5/N6
L
36
L025
.96
2009

UNIVERSITY OF LAGOS

Inaugural Lecture Series 2009

UNIVERSITY OF LAGOS

TOPIC:

MATHEMATICS: IS IT ALL ABOUT NUMBERS? SOME SELECTED CONTRIBUTIONS AS CASE STUDIES

By
PROFESSOR OLUSOLA OLUFEMI AJAYI

University of Lagos Press

MATHEMATICS: IS IT ALL ABOUT NUMBERS? SOME SELECTED CONTRIBUTIONS AS CASE STUDIES

An Inaugural Lecture Delivered at the University of Lagos
Main Auditorium on Wednesday, 17th of June, 2009

DONATION
University Of Lagos Library
G & E
Donor: Unilag Press
Date: 12/11/2009

By

PROFESSOR OLUSOLA OLUFEMI AJAYI
B.Sc.(Special)(Hons), M.Phil (London), Ph.D(Sheffield)
Professor of Mathematics

Department of Mathematics
Faculty of Science
University of Lagos
Akoka – Yaba,
Lagos, Nigeria

University of Lagos Press

UNIVERSITY OF LAGOS LIBRARY

© Olusola Olufemi Ajayi, 2009

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise without the prior permission of the author.

Published 2009

DONATION
University of Lagos Library
Donor: Unilag Press
Date: 12/1/2009

By

University of Lagos Press
Unilag P. O. Box 132,
University of Lagos,
Akoka – Yaba,
Lagos, Nigeria

ISSN 1119-4456

UNIVERSITY OF LAGOS LIBRARY

Preamble

The Vice-Chancellor, Deputy Vice-Chancellor (Management Services), Deputy Vice-Chancellor (Academic and Research), the Registrar, Dean of Science and other Deans, Members of Senate, Distinguished Ladies and Gentlemen.

I find myself here before you today because in my early years I was, it would seem, quite familiar with numbers. The words "this boy is a mathematician, a Chike Obi" kept ringing in my ears. Now, as I approach the winter of my sojourn on this planet I find myself asking, mathematics: is it all about numbers? Please come along with me and let us find out.

Man and Numbers

There is a general consensus that Mathematics is a language. Nevertheless, there is some difficulty in speaking mathematics like one would speak some other languages. This difficulty arises from the fact that in mathematics we express truths only, and nothing but the truth. Indeed, in mathematics there is no room for laxity in the expression of truths.

As a rule, we use symbols in expressing these truths. Even though we also use words to express these truths it does not in any way diminish the difficulty in speaking mathematics. In spite of the difficulties in speaking mathematics, it may still be used to explore our world. In this lecture we will attempt to show how.

The basic foundation for mathematics is the number and the story of its development is quite exciting. At a stage in man's development he was confronted with the need to express his thoughts about quantities. For instance, he needed to know how many sheep he took out for grazing and how many he brought back home. Having developed a means of expressing quantities he soon felt the need of comparing them. These challenges brought man's ingenuity into play even in his early days. The ingenuity persists till today. Man progressed from his familiarity with the basic idea of counting, that is, the counting numbers, to the present day state of advanced mathematics. The history of

UNIVERSITY OF LAGOS

the development of the use of numbers is fascinating but it is not the subject matter of this lecture. We will therefore not pursue the matter further.

Let us take a trip back to our early days in the primary school. I remember, with nostalgia, the chorus that reverberated throughout the classroom when the teacher asked the very simple and elementary question: "what is *2 - 3*?" With the air of impeccable confidence the answer rang out from every corner of the classroom, *ko ṣe ṣe* (it is not possible). Indeed, as long as we confine ourselves to the set of counting numbers {1, 2, 3, . . . } then indeed *2 - 3* has no answer! Man soon discovered that there was, in fact, an answer. All he did was to figure out the negative counting numbers. One may ask the irreverent question: did God create these negative numbers? Before too long the inadequacy of these numbers (the counting numbers and the negative counting numbers) stared man in the face. It was a great challenge but he didn't give up. He merely went on to discover the set of rational numbers. Or should one say that the set of rational numbers was revealed to him?. By the way, the set of rational numbers is composed of numbers of the form $\frac{a}{b}$ where *a* and *b* are integers. For example $\frac{1}{2}$ is a rational number. We can go on in this manner to see how man was confronted at each stage of his development with difficult situations and how man's ingenuity came to the rescue but space and time do not permit. Nonetheless, even the shortest of the history of numbers will be incomplete without a mention of the great mathematician, Betrand Russell. Not too long after the discovery of the set of rational numbers man realised that other kinds of numbers were still lurking in the dark. One thing that was not in doubt was that not all numbers can be written in the form $\frac{a}{b}$ but the proof was rather elusive. It took the genius of Betrand Russell to provide a proof, a proof so short and elegant that every one wondered why they did not think of it first. It is said that the proof occurred to

Russell at the close of work, on his way home. You know what he did? In his own words: "I held my head with my two hands to ensure that my head did not fall down and I walked far away from the road to ensure that I did not get knocked down until I have put down the proof".

Let us cut a long story short but not before we take one more example. I am sure if I ask this august audience the solution of the equation

$$x^2 - 1 = 0 \qquad (0.1)$$

I will be overwhelmed with the answer, namely

$$x = +1 \text{ or } x = -1.$$

Yes, indeed! you are all correct. Now let me change this question slightly to have

$$x^2 + 1 = 0. \qquad (0.2)$$

and ask the question again: what is *x*? Notice that I have only changed the – in equation (0.1) to + to get Equation (0.2). Do I hear the chorus again? *ko ṣe ṣe*! Yes, it would appear it is not possible to find an answer to the question after such a minor, seemingly insignificant change. The fact, however, is that Equation (2) led to the postulation of what we now call complex numbers, thus opening up a vast new area of mathematics. Man, once more, had a revelation and made an impossible solution possible. So much for numbers! Is mathematics all about numbers? I will try to provide an answer in the following three sections.

As mentioned earlier, the truths of mathematics are expressed using symbols and words. It is difficult to express them in words only and it is also difficult to express them in symbols only. We have to use an appropriate mix of words and symbols. I will certainly use an appropriate mix of words and symbols. But I am not going to start teaching you the symbols that we need in this lecture. Do I hear "thank God"? The lecture has been written such that you may read the words only and ignore most of the symbols without much difficulty in comprehending the subject matter. Is mathematics all about numbers?

1. Mathematics and Welding

Each one of us must have at one time or the other seen the panel beater or the welder joining two metals together by the process of oxy-acetylene welding in the case of the panel beater or arc welding in the case of the welder. The process of joining two metals together is simple and straightforward. In the case of arc welding which is the subject matter of this section, the two metals are heated by the discharge of electric current through an electrode melting the metals in the process. The motion of the molten metals in the weld pool determines the heat and mass transfer in it. Consequently, the fluid motion in the weld pool is an important factor in determining the chemical reactions between and the fusion of the metals being joined together. It has been demonstrated that, primarily, the motions in the weld pool are induced by the electromagnetic force arising from the discharge of an electric current through the electrode. We will see shortly that the force that drives the motion of the fluid is due to the interaction of the electric field and its associated magnetic field which gives rise to the ($J \times B$) force, commonly known as the Lorentz force. There have been many attempts to throw light on the role played by the Lorentz force in the motion of the fluid within the weld pool but these attempts were rather too idealized to successfully describe the practical welding situation. The most realistic attempt was made by Sozou & Pickering (*Journal Fluid Mechanics*, 1976, 73) who assumed the weld pool to be hemispherical in shape and supposed the electric current to be discharged into the fluid through a point. Their approach predicted a flow pattern which is compatible with observations in the weld pool but quantitative agreement between the theory and practice was not quite good. This situation is not altogether surprising because, as stated earlier on, the electric current is discharged into the weld pool through an electrode which differs quite significantly from a point. Therefore, in order to correctly understand the motion of the fluid within the weld pool it is necessary to account for the finiteness of the area through which the current is discharged into the fluid. We shall in the following take account of the finiteness of the electrode in our attempt to

obtain a deeper and more realistic insight into the welding process. Is mathematics all about numbers?

