage in fights by butting one another's head [7].





Fig. 1. Head load impact in sports: (a) boxing, (b) American Football



Fig. 2. Rigid linkage head neck manipulator: (a) model, (b) model schematic

A major factor responsible for traumatic brain injuries is the tissue shearing as well as stretching of neutral tissue caused by head rotation [8-9]. It is postulated that the neck muscle architecture has an important role in the placement and stability of the head and that soft tissue loads on the neck can lower the possibility of TBI which may influence sport performance [10]. During impact, the head's stability is also dependent on cervical stiffness, the angular displacement of the neck and loading of the head. As the cervical stiffness increases and angular displacement decreases, the ability to sustain higher magnitude head impacts reduces, while loading the head reduces the peak head angular velocity. This is a result of the change in muscle stiffness and cervical spine viscosity [11], and they both have a strong correlation with muscle strength and muscle activation level.

A crucial component in stabilizing the head is the viscoelastic characteristics of the head-neck system and the critical neck stiffness needed to maintain the head stability against gravitational force is approximately 10Nm/rad [12]. This value is considered large as compared with passive stiffness used for minimal angles of extension or flexion which is approximately 2Nm/rad [13]. The advantages of neck stiffening in its protective effects on the brain are seemingly credible but the results obtained from past studies have not yielded pure outcomes. There is strong evidence that links neck strength with brain injuries as shown in a study that found correlations between measurements of the neck strength and concussion. The overall neck strengths were significant predictors of concussions [14]. More studies have found that the neck strength attenuates the response of the head's dynamics to external forces. Furthermore, cervical muscle activation does not depend on neck strength [15].

A negative correlation has been observed between neck stiffness and other angular motion such as the peak angular velocity or the peak angular acceleration during head impact. Among soccer players, the training of the cervical resistance had no effect on stabilizing the head-neck segment dynamics during the application of force and the effects of kinematics and neck stiffness training were absent. [16-17]. In addition, the findings showed that no player with stronger and larger neck muscles escaped the severity of head impact [18]. In the study to determine the role of neck muscles on mild TBI in football, it was found that neck muscles are ineffective in the resistance of impacts made along the transverse plane. Furthermore, muscle activation caused no reduction in the translational acceleration but did in the rotational velocity [19-21]. These findings confirmed what was reported in previous studies that activating the muscle reduces the possibilities of concussion in the brain as a result of a head impact during sporting activities [22-23]. Hence the neck strength has a high potential of reducing the risk involved in head injury [19].

Head positioning through the cervical spine also affects the rotation of the head thereby increasing the risk of head injury, even though this has not been thoroughly and widely investigated. A study that was done on head-down contact and spearing in tackle football showed that keeping the head up and initiating contact with the shoulder or chest decreases the risk of these injuries. It was found that when a player launches out while his head position is down, he is



likely to suffer from quadriplegia. And when the player launches out first with his head, the possibility of concussion is high [24]. There are several ways to reduce the risk of concussion in head impacts. One such way is to keep the head in an upward position during tackling. By this technique, the torso inertia of the collision of the striking player and the impact force will be reduced [25]. Although, there is clinical evidence for the effects of head positioning in the risk of head injury, the properties of the head inertia and the resulting kinetics have not been quantified for varying configurations of the head-neck system.

The head-neck system is very complex and hence was reduced to a rigid linkage model. This rigid linkage model which may be referred to as a biomechanical model is represented by a dynamical system of equations. Therefore, the main objective of this study is to theoretically consider the dynamic analysis of the head-neck linkage model a nd investigate the effects of various parameters in the model as they affect the biomechanical model. The biom echanical model which is a dynamical system of coupled nonlinear ordinary differential equations does not have closed-form solutions but requires some special techniques for its solution. The computational method such as the Differential transformation method (DTM) provides a direct relationship between the model parameters and provides good and continuous insights into the significance of various parameters affecting the governing model.

The Differential transformation method (DTM) was developed by Zhou in 1986 [27]. It is a semi-analytical method used in providing solutions to both linear and nonlinear Partial Differential Equations [29]. The computational intensity is lower than observed in other methods, [29] and yet more accurate than the methods [28, 30]. The characteristics of DTM have been highlighted in various studies [26]. The results of DTM have been compared with results from other methods which include the Adomian Decomposition Method (ADM) [31] and Finite Element Method (FEM) [28]. The efficiency of the DTM has been combined with other methods [30] to obtain better results. Hence, we apply DTM to our biomechanical model to obtain its solution. The obtained solution was compared with existing experimental results and validated with the fourth-order Runge-Kutta numerical method (RK4). A good agreement was reached between the methods of solution and the experimental results.

