Analysis of Nigerian Stock Market Returns Volatility Using Skewed ARMA-GARCH Model

[1]Isenah, M. Godknows, [2]Agwuegbo, S. O. N., [3]Adewole, A. P.
[1]Department of Statistics, University of Ibadan, Nigeria. isenahgodknows@yahoo.com
[2]Department of Statistics, Federal University of Agriculture, Abeokuta.
[3]Department of Computer Sciences, University of Lagos, Nigeria.

Abstract

This study used ARMA-GARCH type volatility models for predicting future values of the Nigerian stock market's percentage nominal returns and volatility. The data used in the study are time series data of the monthly Nigerian Stock Exchange All-Share-Index for the period of January 1990 to December 2012. The data was further segmented into in-sample and out-sample data sets for model building and out-of-sample forecast comparisons. Three ARMA(1, 2)-GARCH(1, 1) models with skewed normal distribution (SNORM), skewed Student-t distribution (SSTD) and skewed generalized error distribution (SGED) were fitted. In-sample model selections were based on the Akaike Information Criterion (AIC), Bayes Information Criterion (BIC), Schwarz Information Criterion (SIC) and the Hannan – Quinn Information Criterion (HQIC), while out-sample forecast evaluations were based on the Forecast Root Mean Square Error (FRMSE) and Forecast Mean Absolute Error (FMAE) metrics. The results of the study revealed the asymmetry inherent in the stock market returns distribution with kurtosis that exceeds that of normal distribution. The ARMA (1, 2)-GARCH (1, 1) model with skewed normal error distribution slightly outperformed the other models in the out-sample forecast evaluations, but for short-run forecasts the three models are quite adequate.

Keywords: GARCH model, Skewed distribution, Conditional mean and Conditional variance

1.0 Introduction

Forecasting of volatility of financial assets is fundamental in risk management, derivative pricing, hedging and estimation of risks associated with investment portfolios. Volatility is a central parameter for many financial decisions and is simply the random and autocorrelated changes in the variance exhibited by financial time series. The analysis of risks and uncertainty in financial markets has given rise to methods that allow for the modeling of temporal dependencies in the variances and covariances of financial variables.

Most of the volatility models presented in the empirical literature are based on the assumption that volatility is time-varying and that periods of high volatility tend to cluster (Ané, 2006). The Autoregressive Conditional Heteroscedasticity (ARCH) models introduced by Engle (1982) and later extended to Generalized ARCH or GARCH models in Bollerslev (1986) have proven to be useful means for empirically capturing well observed features in financial or economic time series such as fat tails, large kurtosis, leverage effects, co-movements in volatility, and volatility clustering. The insight offered by the ARCH or generalized ARCH models according to Hamilton (2010) lie in the distinction between the conditional and unconditional second order moments. While the unconditional variance-covariance matrix for the variables of interest may be independent of time, the conditional variances and covariances often depend on the past states of the data generating process. Thus, assumptions of homoscedasticity can lead to loss of asymptotic efficiency.

Many studies have shown the existence of direct relationship between the volatility of an asset's return and its market capitalization or its liquidity (Ané, 2006). And it has been observed that most of the empirical applications of ARCH or GARCH models have been in the study of financial time series such as stock prices/ or returns, interest rates and exchange rates (Bollerslev and Woolridge (1992), Hamilton (2010)). Cohen *et al.* (1978) also asserted that variance is inversely related to the market value of a stock. Cheung and Ng (1992) explained that although the strength of the relations between stock price dynamics and firm size appear to change over time, the essential characteristics of such relationships are stable.

In Nigeria, Ogum *et al.* (2005) examined the emerging stock market volatility using Nigerian stock market returns series. They fitted EGARCH model to the series, and the result of their analysis showed the presence of asymmetric volatility in the Nigerian stock market. Also Olowe (2009) investigated the relationship between stock market returns and volatility using an EGARCH-M model based on insurance and banking reforms, stock market crash and the global financial crisis. His results showed some evidence of relationship between volatility and stock returns, the impacts of banking reforms and market crash was found to be negative, and insurance reforms and financial crisis have no effect on stock returns. Emenike (2010) fitted GARCH (1, 1) and GJR-GARCH (1, 1) models to examine volatility persistence, leverage effects and asymmetries of returns on the monthly NSE All-Share-Index. The results of his study showed that the returns process is characterized by fat-tail, leverage effects and volatility persistence. Suleiman (2011) used daily market capitalization index of the Nigerian stock exchange to assess the robustness of stock market returns volatility and its effect on the performance on the capital market. His study employed ARCH and GARCH-type models for the estimation of the conditional returns variance.

The findings from his study showed the presence of volatility in the conditional variance as well as the long-term volatility persistence in the stock market indicating that Nigerian stock market is inefficient. Idris (2012) also examined the impact of stock market liberalization on the size and liquidity of the Nigerian stock exchange using data for the period 1986 to 2010. Fitting multiple regression and ARCH models, the study revealed that foreign portfolio investment has no significant effect on the size and liquidity of the Nigerian stock market. He further recommended that the market structure be strengthened so as to create a sound environment that will encourage foreign portfolio investments. Other studies conducted by Jayasuriya (2002), Terfa (2010), Kehinde (2011), Oke and Adewusi (2012), and Osisanwo and Atanda (2012) as well as those already cited based their models and analyses on the assumption that the returns distribution is Gaussian. This distributional assumption is usually violated by most economic and financial returns time series. Thus, this study intends to determine first the distribution of the NSE All-Share-Index returns process, and in the case of non-normality, fit skewed ARMA-ARCH or ARMA-GARCH models to the returns data. A good exposition to the theory on the ARCH, GARCH, EGARCH and TGARCH models can be found in Franke et al (2004), and in econometric or financial time series textbooks such as Hamilton (1994), Cryer et al (2008), Tsay (2010) and Enders (2010).

2.0 Statistical preliminaries

Let r_t denote the stock market returns at time t. Then, the returns process can be defined as:

$$r_t = \mu_t + \varepsilon_t \tag{1}$$

where $\mu_t = E(r_t|\mathcal{F}_{t-1})$ is the conditional expected returns function which may be time variant; ε_t corresponds to innovation in the return at time t, and $\mathcal{F}_{t-1}$ is a set of conditioning information based on past history of r_t. In the classical time series analysis framework, it is common to model the conditional mean returns process using a multiple regression, nonlinear regression or a stationary Autoregressive-moving-average (ARMA) model. The general formulation of a stationary ARMA(m, n) models is:

$$r_t = \alpha_0 + \sum_{i=1}^{m} \alpha_i r_{t-i} + \sum_{j=0}^{n} \beta_j \varepsilon_{t-j} \tag{2}$$

where $\alpha_i, i = 1, 2, \dots, m$ and $\beta_j, j = 0, 1, 2, \dots, n$ are real constants, ($\beta_0 = 1, \alpha_m \neq 0, \beta_n \neq 0$) and ε_t is a white noise process with constant variance σ^2. An ARMA(m, 0) model is referred to as an autoregressive model of order m denoted by AR(m); while an ARMA(0, n) model is referred to as a moving average model of order n and is denoted by MA(n). The disadvantage of the ARMA models for modeling financial time series is the assumption of constancy of the innovations variance. Since most financial time series exhibit changes in volatility, the series cannot be adequately captured or modeled by the assumption of constant variance. According to Aydemir (1998), the limitations of the ARMA models definitely lead to choices of models where either the general ARMA framework is retained and allowing ε_t as non-Gaussian white noise, or abandoning the linearity assumptions of the ARMA models.

