OPTIMIZATION OF NATURAL GAS TRANSPORTATION IN PIPELINE

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ABSTRACT

Transportation of natural gas is a very important aspect of the oil and gas industry and as such, it must be done with a much efficiency.Pipelines has been recognized as the most economic, effective and safest way of transporting natural gas.

A lot of capital is needed, due to cost of pipeline, compressor stations and also in its maintenance. Therefore in order to minimizing cost, optimization of natural gas transportation processes is necessary.

In this study, optimization procedure of natural gas transportation network was developed using a workable procedure adapting the Generalized Reduced Gradient algorithm. It determines the optimum economical conditions natural gas can be transported through series of pipeline and compressors station. The model developed when applied to the Excravos Lagos pipeline network showed that total cost of flowing natural gas depends on the amount of gas to be transported and also on the outlet pressure required. Results show that depending on the require flow rate, some installed compressors need to be inactive for effective cost reduction. The required diameters to meet the corresponding demand (Flow rates) are presented for the future upgrade of the facilities. Also comparison of different gas flow equations showed that the optimum network configuration for panhandle A and B are almost the same, but for Weymouth equation, it widely varies. The developed model can be extended to treat much larger and more complex network.

Keywords: Gathering pipelines, Compressors station, Reduced Gradient Method, Investment cost, Gas floe equations.

1.0 INTRODUCTION

Natural gas is a gas consisting primarily of methane. It is found associated with other fossil fuels, in coal beds, as methane clathrates, and is created by methanogenic organisms in marshes, bogs, and landfills. It is an important fuel source, a major feedstock for fertilizers, and a potent greenhouse gas. Before natural gas can be used as a fuel, it must undergo extensive processing to remove almost all materials other than methane. The by-products of that processing include ethane, propane, butane, pentane and higher molecular weight hydrocarbon, elemental sulfur, carbon dioxide, water vapor and sometimes helium and nitrogen. Natural gas is often informally referred to as simply gas, especially when compared to other energy sources such as oil or coal.

The Nigerian natural gas can be described as “solution gas” because it dissolves naturally as the oil is been produced and occurs in a large number of small, widely scattered reservoirs. It is concentrated in the Niger Delta which covers an area of about 41000 sq. Miles (106189.50 km^2). Nigeria’s proven and probable reserves form about 1.1 percent of the worlds proven reserves. It is estimated that Nigeria’s proven and probable resrves are in the order of about 85 E scf (2.407 E m^3). This is about 17 billion barrels of oil equivalent. Of the total, Nigeria’s proven reserves, 70% is located on land and 30% is off-shore. About 60% are located east of River Niger while the rest are to the west of River Niger. Nigeria has an undiscovered reserves of oil/gas of about 65 E scf (1.841 E m^3). Associated gas counts for about 50% of the proven reserves. Of these, about 75% exist as gas caps. Experts estimate that the reserves locked in the Nigerian soil is enough to last about 500 years, fuelling our industries, homes and for export.

The major difficulty in the use of natural gas is transportation and storage because of its low density. Natural gas pipelines are economical, but are impractical across oceans. Natural gas which was once an almost embarrassing and unwanted by-product or more correctly a co-production of crude oil production now provides about 1/5th of the entire world primary energy requirement. This remarkable development has taken place in only a few years with increased availability of the gas resources of the country and the construction of long distance, large diameter steel pipeline which have brought these sample supplies of gaseous fuel to domestic, commercial and industrial users many miles away from the field themselves. In the early years of the natural gas industry when gas accompanies crude oil, it has to find a market or be flared in the absence of effective conservation practice. Oil well gas was often flared in huge quantity.

Consequently, gas production at time was often short lived and gas could be purchased for one or two cents per 1000 cubic ft in the field.

Natural gas is useless if it is not consumed and for this to happen it needs to be transported via different means such as pipeline, tankers but the most economical and widely used is pipeline transportation. Transportation of gas by pipeline is a very vital commercial activity which happens on a day to day basis and is a somewhat tricky business because it expensive and poor management of cost could lead to bankrupcy. The gas dispatcher must balance supply and demand under certain circumstances through a proper sequencing of equipment which is both expensive to run and maintain.

Pipeline transportation has become an important means of moving natural gas and with the expansion of market and large demand, millions of pipeline have been laid. Therefore, in moving large quantity of these fuel from the gathering station to the refinery and to transportation and distribution company and finally to the consumers, it can be moved through pipeline.

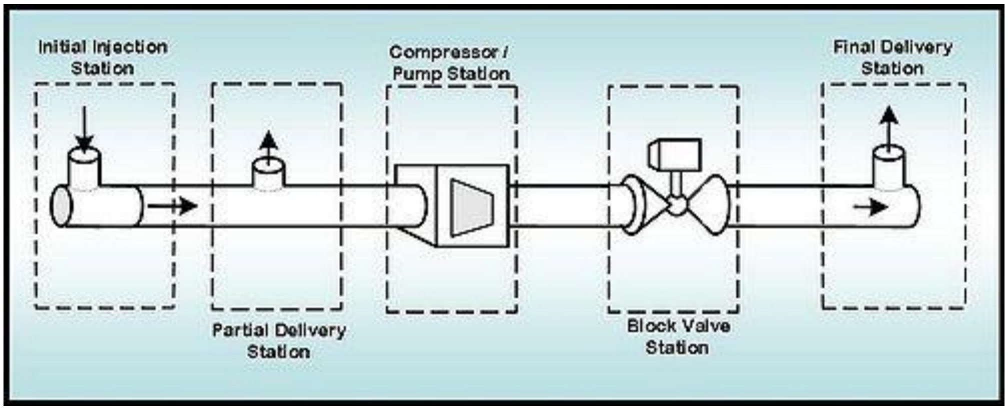
In its development, large input of capital and investment cost is required. Most of these cost are related to two main component, pipeline system and cost related to compressor station. The cost of pipeline depends on its length and diameter and the cost of the pipeline is proprotional to the diameter, while the cost of compressor stration depends on the operating power which is function of both suction and discharge pressure. The use of small pipe will increase the pressure drop and consequently will need compresssor with high power.

Therefore, in order to minimize cost or maximize profit, we have to obtain a proper balance between pipeline cost and compressor cost. Due to the cost complexities, optimization of the pipeline network is necessary. Cost effective design of gas pipeline and its operation, cost of gas pipeline transportation must be low enough to provide adequate profit in financial investment.

Pipeline Components

Pipeline networks are composed of several pieces of equipment that operate together to move products from location to location.