2. Description of Problem and Governing Equation

The human body is modeled as a system. If the system is assumed to be in one degree of freedom (acting at the center of mass) and utilizing Newton's second law, the equation of the particle is

$$m\ddot{y} = f - mg \tag{1}$$

The left position equation (1) can be written as

$$m\ddot{y} = \frac{d}{dt}(m\dot{y}) = \frac{d}{dt}\frac{d}{d\dot{y}}\left(\frac{1}{2}m\dot{y}^2\right) = \frac{d}{dt}\frac{\partial K}{\partial \dot{y}}$$
(2)

where $K = m\dot{y}^2 / 2$ is the Kinetic Energy. Similarly

$$mg = \frac{\partial}{\partial y} (mgy) = \frac{\partial P}{\partial y}$$
(3)

In which P = mgy is the Potential Energy. Let

$$L = K - P = \frac{1}{2}m\dot{y}^2 - mgy \tag{4}$$

Note that $\partial L/\partial \dot{y} = \partial K/\partial \dot{y}$ and $\partial L/\partial y = -\partial P/\partial y$. Then the Euler – Lagrange Equation is

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = f \tag{5}$$

Consider a system of k particles. If the particles are relinquished to move minus any restriction, where $r_1, ..., r_k$ represent position vector at each point. For the generalized coordinate and taking $r_1 = v_1$ and replacing $v_2 = a_1$ in the kinetic equation. The generalized Euler

For the generalized coordinate and taking $r_i = v_i$ and replacing $y_j = q_j$ in the kinetic equation. The generalized Euler – Lagrange equation becomes

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_{j}} - \frac{\partial L}{\partial q_{j}} = \tau_{j}$$
(6)

The Kinetic Energy of the head/neck system consists of translational energy and rotational energy about its joints. Hence

$$K = \frac{1}{2}mv^{T}v + \frac{1}{2}\omega^{T}I\omega$$
⁽⁷⁾



Journal of Applied and Computational Mechanics, Vol. 6, No. 4, (2020), xx-xx

where *m* is the mass, v is the velocity, ω is the angular velocity and *I* is the Inertia Tensor. The head/neck system consists of *n* number of links and the linear and angular velocities of whatever part of the link can be demonstrated in terms of joint variables derivatives and the Jacobian matrix as:

$$v_i = J_{vi}(q)\dot{q}\omega_i = J_{\omega i}(q)\dot{q}$$
(8)

Let the mass of link *i* be m_i and the inertia matrix of link *i* resolved about the coordinate frame that is parallel to frame *i* be I_i , then the Kinetic Energy of the head/neck system is given as

$$K = \frac{1}{2} \dot{q}^{T} \sum \left[m_{i} J_{\nu i} \left(q \right)^{T} J_{\nu i} \left(q \right) + J_{\omega i} \left(q \right)^{T} R_{i} \left(q \right) I_{i} R_{i} \left(q \right)^{T} J_{\omega i} \left(q \right) \right] \dot{q}$$
(9)

or

$$K = \frac{1}{2}\dot{q}^{T}M(q)\dot{q}$$
⁽¹⁰⁾

3. Methods of Solution

The Kinetic Energy is given as

$$K = \frac{1}{2} \sum_{ij}^{n} M_{ij}(q) \dot{q}_{i} \dot{q}_{j} = \frac{1}{2} \dot{q}^{T} M(q) \dot{q}$$
(11)

and Potential Energy is PE = P(q). Therefore the

$$\mathbf{L} = \mathbf{K} - \mathbf{P} = \frac{1}{2} \sum_{ij}^{n} M_{ij} \left(q \right) \dot{q}_{i} \dot{q}_{j} - P(q)$$
(12)

Substituting equation (11) into the Euler – Lagrange equation

$$\sum_{j}^{n} M_{kj} \ddot{q}_{j} + \sum \left(\frac{\partial M_{kj}}{\partial q_{i}} - \frac{1}{2} \frac{\partial M_{ij}}{\partial q_{k}} \right) \dot{q}_{i} \dot{q}_{j} - \frac{\partial P}{\partial q_{k}} = \tau_{k}$$
(13)

But

$$\sum \left(\frac{\partial M_{kj}}{\partial q_i} - \frac{1}{2} \frac{\partial M_{ij}}{\partial q_k} \right) \dot{q}_i \dot{q}_J = \sum_{ij} \frac{1}{2} \left(\frac{\partial M_{kj}}{\partial q_i} + \frac{\partial M_{ki}}{\partial q_j} - \frac{1}{2} \frac{\partial M_{ij}}{\partial q_k} \right) \dot{q}_i \dot{q}_J$$
(14)

Let

$$C_{ijk}(q) = \frac{1}{2} \left(\frac{\partial M_{kj}}{\partial q_i} + \frac{\partial M_{ki}}{\partial q_j} - \frac{1}{2} \frac{\partial M_{ij}}{\partial q_k} \right)$$
(15)

Then the equation of motion is written as

$$\sum_{i} M_{kj}(q) \ddot{q}_{l} + \sum_{ij} C_{ijk}(q) \dot{q}_{l} \dot{q}_{j} + Q_{k}(q) = \tau_{k}$$
(16)

or

$$M(q)\ddot{q} + C(q.\dot{q}) + Q(q) = F$$
(17)

3.1 Musculoskeletal System

The body structure for the skeletal system model is modeled as an articulated multi-body scheme. Disks are inserted between the next vertebrae in the spine of a human, allowing 6 degrees of freedom motion (DOF). Each of the joints is simplified to a 3 degrees of freedom rotational joint. In order to model the stiffness of the disks and ligaments a rotational damped spring was attached as follows:

$$\tau_s = -k_s (q - q_0) - k_d \dot{q} \tag{18}$$

where k_d is the Damping Coefficient and k_s is the spring stiffness. The forces of the muscle are partitioned into



inactive elastic forces f_p represented as the muscles elastic characteristics as they are extended, and active contractile forces f_c produced by the muscles because of the neural control signal

$$\tau_m = P(q)f_c + P(q)f_p \tag{19}$$

P(q) is the moment arm matrix that maps the muscle forces to the joint torque. The Jacobian matrix that transforms the external force f_e into joint Torques is given as J(q).