In this study, the ARMA framework for modeling the conditional mean is retained, and we allow $\varepsilon_t = \sigma_t z_t$ denote an ARCH or GARCH process which may have Gaussian or non-Gaussian distribution. σ_t denotes the standard deviation of the innovations at time t, and z_t is a sequence of independent and identically distributed random variables with mean zero and unit variance. Thus, Equation (1) can be rewritten as:

$$r_t = \mu_t + \varepsilon_t \tag{3}$$

$$\varepsilon_t = \sigma_t z_t; \quad z_t \sim iid(0, 1) \tag{4}$$

The Equation (4) suggests modeling the innovations process using ARCH or GARCH-type models.

2.1 Testing for ARCH Effects

The Lagrange Multiplier (LM) test can be constructed for testing the presence of ARCH effects based on regressing ε_t^2 on ε_{t-i}^2 for $i = 1, 2, \ldots, p$. Under the null hypothesis of no ARCH effects, the test statistic

$$LM = N * R^2 \sim \chi_p^2 \tag{5}$$

where N is the length of the time series used and R^2 is the coefficient of determination obtained from the regression. For significant values of LM statistic, the null hypothesis of no ARCH effects is rejected. Similarly, the Ljung and Box (1978) Q-statistic can be used to diagnose serial correlation in the residuals obtained from the estimated conditional mean returns function. This statistic is given by:

$$Q_k = N(N+2) \sum_{k=1}^{K} \frac{\rho_k^2(\hat{\varepsilon})}{N-k} \sim \chi_k^2 \tag{6}$$

The McLeod and Li (1983) test statistic for testing autocorrelation of the squared residuals, with the same distribution and degrees of freedom as the Ljung and Box Q-statistics is given by:

$$McL(k) = N(N+2) \sum_{k=1}^{K} \frac{\rho_k^2(\hat{\varepsilon}_t^2)}{N-k} \sim \chi_k^2 \tag{7}$$

For significant values of McL statistic, the null hypothesis of no ARCH effects in the squared residuals is rejected.

2.2 The GARCH models

The serial correlations in the squared returns or conditional heteroscedasticity may be modeled using an ARMA(0, q) process for the squared residuals. To allow for conditional heteroscedasticity in the residuals, we assume that $var(r_t|\mathcal{F}_{t-1}) = \sigma_t^2$ and

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 \tag{8}$$

Since ε_t is a white noise process, $var(\varepsilon_t|\mathcal{F}_{t-1}) = E(\varepsilon_t^2|\mathcal{F}_{t-1}) = \sigma_t^2$. Thus, Equation (8) can be rewritten as:

$$\varepsilon_t^2 = \omega + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + v_t \tag{9}$$

where $v_t = \varepsilon_t^2 - E(\varepsilon_t^2|\mathcal{F}_{t-1})$ is a white noise process. The models (3) and (8) constitute the ARCH(q) model of Engle (1982). A formulation that extends Engle's model is the more parsimonious GARCH(p, q) models proposed by Bollerslev (1986). This formulation is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} b_j \sigma_{t-j}^2 \tag{10}$$

The coefficients $\omega \geq 0$, $a_i: i = 1, 2, \ldots, q$ and $b_j: j = 1, 2, \ldots, p$ are all assumed to be positive to ensure that the conditional variance σ_t^2 is always positive. The Equations (3) and (10) specify the ARMA-GARCH models which assume a Gaussian distribution for the residuals obtained from the conditional expected returns function.

2.2.1 Skewed distributions

Fernandez and Steel (1998) proposed a general approach that allows the introduction of skewness into continuous unimodal and symmetric probability distribution functions using inverse scale factors in the positive and negative real half lines (see Zhang (2009) and Alexios, (2011) for details). Given the asymmetry parameter, ξ, the probability density function of a continuous random variable z can be written as:

$$f(z|\xi) = \frac{2\xi}{\xi^2+1}\left[g(\xi z)H(-z) + g\left(\frac{z}{\xi}\right)H(z)\right] \tag{11}$$

where $0 < \xi < \infty$; $g(.)$ is a symmetric density function and $H(z) = \frac{1}{2}[1 + sgn(z)]$ is the heavy-side function. For $\xi = 1$, the probability density function, $f(z|\xi = 1) = g(z)$ is symmetric. The mean and variance of z are given respectively by:

$$E(z) = M_1\left(\frac{\xi^2-1}{\xi}\right) \tag{12}$$

$$Var(z) = (M_2 - M_1^2)\left[\frac{\xi^4+1}{\xi^2}\right] + 2M_1^2 - M_2 \tag{13}$$

where M_r is the r^{th} absolute moments of z derived from the relation:

$$M_r = 2\int_0^{\infty} z^r g(z)dz \tag{14}$$

The Skewed Normal (SNORM), Skewed Student-t (SSTD) and the skewed Generalized Error Distribution (SGED) are variants of the Normal, Student-t and Generalized Error Distribution that have been standardized to have mean zero and unit variance using (12) and (13). Lambert and Laurent (2001) extended Fernandez and Steel (1998) density function to include the conditional mean as well as the conditional variance in a manner that the innovations have mean zero and unit variance. The skewed distribution so obtained is a standardized skewed distribution function. The probability density function of a standardized skewed distribution (see Würtz and Chalabi (2012)), is defined as:

$$f^*(z|\xi\theta) = \frac{2\sigma\xi}{\xi^2+1} g^*\left(z_{\mu_\xi \sigma_\xi}|\theta\right) \tag{15}$$

with

$$z_{\mu_\xi \sigma_\xi} = [\sigma_\xi z + \mu_\xi]\xi^{sgn(\sigma_\xi z + \mu_\xi)} \tag{16}$$

where $g^*(z|\theta)$ may be any standardized symmetric unimodal distribution function. The parameters μ_ξ and σ_ξ are calculated via the moments given in (13).

The probability density function of *skewed Normal distribution* (SNORM) for the random variable z is given by:

$$f(z;\xi) = 2\phi(z)\Phi(\xi z) \tag{17}$$

with ξ is fixed, and $\phi(z)$ is the probability density function of the standard normal distribution, and

$$\Phi(\xi z) = \int_{-\infty}^{\xi z} \phi(x)dx \tag{18}$$

Similarly, the probability density function of the *skewed Student-t distribution* (SSTD) for the random variable is given by:

$$f(z|\xi, \nu) = \begin{cases} \frac{2\sigma\xi}{\xi^2+1} g(\xi(sz+\mu)|\nu) \; for \; z < -\frac{\mu}{\sigma} \\ \frac{2\sigma\xi}{\xi^2+1} g\left(\frac{sz+\mu}{\xi}|\nu\right) \; for \; z \geq -\frac{\mu}{\sigma} \end{cases} \tag{19}$$

