An alternative approach for computing the union and intersection of fuzzy sets: a basis for design of robust fuzzy controllers

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An Alternative Approach for Computing the Union and Intersection of Fuzzy Sets: A Basis for Design of Robust Fuzzy Controllers

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Abstract:
The capability of fuzzy sets to express gradual transition from membership to non-membership and vice versa has broad utility. The overlap between two or more adjoining fuzzy sets create very important fuzzy "patch" that deserves special attention. In the existing literature, Zadeh, Mamdani and Tagaki-Sugeno have established that the Min- and Max- operators can effectively be used for fuzzy interpolative inference. But what has not been done is to define an explicit expression for determining membership function in the overlap region. This paper addresses that concern, as it proposes an alternative approach to determination of membership function based on the Fourier series representation of the envelope of the fuzzy "patch".

Key-Words: Membership functions, Fourier series, Fuzzy overlap, Triangular pulses, Polygonal waveform.

1 Introduction

George Cantor, the founder of set theory, used the comprehension principle to create sets [1]. In line with Cantor's claim, the objects that form the set are definite and distinct from each other. Clearly, this is an application of the law of the Excluded Middle. Fuzzy concepts cannot be rigidly described by Cantor's notion of sets or indeed by the binary bivalent logic of "True/False", which are regarded as logical values and which can be respectively denoted as 1 and 0. Fuzzy set theory was introduced by Zadeh in his seminal work *Fuzzy Sets* [2], in order to represent and manipulate data and information possessing non-statistical uncertainties. The notion of an infinite-valued logic was also introduced by Zadeh in [3], where he described the mathematics of fuzzy set theory, and by extension, fuzzy logic. This theory proposed making the membership functions (or the values F and T) operate over the range of real numbers [0.0, 1.0]. The construction of fuzzy sets generally depends on two things namely: the identification of suitable universe of discourse and the specification of an appropriate membership function. However, the specification of membership functions is, necessarily, somewhat subjective, as different persons may specify the membership functions for the same concept in different terms. Such subjectivity comes from individual differences in expressing abstract concepts and has little to do with randomness. To be sure, the subjectivity and non-randomness of fuzzy sets constitute the primary difference between it and probability theory which is concerned with objective treatment of random phenomena.

2 Fuzzy Set Universe

A fuzzy set A on the given universe U implies that, for any $u \in U$, there is a real number $\mu_A(u) \in [0,1]$ corresponding to u where $\mu_A(u)$ is called the grade of membership of u belonging to A. That is, there is a mapping:

$$\mu \rightarrow [0,1]\ ,\ u \rightarrow \mu_A(u) \qquad (1)$$

and this mapping is called the membership function of A. If the range of μ_A admits only two values, 0 and 1, then μ_A degenerates into a usual set characteristics function.