The main elements of a pipeline system are:



A pipeline schematic

A. Initial injection station: Known also as *supply* or *inlet* station, is the beginning of the system, where the product is injected into the line. Storage facilities, pumps or compressors are usually located at these locations.
B. Compressor/pump stations: Pumps for liquid pipelines and Compressors for gas pipelines, are located along the line to move the product through the pipeline. The location of these stations is defined by the topography of the terrain, the type of product being transported, or operational conditions of the network.
C. Partial delivery station: Known also as *intermediate* stations, these facilities allow the pipeline operator to deliver part of the product being transported.
D. Block valve station: These are the first line of protection for pipelines. With these valves the operator can isolate any segment of the line for maintenance work or isolate a rupture or leak. Block valve stations are usually located every 20 to 30 miles (48 km), depending on the type of pipeline. Even though it is not a design rule, it is a very usual practice in liquid pipelines. The location of these stations depends exclusively on the nature of the product being transported, the trajectory of the pipeline and/or the operational conditions of the line.
E. Regulator station: This is a special type of valve station, where the operator can release some of the pressure from the line. Regulators are usually located at the downhill side of a peak.
F. Final delivery station: Known also as outlet stations or terminals, this is where the product will be distributed to the consumer. It could be a tank terminal for liquid pipelines or a connection to a distribution network for gas pipelines.

MODEL DEVELOPMENT AND METHODOLOGY

The pipeline segment has five variables associated with it and they are,

- The flow rate, Q.
- The discharge pressure p_d from the upstream compressor
- The suction pressure p_s of the downstream compressor
- The pipe diameter, D
- The length of the pipeline, L.

The total cost function model fro the optimization of the design of the pipeline system will be developed here with the associated variables.

MODEL ASSUMPTION

In building a model, one cannot cover every basis and as such errors of quite a significant magnitude may be gotten. In order to reduce these errors, assumptions are made to cover grounds to accommodate these errors and keep them at a minimum. This minimum however, is determined largely by the constraint of this work. The simplifying assumptions chosen for this model are

- The flow of the gas in the pipeline is in a single phase.
- The flowing gas in the pipeline is isothermal, that is, the temperature is constant.
- The gas compressibility factor, z, is constant before and after passing through the compressor
- The type of compressor is a centrifugal compressor
- The flow is a steady-state flow.

TOTAL COST FUNCTION MODEL

The total cost for the pipeline network development can be predicted as follows

$$[TOTAL\ COST] = [\ INVESTMENT\ COST + OPERATING\ COST]_{PIPELINE} + INVESTMENT\ COST + OPERATION\ COST + ENERGY\ COST]_{COMPRESSOR}$$

The gas transmission system is optimized based on a period of time.

Investment Cost

Capital recovery or depreciation method are the two approaches that can be used to able the annual cost of investment. Capital recovery however considers a discount rate while depreciation does not. We employ the capital recovery in this paper for determining the annual investment cost for the entire life of the system. The formulas is as follows

$$A = p\,\frac{(1+r)^n\,r}{(1+r)^n - 1} \qquad 3.1$$

Investment Cost for pipeline

Model for the investment cost of pipeline is as follows

$$C_{pipe} = (\,1 + R_p\,)\,C_p\,L^l\,d^m \qquad 3.2$$

where C_p, l, m values can be found easily by regression from pipe data. Base on capital recovery that is indicated in Eq. 3.1, the annual cost of pipeline investment can be expressed as follows.

$$CIP = \frac{(1+r)^n\,r}{(1+r)^n - 1}\,(\,1 + R_p\,)\,c_p\,L^l\,d^m \qquad 3.3$$

Investment cost for compressor

The investment cost of the compressor depends on its power. The formula for calculating the compressor power for centrifugal compressor is

$$W_{hp} = 3.032qQP_bTZ \frac{\left[\left[\frac{P_{di}}{P_{si}}\right]^{\frac{k-1}{kE_p}} - 1\right]k}{T_b\ (k-1)}$$, k is a constant. 3.4

Where

Q = flow rate

P_b = base pressure

T_b = base temperature

T = flowing temperature

Z = gas compressibility

P_{di} = discharge pressure

P_{si} = suction pressure

We approach investment cost as follows

$$c_{comp} = c_{hp}\ (W_{hp})^b$$ 3.5

c_{hp} *and* b can be obtained by regression data

Base on the recovery and Eq. 3.5, the annual cost for compressor system can be expressed by the following formula

$$CIC = \frac{(1+r)^n r}{(1+r)^n - 1} c_{hp}\ (W_{hp})^b$$ 3.6

Operating Cost

(a) Pipe operating cost

The cost of maintenance of the pipeline is the main determinant for the operating cost and it is assumed that the operating cost is proportional to the annual investment cost.

The operating cost of pipeline is

$$OC_{pipe} = C_{fp}\ X\ CIP \qquad 3.7a$$

$$OC_{pipe} = C_{fp}\frac{(1+r)^{n} r}{(1+r)^{n}-1}\left(1+ R_{p}\right) c_{p}\ L^{l}\ d^{m} \qquad 3.7b$$

(b) Compressor operating cost

The cost of electricity, maintenance cost and cost involved in the compressor system are the determinant for the operating cost of the compressor. The operating cost is assumed proportional to the cost of electricity, hence, it is expressed as

$$OC_{comp} = xE_{LC} \qquad 3.8$$

Where x > 1. For convenience, x is written as

$$x = 1 + C_{OP}$$

With $C_{OP}\ X\ E_{LC}$ as the operating cost of the compressor excluding electric cost. In order to determine the electric cost, we must convert the unit compressor power (Eq. 3.4) which is in horsepower to kwh, thus we obtain

$$W_{hp} = 19809.302QP_{b}TZ\frac{\left[\left[\frac{P_{di}}{P_{si}}\right]^{\frac{k-1}{kE_{p}}} - 1\right]k}{T_{b}\ (k-1)} \qquad 3.9$$

$$E_{LC} = 1.14^{-4}\ W_{hp}\ C_{e}\ H_{y} \qquad 3.10$$

Therefore Eq.3.8 becomes

$$OC_{comp} = 1.14^{-4}\,(1 + C_{OP})W_{hp}\ C_{e}\ H_{y} \qquad 3.11$$

CONSTRAINT MODEL

Constraint are specified when designing a pipeline network. In this paper, we consider two of them which are the inequality and equality constraints.