In the next few paragraphs it will be difficult to communicate without expressing the truths of the analysis in mathematical symbols. I will, however, endeavour to keep such symbols to the minimal level.

Ajayi, Sozou and Pickering (Journal Fluid Mechanics, 1984, 148) considered the fluid motions within a weld pool whose shape was assumed hemispheroidal such that the plane boundary of the fluid forms a free surface, except for a circular electrode of radius k whose centre is coincident with the centre of the equatorial plane of the hemispheroid. We shall in the next paragraph define the configuration of the problem and the coordinate system which we employed.

On account of the geometry of the problem the oblate spheroidal coordinates were employed in tackling the problem. The oblate spheroidal coordinates (μ, ζ, ϕ) form a curvilinear coordinate system in its own right but for the purpose of this lecture we find it desirable to relate them to the cylindrical polar coordinates (x, ϖ, ϕ) so that we may all be carried along. The relationships are as follows:

$$x = k\mu\zeta$$

$$\varpi = k(1-\mu^2)^{\frac{1}{2}}(\zeta^2+1)^{\frac{1}{2}},$$

$$\phi = \phi$$

where $-1 \le \mu \le 1,\ 0 \le \zeta \le \infty,\ 0 \le \phi \le 2\pi$.

The origin of the coordinate system is at the centre of the circular electrode, and the positive x-axis points into the fluid along the axis of symmetry. The surface of the bowl is given by $\zeta = \zeta_0,\ 1 \ge \mu \ge 0$; the electrode is described by $\zeta = 0,\ 1 \le \mu \le 0$; and the free surface is given by $\mu = 0,\ 0 < \zeta \le \zeta_0$. It may be noted that large values of ζ_0 are associated with a small electrode while small values correspond to a large electrode. Figure 1.1 shows an axial section of the configuration studied for the case $\zeta_0 = 1$.

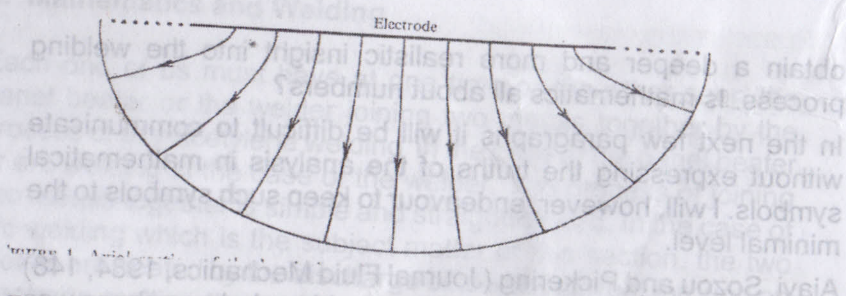

Figure 1: *Axial section of the hemispheroidal bowl for the case $\zeta_0 = 1$. The electrode is labelled, the dotted lines represent the free surface and the curves show the direction of current flow for the case where the electrode is at a fixed potential.*

The equation of motion of the problem is given by (1.1):

$$-\rho v\times \operatorname{curl} v = -\nabla\left(p+\tfrac{1}{2}\rho v^2\right)+\nu\rho\nabla^2 v + j\times B \qquad (1.1)$$

where ρ is the fluid density, v the velocity and p the pressure. Equation (1.1) must be supplemented by an equation which expresses the fact that during the motion of the fluid no matter is created or destroyed. This requirement may be satisfied by defining the velocity in terms of the stream function ψ as in equation (1.2):

$$v = \frac{e_\mu}{k^2\left(\zeta^2+\mu^2\right)^{\frac{1}{2}}\left(\zeta^2+1\right)^{\frac{1}{2}}}\frac{\partial\psi}{\partial\zeta} - \frac{e_\zeta}{k^2\left(\zeta^2+\mu^2\right)^{\frac{1}{2}}\left(\zeta^2+1\right)^{\frac{1}{2}}}\frac{\partial\psi}{\partial\mu} \qquad (1.2)$$

We will shortly say a word about the term on the left hand side of equation (1.1). Meanwhile, let us note that the last term on the right hand side of that equation is the Lorentz force. This is the driving force and so it is easy to see that the induced fluid motion arises from the interaction of the electric current discharged through the electrode with its associated magnetic field. If we suppose that the electrode is raised to a certain potential thus allowing electric current to flow into the fluid and we assume that

the electromotive force induced by the motion of the fluid is negligible then the current discharged into the fluid is given by Equation (1.4):

$$\Phi = \frac{J_o \cot^{-1}\zeta}{2\pi k\sigma} \qquad (1.3)$$

$$j = -\sigma\nabla\Phi = \frac{J_0}{2\pi k^2\left(\zeta^2+\mu^2\right)^{\frac{1}{2}}\left(\zeta^2+1\right)^{\frac{1}{2}}}e_\zeta \qquad (1.4)$$

where Φ is the electrostatic potential, J_0 the total current discharged into the fluid and σ is the conductivity of the fluid. It is a simple matter to show that the associated magnetic induction B is given by equation (1.5):

$$B = \frac{\chi J_0(1-\mu)}{2\pi k\left(\zeta^2+1\right)^{\frac{1}{2}}\left(1-\mu^2\right)^{\frac{1}{2}}}e_\phi. \qquad (1.5)$$

To determine the velocity field we must solve Equation (1.1) subject to the constraints defined by equation (1.2). The task is not easy. In fact, it poses a formidable problem. Indeed at the present moment there isn't a method for obtaining a general analytic solution to the problem. In the literature, the most popular approach to attack problems of this nature is to replace the differential coefficients of Equation (1.1) by their finite-difference equivalents. In so doing the question of instability of the numerical computation often arises. This is not quite surprising because that equation may change character within the solution space. For example, Sozou & Pickering in their paper on the development of the flow field of the round laminar jet, a problem not unrelated to that under discussion here, found that the governing equations were elliptic within some region but become parabolic in some other region. In the present analysis we adopt a solution method which is different from the usual run of the mill. In order to avoid some discomfort to some members of this august audience we shall omit the details of the derivation of our equations. Suffice it to say that when we use equations (1.2), (1.4) and (1.5) in equation (1.1) we obtain the fourth order nonlinear partial differential equation define by equation (1.6):

$$D^2\left[\frac{D^2\Psi}{z^2+\zeta_0^{-2}\mu^2}\right]+K\left\{\left(z^2+\zeta_0^{-2}\right)\frac{\partial\Psi}{\partial\mu}\frac{\partial}{\partial z}\left[\frac{D^2\Psi}{\left(z^2+\zeta_0^{-2}\mu^2\right)\left(z^2+\zeta_0^{-2}\right)}\right]\right.$$

$$\left.-\left(1-\mu^2\right)\frac{\partial\Psi}{\partial z}\frac{\partial}{\partial\mu}\left[\frac{D^2\Psi}{\left(1-\mu^2\right)\left(z^2+\zeta_0^{-2}\mu^2\right)}\right]\right\}=\frac{z(1-\mu)}{z^2+\zeta_0^{-2}} \tag{1.6}$$

where $D^2=\left(z^2+\zeta_0^{-2}\right)\frac{\partial^2}{\partial z^2}+\left(1-\mu^2\right)\frac{\partial^2}{\partial\mu^2}$. We have in effect turned a second order differential equation into a fourth order differential equation. If you permit me to put this situation in common parlance, it is like having double trouble and then going on to double the double trouble. We have, it would appear, made a difficult problem even more difficult. As earlier stated rather than follow the usual method of solution we introduce a procedure whereby the equations of the problem are solved semi-analytically. In this scheme of things the numerical computation is reduced to finding the essential part of the solution ($F_n(z)$ as we shall see later) by the elementary process of numerical integration. But first let us discuss the groundwork. It is well known that ordinary differential equations are more tractable than partial differential equations. For this reason we shall convert the partial differential equations of equation (1.6) into ordinary differential equation by writing the stream function Ψ in the form given in equation (1.7):

$$\Psi=\sum_{n=1}^{\infty}F_n(z)I_{2n+1}(\mu), \tag{1.7}$$

where $I_{2n+1}(\mu)$ is the Gegenbauer's polynomial which is related to the Legendre's polynomial by equation (1.8):

$$I_{2n+1}(\mu)=\frac{P_{2n-1}(\mu)-P_{2n+1}(\mu)}{4n+1}=\frac{\left(1-\mu^2\right)P'_{2n}(\mu)}{2n(2n+1)}. \tag{1.8}$$