$$A = \{u \in U \mid \mu_A(u) = 1\} \qquad (2)$$

Using the above interpretations, Cantor sets can be described as special cases of fuzzy sets. Mamdani [4,5] suggested that any control scheme that can be carried out by an operator could be implemented by fuzzy logic after having translated the operator's experience into qualitative linguistic terms. Mamdani also introduced a fuzzification, Inference, and defuzzication scheme and developed an inference strategy that is generally known as the Min-Max method. Mamdani's method is currently effectively applied to process control, robotics and other expert systems. It has in fact been successfully used in the control of industrial plants, including chemical, cement and steel plants [6]. Starting with Zadeh's fuzzy sets theory [7], several studies have been carried out to demonstrate the success of fuzzy sets and logic in both theory and application. Specifically, research efforts have focused on development of mathematical formulations for different aspects of fuzzy set theory and application, especially within the industrial setting. Takagi-Sugeno [8] proposed a control methodology that limits the use of fuzzy sets only for the input variables and thereby avoids the need for any defuzzification stage. Following his earlier work, Zadeh [9,10] also contributed several techniques for the application of fuzzy sets, including the development of the calculus of fuzzy *If/Then* rules. Pearson [11] investigated a property of linear differential equation where the initial state is described by a vector of fuzzy numbers. Kosko and Isaka in [12] suggested that fuzzy systems could approximate any continuous mathematical function; in fact, they proved a uniform convergence theorem by showing that enough small fuzzy patches are able to sufficiently cover the fuzzy graph of any function or input/output relation. In his contribution, Finn [13] presented an algorithm for developing fuzzy rules from a collection of data and mapping input antecedents to output consequents. In such a case, the input and output spaces, are first divided into a grid of cells and primitive *if/then* rules formulated with each occupied input cell playing the role of an antecedent, while the associated output cells play the role of the consequent within the context of a fuzzy set composite rule. Lakshmikanthan and Mohapatra [14] in turn developed comparison principles and showed the existence and uniqueness of solutions for fuzzy differential equations under a more general condition than that of Lipschitz. The study also established both the continuity and global existence of the solution assuming local existence. Esogbue and Murrell [15] had, in fact, earlier proposed a fuzzy adaptive controller using reinforcement learning neural networks and demonstrated its basic capability to learn effective control for a simple dynamic system. Tao and Taur [16] also developed a methodology of analyzing the characteristics of the fuzzy *if/then* rules, and the extracted features were utilized to reduce the complexity of the fuzzy controller. Thus instead of partitioning each of the individual input variables of fuzzy controllers into fuzzy sets, the entire space of input variables was partitioned into fuzzy sets so as to ultimately reduce the number of fuzzy *if/then* rules. Castellani and Pham [17] later discussed issues of action aggregation and defuzzification in Mamdani-type fuzzy systems. Their work highlighted the shortcomings of the defuzzification techniques associated with the customary interpretation of the sentence connective "*and*" by means of the set union operation whereas going by the Zadeh definition of fuzzy intersection and union, the minimum and the maximum functions are defined, respectively, as follows:

Intersection: $\quad \text{Min}\,(\mu_A(A)\,,\,\mu_B(B))$

Union: $\quad \text{Max}\,(\mu_A(A)\,,\,\mu_B(B)) \qquad (3)$

In current literature, researchers generally treat the overlap region as intersection or union of two or more fuzzy sets and have invoked the Min and Max Operators, respectively, as needed. Badiru and Arif [18], for example, treated the fuzzy overlap as an intersection of two adjoining sets viz: "Low" and "not Low" and invoked the Min Operator to generate output. Olunloyo, Ajofoyinbo and Badiru, in an earlier paper [19], also proposed another algorithm for the treatment of overlap of adjoining fuzzy sets based on partitioned grids. In view of the importance of this fuzzy overlap region, especially where there is need to monitor and ensure smooth transition between the adjoining fuzzy sets in relation to the design of mission critical applications, we now propose a methodology for determination of the membership function in this region. This method is expected to provide a more robust way to design intelligent and fault-tolerant engineering systems.

2 Problem Definition

The membership function of fuzzy set can be defined as a curve that defines how each point in universe of discourse maps to a membership value (or degree of membership) between 0 and 1. To be sure, a fuzzy set is fully defined by its membership function but for most fuzzy logic control problems, the assumption is that the membership functions are piecewise linear and usually triangular in shape. This means that the values to be determined are the parameters defining the triangles; the parameters, in turn, are usually based on the control engineer's experience. However, for many applications, especially where linguistic terms are involved, triangular membership functions are not the most appropriate, as they do not represent accurately the linguistic terms being modeled. In such applications, the Gaussian and exponential membership functions provide better representations. In this paper, we shall formulate explicit expressions for dynamic determination of membership function in the overlap and non-overlap regions.

