ADAPTATION OF A GLOBAL ORTHOMETRIC HEIGHT MODEL TO LOCAL HEIGHT DATUM USING 'SATLEVEL' COLLOCATION

BY

ALEEM, KAMORUDEEN FOLORUNSO

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ADAPTATION OF A GLOBAL ORTHOMETRIC HEIGHT MODEL TO LOCAL HEIGHT DATUM USING 'SATLEVEL' COLLOCATION

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BY

ALEEM, KAMORUDEEN FOLORUNSO
MATRIC NO: 069045005
DEPARTMENT OF SURVEYING AND GEOINORMATICS
FACULTY OF ENGINEERING
UNIVERSITY OF LAGOS
AKOKA, LAGOS

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CERTIFICATION

This is to certify that the thesis

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is a record of original research carried out by

ALEEM, Kamorudeen Folorunso

Department of Surveying and Geoinformatics, School of Postgraduate Studies
University of Lagos, Akoka, Lagos

K. F. ALEEM		
------------------	-----------------------	----------------------
Author's Name	**Signature**	**Date**
Prof. J. B. OLALEYE		
------------------------	-----------------------	----------------------
First Supervisor's Name	**Signature**	**Date**
Dr. O. T. BADEJO		
--------------------	-----------------------	----------------------
Second Supervisor's Name	**Signature**	**Date**
Dr. J. O. OLUSINA		
--------------------	-----------------------	----------------------
Third Supervisor's Name	**Signature**	**Date**
Prof. D.E. ESEZOBOR		
--------------------	-----------------------	----------------------
Internal Examiner's Name	**Signature**	**Date**
Dr. T. O. Idowu		
--------------------	-----------------------	----------------------
External Examiner's Name	**Signature**	**Date**
Dr. E. E. Ikponmwosa		
--------------------	-----------------------	----------------------
SPGS Representative's Name	**Signature**	**Date**

DEDICATION

This work is dedicated to:

my wife: Alhaja Idayat Omolara Aleem

and

my children: Shakiru Folorunso Aleem
Barakat Modupe Aleem
Habib Ayokunmi Aleem
Abdul-malik Arafah Abidemi Aleem
Aishat Omowunmi Aleem

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ABSTRACT

Three-dimensional (3D) maps are of immense value in the planning and execution of engineering and other projects aimed at harmonious environmental development. In many real-life situations, the knowledge of heights is crucial for understanding the relative vertical positions of neighboring entities in a common reference system called **datum** - an important reference surface for elevations of terrain points. In many countries, the reference surface for heights is the **geoid** which is approximated by the Mean Sea Level (MSL). Conceptually, two approaches are used to determine the geoid - deterministic and empirical; and can also be determined in absolute or in relative terms.

The recent success in the determination of reliable global geoid gave opportunities and preferences to the relative geoid determination using the global Geopotential Earth Model (GEM-XX), as platform. The latest edition named GEM2008 defines the global geoid to submeter accuracy. The global model generalizes the geoid in any locality; and therefore, it is not the best fit for any country or place. The fitness of the global geoid is always tested in the locality for its adequacy by comparing the GNSS derived Orthometric Heights with those obtained by direct geodetic levelling. In practice, the discrepancies are usually attributed to the imperfection of the global geoid model. The direct methods of determining Orthometric Heights by geodetic leveling is physically demanding and even almost impossible in some areas such as the rain forested areas and Niger Delta region of Nigeria due to the presence of swamp and the nature of the terrain. Thus, it is preferable to develop a mathematical model for the global model imperfections with the hope of obtaining Orthometric Heights by applying the geoidal error model to GNSS heights (which are easier to acquire) in other to obtain the Orthometric Heights. The challenge in this approach is to get the appropriate geoidal error model and this is the focus of this research.