Here and in subsequent expressions a prime denotes differentiation with respect to the argument of the function. We may now eliminate the functions of μ from equation (1.6). On substituting (1.7) into (1.8) and making use of the following equation,

$$\left(1-\mu^2\right)I''_{2n+1}(\mu)+2n(2n+1)I_{2n+1}(\mu)=0.$$

we obtain the equation of motion in terms of an ordinary differential equation defined by equation (1.9):

$$\begin{aligned}\sum_{m=1}^{\infty}\{\{(\zeta_0^{\,2}z^2+1)[(z^2+\zeta_0^{-2}\mu^2)G''_m-4zG'_m]-2[(2m+3)(m-1)(\zeta_0^{\,2}z^2+\mu^2)\\+6\mu^2-2]G_m\}I_{2m+1}-4\mu(1-\mu^2)G_mI'_{2m+1}\\+KF_mI'_{2m+1}\sum_{s=1}^{\infty}[(\zeta_0^{\,2}z^2+\mu^2)G'_s-\frac{2\zeta_0^{\,2}z}{\zeta_0^{\,2}z^2+1}(2\zeta_0^{\,2}z^2+\mu^2+1)G_s]I_{2s+1}\\-KF'_mI_{2m+1}\sum_{s=1}^{\infty}[(\zeta_0^{\,2}z^2+\mu^2)I'_{2s+1}+\frac{2\mu}{1-\mu^2}(\zeta_0^{\,2}z^2+2\mu^2-1)I_{2s+1}]G_s\\=\frac{z}{\zeta_0^{\,2}z^2+1}(\zeta_0^{\,2}z^2+\mu^2)^2(1-\mu).\end{aligned} \tag{1.9}$$

where the functions $G_s(\zeta)$ are defined by equation (1.10):

$$G_s=(z^2+\zeta_0^{-2})F''_s-2s(2s+1)F_s. \tag{1.10}$$

Finally, we multiply (1.9) by $\frac{I_{2n+1}(\mu)}{\left(1-\mu^2\right)}$ and integrate both sides of the resulting equation with respect to μ over the interval (0, 1), once more the algebra is omitted, the final result is equation (1.11):

$$\begin{aligned}a_1G''_n+a_2G'_n+a_3G_n+a_4G''_{n+1}+a_5G_{n+1}+a_6G''_{n-1}+a_7G_{n-1}\\+K\sum_{m=1}^{\infty}\sum_{s=1}^{\infty}[F_m(a_8G'_s+a_9G_s)+a_{10}F'_mG_s]=a_{11}\end{aligned} \tag{1.11}$$

where $a_1,\ldots,a_7$ and are functions of ζ_0, n and z, and a_8, a_9 and a_{10} are functions of ζ_0, m, n, s and z. To be sure equation (1.11) is not nearly a simple equation. However, the good news is that it is an ordinary differential equation and fortunately we are able to construct its solution albeit numerically.

In order that the solution we construct may indeed describe the

UNIVERSITY OF LAG

fluid motion of the weld pool that we have in mind, it is necessary that appropriate conditions be imposed on our solution. In other words we need to solve equation (1.6) subject to the appropriate boundary conditions. Since the surface of the bowl, that is z=0, and the part of the electrode in contact with the fluid, that is z=1, are solid we must impose the no slip condition. This means that the fluid in contact with these surfaces must have the same velocity as the surfaces, in this case each of the velocities must be zero. In addition, at the free surface, that is $\mu = 0$, the normal component of the velocity and the shear stress are zero. These conditions may be written in terms of the stream function as given in equations (1.12)-(1.17):

$$\Psi(\mu,0)=0, \tag{1.12}$$

$$\Psi_z(\mu,0)=0, \tag{1.13}$$

$$\Psi(\mu,1)=0, \tag{1.14}$$

$$\Psi(\mu,1)=0, \tag{1.15}$$

$$\Psi(0,z)=0, \tag{1.16}$$

$$\Psi_{\mu\mu}(0,z)=0. \tag{1.17}$$

The judicious choice of Ψ in equation (1.7) ensures that conditions (1.16) and (1.17) are automatically satisfied, and (1.12) – (1.15) will be satisfied provided equations (1.18)-(1.21) hold:

$$F_n(0)=0, F_n'(0)=0, F_n(1)=0, F_n'(1)=0. \qquad (1.18)\text{-}(1.21)$$

The solution method discussed above, we wish to point out, is not at the moment the sort of material to be found in textbooks. It should also be pointed out that the approach adopted here can easily be modified to accommodate the case where the solid electrode is replaced by a free surface where the shear stress is zero, reminiscent of the work of Sozou & Pickering.

It will be pointless to attempt to solve equations (1.10) and (1.11) analytically, moreover a direct numerical solution is also out of the question. We therefore solved these equations by an iterative

UNIVERSITY OF LAG

process. First, we assumed that the functions $F_n(z)$ are known and solved (1.11) for the functions $G_n(z)$; these results are then used to obtain a new approximation to $F_n(z)$. This process is continued until convergence is obtained; here we assume that there is convergence when two successive iterations produced the same output to the third significant figure for all G_n at all nodal points.

For the purpose of this lecture, a detailed discussion of the results will be out of place. Nevertheless, it is necessary to say a word or two about what all this mathematics has achieved. We carried out computations for values of ζ_0 ranging from 0.5 to 100. These values correspond to the situations where the size of the electrode ranges from large to small thus giving us an insight into the fluid motions of the weld pools over a wide range of the electrode size. Details of the structure of the velocity fields are shown in Figures 1.2, 1.3 and 1.4 the arrows on the solid curves show the direction of motion of the fluid.