Inequality Constraints

The operation of each compressor station is constrained so that the discharge pressure is greater or equal to the suction pressure

$$\frac{P_{di}}{P_{si}} \geq 1 \qquad i = 1, 2, 3..............n \qquad 3.12$$

And the compression ratio does not exceed some pre-specified maximum limit k

$$\frac{P_{di}}{P_{si}} \leq K_i \qquad i = 1, 2, 3..............n \qquad 3.13$$

In addition, upper and lower bounds are placed on each of the four variables

$$P_{di}^{min} \leq P_{di} \leq P_{di}^{max} \qquad 3.14a$$

$$P_{si}^{min} \leq P_{si} \leq P_{si}^{max} \qquad 3.14b$$

$$l_i^{min} \leq l_i \leq l_i^{max} \qquad 3.15$$

$$D_i^{max} \leq D_i \leq D_i^{min} \qquad 3.16$$

Equality constraint

In this, a simplified constraint equation was used. There are many equation used to depict flow in the gas industry, we would only consider Weymouth equation. Certainly, it will not change the total cost model that is derived in the previous section if we use another pipeline equation.

$$Q = 18.063 \left(\frac{T_b}{P_b}\right) D_j^{2.6182} E \left(\frac{P_d^2 - e^s P_s^2}{L_j \gamma Z T}\right)^{0.5} \quad 3.17$$

Where

e = base of natural logarithm = 2.718

s = 0.0375 γ_g Δh

Δh = oulet elevation minus inlet elevation

P_d = discharge pressure at entrance of pipeline segment

P_s = suction pressure at exit of pipeline segment

SOLUTION TECHNIQUES

The total cost function developed for system of pipeline network and various constraint associated with it given below

$$f(x) = \sum_{j=1}^{m} \frac{(1+r)^n r}{(1+r)^n - 1} (1 + R_p) c_p L^l d^m (1 + C_{fp}) + QP_b TZ \frac{\left[\left[\frac{P_{di}}{P_{si}}\right]^{\frac{k-1}{kE_p}} - 1\right] k}{T_b (k-1)} \left[19809.302 \frac{(1+r)^n r}{(1+r)^n - 1} c_{hp} + 9.14 (1 + C_{OP}) C_e H_y\right]$$

Subjected to;

$$Q = 18.063 \left(\frac{T_b}{P_b}\right) D_j^{2.6182} E \left(\frac{P_d^2 - e^s P_s^2}{L_j \gamma Z T}\right)^{0.5}$$ for each pipeline segment

Or

$h(x)$ = $326.23\, T_b^2 D_j^{5.33} (P_d^2 - e^s P_s^2) - Q_j^2 L_j \gamma Z T P_b^2 = 0$

And

$$1 \leq \frac{P_{di}}{P_{si}} \leq K_i$$

$P_{di}{}^{min} \leq P_{di} \leq P_{di}{}^{max}$

$P_{si}{}^{min} \leq P_{si} \leq P_{si}{}^{max}$

$l_i{}^{min} \leq l_i \leq l_i{}^{max}$

$D_i^{max} \leq D_i \leq D_i^{min}$

OPTIMIZATION METHOD

From the objective function (i.e. total cost function) that we derived above and from the constraint functions, we see that both function are non linear. Therefore, we deal with constrained optimization problem with non linear objective function and non linear constraints.

In order to optimize this problem, we use generalized reduced gradient minimization technique, which is well known for its accuracy to handle non linear constraint optimization problems.

Computer Simulation for Optimal Pipeline System

The algorithm for computer simulation of complex gas pipeline network system using GRG method is presented as follows:

- Subdivide the pipeline network into a number of network loops. Be sure that all pipes are included in at least one loop
- Input the pipeline, flow data, compressibility, PVT data and other necessary information into the database subroutine.
- Add slack or surplus variables to formulate all non equality constraint to equalities.
- Choose a starting point *h(x)* = 0 with $x^l \leq x_0 \leq x^u$
- Perform Gaussian elimination using pivot search
- Determine basic and non basic variables and also evaluate reduced gradient **R**
- Determine the direction vector **d** to be the descent direction in Z-space. If **d** = 0 stop; the current point is a **KKT** point
- Determine the step size α_k and determine x_{k+1} using $x_{k+1} = x_k + \alpha_k \boldsymbol{d}$

- If x_{k+1} is feasible, then set $x_{k+1} = x_k$ and go to step 2. Otherwise perform Newton iteration to return to the feasible surface to obtain a corrected point $\boldsymbol{x}_{k+1^c}$. if $f(\boldsymbol{x}_{k+1^c})$ < $f(x_k)$ then set $x_k = \boldsymbol{x}_{k+1^c}$
- If $f(\boldsymbol{x}_{k+1^c}) < f(x_k)$ then set $\alpha_k = \alpha_k/2$.obtain a new x_{k+1} and go to the beginning of this step.

CASE STUDY (EXCRAVOS LAGOS PIPELINE, (ELP))

The developed model was applied to Excravos Lagos Pipeline to predict the optimum diameter needed for the upgrade of the facility to meet the future demand of natural gas. Excravos Lagos Pipeline is an onshore transmission pipeline network in Warri, with its distance, about 516km, we choose it as our case study in optimizing the gas transmission system. The ELP has the capacity to deliver 31.5mmscf/day of natural gas. The ELP serve as the gateway of supply to NEPA Egbin. Currently less than 60% of the ELP capacity is utilized. Gas demand on the ELP is inclusive of gas demand of WAPCO's two cement plant of 849.6mmscf/day, while gas balance is still in excess of 12mmscf/day. Gas supply to this line is provided from shell and Chevron fields in Excravos and its environs.

TABLE 1. GAS SUPPLY AND DEMAND BALANCE ON THE ELP

year		2000	2001	2002	2003	2006	2008
1	Gas Supply mmscf/d	31.15	31.15	31.15	31.15	31.15	31.15
2	Gas Demand mmscf/d	8.96	13.92	13.97	16.26	16.31	17.03
3	Gas Balance mmscf/d	22.19	17.23	17.28	14.89	14.84	14.12

Actually, the development of ELP network was finished some years back. But recently, there is an increasing demand for its rate and pressure. A revised present gas demand includes olokola free trade zone, IPP's, Ibese cement and the ongoing west African pipeline project which is meant to supply natural gas to the west African countries. This revised estimate gave a total demand of about 406,096scf/hr. The demand is the most definite demand which as been firmly established. The revised with future demands are possible projection only which is mainly an intelligent guess.