3 Determination of Membership Functions

According to Watanabe [20], the determination of membership function can be either manual or automatic. Manual statistical techniques for determining membership functions fall into two broad categories: use of frequencies and direct estimation. However, the manual techniques can be deficient since they usually rely on subjective interpretation of words, are subject to the inadequacies of human experts, and generally suffer from other documented problems associated with the knowledge-acquisition process. The automatic generation of membership function differs significantly from the manual methods in that either the expert is completely removed from the process, or the membership functions are 'fine-tuned' based on an initial guess by the expert. The emphasis in this case is on the use of Genetic Algorithm (GA) and Artificial Neural Networks (ANN). Takagi and Hayashi [21] first pointed out that fuzzy reasoning presents two specific challenges, namely (1) the lack of a definite method for determining the membership function, and (2) the lack of a learning function. Takagi and Hayashi then went ahead to describe an approach for using ANNs to overcome these problems. The method investigates *if/then* rules by using neural networks to determine the membership functions of the antecedent and then determine the consequent component at the output for each rule. The approach used is to take a raw data (say, in a control problem), apply a conventional clustering algorithm to group the data into clusters, and thereafter apply an ANN to this clustered data to determine the membership of a pattern within a particular fuzzy set. Wang [22] in an alternative approach builds on the information provided by an expert and uses ANNs to fine-tune the membership function. In other words, the pairs (x, y) that describes the relationship between X and Y is presented to the neural network, which fits a function to the points. In an interesting contribution, Meredith et al [23] applied GA to the fine-tuning of membership functions in a fuzzy logic controller for a helicopter. An initial guess for the membership function is made by the Control Engineer, and then the GAs adjust the parameters that define the functions by using them to minimize the movement of a hovering helicopter. For this case, triangular membership functions were used. Karr [24] also applied GA to the design of Fuzzy Logic Controller for the "Cart Pole" problem. The membership function used here is Gaussian in nature, and the objective is to minimize an objective function that minimizes the squared difference between a Cart and the centre of the track that the Cart is on, while keeping the pole balanced at the same time. Lee and Takagi [25] also tackled the Cart problem, adopting a holistic approach by using GA to design the whole system (i.e. determine the optimal number of rules as well as the membership functions). Ross [26] reported on six popular methods for developing membership functions namely: Intuition, Inference, Rank Ordering, Neural Networks, Genetic Algorithms and Inductive Reasoning. Olunloyo and Ajofoyinbo have, in a set of recent papers [27][28][29], suggested an alternative approach for treatment of union and intersection of fuzzy sets based on Fourier series representation of the membership functions.

4 Fourier series Representation

Both techniques (manual and automatic) for determining membership functions as described above are non-systematic and suffer from certain deficiencies. On one hand, the automatic techniques are heuristic in nature, which implies that different values can be obtained for same input values presented at different times. On the other hand, the manual techniques suffer from the deficiency that they rely on very subjective interpretation of words and the peculiarities of the engaged human expert. In this study, we shall model the union and intersection of fuzzy sets as polygonal waves and triangular pulses, respectively, and then formulate an explicit Fourier series representation for determining grade of membership of fuzzy sets in both the overlap and non-overlap regions. To be sure, Trigonometric series of the form

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k Coskwt + b_k Sinkwt\right) \qquad (4)$$

where a_k , b_k are coefficients and the period $p = \frac{2\pi}{w}$, are encountered in the treatment of many physical problems. One fundamental feature of the series is that it has a period of 2π spanning over the range $-\pi \le t \le \pi$ or $0 \le t \le 2\pi$, while outside this interval, f(t) is determined by the periodicity condition viz: $f(\alpha + 2\pi) = f(\alpha)$. Traditionally, the universe of discourse in a fuzzy plane consists of one or more data points. However, each of the data points in a given universe of discourse has some form of data distribution around it in the form of some shape, whether Gaussian, exponential, cosine, triangular or another. Since all data points in the universe of discourse would have same form of data distribution around every data point, we could therefore derive an explicit Fourier series expression for the envelope of the fuzzy patch since we can be assured of the repetition of the distribution pattern around each data point.