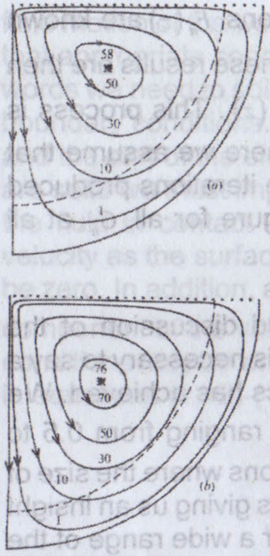

Figure 1.2: *Meridional section of the flow field for the case $\zeta_0 = 100$; (a) linear case; (b) K=80.*

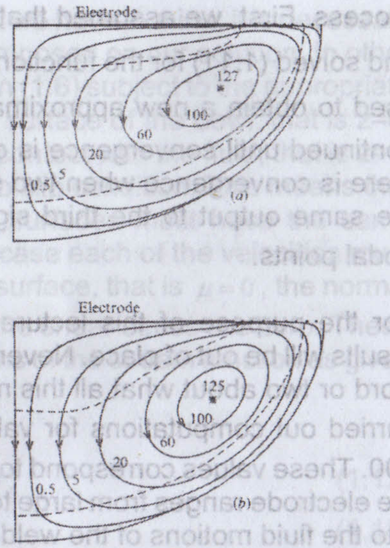

Figure 1.3: *Meridional section of the flow field for the case $\zeta_0 = 1$; (a) linear case; (b) $K = 10^5$.*

If vortices exist within the weld pool, this will be a fortunate occurrence because they have the potential to enhance heat and mass transfer within the pool. With regard to the existence of vortices, our results are not conclusive even though in one of our computations vortices were present but as K and the nonlinearities of the flow field increase, they become smaller and eventually disappear. Any interested researcher may wish to note that the analysis presented here is based on the assumption that the free surface is flat and thus our solution is valid provided the deviation of the free surface from the flat position is small. The calculation of the deformation when the flow field is nonlinear is rather involved and has not been pursued here. Estimates of the deformation have, however, been given by Craine & Weatherill

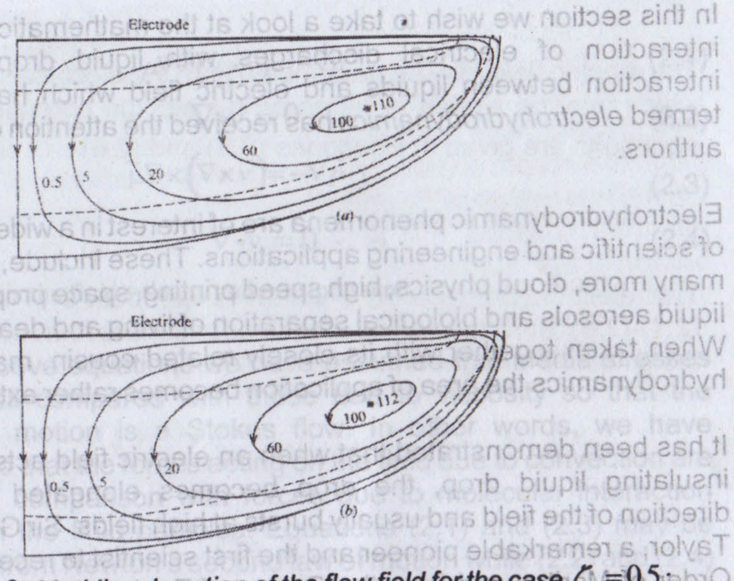

Figure 1.3: *Meridional section of the flow field for the case $\zeta_0 = 0.5$; (a) linear case; (b) $K = 10^6$.*

(*Journal of Fluid Mechanics*, 1980). They found that for the sort of currents considered in this work the maximum deformation is not large. This gives us some assurance that the above analysis is a close model of the electro-magnetohydrodynamics of the weld pool.

To summarise, we have shown above how mathematics can be used to investigate some basic principles of welding. It is hoped that this work we contribute to a better understanding of the welding process. Is mathematics all about numbers?

2. Mathematics and Drops

We are all very familiar with drops and bubbles because they exist in numerous phenomena in everyday life. The bubbles in our fizzy drinks or beer, and drops that herald rainfall are common examples. With respect to rainfall the frequently accompanying lightning is also well-known to all.

UNIVERSITY OF LAGO
LIBRARY

In this section we wish to take a look at the mathematics of the interaction of electrical discharges with liquid drops. The interaction between liquids and electric field which has been termed *electrohydrodynamics* has received the attention of many authors.

Electrohydrodynamic phenomena are of interest in a wide variety of scientific and engineering applications. These include, among many more, cloud physics, high speed printing, space propulsion, liquid aerosols and biological separation of living and dead cells. When taken together with its closely related cousin, magneto-hydrodynamics the area of application becomes rather extensive.

It has been demonstrated that when an electric field acts on an insulating liquid drop, the drop becomes elongated in the direction of the field and usually bursts at high fields. Sir Geoffrey Taylor, a remarkable pioneer and the first scientist to receive the Order of Merit award from the Queen of England, predicted in one of his pioneering and illuminating scientific contributions that when the liquid in which the drop is immersed is conducting, the tangential stress of the electric field stress on the surface of the drop generates a flow field within the drop as well as in its surroundings (Proc. Roy Soc. Lond. A **111**, 1966). This prediction was later confirmed experimentally by Torza, Cox and Mason (Phil. Trans. A **269**, 1971) but these authors found that in most cases the measured deformations were greater than the theoretical prediction. There was, at that time, no accepted explanation of the discrepancy. For this reason, Ajayi (Proc. Roy Soc. Lond., A. **364**, 1978) re-examined the basis of Taylor's theory by paying particular attention to the boundary conditions to be satisfied on the deformed drop. We noted that in the first instance the electric field induced a deformation of the drop as well as a fluid motion within and outside the drop. Surely, it is unlikely that the interaction of the deformed drop with the induced motion will be negligible. Let us take a brief look at the mathematics in as simple a form as one can make it: The relevant equations of motion are given in Equations (2.1) – (2.4):

$$\mu\nabla\times(\nabla\times v) = -\nabla p \tag{2.1}$$

$$\nabla\bullet v = 0 \tag{2.2}$$

$$\tilde{\mu}\nabla\times\left(\nabla\times\tilde{v}\right) = -\nabla\tilde{p} \tag{2.3}$$

$$\nabla\bullet\tilde{v} = 0 \tag{2.4}$$

where v is the fluid velocity and p its pressure .

In the above equations we have assumed that inertia stresses are small compared with those due to viscosity so that the induced motion is a Stokes flow. In other words, we have assumed that the forces acting on the fluid due to convection are small in comparison with forces due to molecular interaction between the fluid particles. Equations (2.1) and (2.3) may be derived from Newton's second law of motion while (2.2) and (2.4) may be considered as a mathematical representation of the assumption that matter is neither created nor destroyed during the motion.

The motion of the fluid is determined by solving these four equations for the velocities inside and outside the drop. Thereafter we may obtain the pressure within the fluid from equations (2.1) and (2.3). It will be out of order and perhaps unkind to bother the greater percentage of this audience with the detailed solution of these equations. Suffice it to say that these equations remain valid for all fluid problems as long as the fluid is viscous and incompressible. A fluid is said to be viscous if it is more like a syrup than water and is incompressible if its density never changes. When we have obtained the solutions to these equations for a particular geometry, these solutions remain valid for all fluid motions having that geometry. In other to discuss a particular problem it is important, indeed necessary, to specify the conditions that distinguish that problem from any other one. For the problem at hand the conditions, called *boundary conditions*, will be stated later.

UNIVERSITY OF LAGOS
LIBRARY

The electric field may be determined from the electrical equations which may be written in terms of the electric potential, ϕ, the equations are given in Equations (2.5) and (2.6)

$$\nabla^2 \phi = 0 \tag{2.5}$$

$$\nabla^2 \tilde{\phi} = 0 \tag{2.6}$$

The electrohydrodynamic problem is composed of equations (2.1)-(2.6). Solving the fluid motion equations is relatively simple, the solutions are given in Equations (2.7) and (2.8):

$$v = \sum_n b_n \{(2-n) r^2 \nabla \pi_n + 2(n+1) r \pi_n\} - a^2 \sum_n n c_n \nabla \pi_n , \tag{2.7}$$

$$\tilde{v} = \sum_n B_n \{(n+3) r^2 \nabla \omega_n - 2 n r \omega_n\} + a^2 \sum_n (n+1) C_n \nabla \omega_n , \tag{2.8}$$

but satisfying the boundary conditions is more formidable, and it is here that our analysis goes beyond that of Sir Geoffrey Taylor.