Substituting equation (15) and (16) and the external force matrix into equation (14), we obtain the equation of motion as

$$M(q)\ddot{q} + C(q.\dot{q}) + Q(q) + k_{d}\dot{q} + k_{s}q = P(q)f_{c} + P(q)f_{p} + J(q)^{T}f_{e}$$
⁽²⁰⁾

This is similar to a mass-spring-damper model with an excitation. However, the dependent variable q is a function of three variables as shown below:

$$q = f\left(\xi, \psi, \chi\right) \tag{21}$$

For a single degree of freedom system, we have q = f(q). Substituting into equation (21), we have

$$M\ddot{q} + C\dot{q} + Kq = Pf_p + Pf_c + J^T f_e$$
⁽²²⁾

Grouping the forcing functions

$$M\ddot{q} + C\dot{q} + Kq = \underbrace{Pf_p + Pf_c + J^T f_e}_{F(t) = Generalized force ing function} \quad \text{and} \quad \tau = \frac{2\pi}{\omega}$$
(23)

Using Fourier series approximation to represent the forcing function, we obtain

$$M\ddot{q} + C\dot{q} + Kq = F(t) = \frac{a_0}{2} + \sum_{j=1}^{\infty} a_j \cos j\omega t + \sum_{j=1}^{\infty} b_j \sin j\omega t$$
(24)

where

$$a_j = \frac{2}{\tau} \int_0^\tau F(t) \cos j\omega t \, dt \tag{25a}$$

$$b_j = \frac{2}{\tau} \int_0^\tau F(t) \sin j\omega t \, dt \tag{25b}$$

The solution of equation (25) will have both complimentary function (CF) and particular Integral (PI) as expressed below:

$$q = q_{CF} + q_{PI} \tag{26}$$

The complimentary function is obtained by setting the directed toward the right of equation (20) to zero and solving.

$$p_{CF} = A\sin\omega t + B\cos\omega t \tag{27}$$

For the particular integral, the right-hand side is split into three, solved separately and combined as shown below:

$$M\ddot{q} + C\dot{q} + Kq = F(t) = \frac{a_0}{2}$$

$$M\ddot{q} + C\dot{q} + Kq = F(t) = a_j \cos j\omega t$$

$$M\ddot{q} + C\dot{q} + Kq = F(t) = b_j \sin j\omega t$$
(28)

The solutions are

$$q_{PI}^{1} = \frac{a_{0}}{2K}$$

$$q_{PI}^{2} = \frac{a_{j} / K}{\sqrt{\left(1 - j^{2}r^{2}\right)^{2} + \left(2\varsigma jr\right)^{2}}} \cos(j\omega t - \phi)$$
(29)



$$q_{PI}^{3} = \frac{b_{j} / K}{\sqrt{\left(1 - j^{2}r^{2}\right)^{2} + \left(2\varsigma jr\right)^{2}}} \sin(j\omega t - \phi)$$

where

$$\phi = \tan^{-1}\left(\frac{2\varsigma jr}{1-j^2r^2}\right)$$
 and $r = \frac{\omega}{\omega_n}$ (30)

The steady-state solution thus becomes

$$q_{PI} = \frac{a_0}{2K} + \frac{a_j / K}{\sqrt{\left(1 - j^2 r^2\right)^2 + \left(2\varsigma jr\right)^2}} \cos(j\omega t - \phi) + \frac{b_j / K}{\sqrt{\left(1 - j^2 r^2\right)^2 + \left(2\varsigma jr\right)^2}} \sin(j\omega t - \phi)$$
(31)

while the general solution for the single degree of freedom system is

$$q(t) = A\sin\omega t + B\cos\omega t + \frac{a_0}{2K} + \frac{a_j/K}{\sqrt{(1-j^2r^2)^2 + (2\varsigma jr)^2}}\cos(j\omega t - \phi) + \frac{b_j/K}{\sqrt{(1-j^2r^2)^2 + (2\varsigma jr)^2}}\sin(j\omega t - \phi)$$
(31)

For the three degrees of freedom system, the dependent variable becomes

$$q = f\left(\xi, \psi, \chi\right) \tag{32}$$

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Matrices that contain three equations will be generated instead of an equation as it was in the single degree of freedom system.