ELP Network Data

The corresponding data for the existing pipes given below are for outside diameter. The data is as follows

Table 2. ELP NETWORK DATA

DATA TYPE	VALUES
Inlet pressure at Excravos (psig)	1060
Allowable maximum pressure (psig)	1060
Final outlet pressure (psig)	550
Flowing gas temperature (^{o}F)	95
Specific gravity of gas	0.617
Total length of pipeline (km)	514
Average compressibility factor (Z)	0.95
Compression ratio(K)	1.24
Base temperature (R)	520
Base pressure (psia)	14.7

Also various economic data required for the purpose of this study have been estimated base on intelligent assumption and prevailing condition. These include:

Discounted rate of return (r) = 12%

Electric price C_e = 0.055 US$/kwh

Pipe unit price c_p = 0.569 US$/ft.inch

Compressor price c_{hp} = 1500 US$/hp

Polytropic efficiency E_p = 1.0 centrifugal type compressor

Operating compressor hours in year H_y = 8760 hr

Fraction of compressor cost excluding electric cost to run the compressor C_{OP} = 0.75

Fraction of pipe operating cost to annual investment cost of pipeline C_{fp} = 0.2

RESULTS

This chapter provides the numerical results of the optimal model applied to the ELP by using data in the previous section. The results were generated using the Excel solver which incorporated the GRG algorithm as its solution engine. Note that the diameter in the model is inside diameter.