The Boundary Conditions

Taylor assumed that the drop departs only slightly from a spherical shape, and so the boundary conditions which strictly ought to be satisfied at the surface of the drop, $r = a(1 + f(\theta))$, may be applied as if the surface were the sphere r=a. To proceed beyond Taylor's analysis we assumed that in the steady state the drop surface is given by Equation (2.9):

$$r = a\left(1 + \varpi f_1(\theta) + \varpi^2 f_2(\theta) + \ldots\right) \tag{2.9}$$

where ϖ is a small unknown parameter. The introduction of this parameter further complicates the problem because equations (2.1)-(2.6) are no longer sufficient to determine the solution to the problem. This difficulty may be resolved by introducing an additional equation which addresses the question of the balance of the normal stress on the surface of the drop. Here we do not, for the sake of prudence, give a detailed mathematical exposition of these equations, indeed some would say it will be unkind to do

so. However, it is worthy to note that while the tangential stresses are responsible for the induced fluid motions within the drop and outside it, the normal stresses are in fact responsible for the deformation of the drop. The satisfaction of the boundary conditions to be imposed at the surface given by equation (2.9) is a next to impossible task and some ingenuity is required to make the problem tractable. We overcome this difficulty by transforming all boundary conditions imposed at the surface (2.9) into equations satisfied at r=a. It must be stressed that this is not the same as replacing the actual surface by a sphere as was done by Taylor.

Deformation of the Drop

Care must be exercised in determining the deformation of the drop since the zeroth-order normal stress determines the first-order displacement, $r = a\varpi f_1$, of the surface, and the first-order stress similarly determines $r = a\varpi f_2$. It may be readily shown that, correct to terms of order ϖ^2 the curvature of the drop is given by Equation (2.10):

$$p_1^{-1} + p_2^{-1} = \left(2 - \varpi L f_1 - \varpi^2 \{L f_2 - 2 f_1 (L f_1 - f_1)\}\right)/a , \tag{2.10}$$

where the operator L is defined in (2.11):

$$Lf = \frac{d}{d\eta}\left\{(1-\eta^2)\frac{df}{d\eta}\right\} + 2f . \tag{2.11}$$

To the lowest order in ϖ the boundary condition is given by Equation (2.12):

$$(P_{0n})_E + (P_{0rr})_H - (\tilde{P}_{0rr})_H = -T\varpi a^{-1} L f_1 , \tag{2.12}$$

and, to the next order we have Equation (2.13):

$$\begin{aligned} &(P_{1n})_E + (P_{1rr})_H + a f_1 \frac{\partial}{\partial r}(P_{0rr})_H - 2\frac{\partial f1}{\partial \theta}(P_{0r\theta})_H - (\tilde{P}_{1rr})_H \\ &- a f_1 \frac{\partial}{\partial r}(\tilde{P}_{0rr})_H - 2\frac{\partial f_1}{\partial \theta}(P_{0r\theta})_H = -T\varpi a^{-1}\left(L f_2 - 2 f_1 (L f_1 - f_1)\right) \end{aligned} \tag{2.13}$$

The terms on the left of (2.12) involve only the Legendre function

$P_2(\eta)$; it follows that f_1 is of the form $\lambda P_2(\eta)$. With further analysis (details omitted) λ is found to be given by the next equation:

$$4T\varpi\lambda a^{-1} = \Delta\left\{\tfrac{1}{3}\left(1+R^2-2QR^2\right)+\tfrac{1}{5}R\left(1-QR\right)\left(2\mu+3\tilde{\mu}\right)/\left(\mu+\tilde{\mu}\right)\right\}.$$

The Taylor's discriminant Λ may be retrieved from our equation (2.14):

$$\varpi = \tfrac{1}{12}a\Delta/T = \tfrac{1}{12}\left(\varepsilon a/T\right)\left(3Z/(1+2R)\right)^2 \qquad (2.14)$$

After some manipulation we find that f_2 is given by Equation (2.15):

$$f_2 = \Lambda^2\left\{-\tfrac{1}{5}+\left(6K-\tfrac{1}{7}\right)P_2+\tfrac{26}{35}P_4\right\}$$
$$+\tfrac{1}{5}\Lambda R(1-QR)\left\{\left(3\tfrac{5}{2}-\tfrac{3}{2}W-\tfrac{3}{35}W^2P_2\right)+\tfrac{2}{5}\left(1+\tfrac{11}{21}W\right)P_4\right\}. \qquad (2.15)$$

Equations (2.14) and (2.15) together with (2.9) and $f_1 = \Lambda P_2$ give the shape of the deformed drop correct to terms of second degree in the perturbation parameter ϖ. In view of Equation (2.16),

$$\Lambda = (1-R)^2 + R(1-QR)\left\{2+\tfrac{3}{5}(2+3M)/(1+M)\right\} \qquad (2.16)$$

the deformed drop is certainly prolate if QR<1, but may be oblate if QR is sufficiently large. This is what was confirmed experimentally by Torza *et al.*(*Phil. Trans. Royal Soc.* Lond. A. **269**, 1971).

In conclusion, we have demonstrated that the discrepancy between Taylor's electrohydrodynamic theory and experiment may be partly explained by satisfying the boundary condition on the surface of the deformed drop more accurately than was previously done. The smallness of the expansion parameter ϖ precludes any great reduction in the discrepancy. Retaining higher order terms in the expansions for the field and motion is likely to improve the accuracy between theory and experiment but is unlikely to succeed in curing it. One way out of the dilemma would be to employ numerical methods.

In a related paper, Ajayi & Sozou (*Journal of Electrostatics*, **9**, 1981) investigated the stability of the axisymmetric equilibrium configuration of two oppositely charged membranes. They demonstrated that when the membranes are sufficiently close together and are charged beyond a certain level they stretch continuously until they touch at their centres. These results give us an insight into how two raindrops coalesce in rain clouds to form larger drops until they fall as raindrops. When these results are taken together with an earlier analysis of Ajayi (1976, J. West Afri. Sci. Assoc.) who demonstrated that the fluidity of two interacting drops in rain cloud enhances an increase in the collision efficiency of raindrops then it is easy to see how these results provide an illuminating insight into the formation of raindrops in rainclouds.

In this section we have been able to use simple mathematics to explore the interaction between discharges and drops/bubbles. *Is mathematics all about numbers?*

3. A Novel Approach to Finite Differences

In this section we wish to discuss a novel method for solving differential equations numerically. To avoid mathematical indigestion on the part of many, we shall only give a synopsis of the method but we will illustrate its use with two mathematical models of population growth. These models will be simple enough for anyone to follow and are chosen for relevance and clarity. In the process we shall give an opinion on the Nigerian Census results of the past half a century or so.

Differential equations may be used to describe many phenomena. Once the theory behind a phenomenon is known, it is often, but not always, possible to derive a differential equation which describes that phenomenon. But the solution of the resulting differential equation is, at times, a different matter entirely. As an illustration, let us consider the classical simple pendulum.

The simple pendulum consists of a small particle of mass m attached to one end of a light, inextensible string of length a while the other end of the string is fixed at some point O and the particle swings freely in a vertical plane, see Figure 3.1.