$$M\ddot{q} + C\dot{q} + Kq = Pf_p + Pf_c + J^T f_e$$
(33)

$$\begin{bmatrix} M_{11} & 0 & 0 \\ 0 & M_{22} & 0 \\ 0 & 0 & M_{33} \end{bmatrix} \begin{bmatrix} \ddot{\xi} \\ \ddot{\psi} \\ \ddot{\chi} \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \dot{\xi} \\ \dot{\psi} \\ \dot{\chi} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & M_{33} \end{bmatrix} \begin{bmatrix} \xi \\ \psi \\ \chi \end{bmatrix} = P \begin{bmatrix} f_{p1} + f_{c1} \\ f_{p2} + f_{c2} \\ f_{p3} + f_{c3} \end{bmatrix} + \begin{bmatrix} \frac{\partial}{\partial t} f_{e1} \\ \frac{\partial}{\partial t} f_{e2} \\ \frac{\partial}{\partial t} f_{e3} \end{bmatrix}$$
(34)

In this work, the spring and damping parameters are modeled as a function in a vector C.

$$\begin{bmatrix} M_{11} & 0 & 0 \\ 0 & M_{22} & 0 \\ 0 & 0 & M_{33} \end{bmatrix} \begin{bmatrix} \ddot{\xi} \\ \ddot{\psi} \\ \ddot{\chi} \end{bmatrix} + \begin{bmatrix} C_{11} \\ C_{21} \\ C_{31} \end{bmatrix} = P \begin{bmatrix} f_{p1} + f_{c1} \\ f_{p2} + f_{c2} \\ f_{p3} + f_{c3} \end{bmatrix} + \begin{vmatrix} \frac{\partial}{\partial t} f_{e1} \\ \frac{\partial}{\partial t} f_{e2} \\ \frac{\partial}{\partial t} f_{e3} \end{bmatrix}$$
(35)

-

where

$$M_{11} = H_{zz} + N_{zz} + m_H \left(L^2 + h^2 + 2hL\cos(\psi(t)) + \frac{1}{4}m_N L^2 \right)$$
(36)

$$M_{22} = h^2 m_H + H_{zz} aga{37}$$

$$M_{33} = \mathbf{m}_H + \mathbf{m}_B + \mathbf{m}_N \tag{38}$$

$$C_{11} = m_H hL \sin(\psi(t)) \left[\left(\frac{\mathrm{d}}{\mathrm{d}t} \xi(t) \right)^2 - \left(\frac{\mathrm{d}}{\mathrm{d}t} \psi(t) + \frac{\mathrm{d}}{\mathrm{d}t} \xi(t) \right)^2 \right]$$
(39)



$$C_{22} = m_H h L \sin\left(\psi(t)\right) \left(\frac{\mathrm{d}}{\mathrm{d}t}\xi(t)\right)^2 \tag{40}$$

$$C_{33} = m_H h \sin\left(\psi\left(t\right) + \xi\left(t\right)\right) \left(\frac{\mathrm{d}}{\mathrm{d}t}\psi\left(t\right) - \frac{\mathrm{d}}{\mathrm{d}t}\xi\left(t\right)\right)^2 + \frac{1}{2} (m_N + 2m_H) \left(\frac{\mathrm{d}}{\mathrm{d}t}\xi\left(t\right)\right)^2 \sin\left(\xi\left(t\right)\right)$$
(41)

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3.2 Using Differential Transform Method (DTM) for the solution

Equation (35) is the desired fully coupled three degrees of freedom model that will be solved using DTM. To apply DTM, we recall the model in expanded form as

$$M_{11}\ddot{\xi} + C_{11} = P(f_{p1} + f_{c1}) + \frac{\partial}{\partial t} f_{e1}$$

$$M_{22}\ddot{\psi} + C_{11} = P(f_{p2} + f_{c2}) + \frac{\partial}{\partial t} f_{e2}$$

$$M_{33}\ddot{\chi} + C_{11} = P(f_{p3} + f_{c3}) + \frac{\partial}{\partial t} f_{e3}$$
(42)

Making necessary substitutions,

substitutions,

$$F_{1} = \left[H_{zz} + N_{zz} + m_{H}\left(L^{2} + h^{2} + 2hL\cos\left(\psi(t)\right) + 1/4m_{N}L^{2}\right)\right]\frac{d^{2}}{dt^{2}}\xi(t) + m_{H}hL\sin\left(\psi(t)\right)\left[\left(\frac{d}{dt}\xi(t)\right)^{2} - \left(\frac{d}{dt}\psi(t) + \frac{d}{dt}\xi(t)\right)^{2}\right]\right]$$

$$F_{2} = \left[h^{2}m_{H} + H_{zz}\right]\frac{d^{2}}{dt^{2}}\psi(t) + m_{H}hL\sin\left(\psi(t)\right)\left(\frac{d}{dt}\xi(t)\right)^{2}$$

$$F_{3} = \left[m_{H} + m_{B} + m_{N}\right]\frac{d^{2}}{dt^{2}}\chi(t) + m_{H}h\sin\left(\psi(t) + \xi(t)\right)\left(\frac{d}{dt}\psi(t) - \frac{d}{dt}\xi(t)\right)^{2} + 1/2(m_{N} + 2m_{H})\left(\frac{d}{dt}\xi(t)\right)^{2}\sin\left(\xi(t)\right)$$
(43)