The function $g(.)$ in equation (19) is the standard Student-t density function; (ξ, ν) denote the asymmetric and shape parameters, while (μ, s) denote the mean and standard deviation of the skewed Student-t distribution. The mean μ and standard deviation s are defined respectively by:

$$\mu = \frac{\Gamma\left(\frac{\nu+1}{2}\right)\sqrt{\nu-2}}{\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)} \left[\frac{\xi^2-1}{\xi}\right] \tag{20}$$

$$s = \sqrt{\xi^2 + \xi^{-2} - \mu^2 - 1} \tag{21}$$

The probability density function of the standard *skewed generalized error distribution* (SGED) is given by:

$$f(z|\nu, \xi) = \frac{\nu}{2\theta\Gamma(\nu^{-1})} exp\left[\frac{|z-\delta|^\nu}{[1-sgn(z-\delta)]^\nu \theta^\nu}\right] \tag{22}$$

where

$$\theta = \sqrt{\frac{\Gamma(\nu^{-1})}{\Gamma\left(\frac{3}{\nu}\right)}} * \frac{1}{S(\xi)} \tag{23}$$

$$\delta = 2 * \frac{A\xi}{S(\xi)} \tag{24}$$

$$S(\xi) = \sqrt{1 + (3 - 4A^2)\xi^2} \tag{25}$$

and

$$A = \frac{\Gamma\left(\frac{2}{\nu}\right)}{\sqrt{\Gamma\left(\frac{1}{\nu}\right)\Gamma\left(\frac{3}{\nu}\right)}} \qquad (26)$$

The parameters ξ and $\nu > 0$ control the skewness and fat tails of the density function. For $\xi = 0$ and $\nu = 2$, the SGED becomes the standard normal distribution.

2.3 Parameter estimation

The skewed Student-t distribution was extended by Lambert and Laurent (2000 and 2001) to accommodate the GARCH-type models proposed by Fernandez and Steel (1998). Its log-likelihood function for the sequence $\{z_t(\vartheta)\}$ is given by:

$$L(z_t, \vartheta) = N\left[\log_e \Gamma\left(\frac{\nu+1}{2}\right) - \log_e\left(\frac{\nu}{2}\right) - \frac{1}{2}\log_e \pi(\nu-2) + \log_e\left(\frac{2}{\xi+\xi^{-1}}\right) + \log_e s\right] - \frac{1}{2}\sum_{t=1}^{N}\left[\log_e \sigma_t^2 + (\nu+1)\log_e\left(1 + \frac{(sz_t+\mu)^2}{\nu-2}\xi^{-2I_t}\right)\right] \qquad (27)$$

with the asymmetric parameter denoted by ξ and ν denoting the degree of freedom of the distribution, and

$$s = \sqrt{\xi^2 + \xi^{-2} - \mu^2 - 1} \qquad (28)$$

$$\mu = \frac{\Gamma\left(\frac{\nu+1}{2}\right)\sqrt{\nu-2}}{\sqrt{\pi}\,\Gamma\left(\frac{\nu}{2}\right)}(\xi - \xi^{-1}) \qquad (29)$$

$$I_t = \begin{cases} 1, if\ z_t \geq -\mu/s \\ -1, if\ z_t < -\mu/s \end{cases} \qquad (30)$$

The values of the parameter vector ϑ that optimizes equation (27) are the maximum likelihood estimate of the log-likelihood function. In a similar manner, the maximum likelihood estimates of the parameter vector of the skewed normal and skewed generalized error distribution can be obtained, (see Lambert and Laurent (2001) and Dima et al (2008) for details). For this study, the quasi-maximum likelihood estimator as implemented in the R - fGARCH package developed by Wuertz and Chalabi (2012) will be used for estimating the parameter vectors of the models. Though, it has been reported that the QMLE estimates are consistent and have limiting normal distribution that provides asymptotic standard errors that are valid under non-normality, however, the estimates are not efficient and the efficiency loss can be marked under asymmetric distributions (see Bollerslev and Woolridge (1992), Alexios (2010), and Wuertz and Chalabi (2012)).

2.4 Forecasting

The ability of the GARCH models for forecasting has been discussed and this can be found in the review paper by Poon and Granger (2003). The forecast of the future values of the conditional mean and conditional variance of the percentage nominal returns series will be carried out using the R-fGarch package. The documentation on forecasting using this package can be found in Wuertz and Chalabi (2012).

3.0 Methodology

The monthly percentage nominal return at time t denoted by r_t is defined as:

$$r_t = 100 * \left[\log_e\left(\frac{ASI_t}{ASI_{t-1}}\right)\right] \qquad (31)$$

While the monthly percentage nominal return in deviation form at time t is defined as:

$$R_t = r_t - \bar{r} \qquad (32)$$

with ASI and $\bar{r}$ denoting the *All-Share-Index* and sample mean of respectively. Figure 1 shows the time series, histogram (*empirical density plot superimposed*) and the quantile-quantile normal plots of the percentage monthly stock market nominal returns for the period January 1990 through December 2010 (see the Appendix). The sample descriptive statistics of the returns for this period and the normality tests are presented in Tables 1 and 2 respectively. It is obvious from the results reported in Tables 1 and 2 and the plotted graphs in Figure 1 that the returns process is asymmetric and non-normal with kurtosis that exceeds that of the normal distribution. Also, the time series plot shows that the variance of the returns process changes over time.

Minimum	Mean	Median	Maximum	Variance	Skewness	Kurtosis
-36.58828	1.70505	1.69188	32.5158	40.33999	-0.7191997	7.955137

Table 1: Sample descriptive statistics of the percentage nominal returns series (Jan. 1990 - Dec. 2010).

test	Statistic	Degree of freedom	p-value
Shapiro - Wilks	0.9054	-	1.756e - 11
Jarque - Bera	653.3879	2	2.2e - 16

Table 2: Normality tests of the log - returns time series (Jan. 1990 - Dec. 2010)

3.1 Data segmentation

The monthly stock market returns series were segmented into in-sample and out-sample data sets respectively. The in-sample data set used for fitting the models comprise of data points from January 1990 through December 2010, while the out-sample data set used for out-of-sample evaluation comprise of data points from January 2011 through December 2012.

3.2 Model evaluations

3.2.1 In-sample evaluation

Model selection criteria for the ARMA (m, n) models have the form:

$$MSC(m,n) = \log_e \tilde{\sigma}^2(m,n) + C_N * \varphi(m,n) \tag{33}$$

where $\tilde{\sigma}^2(m,n)$ is the maximum likelihood estimate of the variance of the residual errors from the fitted ARMA (m, n) model, and C_N is a sequence indexed by the length of the series N, and $\varphi(m,n)$ is the penalty function which penalizes large ARMA (m, n) models. The following information criteria will be applied in the study for in-sample evaluations and selection of the best fitted models.

Akaike's Information Criterion (AIC); Akaike (1973):

$$AIC(k) = N \log_e \left[\frac{SSE}{N}\right] + 2k \tag{34}$$

Bayesian Information Criterion (BIC); Sawa (1978):

$$BIC(k) = N \log_e \left[\frac{SSE}{N}\right] + \frac{2(k+2)N\sigma^2}{SSE} + \frac{2N^2\sigma^4}{SSE^2} \tag{35}$$

Hannan-Quin Information Criterion (HQIC); Hannan and Quinn (1980):

$$HQIC(k) = N \log_e \left[\frac{SSE}{N}\right] + k \log_e(\log_e N) \tag{36}$$

And the Schwarz Information Criterion (SIC); Schwarz (1978):

$$SIC(k) = N \log_e \left[\frac{SSE}{N}\right] + k \log_e N \tag{37}$$

The constant k denotes the number of estimated parameters in the fitted model, $SSE = \sum_{t=1}^{N}(R_t - \hat{R}_t)^2$ denotes the residual sum of squares, while σ^2 denotes the pure error variance fitting the full model. The models in which all of the information criteria simultaneously agree will be selected as tentative for further analysis.