5 Assumptions

The membership function of fuzzy sets can take on any shape. A literature survey indicates that the following shapes of fuzzy membership functions are commonly used: Triangular, Trapezoidal, Exponential, Gaussian, and Cosine [30]. Although various functional profiles of membership functions could be used, the triangular and trapezoidal give approximations of the other forms and will thus receive further consideration in this work. The trapezoidal form can also be approximated by the triangular forms since the end-points of the tolerance interval have the same grade of membership and could therefore be assigned a point value that represents the peak of the triangular profile. In any case, our methodology can also be adopted for other functional forms; as will be shown in a future paper. In general, we assume that the fuzzy sets are triangular and symmetric. In particular, the overlap domains are triangular and symmetric. For purposes of modeling, we adopted the following relations: The grade of membership function of fuzzy set X, μ_x, is mapped to $f(\bar{x})$, and the data values (i.e. the universe of discourse) are mapped to $\bar{x}$ for the intersection and union as illustrated in Fig. 1 and Fig. 2, respectively, below:

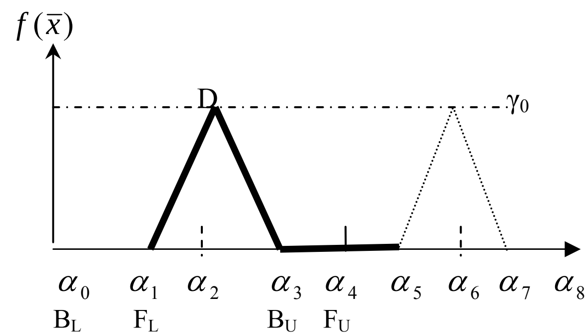

Fig. 1: Triangular Pulses

5.1. Intersection of Fuzzy Sets

Here we introduce the cross labeling as follows:

$$B_L = \alpha_0;\ F_L = \alpha_1;\ m = \alpha_2 \qquad (5)$$
$$B_U = \alpha_3;\ F_U = \alpha_4$$

Since the period of this Fourier series is 2π, we assign values to the parameters in Equation (5) for the first period as follows:

$$\alpha_0 = 0;\ \alpha_1 = \frac{\pi}{2};\ \alpha_2 = \pi \qquad (6)$$
$$\alpha_3 = \frac{3\pi}{2};\ \alpha_4 = 2\pi$$

The values of the parameters in Equation (6) can also be extended to the second period as follows:

$$\alpha_5 = \frac{5\pi}{2};\ \alpha_6 = 3\pi \qquad (7)$$
$$\alpha_7 = \frac{7\pi}{2};\ \alpha_8 = 4\pi$$

5.2 Union of Fuzzy Sets

According to the Zadeh proposition, the membership function of the union of the two fuzzy sets B and F with membership functions μ_B and μ_F, respectively, is defined as the maximum of the two individual membership functions. This is the maximum criterion:

$$\mu_{B \cup F} = Max\,(\mu_B\,,\mu_F) \qquad (8)$$

Conversely, the membership of the intersection of the two fuzzy sets B and F with membership function μ_B and μ_F, respectively, is defined as the minimum of the two individual membership functions. This is called the minimum criterion:

$$\mu_{B \cap F} = Min\,(\mu_B\,,\mu_F) \qquad (9)$$

The use of Min and Max operators is based on the fact that grade of membership of data values in the fuzzy set is presumed as known and specified. That is, the grade of membership that corresponds to each data value is pre-determined and specified. In practice, data values are generated dynamically by systems during operation and often vary with time. In such cases, there is need to continuously generate grade of membership for every data value in the fuzzy sets (or universe of discourse). Thus if we can generate corresponding expressions for the triangular pulse illustrated in Fig. 1 and the polygonal waveform in Fig. 2 and normalize them appropriately, we will then have developed a methodology to dynamically generate membership function in the overlap and non-overlap domains and associate same with the corresponding linguistic variables.