Using series approximation on the trigonometric functions, we have

$$F_{1} = \begin{bmatrix} (H_{x} + N_{x}) \frac{d^{2}}{dt^{2}} \xi(t) + m_{H} \begin{pmatrix} (L^{2} + h^{2}) \frac{d^{2}}{dt^{2}} \xi(t) + 2hL \begin{bmatrix} \frac{d^{2}}{dt^{2}} \xi(t) - \frac{1}{2} \psi^{2} \frac{d^{2}}{dt^{2}} \xi(t) \end{bmatrix} \\ + 1/4m_{N}L^{2} \frac{d^{2}}{dt^{2}} \xi(t) \\ + m_{H}hL \Big[\psi - \frac{1}{6} \psi^{3} \Big] \Big[- \Big[\frac{d}{dt} \psi(t) \Big]^{2} + 2\frac{d}{dt} \psi(t) \frac{d}{dt} \xi(t) \Big] \\ F_{2} = \Big[h^{2}m_{H} + H_{x} \Big] \frac{d^{2}}{dt^{2}} \psi(t) + m_{H}hL \Big[\psi - \frac{1}{6} \psi^{3} \Big] \Big[\frac{d}{dt} \xi(t) \Big]^{2} \\ F_{3} = \Big[m_{H} + m_{B} + m_{N} \Big] \frac{d^{2}}{dt^{2}} \chi(t) + m_{H}h \Big[\xi - \frac{1}{6} \xi^{3} + \Big[\psi - \frac{1}{2} \xi^{2} \psi \Big] \Big] \Big[\frac{d}{dt} \psi(t) - \frac{d}{dt} \xi(t) \Big]^{2} \\ + \frac{1}{2} (m_{N} + 2m_{H}) \Big[\frac{d}{dt} \xi(t) \Big]^{2} \Big[\xi - \frac{1}{6} \xi^{3} \Big] \end{bmatrix}$$

$$(44)$$

with initial conditions

$$\xi_{1} = 0, \qquad \xi_{0} = a \cos(\omega t)$$

$$\psi_{1} = 0, \qquad \psi_{0} = b \cos(\omega t)$$

$$\chi_{1} = 0, \qquad \chi_{0} = c \cos(\omega t)$$
(45)

The DTM recursive relations for the solution of Equation (44) becomes:



$$\xi_{k+2} = \frac{2B}{3A} \tag{46a}$$

where

$$\begin{aligned} A &= m_{H} L^{2} k^{2} m_{N} + 4m_{H} L^{2} k^{2} + 3m_{H} L^{2} km_{N} + 8m_{H} Lhk^{2} + 4m_{H} h^{2} k^{2} + 12m_{H} h^{2} k \\ &+ 2m_{H} m_{N} L^{2} + 24m_{H} Lhk + 12m_{H} h^{2} k + 4H_{L} k^{2} + 8m_{H} L^{2} + 16m_{H} hL \\ &+ 4N_{L} k^{2} + 8m_{H} h^{2} + 12H_{L} k + 12N_{L} k + 8H_{L} k^{2} + 8m_{L} L^{2} + 16m_{H} hL \\ &+ 4N_{L} k^{2} + 8m_{H} h^{2} + 12H_{L} k + 12N_{L} k + 8H_{L} k^{2} + 8m_{L} L^{2} + 16m_{H} hL \\ &+ 4N_{L} k^{2} + 8m_{H} h^{2} + 12H_{L} k + 12N_{L} k + 8H_{L} k^{2} + 8m_{L} \\ B &= 6m_{H} hL \sum_{p=0}^{k} \left[\sum_{l=0}^{p} (1+l)(2+l)\xi_{l+2}\psi_{p-l}\psi_{k-p} \right] + 6m_{H} hL \sum_{p=0}^{k} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\psi_{l+1}\psi_{p-l+1}\psi_{k-p} \right] \\ &- 12m_{H} hL \sum_{p=0}^{k} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\psi_{l+1}\xi_{p-l+1}\psi_{k-p} \right] \\ &- m_{H} hL \sum_{p=0}^{k} \left[\sum_{p=0}^{p} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\psi_{l+1}\xi_{p-l+1}\psi_{u-p}\psi_{v-u}\psi_{k-v} \right] \right] \right] \\ &+ 2m_{H} hL \sum_{p=0}^{k} \left[\sum_{u=0}^{p} \left[\sum_{p=0}^{p} (1+l)(1-l+p)\psi_{l+1}\xi_{p-l+1}\psi_{u-p}\psi_{v-u}\psi_{k-v} \right] \right] + 6F_{1} \\ &\psi_{k+2} = \frac{1}{6(h^{2}m_{H} + H_{L})(k+1)(k+2)} \left[-6m_{H} hL \sum_{p=0}^{k} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\xi_{l+1}\xi_{p-l+1}\psi_{k-p} \right] \\ &+ m_{H} hL \sum_{v=0}^{k} \left[\sum_{u=0}^{v} \left[\sum_{p=0}^{u} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\xi_{l+1}\xi_{p-l+1}\psi_{u-p}\psi_{v-u}\psi_{k-v} \right] \right] \right] + 6F_{2} \right] \\ &\chi_{k+2} = \frac{C}{12(m_{H} + m_{B} + m_{N})(k+1)(k+2)} \end{aligned}$$