3.2.2 Out-sample evaluation

Comparisons of the out-of-sample forecast performance of selected models will be based on the following metrics:

Forecast Root Mean Square Error (FRMSE) defined by

$$FRMSE = \left[\frac{1}{\tau^*}\sum_{\tau=1}^{\tau^*}(R_\tau - \hat{R}_\tau)^2\right]^{1/2} \tag{38}$$

Forecast Mean Absolute Error (FMAE) defined by

$$FMAE = \frac{1}{\tau^*}\sum_{\tau=1}^{\tau^*}\left|R_\tau - \hat{R}_\tau\right| \tag{39}$$

where τ^* is the number of observations in the out-sample data set and the sequence $\{\hat{R}_\tau\}$.

4.0 Data analysis and results

Figure 2 presents the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the centred percentage returns time series of the in-sample data. These functions suggest maximum orders of $m = 4$ and $n = 3$ respectively for the AR and MA parts for the expected conditional mean function. Fitting different ARMA models to the centred percentage returns by varying the order combinations give optimal orders $(m, n) = (1, 2)$ for the expected conditional mean function, and $(p, q) = (1, 1)$ for the conditional variance function. These model orders are optimal for the SNORM, SSTD and SGED GARCH models respectively. The ARCH test results on the squared residuals of the fitted $ARMA(1, 2)$ model using Equation (7) are shown in Table 3, while Table 4 shows the summary of the models selected using the information criteria. The McLeod and Li tests show that ARCH effects exist in the residuals at lags 10, 20, 30, and 40 respectively; thus the choice of ARMA-GARCH functions for modeling the log-returns series is appropriate.

Type of Test	Lag	Chi-square statistic	Degree of freedom	p-value
McLeod and Li	10	85.2481	10	4.663e − 14
	20	92.7943	20	2.397e − 11
	30	94.906	30	1.163e − 08
	40	96.3237	40	1.51e − 06

Table 3: ARCH tests on squared residuals of the fitted ARMA (1, 2) model using McLeod and Li test.

	Model	AIC	BIC	SIC	HQIC
Skewed Normal Distribution (SNORM)	ARMA(1, 0)-GARCH(1, 1)	6.127231	6.197460	6.126458	6.155493
	ARMA(1, 1)-GARCH(1, 1)	6.104684	6.188958	6.103576	6.138598
	ARMA(1, 2)-GARCH(1, 1)	6.061721	6.160041	6.060221	6.101287
Skewed Student-t Distribution (SSTD)	**ARMA(1, 2)-GARCH(1, 1)**	5.952621	6.064986	5.950671	5.997839
Skewed Generalized Error Distribution (SGED)	ARMA(1, 0)-GARCH(1, 1)	5.972672	6.056946	5.971564	6.006586
	ARMA(1, 1)-GARCH(1, 1)	5.962330	6.060650	5.960830	6.001896
	ARMA(1, 2)-GARCH(1, 1)	5.933623	6.045988	5.931674	5.978842

Table 4: Summary of the fitted models adequacy by information criterion (*selected models for each distribution indicated in bold* ***blue*** *colour*).

The details of the estimated parameters and the residual diagnostics and fitted values for each of the estimated models are given in Table 5, Figures 3, 4, 5 and 6 respectively (see the Appendix). The parameters of the fitted models are all highly significant, except for the intercept term of the SSTD conditional variance function. Also the residual diagnostics show that there are no inadequacies found in the fitted models. The estimated parameters for skewness and shape are also significant for all of the models, except for SNORM which has no shape parameter. This is an indication that the returns distribution is asymmetric. The Jarque-Bera and Shapiro-Wilks statistics are significant, indicating the non-normality of the error distribution. Also, the LM Arch-test, Ljung-Box and McLeod tests for the residuals and squared residuals indicate that the fitted ARMA(1, 2)-GARCH(1, 1) models are all adequate for forecasting purposes. The time series plots the fitted values of the estimated models have almost equal goodness-of-fit to the data. The fitted ARMA(1, 2)-GARCH(1, 1) models for each error distribution type are:

Skewed Normal Distribution (SNORM):

$$\hat{R}_t = 0.9841R_{t-1} - 0.6146\hat{\varepsilon}_{t-1} - 0.2719\hat{\varepsilon}_{t-2} \tag{40}$$

$$\hat{\sigma}_t^2 = 1.7149 + 0.4361\hat{\varepsilon}_{t-1}^2 + 0.6062\hat{\sigma}_{t-1}^2 \tag{41}$$

Skewed Student-t Distribution (SSTD):

$$\hat{R}_t = 0.9687R_{t-1} - 0.5786\hat{\varepsilon}_{t-1} - 0.2501\hat{\varepsilon}_{t-2} \tag{42}$$

$$\hat{\sigma}_t^2 = 1.7008 + 0.4974\hat{\varepsilon}_{t-1}^2 + 0.6411\hat{\sigma}_{t-1}^2 \tag{43}$$

Skewed Generalized Error Distribution (SGED):

$$\hat{R}_t = 0.9724R_{t-1} - 0.6158\hat{\varepsilon}_{t-1} - 0.2594\hat{\varepsilon}_{t-2} \tag{44}$$

$$\hat{\sigma}_t^2 = 1.8139 + 0.4363\hat{\varepsilon}_{t-1}^2 + 0.6058\hat{\sigma}_{t-1}^2 \tag{45}$$

The summary of forecast performance of the fitted models is presented in Table 5. The results show that the ARMA(1, 2)-GARCH(1, 1) model with skewed normal error distribution slightly outperformed the other models. The difference in the values of the performance metrics between SNORM and SGED are negligible. Figures 7, 8 and 9 show the out-of-sample forecasts for the three models, where the forecast confidence intervals are computed using conditional mean square error. It is also clear from these forecasts values that as the forecast horizon increases, the forecast intervals also increases or widens.

Model	FRMSE	FMAE
ARMA(1, 2)-GARCH(1, 1) (*SNORM*)	0.513805	1.780493
ARMA(1, 2)-GARCH(1, 1) (*SSTD*)	0.529947	1.806737
ARMA(1, 2)-GARCH(1, 1) (*SGED*)	0.519722	1.790928

Table 5: Values of the forecast performance metrics of the fitted models in the out-of-sample data.