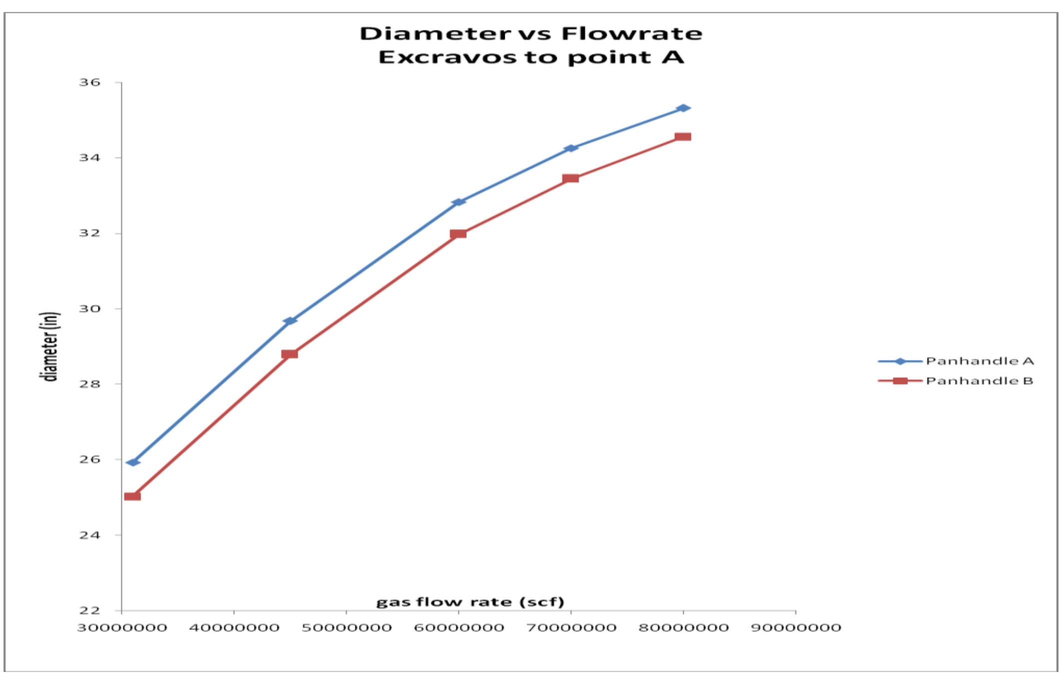


Fig. 1: Plot of Flow rate and Required Diameter from Excravos to point A

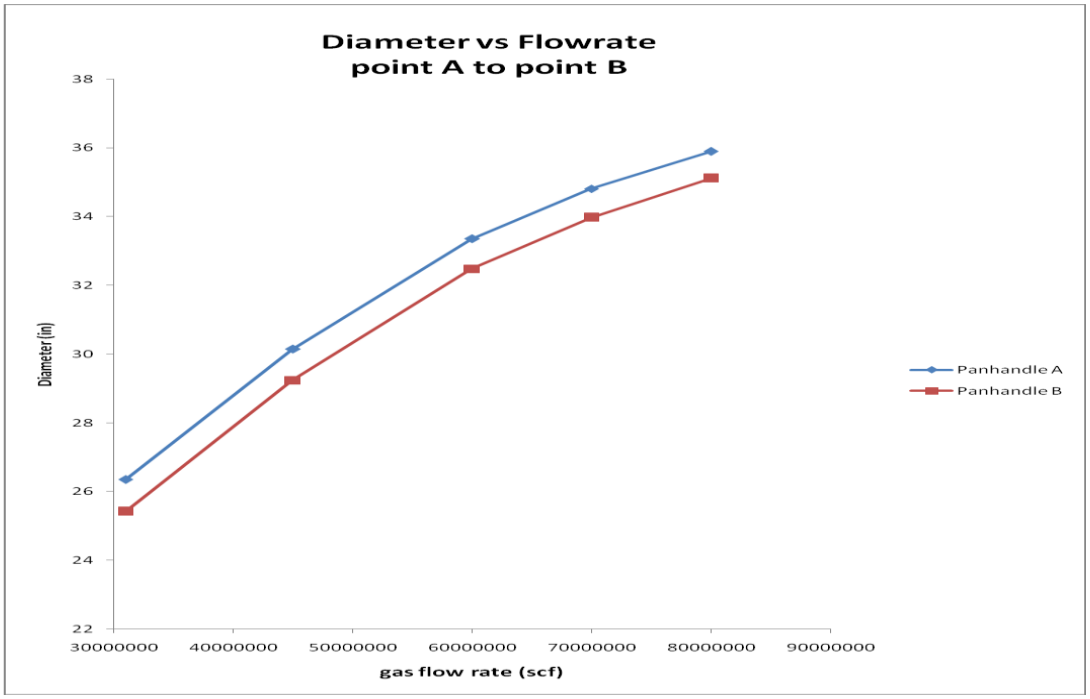


Fig. 2: Plot of Flow rate and Required Diameter from point A to point B

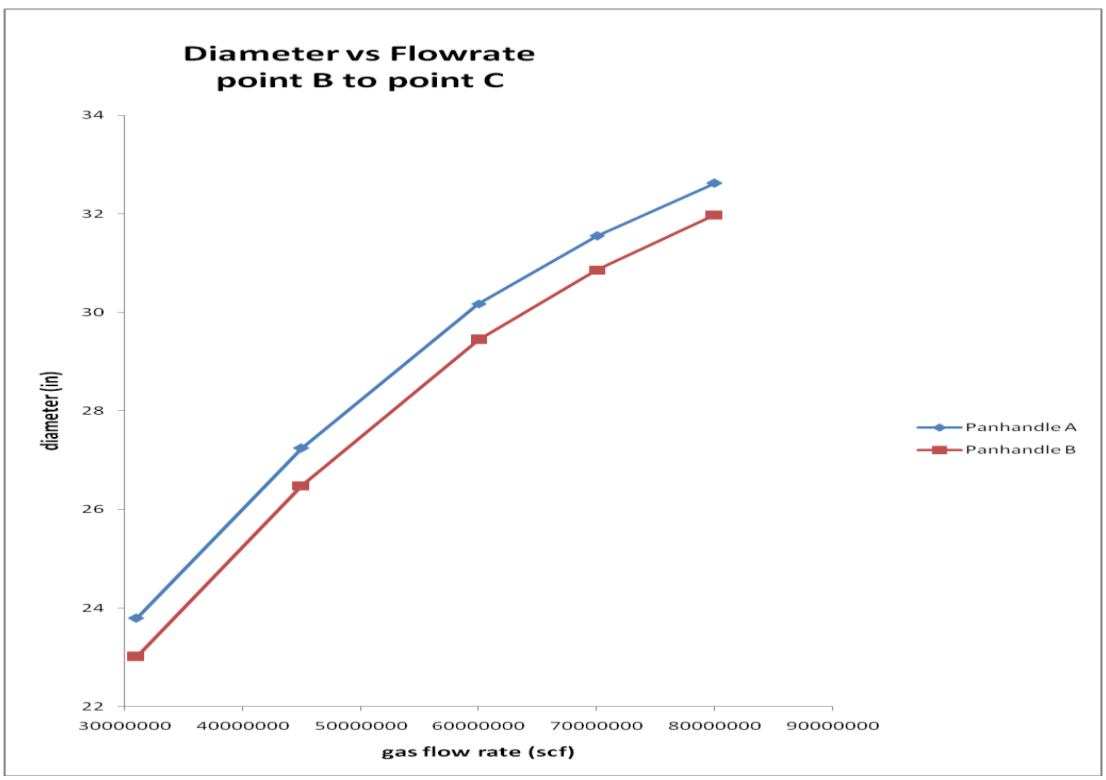