$$(47a)$$

where

$$C = -6m_{H}hm_{N}\sum_{p=0}^{k} \left[\sum_{l=0}^{p} (1+l)(p-l+1)\xi_{l+1}\xi_{p-l+1}\xi_{k-p} \right] - 12m_{H}hm_{h}\sum_{p=0}^{k} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\xi_{l+1}\xi_{p-l+1}\xi_{k-p} \right] + 2m_{H}hm_{h}\sum_{\nu=0}^{k} \left[\sum_{u=0}^{\nu} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\psi_{l+1}\psi_{p-l+1}\xi_{u-p}\xi_{\nu-u}\xi_{k-\nu} \right] \right] \right] - 12m_{H}h\sum_{p=0}^{k} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\psi_{l+1}\psi_{p-l+1}\xi_{k-p} \right] + 2m_{H}h\sum_{\nu=0}^{k} \left[\sum_{u=0}^{\nu} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\psi_{l+1}\psi_{p-l+1}\xi_{u-p}\xi_{\nu-u}\xi_{k-\nu} \right] \right] \right] - 12m_{H}h\sum_{p=0}^{k} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\psi_{l+1}\psi_{p-l+1}\xi_{k-p} \right] + 2m_{H}h\sum_{\nu=0}^{k} \left[\sum_{u=0}^{\nu} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\psi_{l+1}\psi_{p-l+1}\xi_{u-p}\xi_{\nu-u}\xi_{k-\nu} \right] \right] \right] - 12m_{H}h\sum_{p=0}^{k} \left[\sum_{l=0}^{p} (1+l)(p-l+1)\psi_{l+1}\psi_{p-l+1}\xi_{k-p} \right]$$

$$(48b)$$

$$+ 6m_{H}h\sum_{\nu=0}^{k} \left[\sum_{u=0}^{\nu} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\psi_{l+1}\psi_{p-l+1}\xi_{u-p}\xi_{\nu-u}\psi_{k-\nu} \right] \right] \right] + 12F_{3}$$

$$+ m_{H}hm_{N}\sum_{\nu=0}^{k} \left[\sum_{u=0}^{\nu} \left[\sum_{l=0}^{p} (1+l)(1-l+p)\xi_{l+1}\xi_{p-l+1}\xi_{u-p}\xi_{\nu-u}\xi_{k-\nu} \right] \right] \right]$$

Using the transformed conditions on the transformed governing equation, the term by term DTM solution becomes as presented below:

$$\xi_{0} = a\cos(\omega t)$$

$$\xi_{1} = 0$$

$$\xi_{2} = \frac{2F_{1}}{-4m_{H}L(\cos(\omega t))^{2}b^{2}h + m_{H}m_{N}L^{2} + 4m_{H}L^{2} + 8m_{H}Lh + 4m_{H}h^{2} + 4H_{zz} + 4N_{zz}}$$

$$\xi_{3} = \frac{2F_{1}}{3(-4m_{H}L(\cos(\omega t))^{2}b^{2}h + m_{H}m_{N}L^{2} + 4m_{H}L^{2} + 8m_{H}Lh + 4m_{H}h^{2} + 4H_{zz} + 4N_{zz})}$$
(49)

also,

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()

$$\psi_{0} = b \cos(\omega t)$$

$$\psi_{1} = 0$$

$$\psi_{2} = \frac{F_{2}}{2(m_{H}h^{2} + H_{zz})}$$

$$\psi_{3} = \frac{F_{2}}{6(m_{H}h^{2} + H_{zz})}$$
(50)

and finally,

$$\chi_{0} = c \cos(\omega t)$$

$$\chi_{1} = 0$$

$$\chi_{2} = \frac{F_{3}}{2(m_{H} + m_{B} + m_{N})}$$

$$\chi_{3} = \frac{F_{3}}{6(m_{H} + m_{B} + m_{N})}$$
(51)

From the principle of DTM inversion, the series solution is generally represented as

$$\xi(t) = \sum_{\zeta=0}^{N} \xi_{\zeta} t^{\zeta}$$

$$\psi(t) = \sum_{\zeta=0}^{N} \psi_{\zeta} t^{\zeta}$$

$$\chi(t) = \sum_{\zeta=0}^{N} \chi_{\zeta} t^{\zeta}$$
(52)

which in expanded form becomes:

$$\xi(t) = \xi_0 + \xi_1 t + \xi_2 t^2 + \xi_3 t^3 + \dots$$

$$\psi(t) = \psi_0 + \psi_1 t + \psi_2 t^2 + \psi_3 t^3 + \dots$$

$$\chi(t) = \chi_0 + \chi_1 t + \chi_2 t^2 + \chi_3 t^3 + \dots$$
(53)