In this study, an empirical error models code named **'Satlevel' Collocation model** is developed, and compared with other existing predictive models developed and used by researchers in other countries. The 'Satlevel' uses an optimal numerical model which transforms the global model to local datum using geoid modelling techniques aided by GNSS measurements. GNSS is used to obtain ellipsoidal heights at some selected control points, while the Orthometric Heights of those points were determined by geodetic levelling. Orthometric and ellipsoidal Heights were related by **Geoidal Undulation** (N) for which empirical error model was sought. Spherical and Rectangular 'Satlevel' models were explored. GEM2008 Orthometric Heights were computed in the two selected study areas (Port Harcourt, Rivers State and Lagos State of Nigeria). The results show that the geoidal undulation determination with Spherical 'Satlevel' models gave average values of 18.946m and 22.854m for Port Harcourt and Lagos State respectively, while Rectangular 'Satlevel' models gave average values of 18.948m and 22.857m for the two areas. The Mean Square Error (MSE) for Spherical 'Satlevel' were 6.151mm and 0.0033mm; while Rectangular 'Satlevel' gave 1.728mm and 0.00032mm for Port Harcourt and Lagos State respectively. The statistics show that there are no significant differences between the observed Geoidal Undulations and the computed Geoidal Undulations from 'Satlevel' collocation models. Also, the goodness of fit of the model was checked and satisfied 95% significant level. The geoidal map and 3D surface models of the areas were produced. 'Orthometric height on the fly', a user-friendly interactive program was developed to compute the local Geoidal Undulation and Orthometric Height from "Satlevel' collocation model. Although, all the models considered give comparable accuracy of the orders of millimeters, Spherical 'Satlevel' model is more preferable than the Rectangular 'Satlevel' because of its advantage of accepting data format that is most common on maps and GNSS devices over other models. GEM2008 fits perfectly in the Port Harcourt Coastal area of Nigeria and therefore adapted for Orthometric Height with the use of 'Satlevel' Collocation Models developed in this research. The study recommends re-observation of the Nigerian Vertical control network and its integration with GNSS observations so that a general model applicable to the entire country can be determined.

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LIST OF ABBREVIATIONS

AGP - African Geoid Project

AHD - Australian Height Datum

AIS - Automatic Identification System

ANN - Artificial Neural Networks

ANOVA- Analysis of Variance

BGI - International Bureau of Gravimetric Service

C/A - Coarse / Acquisition code

CNSS - Compass Navigation Satellite System of China

CR - Coefficient of Representativity

DGPS - Differential Global Positioning System

DoD - Department of Defence

DORIS - Doppler Orbitography by Radio-positioning Integrated on Satellite

DOT - Dynamic Ocean Topography

DTM - Digital Terrain Modelling

EGM96 - Earth Geopotential Model 1996

ESA- European Space Agency

ESSP - Earth System Science Pathfinder Programme

FFT - Fast Fourier Transformation

FOC - Full Operational Capability

GBAS - Ground Based Augmentation System

GBVP - Geodetic Boundary Value Problem

GEM - Geopotential Earth Model

GEOSS - Global Earth Observation System of Systems

GIS – Geospatial Information System

GLONASS - Global Navigation Satellite System

GOCE - Gravity Field and Ocean Circulation Experiment

GOCE - Gravity Field and Steady-State Ocean Circulation Explorer

GPS – Global Positioning System

GRACE - Gravity Recovery and Climate Experiment

GRF - Geodetic Reference Frames

IAG - International Association of Geodesy

ICSU - International Council for Science

IGFS - International Gravity Field Service

INS – Inertial Navigation System

IRNSS - Indian Regional Navigation Satellite System

IUGG - International Union of Geodesy and Geophysics

JAXA - Japan Aerospace Exploration Agency

L2C - Second Civilian Signal

MCS - Master Control Station

MGM-Net - Multi-GNSS Monitoring Network

MRE -Multiple Regression Equation

MSL Mean Sea Level

MSS - Mean Sea Surface

NASA - National Administration of Space Agency

NAT - North Aegean Trough

NGA - National Geospatial-Intelligence Agency

NPA - Nigeria Port Authority

OC- Orthometric Correction

OHC- Orthometric Height Correction

OSU91 – Ohio State University 1991

PRN - Pseudo-Random Number

QZSS - Quasi Zenith Navigation Satellite System

RINEX- Receiver Internet Exchange

RKHS - Reproducing Kernel Hilbert Space

RNP - Required Navigation Performance

RTM - Residual Terrain Modelling

SBAS - Satellite- Based Augmentation System

SPDC – Shell Petroleum and Exploration Development Company

SPS - Standard Positioning Service

SRTM - Shuttle Radar Topographic Mission

SSH – Sea Surface Height

SST – Sae Surface Topography

TEC - Total Electron Content

UTC - Coordinated Universal Time.

VTS - Vessel Traffic Services

WAAS - Wide Area Augmentation System

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