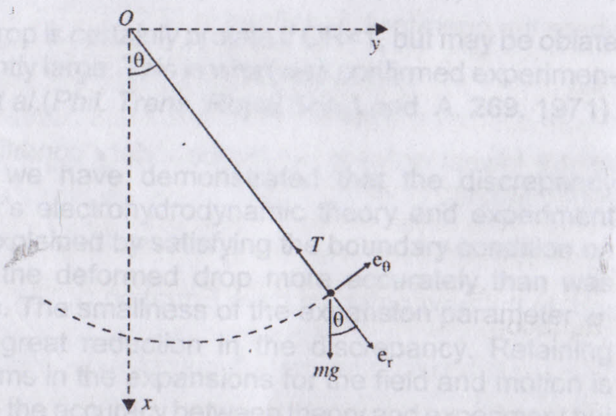

Figure 3.1: A *simple pendulum*

If we employ the plane polar coordinates (r, θ) and use the Newton's second law of motion we find,

$$ma\dot{\theta}^2 = -mg\cos\theta + T \qquad (3.1)$$

$$ma\ddot{\theta} = -mg\sin\theta. \qquad (3.2)$$

where T is the tension in the string and a dot denotes differentiation with respect to time t. It follows from Equation (3.2) that the motion of the pendulum is described by the differential equation,

$$\ddot{\theta} = -\omega^2 \sin\theta. \qquad (3.3)$$

I hope it does not surprise many to hear that Equation (3.3) is the equation of motion of a simple pendulum. If it does, then it needs to be pointed out that the very popular equation of motion of a simple pendulum, that is,

$$\ddot{\theta} = -\omega^2\theta, \qquad (3.4)$$

may be derived as an approximation of (3.3) if we assume that the movement of the particle from the vertical position is always small.

Even though Equation (3.3) looks quite innocuous, it is not possible to determine its general solution in terms of elementary functions. This difficulty is what gives rise to the need to use the approximate equation (3.4). We hope that with this simple example, we have made the point that it is not always possible to solve a differential equation. When faced with the situation in which an analytic solution of a differential equation is unavailable one way out of the difficulty is to use numerical methods. In this section we shall discuss a novel approach for solving differential equations numerically.

First, let us consider the present state of things. A popular approach for solving differential equations numerically is the use of finite differences which we illustrate with the following example. Suppose we wish to solve the equation,

$$a_2 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_0 y = f(x) \qquad (3.5)$$

in some interval of interest, (x_0, x_n) say, where, in general, a_2, a_1 and a_0 are functions of *x*. But, for clarity and simplicity, we shall assume each of those functions to be constant. We replace the differential coefficients with their equivalent finite differences and solve the resulting finite difference equation. It will be out of place to give details of the procedure. The essential point to note is that certain terms in the given equation are replaced by some other equivalent expressions.

We now wish to discuss a new approach to solving the equation numerically. We shall, for brevity and simplicity, only give a synopsis of the method omitting all details but we shall later illustrate its use and compare our result with analytical ones to demonstrate its accuracy.

Suppose we need to solve Equation (3.4) in the interval (x_0, x_n) subject to two boundary conditions

$$y(x_0) = \alpha, \quad y(x_n) = \beta. \qquad (3.5)$$

where α and β are specified constants. We, as usual, divide the interval (x_0, x_n) into *n* sub-intervals, as shown in Figure 3.2.

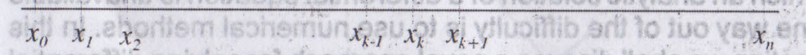

Figure 3.2

Instead of solving (3.4) in the interval (x_0, x_n) we replace the equation by

$$a_2 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_0 y = f(x_k), \qquad x_{k-1} \le x \le x_{k+1} \qquad (3.6)$$

and solve it in each of the n sub-intervals as defined in Figure 3.2 where the function *f*(*x*) is replaced by $f(x_k)$. It is elementary to solve Equation (3.6). In this manner we have made an otherwise intractable problem tractable in any chosen interval. Of course, what we now have is an approximation to the given problem. But, take note that *f*(*x*) is not being replaced by $f(x_k)$ over the entire interval of solution but in a specified interval only. In other words the approximation involved is local and not global. The error involved in the approximation can be cured by taking our sub-intervals small enough. One other obvious improvement is to replace *f*(*x*) by a refined approximation in the specified sub-intervals. It may be observed that, in effect, what we have is a generalized finite difference method and it is not too difficult to show that the age-old approach of using finite differences to replace differential coefficients is an approximation of our method.

The method discussed above has been successfully employed to tackle problems that are even nonlinear in character. One or two examples will suffice. Ajayi, O.O. and Ajose, S.O (*IEE Proceedings*, **135**, 1989) used the method to derive a semi-analytical procedure for computing the input impedance of a general, non-uniform transmission line; it was also employed by Uwanta (*Ph.D Thesis*, University of Lagos, 2005) to determine the unsteady nonlinear fluid motion induced by a heat point source in an incompressible, viscous fluid of infinite extent.

We have in the above paragraphs merely sketched the procedure involved in using our new method. We shall now apply the method to take a look at the perennial problem of the acceptability of Nigerian Census results and proffer a mathematical opinion.

The Nigerian Census Results

Our nation, *Good people, Great nation*, has on numerous occasions embarked, like other nations, on the arduous task of determining how many of us inhabit our good land. Each time the outcome of the exercise was greeted with dissatisfaction and complaints. I wish in these closing remarks to give a mathematical opinion of the past Nigerian Census results.

There has, for a long time, been keen interest in modelling the growth of populations. One of the earliest workers in this area was a reverend gentleman and economist, T.R. Malthus. His model was simple and straightforward. Let us take a look at it: Let $p(t)$ denote the population of a given species at any time t and $r(p, t)$ the difference between its birth rate and death rate. Malthus supposed that the rate of growth of the population is directly proportion to the population, that is,

$$\frac{dp}{dt} \propto p. \qquad (3.7)$$

Mathematically, we may write Equation (3.7) as

$$\frac{dp}{dt} = kp, \qquad (3.8)$$

where k measures the average growth rate per unit time per unit population. It follows that

$$\frac{dp}{dt} = rp. \qquad (3.9)$$

When r is a constant, that is, the growth rate is constant, then we have

$$\frac{dp}{dt} = ap(t), \quad x_0 \le x \le x_n, \qquad (3.10)$$

where a is the constant growth rate and (x_0, x_n) is the interval of interest. The last equation is the famous *Malthusian law of population growth*.

Let us look a little more closely at what Malthus had in store for us. Suppose that Adam and Eve *begat* their children, who in turn *begat* their own children and their children's children begat their own children and so on until the earth's human population was 2.518 billions in 1950 (UN figures). If we assume as has been well-documented that the world population has been increasing at the rate of about 2% per year since 1950. To obtain the subsequent human population all we need do is to solve Equation (3.10) subject to these known values. The solution is

$$p = (2,518,000,000)e^{a(t-1950)}. \qquad (3.11)$$

Equation (3.11) gives the world human population at any t after 1950. It is easy to see from this solution that the world population will grow without end.

Since we have obtained the analytic solution to our equation it will be superfluous to employ a numerical method to find the solution. But, as a simple illustration of our new method discussed earlier we shall determine a numerical solution and compare the solution with the analytic one. For this purpose we may in the interval of interest (1950, 2010), approximate Equation (3.10) by

$$\frac{dp}{dt} = ap(t_k), \quad x_k \le x \le x_{k+1}. \qquad (3.12)$$

On solving Equation (3.12) and eliminating the constant of integration involved, we find

$$p_{k+1} = p_k + a\left(x_{k+1} - x_k\right)p_k. \qquad (3.13)$$

The solution in the interval of interest (x_0, x_n) may be generated using (3.13). A comparison of the analytic solution and the numerical solution is given in Table 3.1. There is good agreement between the numerical and analytical values if the ratios of the two quantities are close to unity. It will be noticed in column 4 of Table 3.1 that our numerical solution agrees excellently with the analytical ones.