The desired angular motion of the neck region, angular motion of the head region and linear motion of the head thus becomes:

$$\xi(t) = a\cos(\omega t) + \left(\frac{2F_1}{-4m_H L(\cos(\omega t))^2 b^2 h + m_H m_N L^2 + 4m_H L^2 + 8m_H Lh + 4m_H h^2 + 4H_{zz} + 4N_{zz}}\right) t^2 + \left(\frac{2F_1}{3\left(-4m_H L(\cos(\omega t))^2 b^2 h + m_H m_N L^2 + 4m_H L^2 + 8m_H Lh + 4m_H h^2 + 4H_{zz} + 4N_{zz}}\right)} t^3 + \dots$$
(54)

$$\psi(t) = b\cos(\omega t) + \left(\frac{F_2}{2(m_H h^2 + H_z)}\right) t^2 + \left(\frac{F_2}{6(m_H h^2 + H_z)}\right) t^3 + \dots$$
(55)

$$\chi(t) = c\cos(\omega t) + \left(\frac{F_3}{2(m_H + m_B + m_N)}\right) t^2 + \left(\frac{F_3 t^3}{6(m_H + m_B + m_N)}\right) t^3 + \dots$$
(56)

4. Results and Discussion

Using the values of parameters in Table 1 as obtained from literature, the results of this study from the simulations of the model using the Differential Transform Method (DTM) is compared with the results obtained in the laboratories (See Figure 5) through experiments [31-32] and validated using Runge-Kutta method numerical solution. The result shows a



very minimal error when compared with the numerical results as shown in Table 2.

Symbol	Parameter	Value	
m_H	Head mass	4.0 kg	
L	Lower linkage length	0.12m	
H	Upper linkage length	0.06m	
m_N	Neck mass	1.2 kg	
H_{ZZ}	Head moment of inertia	0.025 kgm ²	
N_{ZZ}	Neck moment of inertia	0.003 kgm ²	
m_B	Body mass	100 kg	
8 7 - 6 - (ψ) 7 - 7 - 8 - 7 - 6 - 7		i chael et al., 2018 resent study (DTM)	
-10 -2 -			
0 5	io 100 150 200 250 300 3	50 400 450 50	0

Table 1. The Symbols used in the System Model and Values of the Parameters

Fig. 3. Model DTM solution compared with experimental data for angular motion of the neck region for a long period of time

Table 2. Table of comparison of results of angular motion of the neck region for a short time

The results of DTM and numerical methods for $\psi(t)$				
Т	DTM	NUM	Error of DTM	
0.00	0.36012341	0.36012341	0.00000000	
1.00	-2.33906559	-2.33906559	0.00000000	
2.00	9.48614059	9.48614059	0.00000000	
3.00	-2.38724438	-2.38724438	0.00000000	
4.00	-28.98446950	-28.98443350	0.00000000	
5.00	43.94610222	43.94610222	-0.00003600	
6.00	10.88880610	10.88880610	0.00000000	
7.00	-111.78985300	-111.78985300	0.00000000	
8.00	103.62338500	103.62332100	0.00000000	
9.00	87.30854565	87.30854565	0.00006400	
10.00	0.36012341	0.36012341	0.00000000	

Table 3. Table of comparison of results of angular motion of the head for a short time

	The results of DTM and numerical methods for $\xi(t)$			
	Т	DTM	NUM	Error of DTM
	0.00	3.00000000	3.00000000	0.00000000
	1.00	-1.06476400	-1.06476406	0.00000006
	2.00	-0.61421181	-0.61421182	0.00000001
	3.00	-4.77003930	-4.77003931	0.00000001
)	4.00	-6.32609021	-6.32609023	0.00000001
	5.00	6.67096512	6.67096510	0.00000001
	6.00	16.41064723	16.41064763	-0.00000040
	7.00	18.57538245	18.57538275	-0.00000030
	8.00	-8.94736384	-8.94736385	0.00000000
	9.00	-109.47056040	-109.47056040	0.00000000
	10.00	-79.39932500	-79.39932500	0.00000000

4.1 Dynamic Response of Angular and Linear Displacement of the Head and Neck Region

Figures 4-7 show the simulated dynamic response of angular and linear displacements of the head and neck regions. The behavior of these displacements shows that an increase in the amplitude which is a result of the presence of a forcing function for the angular displacements and axial force for the linear displacement.



 \mathbf{x}

The results of DTW and numerical methods for $\chi(t)$					
Т	DTM	NUM	Error of DTM		
0.00	-1.00000000	-1.00000000	0.00000000		
1.00	-0.18006171	-0.18006171	0.00000000		
2.00	1.16953280	1.16953280	0.00000000		
3.00	-4.74307030	-4.74307030	0.00000000		
4.00	1.19362219	1.19362219	0.00000000		
5.00	14.49223475	14.49221675	0.00001800		
6.00	-21.97305111	-21.97305111	0.00000000		
7.00	-5.44440305	-5.44440305	0.00000000		
8.00	55.89492650	55.89492650	0.00000000		
9.00	-51.81169250	-51.81166050	-0.00003200		
10.00	-43.65427283	-43.65427283	0.00000000		

Table 4. Table of comparison of results of linear motion of the head for a short time

The results of DTM and received mixed mother deform $\omega(t)$



Fig. 4. Angular displacement of the neck region



Fig. 6. Linear displacement of the head region



Fig. 5. Angular displacement of the head region



Fig. 7. Super-imposed plot of the responses

These parameters are very important in the study of the biomechanical modeling of head loads because they directly affect the organs which may either cause serious pains or even fracture as a result of excessive loading.