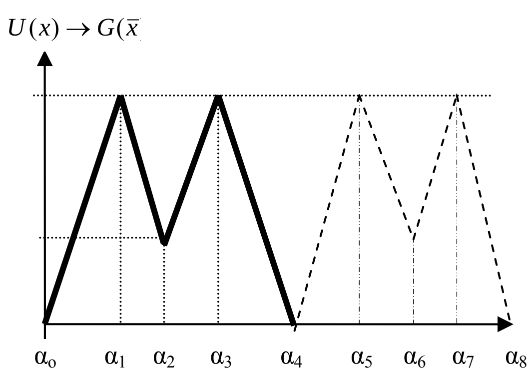

Fig. 2: Union of Fuzzy Sets

6 Problem Formulation

For the purpose of this work and in compliance with the requirement of membership function, we normalize f(x) by writing

$$f(\bar{x}) = \frac{f(x)}{\gamma_0} = 1 \;\; ; \;\; i.e \;\; f(\bar{x})_{\max} = 1 \qquad (10)$$

The intersection of the normalized fuzzy sets is described by the function $f(\bar{x})$, where

$$f(\bar{x}) = \begin{cases} 2\left(\frac{\bar{x}}{\pi} - 1\right) & ; \; if \quad \pi \le \bar{x} \le \frac{3\pi}{2} \\ 4 - \frac{2}{\pi}\bar{x} & ; \; if \quad \frac{3\pi}{2} \le \bar{x} \le 2\pi \\ 0 & ; \; if \quad 2\pi \le \bar{x} \le \frac{3\pi}{2} \\ 0 & ; \; if \quad \frac{3\pi}{2} \le \bar{x} \le 3\pi \end{cases} \qquad (11)$$

In writing the above expression, we have taken advantage of the periodicity condition of our Fourier series representation to pick any convenient cycle of our choice; for this example we have chosen as our period the interval $\pi \le \bar{x} \le 3\pi$ i.e. $(\alpha_2 \le \bar{x} \le \alpha_6)$. Of course, if we had elected to use the interval $(\alpha_0 \le x \le \alpha_4)$, our results from the Fourier series would have been the same. Similarly, we define the union of the fuzzy sets as represented by the function $G(\bar{x})$ where

$$G(\bar{x}) = \begin{cases} \frac{2}{\pi}\bar{x} & ; \; if \quad 0 \le \bar{x} \le \frac{\pi}{2} \\ \frac{2(\gamma_0 - 1)}{\pi}\bar{x} + (2 - \gamma_0) & ; \; if \quad \frac{\pi}{2} \le \bar{x} \le \pi \\ \frac{2(1-\gamma_0)}{\pi}\bar{x} + (3\gamma_0 - 2) & ; \; if \quad \pi \le \bar{x} \le \frac{3\pi}{2} \\ 4 - \frac{2}{\pi}\bar{x} & ; \; if \quad \frac{3\pi}{2} \le \bar{x} \le 2\pi \end{cases} \qquad (12)$$

but for this case we have used the interval $(0 \le \bar{x} \le 2\pi)$ i.e., $(\alpha_0 \le x \le \alpha_4)$.

7 Fourier Series Representation

The Fourier representations are obtained as follows:

7.1 Fuzzy Set Intersection

Lower Boundary Value $: \alpha_2 = \pi$

Upper Boundary Value $: \alpha_6 = 3\pi$

Number of terms in truncated Fourier series: N

Normalisation of Input Values:

$$\bar{x} = \left(\frac{(x_i - \alpha_2)}{(\alpha_6 - \alpha_2)}\right) * 2\pi \tag{13}$$

where x_i is any data value in the universe of discourse. The Fourier series representation for the normalized fuzzy set intersection $f(\bar{x})$ of the triangular pulse described by Equation (11) is given by:

$$f(\bar{x}) = \frac{a_o}{2} + \sum_{k=1}^{\infty} [a_k \cos(k\bar{x}) + a_k \sin(k\bar{x})] \tag{14}$$

where

$$\frac{a_0}{2} = \frac{1}{4} \tag{15}$$

and

$$a_k = \begin{cases} 0 & ; \quad k \quad odd \\ \frac{4}{\pi^2 k^2}\left((-1)^{\frac{k}{2}} - 1\right) & ; \quad k \quad even \end{cases} \tag{16}$$

while

$$b_k = \begin{cases} \frac{-4(-1)^{\frac{(k-1)}{2}}}{\pi^2 k^2} & ; \quad k \quad Odd \\ 0 & ; \quad k \quad Even \end{cases} \tag{17}$$

On plugging in the values for $\frac{a_0}{2}$, a_k and b_k we obtain the Fourier series expression as follows:

$$f(\bar{x}) = \frac{a_0}{2} + \sum_{k=1}^{\infty} [a_k Cos(k\bar{x}) + b_k Sin(k\bar{x})] \tag{18}$$

Where, for the case k odd

$$f(\bar{x}) = \frac{1}{4} - \sum_{k=1}^{N} \frac{4(-1)^{\frac{k-1}{2}}}{\pi^2 k^2} Sin(k\bar{x}) \tag{19}$$

and for the case k even

$$f(\bar{x}) = \frac{1}{4} + \sum_{k=1}^{N} \frac{4}{\pi^2 k^2}\left((-1)^{\frac{k}{2}} - 1\right) Cos(k\bar{x}) \tag{20}$$

7.2 Fuzzy Sets Union

Lower Boundary Value : $\alpha_0 = 0$

Upper Boundary Value : $\alpha_4 = 2\pi$

Number of terms in truncated Fourier series: N

Normalisation of Input Values:

$$\bar{x} = \left(\frac{(x_i - \alpha_0)}{(\alpha_4 - \alpha_0)}\right) * 2\pi \tag{21}$$

where x_i is any data value in the universe of discourse. The Fourier series representation for the union of the normalized fuzzy sets $G(\bar{x})$ of the polygonal waveform described by Equation (12) is given by:

$$G(\bar{x}) = \frac{a_o}{2} + \sum_{k=1}^{\infty} a_k \cos(k\bar{x}) + b_k \sin(k\bar{x}) \tag{22}$$

where

$$\frac{a_0}{2} = \frac{1}{2} + \frac{\gamma_0}{4} \tag{23}$$

and

$$a_k = \begin{cases} -\frac{4\gamma_0}{\pi^2 k^2} & ; \quad k \quad odd \\ \frac{1}{\pi^2 k^2}\left((8 - 4\gamma_0)(-1)^{\frac{k}{2}} + 4\gamma_0 - 8\right) & ; \quad k \quad even \end{cases} \tag{24}$$

while

$$b_k = \begin{cases} 0 & ; \quad k \quad odd \\ 0 & ; \quad k \quad even \end{cases} \tag{25}$$

Upon plugging in the values for $\frac{a_0}{2}$, a_k and b_k we obtain the Fourier expression as follows:

$$G(\bar{x}) = \frac{a_0}{2} + \sum_{k=1}^{\infty} [a_k Cos(k\bar{x}) + b_k Sin(k\bar{x})] \tag{26}$$

which can be simplified, for the case k odd, as

$$G(\bar{x}) = \left(\frac{1}{2} + \frac{\alpha_0}{4}\right) - \sum_{k=1}^{N} \frac{4\alpha_0}{\pi^2 k^2} Cos(k\bar{x}) \tag{27}$$

while, for the case k even

$$G(\bar{x}) = \left(\frac{1}{2} + \frac{\alpha_0}{4}\right) + \sum_{k=1}^{N}\left((8 - 4\alpha_0)(-1)^{\frac{k}{2}} + 4\alpha_0 - 8\right) Cos(k\bar{x}) \tag{28}$$

8 Application of the proposed Technique

To test the efficacy of the proposed techniques, we applied the techniques to data obtained from a natural gas distribution company in Nigeria.