5.0 Conclusion

The aim of the study was to fit skewed GARCH-type volatility models that are adequate and that can give precise predictions of future values of the conditional mean nominal returns as well as the associated conditional variance. The nominal returns series was assessed for non-normality and asymmetry, and it was found to be non-normal with skewness to the left and kurtosis that exceeds that of the normal distribution. Analysis of the residuals from the fitted ARMA(1, 2) model for the conditional mean returns function indicates that ARCH effects are present and significant at the 5% level of significance. These tests necessitate fitting of ARMA-GARCH models with skewed error distributions to the nominal returns time series. The nominal returns series was segmented into in-sample and out-sample data sets for model estimation and forecast performance evaluations respectively. Three ARMA(1, 2)-GARCH(1, 1) models were estimated with Skewed Normal (SNORM), Skewed Student-t Distribution (SSTD) and the Skewed Generalized Error Distribution (SGED). Each of the estimated models have adequate fit on the data, but it was observed that the SGED ARMA(1, 2)-GARCH(1, 1) model has the best fit on the data based on the information criteria used for model selection.

The out-sample forecast performance evaluations were conducted using FRMSE and FMAE. The evaluations show that the ARMA-GARCH model with SNORM error distribution slightly outperformed the order models, but the differences in values of the metrics between SNORM and SGED or SGED and SSTD error models are very small. For predicting short-run future values of the conditional returns and the conditional variance or volatility, any of the models can be used. But for long-run forecasts, the forecast confidence intervals widens rapidly which may give imprecise forecast values, especially with the SSTD error distribution model.

References

Akaike, H. (1973). Information theory and an extension of the maximum likelihood principle. In B. N. Petrov and F. Csaki (Eds.), *Second International Symposium of Information theory*, 267 – 281. Budapest: Academiai Kiado.

Alexios, G. (2011). *Introduction to the rugarch package*. http://www.rgarch.r-forge.r.project

Ané, T. (2006). An analysis of the flexibility of asymmetric power GARCH models. *Journal of Computational Statistics & Data Analysis*, ELSEVIER, Vol. **51**, p1293-1311

Aydemir, A. B. (1998). *Forecasting Volatility in the financial markets*.Eds. John Knight and Stephen Satchell. Butterworth-Heinemann.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics*, vol. **31**, p307-327.

Bollerslev, T. and Woolridge, J. (1992). Quasi-maximum likelihood estimation inference in dynamic models with time varying covariance. *Econometric Theory*, vol. **11**, 143-172.

Cheung, Y. W. and Ng, L. K. (1992). Stock price dynamics and firm size: an empirical investigation. *Journal of Finance*, vol. **47**(5), p1985-1997.

Cohen, K. J., Maier, S. S. & Schwartz, R. A. (1978). The return generating process, return variances and the effects of thinness in securities markets. *Journal of Finance*, vol. 33(1), p149-167.

Cryer, J. D. and Chan, K. S. (2008). *Time series analysis: with applications in R, 2nd edition*. Springer Science+Business Media, LLC, New York.

Dima, A., Haim, S. & Rami, Y. (2008). Estimating stock market volatility using asymmetric GARCH models. *Applied Financial Economics*, vol. 18, 1201-1208.

Emenike, K. O. (2010). Modeling stock returns volatility in Nigeria using GARCH models. *MPRA Paper*, No. 23432, posted 5th July 2010 | 19:53

Enders W. 2010. *Applied econometric time series, 3rd edition*. John Wiley and Sons, Inc., New York.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of U.K. inflation. *Econometrica*, Vol. 50, p987-1008.

Fernandez, C. and Steel, M. (1998). On Bayesian modeling of fat tails and skewness. *Journal of the American Statistical Association*, vol. 93, 359-371.

Franke, J., Härdle, W. & Hafner, C. (2004). *Statistics of financial markets: an introduction*. Springer-Verlag, Heidelberg - Germany.

Hamilton, J. D. (1994). *Time series analysis*. Princeton University Press, Princeton, NJ.

Hamilton, J. D. (2010). Macroeconomics and ARCH in Tim Bollerslev, Jeffrey, R. Russel and Mark, W. Watson (Eds), *Volatility and Time Series Econometrics: Essays in Honor of Robert, F. Engle*. Oxford University Press.

Hannan, E. J. and B. G., Quinn (1980). The estimation of the order of an ARMA process. *Annals of Statistics*, vol. 8, 1071 – 1081.

Idris, A. (2012). Impact of stock market liberalization on the performance of the Nigerian stock exchange .A P.hD. dissertation, Ahmadu Bello University, Zaria, Nigeria. http://hdl./123456789/2865[05/06/2013]

Jayasuriya, S. (2002). Does stock market liberalization affect the volatility of stock returns: evidence from emerging market economies. Georgetown University Discussion Series, August.

Kehinde, J. S. (2011). Microstructure change: effect on market turnover in the Nigerian stock market. *Journal of African Macroeconomic Review*, vol 1, p 16-31. http://www.journalsbank.com

Lambert, P. and Laurent, S. (2000). Modeling skewness dynamics in series of financial data. *Discussion Paper, Institut de Statistique, Louvain-la-Neuve.*

Lambert, P. and Laurent, S. (2001). Modeling financial time series using GARCH-type models and skewed student density. *Memeo*, Université de Liège.

Ljung, G. M. and Box, G. E. P. (1978). On a Measure of lack of fit in time series models. *Biometrika* vol. 65, 257-303.

Mcleod, A. I. and Li W. K. (1983). Diagnostic checking of ARMA time series models using squared-residual autocorrelations. *Journal of Time Series Analysis* vol. 4, 269-273.

Ogum, G., Beer, F. & Nouyrigat, G. (2005). Emerging equity market volatility: an empirical investigation of markets in Kenya and Nigeria. *Journal of African Business*, vol. 6, (1 | 2): 139-154

Oke, M. O. and Adewusi, S. O. (2012). Impact of capital market reforms on economic growth: the Nigerian experience. *Australia Journal of Business and Management Research*, vol. 2, No 2 [20-30] | May 2012.

Olowe, R. A. (2009). Stock return volatility and the global financial crisis in an emerging market: the Nigerian case. *International Review of Business Research Papers*, vol. 5, 4: 426-447

Osisanwo, B. G. and Atanda, A. A. (2012). Determinants of stock market returns in Nigeria: a time series analysis. *African Journal of Scientific Research*, vol. 9, No 1, p 447-496. http://creativeconomics.org/licenses/by-nc/3.0/

Poon, S. and Granger, C. (2003). Forecasting financial market volatility: a review. *Journal of Economic Literature*, vol. 41, 478-539.

Sawa, T. (1978). Information criteria for discriminating among alternative regression models. *Econometrica*, vol. 46, 1273 – 1282.

Schwarz, G. (1978). Estimating the dimension of a model. *Annals of Statistics*, vol. 6, 461 – 464.

Suleiman, H. K. (2011). Stock return and volatility persistence in the Nigerian capital market. A P.hD. post-data seminar submitted at Ahmadu Bello University, Zaria, Nigeria.

Terfa, W. A. (2010). Stock market reaction to selected macroeconomic variables in the Nigerian economy.*CBN Journal of Applied Statistics*, vol. 2, No 1, p 61-70

Tsay, R. S. (2010). *Analysis of financial time series, 3rd edition*. Wiley, New York.

Wuertz, D. and Chalabi, Y. (2012). *Package "fGarch": autoregressive conditional heteroscedastic modeling*. http://www.rmetrics.org.

Würtz, D., Chalabi, Y. & Luksan, L. (Draft copy). Parameter estimation of ARMA models with GARCH/APARCH errors: an R and S-PLUS software implementation. *Journal of Statistical Software*, http://www.jstatsoft.org

Zhang, Z. (2009). Analysis skewness in GARCH models. D-Level Thesis, School of Economics and Social Sciences, Högskolan Dalarna, Sweden.