Fig. 3: Plot of Flow rate and Required Diameter from point B to point C

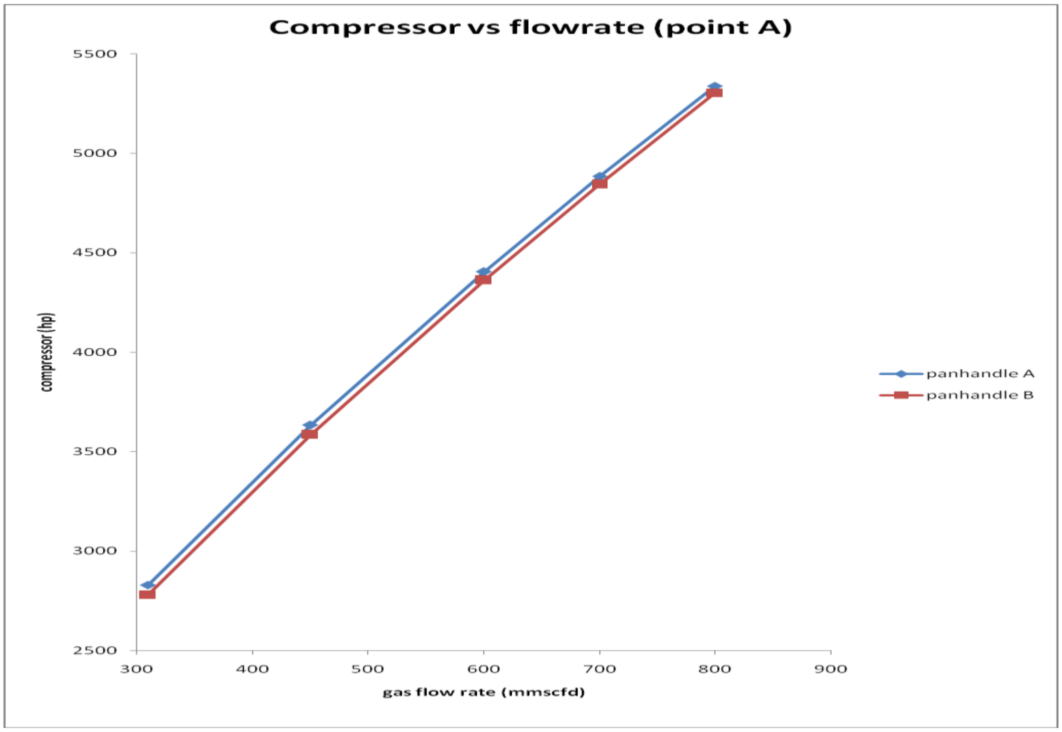


Fig 4: Plot of Required Compressor power against the Flow rate at point A

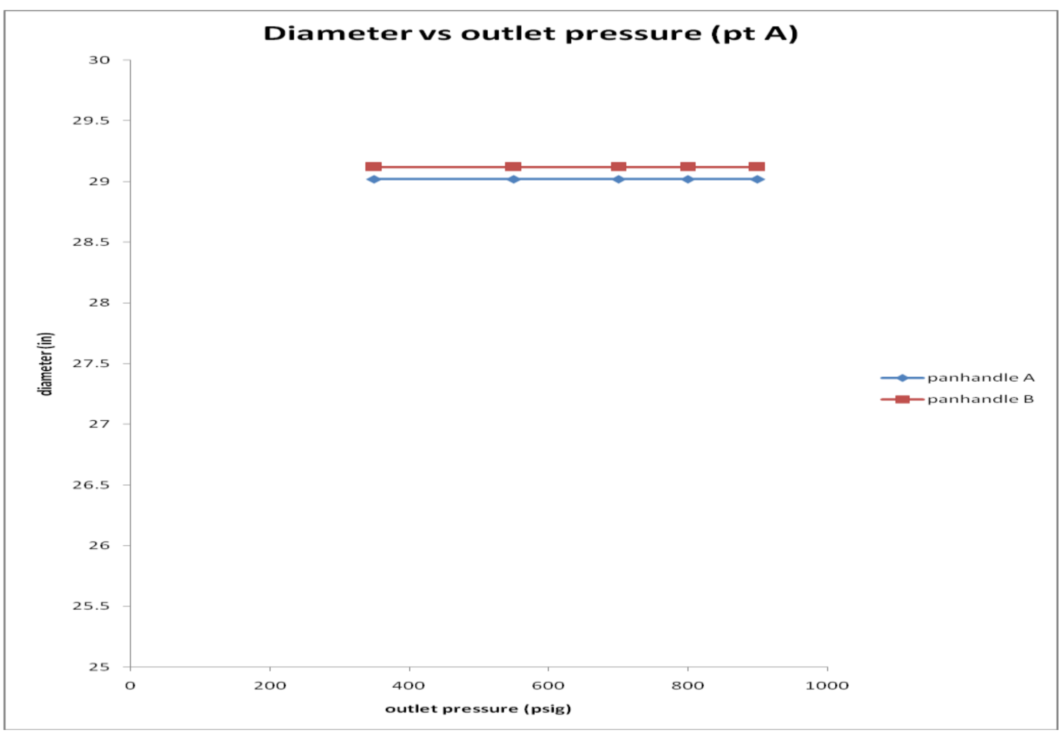


Fig 5: Plot of Required Diameter against the Outlet pressure from excravos to point A