8.1 Algorithm for adopting the new Approach.

Step 1: Map out boundary points of the fuzzy sets
Step 2: Ensure the fuzzy sets overlap
Step 3: For this case, fuzzy set is approximated as triangular in shape.
Step 4: Fuzzy plane can comprise of many overlapping fuzzy sets
Step 5: Invoke the derived Fourier series representation for the union and intersection of fuzzy sets.

Step 6: Membership functions are determined by providing data value, x_i; (Note: the x_i are normalized with respect to the period of the Fourier series representation (i.e. 2π))

8.2 Pre-Processing of Data

For our sample problem, the raw data from the gas distribution company represent mole percentage of Methane in the natural gas stream. For this case, the normal mole percentage of methane in the natural gas stream as per technical specification of the plants is in the range 82–94 %. We consequently took 76% as the lower bound of the universe of discourse and 100% as the upper bound. We normalize input value as follows:

$$\bar{x} = \left(\frac{x_i - Lower\ Bound}{Upper\ Bound - Lower\ Bound} \right) * 2\pi \qquad (29)$$

where x_i are data values within the universe of discourse.

8.3 Sample Output

As a demonstration of the capability of our approach, the results of sample input (mole % of methane) are presented in Tables 2 and 3, which are listed in the expanded version of the Paper. The full code for the Simulation (which was implemented in MATLAB) can also be provided on request.

8.4 Advantages of the proposed Methodology

One of the many advantages of this method is that it treats membership grade as a continuous function over the problem domain rather than as a discontinuous function. With respect to design of embedded fuzzy controllers in engineering systems, this approach will:

a) significantly reduce number of *if/then* production rules and thereby simplify the design of dedicated and general purpose fuzzy controllers;
b) facilitate the design of embedded fuzzy controllers in engineering systems;
c) reduce the cost of fuzzy controllers with respect to the processing subsystems; and
d) ultimately reduce the complexity and volume of implementation codes of fuzzy logic systems.

We may however emphasise that this methodology is not limited to the piecewise linear, polygonal and triangular representations; the advantages of the method are in fact more noticeable when dealing with the Gaussian membership function, which is commonly used in engineering problems involving measurements, as it gives actual representation at every point. In terms of hardware implementation of fuzzy controllers, this approach forms a basis for design of robust fuzzy controllers as the Fourier series representation can be codified in Assembly or C Language.

9 Discussion and Conclusion

Errors arising from vagueness or imprecision are often associated with measurements in engineering, just as they are with many other human endeavours. The capability of fuzzy sets to express a seamless transition from membership to non-membership and vice-versa has broad utility, especially for codifying the types of inaccuracies identified in this paper. Hence, there is a need to organize such pattern of measurements in sets that are fuzzy. The use of the Min and Max operators is based on the fact that grades of membership of data values are presumed as known and specified. In practice however, data values are generated dynamically by systems during operations and often vary with time. In such cases, there is a need to continuously generate the membership grade corresponding to every data value in a fuzzy set. Over the years, the Min and Max operators have effectively been used for fuzzy interpolative inference. But what has not been done is to define an explicit expression for determining membership function in the overlap region (the intersection) and the “unified” fuzzy sets (the union). In this work, we have modeled the union and intersection of fuzzy sets as polygonal waveform and triangular pulses respectively; we have then proceeded to formulate Fourier series representation for determining consistent grade of membership in the corresponding patches of the fuzzy sets. We have also tested and validated the formulated Fourier representation using data from process plants in Nigeria, and the results show that our methodology can effectively be used to generate grade of membership of fuzzy sets. The proposed methodology thus provides a basis for the design of robust fuzzy controller and, by extension, design of robust intelligent engineering systems.

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