Appendix

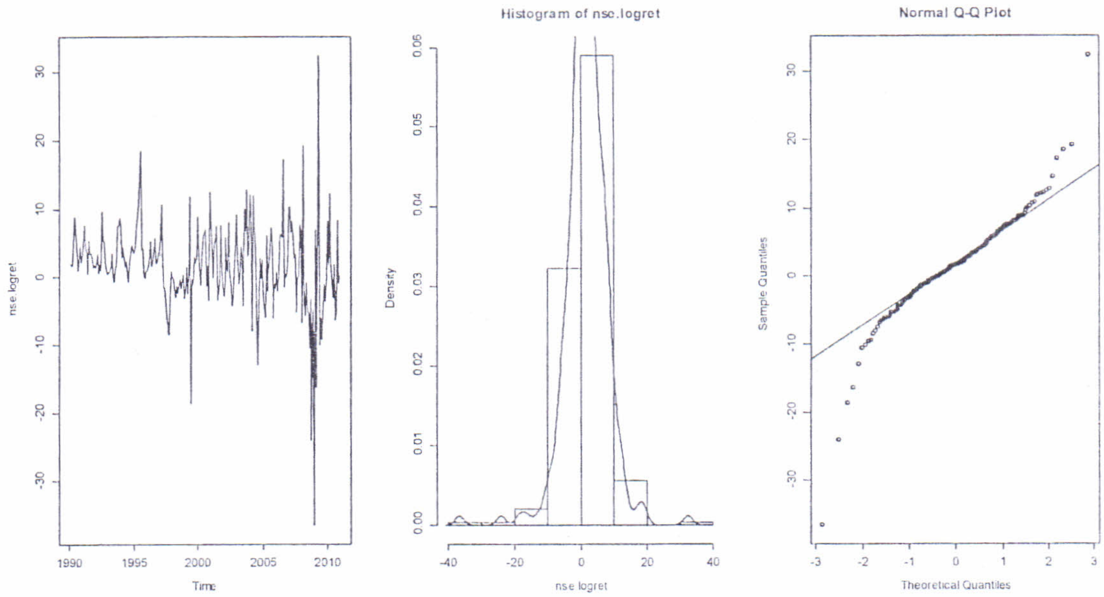

Figure 1: Actual realizations, histogram with superimposed density plot and quantile-quantile normal plot of the percentage nominal log-returns process (Jan. 1990 - Dec. 2010).

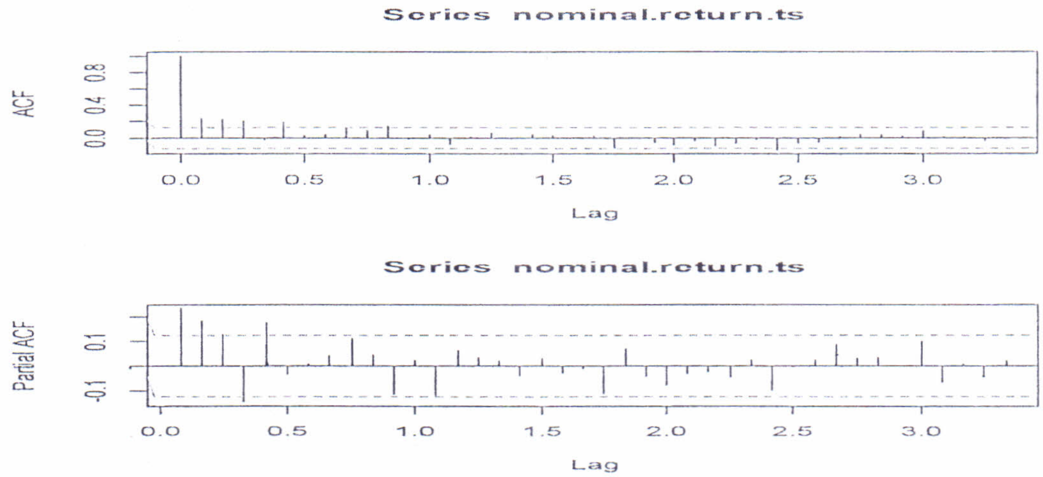

Figure 2: ACF and PACF of the in-sample monthly centred percentage nominal returns time series.

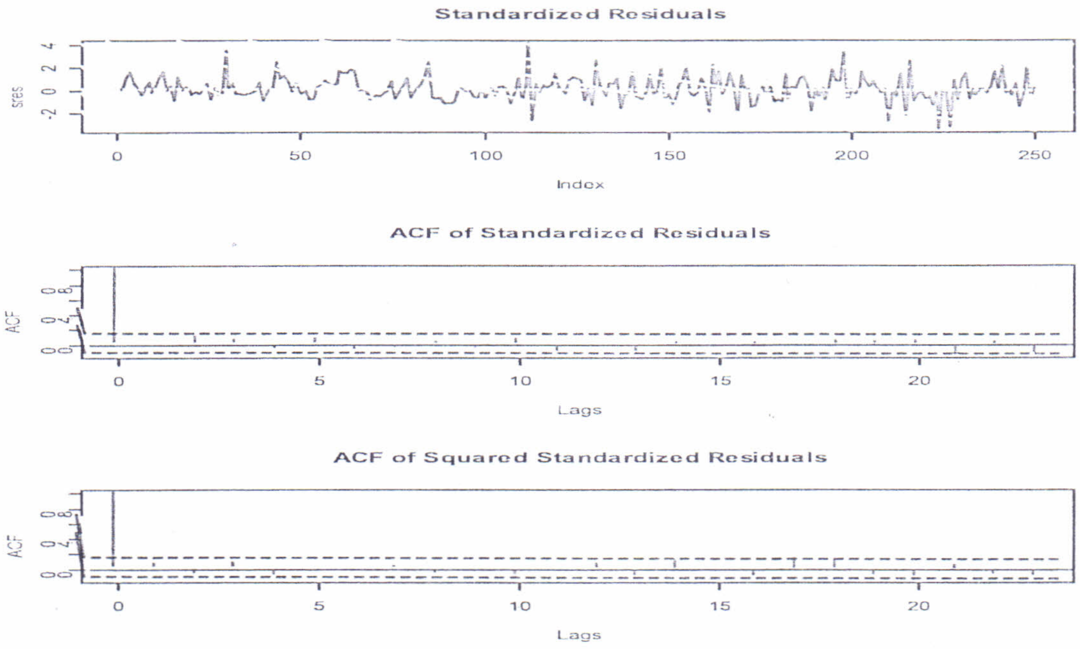

Figure 3: Diagnostics of the fitted ARMA(1, 2)-GARCH(1, 1) model for SNORM.