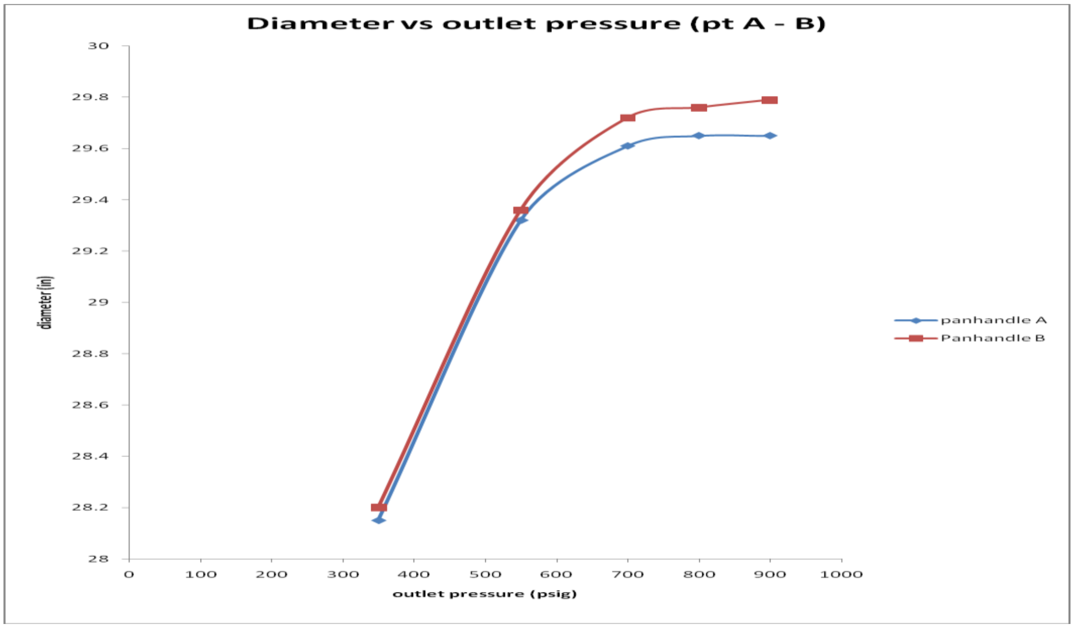


Fig 6: Plot of Required Diameter against the Outlet pressure from point A to B

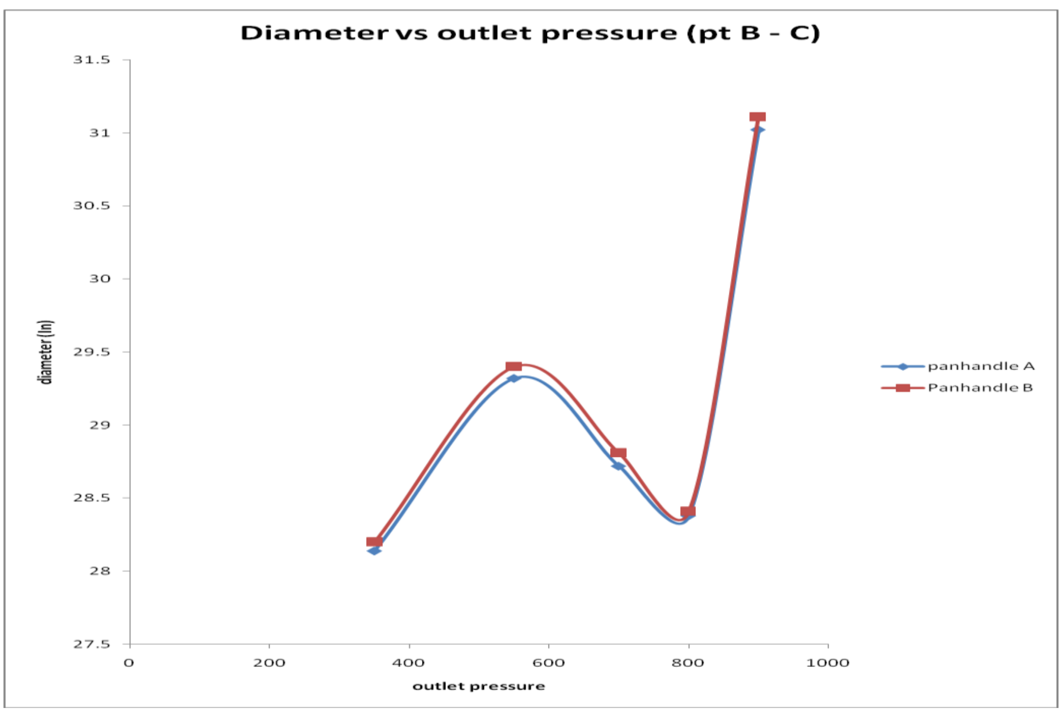


Fig 7: Plot of Required Diameter against the Outlet pressure from point B to C

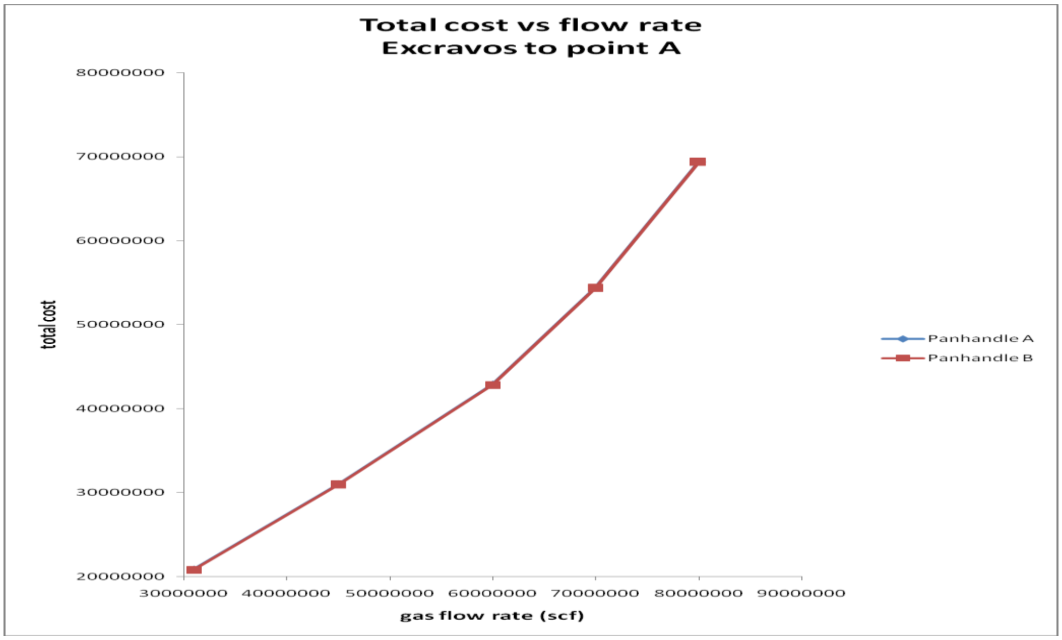


Fig 8: Plot of Total Cost against the Flow rate from excravos to point A Using Panhandle A & B

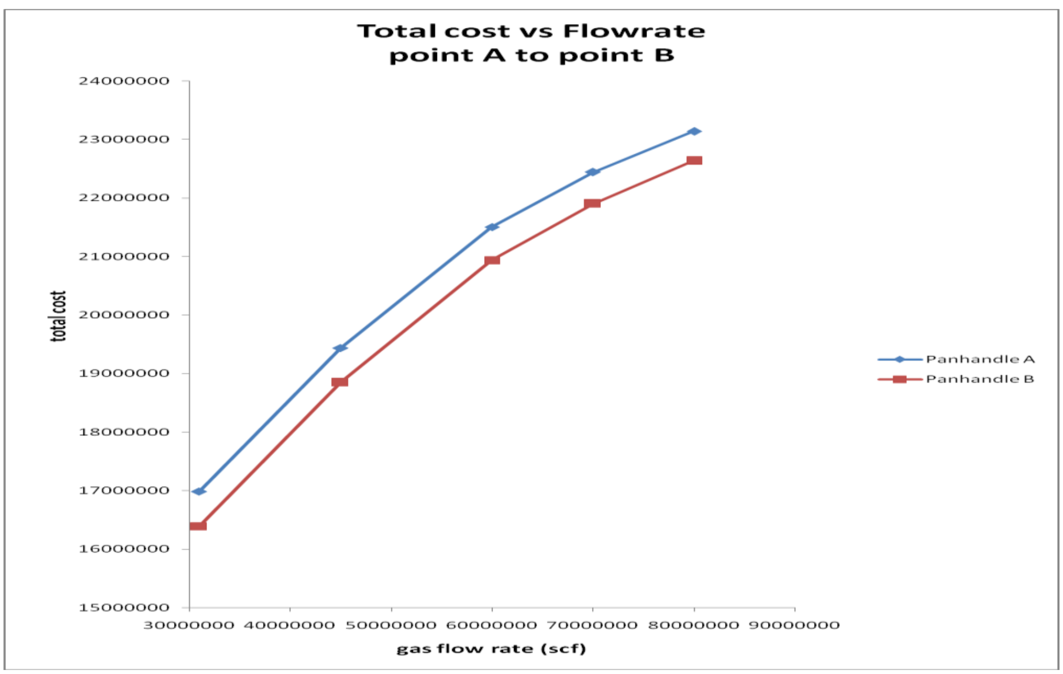


Fig 9: Plot of Total Cost against the Flow rate from point A to point B Using Panhandle A & B

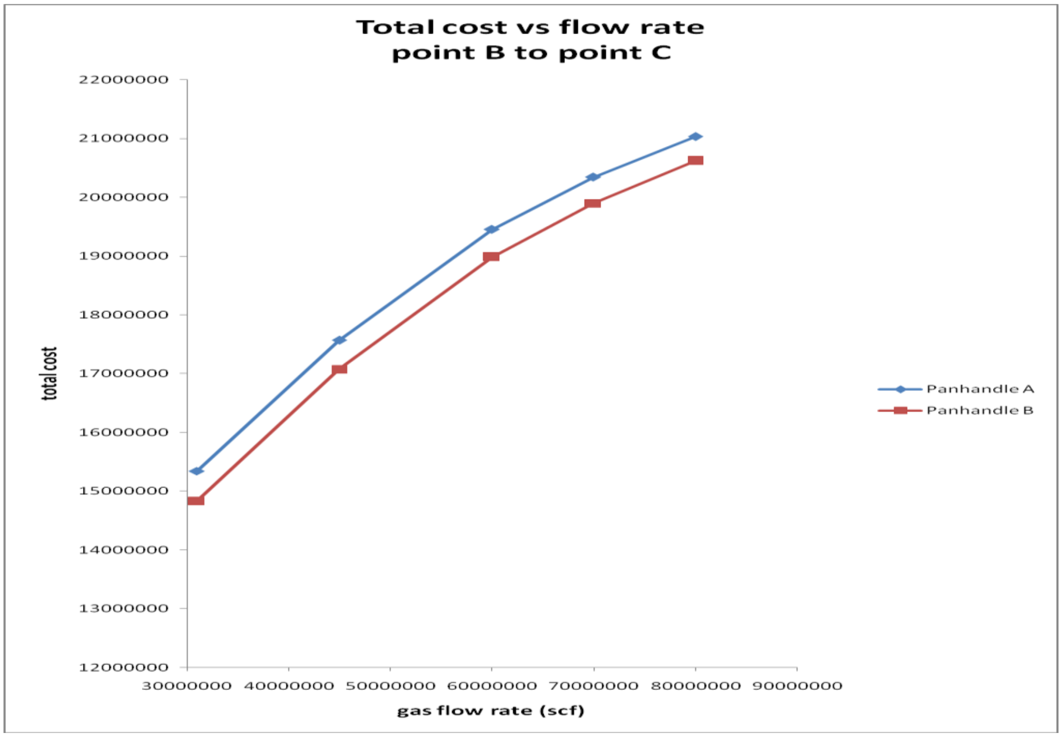


Fig 10: Plot of Total Cost against the Flow rate from point B to point C Using Panhandle A & B

DISCUSSION OF RESULTS

The model equation developed expresses the relationship between the total cost function and the design variable. This section present an analysis of the result obtained from the model to the ELP network using the data from the previous section.