The effects of the forcing function which is caused by neck muscle activation and head loads or head mass are shown in figures 8 and 9. It can be observed that the stiffness and damping joints of the head linkage respond to the minimum and maximum neck muscle activation.

For free vibration, the amplitude of the system is observed to be conserved as minimal variation is noticed throughout the time history. However, as the forcing function is increased, the displacement function also increases. Figure 8 shows the effects of forcing function on the dynamic behavior of the system in concern. For free vibration, the amplitude of the system is observed to be conserved as minimal variation is noticed throughout the time history. However, as the forcing function also increases. The physical implication of this is that when the load on the head continues to increase during impact, it increases the tension and twisting effects on the head-neck region and eventual failure of the affected body systems. This result helps to determine the loading limits once the physical implication of the displacement impact is known.

The impact of force on the Lower linkage length is shown in Figure 10, it is observed that for small loads, the amplitude



of the system is conserved as minimal variation is seen when the time begins the time history, as the forcing function is increased, there is a slight increase in the displacement function. In addition, there is a slight difference in varying lower linkage lengths, consequently, the lower linkage length has some significant effects on the response of the system model.



Fig. 12. Effect of Neck mass m_N



From Figure 11, the response of the system to varying upper linkage length *L* can be observed at varying upper linkage length. There is minimal variation in the response which shows that the varying upper linkage length does not affect the response of the system. The effects of neck mass m_H on the model of the system are shown in Figure 12. At varying neck masses, some slight differences are noticed in the response of the system as shown in the graph. While at the varying head moment of inertia H_{ZZ} made no significant differences in the response of the system as observed in Figure 13.

Similar to the effect of the head moment of inertia H_{ZZ} in the neck moment of inertia N_{ZZ} . There is no significant difference in the response of the linear displacement of the head with varying neck moment of inertia as observed in Figure 14. The effect of body mass on the linear displacement of the head is shown in Figure 15. It is observed that as the body mass increases, the amplitude of the response increases as shown on the graphs.

These results show the efficiency of the governing model which provides an analytic relationship between the angular positioning of the head and its response to input forces in the form of impact loads. These results show that minimal



changes in the angles of the head neck arrangement lead to a large increase in the angular acceleration of the head. This emphasizes the significance of proper angular positioning of the head before contact or impact with loads. This study shows that an optimal position of the head exists which changes in height and which depends on the neck joint configuration. Despite the complexity of the mathematical equation relating the head impact location, neck joint angles, and the input forces, the additional forces of the head from its center of mass resulted in higher angular acceleration of the head. It has been shown from experiments in past studies that Non- centric impacts often lead to notable angular acceleration of the head than the centric impacts [33]. This analysis can be used as an evaluating tool on how head load can affect the rotational acceleration of the head.



From the simulation of the models for the duration of head loads, it can be observed that when the soft tissue forces from neck muscle activation are increased, the angular acceleration is reduced by 20%. This study shows that forcing function from the activation of the neck muscle can reduce neck injury from high impact loads. These results further show that activation of the neck muscles for the forcing function does not reduce angular kinematic substantially during the period as much as the head-neck positioning.

It is observed from all the results obtained in the study that the computational method provides a better understanding of the relationship between the physical quantities in the governing model of the head/neck problem investigated. This cannot be achieved by the conventional numerical methods for nonlinear models. The method also shows a direct relationship between the model parameters and provides good and continuous physical insights into the significance of various parameters affecting the head/neck system. For instance, during impact in sporting activities, it can be observed from the results that the activation (stiffening) of the Neck muscles reduces the risk of head injuries. Similarly, the positioning of the head also has effects on the risk of head injuries during impact. These observations are made possible through the analysis of the computational method. In addition, since impact loads are unpredictable, the head protection devices must be well designed for an adequate shield against sudden impacts that lead to head injuries. The computational method provides a platform for improvement in the design of such head protection devices because the method shows a direct relationship between the model parameters.

5. Conclusion

In this study, the dynamic analysis of the biomechanical modeling of head load impact has been presented. The obtained models were solved using the differential transform method (DTM) and were validated with the fourth-order Runge-Kutta numerical method (RK4). The obtained results were compared with the results obtained in earlier studies. Good agreements were reached in all the results. The effects of head loads, head mass, neck mass, upper and lower linkage lengths, head and neck moments of inertia were investigated. As the head loads increased, there were increases in both axial and angular displacement of the head motion and the neck region. The study provides a theoretical basis for the design and understanding of the effects of head load carriage on vital organs that are susceptible to pains, damages, and even failure.

Author Contributions

O.A. Adeleye initiated the project, developed the theoretical and biomechanical model of the study and suggested the analytical solution; O. Ipinnimo conducted the literature survey and examined the analytical solution relevance and validation; A. Yinusa developed the analytical solution and numerical validation; O.E. Precious developed the results in graphs and tables and provided the experimental validation. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

Conflict of Interest

The authors of this paper have declared that there is no conflict of interest in this study, with respect to the authorship,



and the publication of the research work.

Funding

The authors received no financial support for the research, authorship and publication of this article.

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