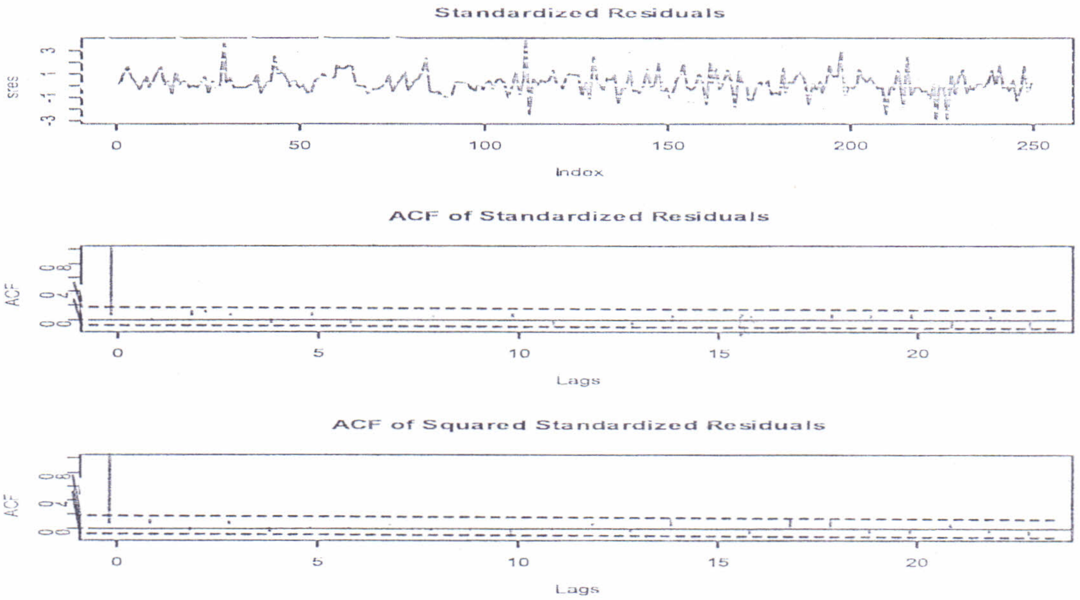

Figure 4: Diagnostics of the fitted ARMA(1, 2)-GARCH(1, 1) model for SSTD.

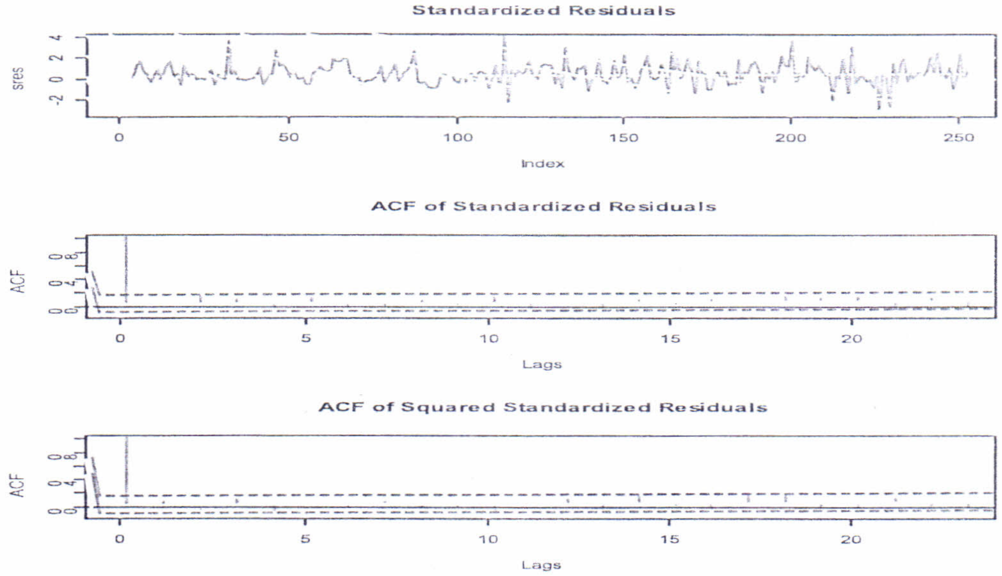

Figure 5: Diagnostics of the fitted ARMA(1, 2)-GARCH(1, 1) model for SGED.

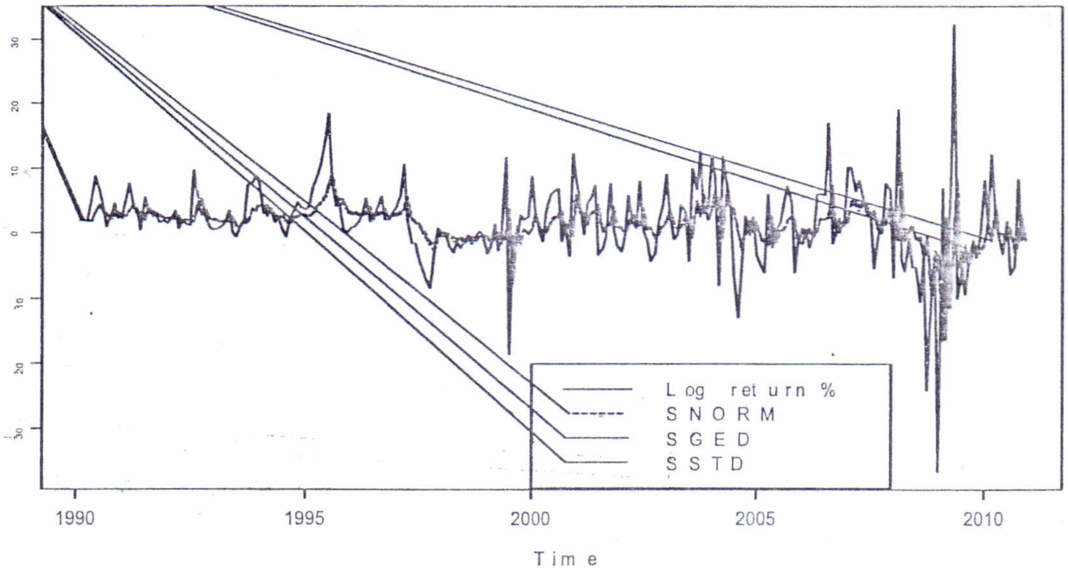

Figure 6: Time series plots of actual realizations of percentage logreturns and the fitted values of SNORM , SSTD and SGED for the period Jan. 1990 to Dec. 2010.

Table 5: Values of the estimated parameters and the standardized residuals test for the ARMA(1, 2)-GARCH(1, 1) models for each distribution.

	Skewed Normal Distribution (SNORM)		Skewed Student-t Distribution (SSTD)		Skewed Generalized Error Distribution (SGED)	
Parameter	Estimate	p-value	Estimate	p-value	Estimate	p-value
$ar1$	0.98408	<2e-16***	0.96874	<2e-16***	0.972399	<2e-16***
$ma1$	-0.61455	<2e-16***	-0.57859	<2e-16***	-0.615794	<2e-16***
$ma2$	-0.27191	0.000218***	-0.25010	1.99e-05***	-0.259425	<2e-16***
$omega$	1.71481	0.008744**	1.70078	0.138866	1.813909	0.0388*
$alpha1$	0.43612	3.44e-05***	0.49735	0.025730*	0.436307	2.07e-06***
$beta1$	0.60623	<2e-16***	0.64114	2.71e-12***	0.605820	<2e-16***
$skew$	1.09363	<2e-16***	1.07794	<2e-16***	1.036855	<2e-16***
$shape$	-	-	3.18990	0.000103***	1.00000	<2e-16***
$J-B\ Test$	48.20384	3.409328e-11	60.51864	7.21645e-14	51.11655	7.946643e-12
$S-W\ Test$	0.962365	3.772147e-06	0.9585427	1.28295e-06	0.9615067	2.946434e-06
$LBRQ(10)$	8.355121	0.5941921	8.212451	0.6080936	8.275416	0.6019543
$LBRQ(15)$	12.23536	0.6611338	12.70352	0.6251862	11.92865	0.6844213
$LBRQ(20)$	14.57433	0.8002248	16.60749	0.6782937	14.60943	0.793031
$LBR^2Q(10)$	7.928778	0.6357937	8.538717	0.5763668	8.105748	0.6185083
$LBR^2Q(15)$	14.68337	0.4744558	14.50729	0.4874542	14.21884	0.5090011
$LBR^2Q(20)$	27.87349	0.1124281	25.66422	0.1771902	26.78799	0.1413349
$LMArchTest$	10.69959	0.554824	10.32239	0.5877007	10.41812	0.5793296