From the figures above and the appendix, we can see that the diameters of the pipes increase in respect to the increase in flow rate, either using Weymouth, panhandle A or panhandle B as our pipeline equation and so do the compressor power and total cost. Also, the optimum model chooses to put to put compressors in point A and point B only when the flow rate is 310mmscf/d. For larger flow rate, it will choose to put single compressor in point A. The result also show that the use of panhandle B equation will pick the bigger pipe diameter than using panhandle A equation and this will make the compressor power of panhandle A slightly higher than that of panhandle B. Weymouth equation gives an outrageously large value for the diameter an as such cannot be a suitable equation use in the optimization model.

Similarly, increasing the outlet pressure at a constant flow rate of 450mmscf, the diameter does not change (remains constant) for the first segment of the network. But for the next segments, the diameters vary. This variation in the pipe diameter has a strong relation with using of the compressor in point A and point B.

From the table, the model chooses to put two compressor when the outlet pressure is more than 550psig for its optimum condition. However, when the outlet pressure is 700psig, the diameter from point B to point C is smaller than the number of compressor stations as increase to two. Also, for each outlet pressure the compressor power at point A is the same contrary to what is attained in point B. This also shows that the increase in outlet pressure will make the total cost more expensive.

CONCLUSION

In the optimization of natural gas transportation network was developed using a workable procedure adapting the Generalized Reduced Gradient algorithm. It determines the optimum condition and pipe diameter. The model developed was applied to the Excravos Lagos pipeline. From the analysis of the result the following conclusions were made:

- The testing result for the validation of the model shows that an increase in the demand for the gas flow rate increases the total cost.
- The use of the model in ELP network shows systems comprising of compressor stations that is able to satisfy the demand at higher outlet pressure and with smaller pipe diameter.
- The optimum diameter that was obtained varies form the existing ELP network pipeline because the simulation is ran at different flow rate and outlet pressure.
- The optimum network configuration for panhandle A and B are the same, but for Weymouth equation, it widely varies and is not applicable to the model. Hence, panhandle A and B are better suited for the model compared to Weymouth.

RECOMMENDATION

- It is necessary to predict the gas necessity in future before developing a natural gas transmission line
- The model can be extended to treat much larger and more complex network, at the expense of considerable computer time.

- **NOMENCLATURE**

- A = uniform annual capital cost

- b = non linear constant between compressor price and its power

- C_p = electric price (US$/Kwh)

- c_{hp} = compressor price (US$/hp)

- C_{fp} = fraction of pipe operating cost to annual investment cost of pipeline

- c_{comp} = compressor investment cost (US$)

- C_{pipe} = pipeline investment cost (US$)

- C_{OP} = fraction of compressor operating cost excluding electric to electric cost

- CIP = annual investment cost of pipeline (US$)

- CIC = annual investment cost of pipeline (US$)

- D = diameer of pipeline (inch)

- E_p = polypropic efficiency

- E_{LC}= electric cost of pipeline

- H_y = operating compressor cost in a year

- l = non linear costant between pipe’s price and pipe length

- L = pipe length (ft)
- m = non linear costant between pipe's price and pipe diameter
- n = life time of pipe eqiupment
- OC_{comp} = operating cost of compressor (US$/yr)
- OC_{pipe} = operating cost of pipeline (US$/yr)
- P = present value of total investment
- P_b = base pressure (psia)
- P_{di} = discharge pressure (psia)
- P_{si} = suction pressure (psia)
- Q= gas flow rate (MMSCFd)
- r = discount rate
- R_p = fraction between pipe installation cost and pipe price
- T_b = base temperature (R)
- T = flowing temperature (R)
- Z = gas compressibility
- W_{hp} = compressor power (hp)

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APPENDIX

Tables of result.

EXCRAVOS TO POINT A

	Q	310000000	450000000	600000000	700000000	800000000
	Pd	1074.7	1074.7	1074.7	1074.7	1074.7
	Ps	950	945	938.9	926.8	910
	d	1860.343081	2141.303548	2387.812946	2535.242794	2673.188899
weymouth	**cost**	191977776.5	290750069.6	407443156.2	521651050	671024644.7
	Pd	1074.7	1074.7	1074.7	1074.7	1074.7
	Ps	950	945	938.9	926.8	910
	d	25.93428505	29.67533447	32.83019655	34.25671599	35.32019653
Pan Handle A	**cost**	208697117.8	309881194.3	428608160.5	543735704	693794904.1
	Pd	1074.7	1074.7	1074.7	1074.7	1074.7
	Ps	950	945	938.9	926.8	910
	d	25.03190837	28.78998108	31.97905111	33.44892217	34.56390633
Pan Handle B	**cost**	208115372.2	309310423.4	428059442.8	543214934.3	693307337.8

POINT A TO POINT B

	Q	310000000	450000000	600000000	700000000	800000000
	Pd	1000	1000	1000	1000	1000
	Ps	875.4	870.3	864.2	852.1	835.3
	d	1914.347793	2203.645153	2457.521099	2609.652629	2752.225016
weymouth	**cost**	1234144569	1420649219	1584318340	1682394720	1774308498
	Pd	1000	1000	1000	1000	1000
	Ps	875.4	870.3	864.2	852.1	835.3
	d	26.35115526	30.14891185	33.35578427	34.80861635	35.89428103
Pan Handle A	**cost**	16988101.78	19436445.12	21503856.38	22440470.32	23140378.23
	Pd	1000	1000	1000	1000	1000
	Ps	875.4	870.3	864.2	852.1	835.3
	d	25.42553754	29.23945447	32.47989726	33.9761079	35.11349964
Pan Handle B	**cost**	16391373.18	18850134.79	20939188.25	21903767.53	22637022.93

POINT B TO POINT C

	Q	310000000	450000000	600000000	700000000	800000000
	Pd	894.1	894.1	894.1	894.1	894.1
	Ps	644.9	634.7	622.5	598.3	564.7
	d	2053.408445	2366.268678	2642.248077	2812.794087	2976.376674
weymouth	**cost**	1323794396	1525489594	1703408401	1813356256	1918814920
	Pd	894.1	894.1	894.1	894.1	894.1
	Ps	644.9	634.7	622.5	598.3	564.7
	d	23.79411169	27.24537593	30.17288896	31.54925247	32.62493863
Pan Handle A	**cost**	15339623.15	17564589.28	19451902.72	20339218.78	21032693.73
	Pd	894.1	894.1	894.1	894.1	894.1
	Ps	644.9	634.7	622.5	598.3	564.7
	d	23.00885491	26.48121366	29.44412788	30.85999114	31.98100363
Pan Handle B	**cost**	14833382.64	17071948.02	18982083.95	19894864.77	20617560.76