UNIVERSITY OF I

Table 3.1: *Comparison of analytic and numerical solutions of the Malthusian equation showing the world population*

Year	Analytic population	Predicted population	Analytic Predicted
1950	2518		
1955	2782.82	2782.79	1.00001
1960	3075.49	3075.43	1.00002
1965	3398.94	3398.84	1.00003
1970	3756.41	3756.26	1.00004
1975	4151.48	4151.26	1.00005
1980	4588.1	4587.81	1.00006
1985	5070.63	5070.26	1.00007
1990	5603.91	5603.45	1.00008
1995	6193.28	6192.7	1.00009
2000	6844.63	6843.92	1.0001
2005	7564.49	7563.62	1.00011
2010	8360.05	8359.01	1.00012
2015	9239.29	9238.04	1.00014
2020	10211.00	10209.5	1.00015
2025	11284.9	11283.1	1.00016
2030	12471.7	12469.7	1.00017
2035	13783.4	13781	1.00018
2040	15233	15230.2	1.00019
2045	16835.1	16831.8	1.0002
2050	18605.6	18601.8	1.00021
2055	20562.4	20557.9	1.00022
2060	22725	22719.8	1.00023
2065	25115	25109	1.00024
2070	27756.4	27749.4	1.00025
2075	30675.5	30667.5	1.00026
2080	33901.7	33892.5	1.00027
2085	37467.2	37456.6	1.00028
2090	41407.6	41395.5	1.00029
2095	45762.5	45748.7	1.0003

We are pleased with this excellent agreement but we need not be hilarious about it because the above example, a first order equation, is not a stringent test of the accuracy or even the efficiency of the new method but, at least in the meantime, it provides us a reassuring start. In view of the above, we shall for the rest of this section use our new method to obtain all numerical solutions employed in this section.

You will notice from Table 3.1 that the world population keeps increasing and with time the population will explode beyond what the human race can handle. When Malthus introduced his theory this scenario almost frightened life out of people because according to Malthus the human population will grow without end and as a result sooner or later, with emphasis on sooner, this earth will no longer be big enough to sustain all of us. Available food and other resources will not be nearly enough to sustain us all. The consequence can only be imagined.

The question of the accuracy of the Malthusian theory does arise. To answer this question let us compare the predicted Malthusian world population with the actual world population. Table 3.2 shows a comparison of the actual world population and the predicted ones using 1950 as our starting point. It will be noticed from column 4 of that Table that there is reasonably good agreement between the actual and predicted values up to about 1990, thereafter the agreement is no longer tenable.

Table 3.2: *Comparison of actual world population(in billions) with corresponding values predicted by the Malthusian theory using our method of solution*

Year	Actual Population	Predicted Population	Actual/ Predicted
1950	2.518	2.518	-
1955	2.755	2.78279	0.990013
1960	2.982	3.07543	0.969621
1965	3.335	3.39884	0.981218
1970	3.6925	3.75626	0.983026
1975	4.068	4.15126	0.979943
1980	4.4347	4.58781	0.966627
1985	4.831	5.07026	0.952811
1990	5.2636	5.60345	0.939351
1995	5.674	6.1927	0.91624
2000	6.0706	6.84392	0.887006
2005	6.4536	7.56362	0.853242
2010	-	8.35901	-

Fortunately, the explanation for this discrepancy seems quite simple. The fact is that the Malthusian model imposes no restriction on the population growth. It does not take account, for instance, of the effect of advances in medical services and improvement in agricultural practices which have led to an increase in life expectancy. Neither did it take into consideration such debilitating factors as war and epidemics. Other factors such as emigration and immigration were not taken into consideration by Malthus. It stands to reason that when these factors are woven into the Malthusian theory then we can expect some improvement in the theory of population growth. The logistic theory of population growth was introduced many years ago to address the shortcomings mentioned above. Our interest is to employ this theory to examine the validity of the various Census exercises carried out in our dear country. For this reason we shall merely state the logistic equation and map out the numerical procedure as stated at the beginning of this section. The logistic equation is given in Equation (3.14):

$$\frac{dp}{dt} = ap(t) - bp^2(t), \qquad x_o \le x \le x_n \tag{3.14}$$

where *b* is a parameter that accounts for factors militating against or encouraging the growth of a population. As previously mentioned, even though we need not solve this equation numerically, we shall do so on this occasion so as to illustrate our new method of solving differential equations numerically. To this end we re-write Equation (3.14) as given in (3.15):

$$\frac{dp}{dt} = ap(t_k) - bp^2(t_k), \qquad x_k \le x \le x_{k+1} \tag{3.15}$$

where we have divided the interval (x_0, x_n) into *n* sub-intervals and *k=0,1,2,. . .,n-1*. We then proceed to solve Equation (3.15) in each of the sub-intervals to determine the solution over the entire interval of interest. On solving Equation (3.15) and eliminating the constant of integration involved in the solution we find,

$$p_{k+1} = p_k + (t_{k+1} - t_k)(a - bp_k)\, p_k \,. \tag{3.16}$$

Equation (3.16) is used to generate the solution over the entire interval of interest. The solution method is as simple as that. We must, however, sound a note of warning. Even though the method is as simple as demonstrated in this example but when we go on to consider equations of higher order then new challenges do crop up. We have so far found these challenges surmountable even when the equation is of the fourth order. We shall now discuss the results.

Table 3.3 shows the values of the actual world population and the corresponding values predicted using our solution derived from equation (3.16). It will be observed that unlike the

Table 3.3: *Comparison of actual world population(in billions) with corresponding values predicted by the Logistic theory using our method of solution*

Year	Actual population	Predicted population	Actual/ Predicted
1950	2.518	2.518	1
1955	2.755	2.75542	0.999847
1960	2.982	3.01244	0.989896
1965	3.335	3.29013	1.01364
1970	3.6925	3.58953	1.02869
1975	4.068	3.91162	1.03998
1980	4.4347	4.25727	1.04168
1985	4.831	4.62725	1.04403
1990	5.2636	5.02218	1.04807
1995	5.674	5.44249	1.04254
2000	6.0706	5.8884	1.03094
2005	6.4536	6.35989	1.01473
2010	-	6.85667	-

Malthusian theory, the logistic model predicts the world population from 1950 to the present moment with acceptable accuracy. You may wish to note, in particular, the value, 6.85 billions predicted for 2010, the latest available world population is 6.706 for 2008. Table 3.4 shows a comparison of the actual population of the United States of America and corresponding values predicted using our solution derived from Equation (3.16)

Table 3.4 *Comparison of the actual population (in millions) of the United States with corresponding values predicted by the Logistic theory using our method of solution*

Year	Actual population	Predicted population	Actual/ Predicted
1790	3.92921	3.92921	1
1800	5.23663	5.33578	0.981417
1810	7.23988	7.22692	1.00179
1820	9.63845	9.75402	0.988151
1830	12.866	13.1034	0.981881
1840	17.0695	17.4949	0.97568
1850	23.1919	23.1716	1.00087
1860	31.4433	30.3767	1.03511
1870	38.5584	39.3124	0.98082
1880	49.3713	50.0813	0.985823
1890	62.9798	62.6206	1.00574
1900	76.2122	76.6492	0.994299
1910	92.23	91.6591	1.00623
1920	106.02	106.972	0.991104
1930	123.2	121.852	1.01107
1940	132.16	135.644	0.974318
1950	151.33	147.878	1.02334
1960	179.32	158.316	1.13267
1970	203.21	166.929	1.21735
1980	226.55	173.841	1.3032
1990	248.71	179.267	1.38737
2000	281.42	183.452	1.53402
2010		186.637	

for the years 1790 to 2000. The Table also gives the predicted population of USA for 2010. It will be observed that our solution is in a very good agreement with the population of America for a total of 160 years!

UNIVERSITY OF LAGOS LIBRARY

The discrepancy in the years after 1950 may be attributed to the fact that there has been a significant decline in the rate of growth of the population of that country which is now about 1.1% instead of about 3% in the years after 1790 which is the value used in our computation.