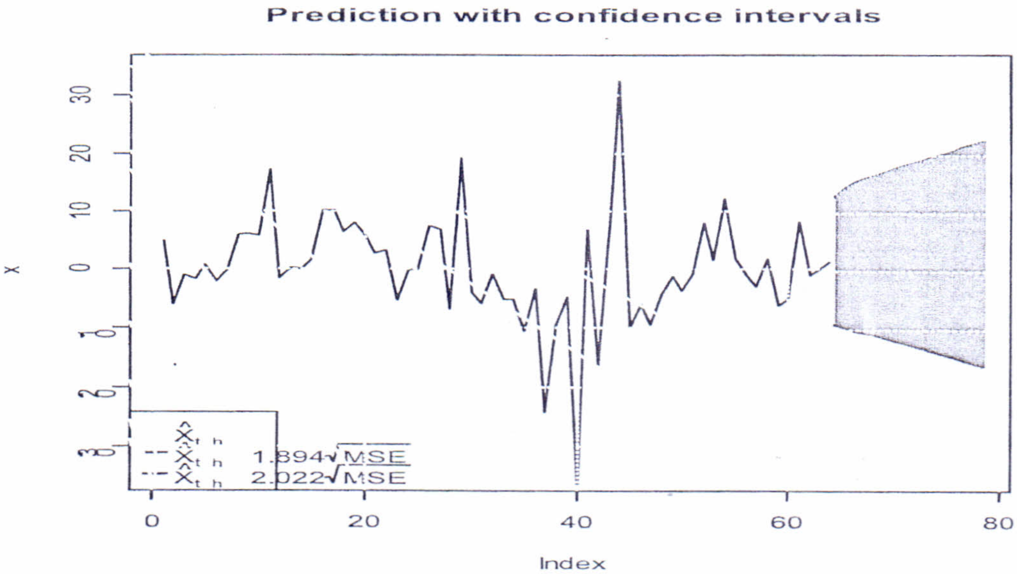

Figure 7: Out-of-sample forecasts of SNORMARMA(1, 2)-GARCH(1, 1)

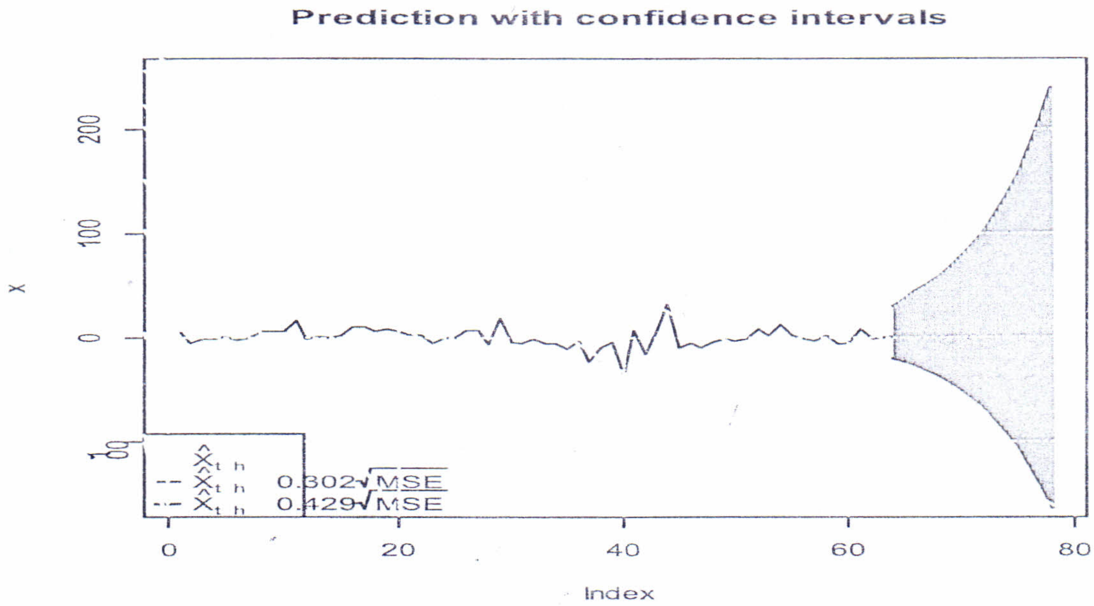

Figure 8: Out-of-sample forecasts of SSTD ARMA(1, 2)-GARCH(1, 1)

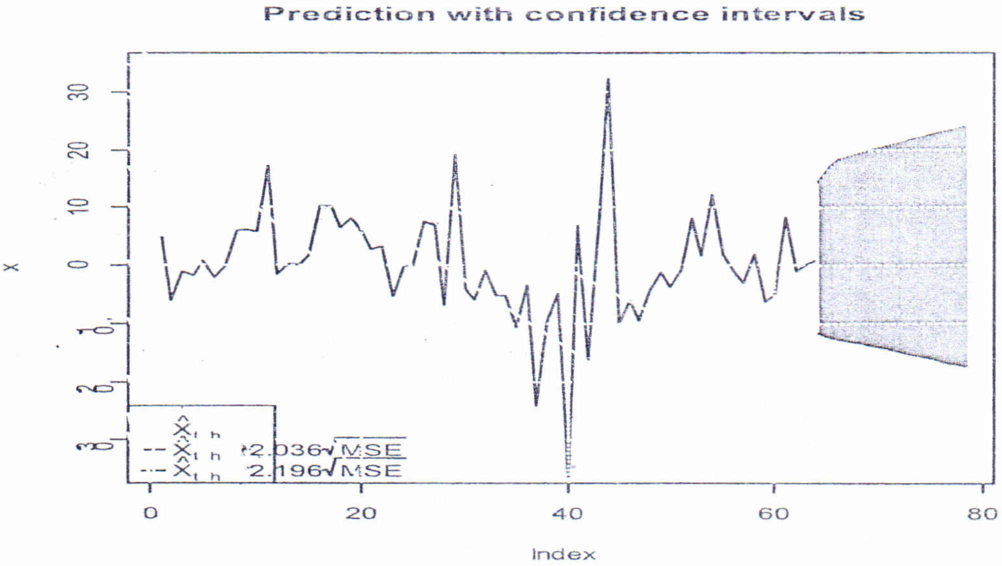

Figure 9: Out-of-sample forecasts of SGED ARMA(1, 2) -GARCH(1, 1) .