Now let us take a look at the case of the Nigerian population. To start with, we note that it would appear that the least controversial Census figure is that of 1953. We accepted that figure as correct and used it as the basis of our computation. Column 3 of Table 3.5 shows the predicted population (in millions) of Nigeria while the second column contains the few available corresponding values. The paucity of data gave us much difficulty in determining the values of the parameters *a* and *b*. Therefore, the results under consideration here must be viewed in that light. The results of our computation are presented in Table 3.5 which shows the predicted population of Nigeria from 1953 to 2012 as well as the few available Census results. It will be observed that with the exception of the 1991 Census result there is not much agreement between the predicted and the Census results. We note further that our predicted Nigerian population for 2009, 148.74 millions is in reasonable agreement with the UN-estimated Nigerian population, 148.24 millions for that year.

Table 3.5 *Comparison of the actual population (in millions) of Nigeria with corresponding values predicted by the Logistic theory using our method of solution*

Year	Actual population	Predicted population	Actual/ Predicted
1953	30.39	-	-
1954	-	31.2759	-
1955	-	32.1873	-
1956	-	33.1251	-
1957	-	34.0899	-
1958	-	35.0825	-
1959	-	36.1037	-
1960	-	37.1544	-
1961	-	38.2352	-
1962	-	39.3471	-
1963	55.7	40.491	1.37562
1964	-	41.6677	-
1965	-	42.8781	-
1966	-	44.1233	-
1967	-	45.4041	-
1968	-	46.7217	-
1969	-	48.0768	-
1970	-	49.4708	-
1971	-	50.9045	-
1972	-	52.3791	-
1973	-	53.8958	-
1974	-	55.4556	-
1975	-	57.0599	-
1976	-	58.7097	-
1977	-	60.4064	-
1978	-	62.1512	-
1979	-	63.9454	-
1980	84.7	65.7904	1.28742
1981	-	67.6876	-
1982	-	69.6383	-

UNIVERSITY LIBRARY

1983	-	71.6441	-
1984	93.7	73.7063	1.27126
1985	-	75.8266	-
1986	-	78.0064	-
1987	-	80.2474	-
1988	-	82.5512	-
1989	-	84.9194	-
1990	-	87.3538	-
1991	88.5	89.8561	0.984908
1992	-	92.4281	-
1993	-	95.0716	-
1994	-	97.7885	-
1995	-	100.581	-
1996	-	103.45	-
1997	-	106.399	-
1998	-	109.429	-
1999	-	112.542	-
2000	-	115.74	-
2001	-	119.027	-
2002	-	122.403	-
2003	-	125.871	-
2004	-	129.434	-
2005	-	133.093	-
2006	-	136.852	-
2007	-	140.712	-
2008	-	144.676	-
2009	-	148.747	-
2010	-	152.927	-
2011	-	157.219	-
2012	-	161.626	-

Our mathematical calculations therefore show that the Census results of 1963, 1980 and 1984 may well be an aberration while the exercise of 1991 was in all probability carried out with a large degree of accuracy. If we are correct in our analysis the accuracy of the 1991 Census exercise persuades us that honest and

patriotic people still inhabit our good land. Therefore, there is hope that indeed Nigeria may yet be a land of *Good people, Great nation!*

Is mathematics all about numbers? I hope we have been able to show in the foregoing that in the beginning were numbers and mathematics was built on numbers and mathematics goes beyond numbers.

Thank you for your kind attention.

Acknowledgements
To start with, let me apologise at the onset if you do not get mentioned in this acknowledgement. If you are not mentioned it is either due to amnesia on my part or because one is trying not to put down everything here. There is a time for everything. Your time will come. For now one wishes to focus largely on factors that contributed to one's academic growth.

At any point in time, our person is a relic of our encounters and interactions. I have been particularly fortunate in my encounters. I feel both a sense of gratitude, joy and satisfaction for having enjoyed a sum total of positive encounters and interactions. Without any sense of immodesty, in my early days, these encounters turned a bad apple into a good one! Right from my childhood one had the good fortune to have crossed the path of unforgettable individuals who provided encouragement which directed my trajectory along the path that had enabled me to stand here before you today. Sometimes the encouragement was in cash only, sometimes in words only and sometimes in cash and words.

In this context, my earliest vivid recollection is *broda Sura.* The encounter happened while I was still in the primary school. How can I forget *broda* Sura, a sign writer, for his six pence! An unsolicited six pence which was a reward for showing him how to write his "tally board" in a more efficient way based on one's knowledge of arithmetic. The impact of that six pence on my life is immeasurable considering the fact that we used to be given only half a penny for fetching water from miles away! Thank you so much *broda* Sura.

The sum total of my encounter with my teachers in the primary school, the famous St. Jude's Primary School, Ebute-Metta, was positively encouraging: Miss. Obviageli, you were wonderful; Mr. Babatunde, you were great; Mr Momah, an unrelenting disciplinarian, you were a builder; and last but not the least, the one and only one headmaster of our school, Mr. Odugbose, thank you so much for your time you spent with us on Saturdays

coaching us for free even though you were not our teacher! My teachers in the secondary school, the Government College, Ibadan, without exception you were all exceptional but permit me to make special mention of Mr. Aluko, I appreciate your words of encouragement and the money you gave me because of my performance in your mathematics examination; Mr. Amazigo (now Prof. J.C. Amazigo), I hope my writing improved because you sat me down to practice writing and you allowed me to use your Parker pen! Your extra lectures and your rice and *dodo* are appreciated; and to our principal D.J (Mr. D.J. Bullock) you have my special gratitude for saving me from the jaws of imminent premature educational death. Confronted with the choice between discipline, fair play and justice you chose fair play and justice. How can I forget you? All you good people have been my role model. I can only hope that I have kept faith with the values I enjoyed from you. Thank you.

Ladies and gentlemen, I hope you now see how truly fortunate I was as a consequence of my encounter with all these good people.

Now, my colleagues in Unilag, it has been so far so good. My encounter with you is still an ongoing project and so I will out of modesty not want to say much now. Suffice it to say that life is full of up and down, I have enjoyed my time with you and I am truly grateful. However, to every rule there is an exception. I must not fail to express my gratitude to the Vice-Chancellor, Prof. Tolu Odugbemi; the Registrar, Mr. Olu Shodimu; the Deputy Vice-Chancellor (Academic & Research), Prof. (Mrs) Dupe Ogunlesi; the Dean of Science, Prof. Wole Familoni, the Head of my Department, Prof. Jerry Adepoju and all you other good people for your many visits to me when I was hospitalised not too long ago. May God continue to bless you. I must also not fail to acknowledge my collaborators who are no longer with us, in the persons of late Prof. Debo Falade and late Prof. Sola Ijaola as well as the living one, Prof. Mide Ajose. It was a joy working with you.

To my siblings, Chief (Mrs) Olabisi Balogun, Prof. S. O. Ajayi and Mrs. Abosede Kujero, I am sure that you will understand quite well if I say that your loving disposition was strong enough to redeem a sinner (no blasphemy intended), because of you one would wish to live forever! Thank you.

My mother, Mrs. Bernice Omodunni Ajayi a philosopher of few words (your words of wisdom still guide me till today) and a most unobtrusive motivator. You nurtured me, molded me and were and still are a rock of Gibraltar. We thank God for your long life and good health. May the good Lord continue to grant you your heart's desire.

My father, late Chief Ezekiel Awoniyi Ajayi, the *Mowo of Ososa*, a complete gentleman, understanding, loving and most responsible, what can I say? Your labour has not been in vain. I have tried to emulate you. Continue to rest in perfect peace.
To my nuclear family, may God continue to bless you, may his love and mercy continue to shine upon you. You have made my live worthwhile. When I come again, I will want you all over again. Thank you.

Thank you all and God bless.

DONATION
University Of Lagos Library
G & E
Donor: Unilag Press
Date: 12/11/2009.
UNIVERSITY OF LAGOS
LIBRARY