CHAPTER ONE
INTRODUCTION

1.1 BACKGROUND TO THE STUDY

In many real-life situations, the knowledge of heights is crucial for understanding the relative positions of neighbouring entities in a common reference system. Perhaps, one of the major realities of the recent occurrence of tsunami which wreaked havoc in some parts of Asia is the fact that those who choose the sea as their neighbour could be swallowed by it. Hundreds of thousands of human lives were washed away by sea wave which jumped coastal barriers and entered into living rooms and recreation parks. The last tsunami occured in Asia part of low altitude coastlines, where more than half of the world's population inhabits. It is a natural phenomenon that nobody knows the time, the magnitude and location of the next visitation. As a result of that, the United Nations has proposed a solution in which tsunami sensors would be placed in the seas to give early warning signals, so that coastal dwellers can have enough time to run to the mountains. However, people need to know how far up the hills would be sufficient for safety. Therefore, the need for height information in all aspect of human activities cannot be overemphasized. In practical terms, height data is needed for:

- infrastructural development such as: construction of building, skyscrappers, roads, bridges, dams, drainages, airports, tunnels, pipelines,
- disaster monitoring and remediation,
- subsidence monitoring,
- study of sea level variation,
- monitoring of high rise buildings,
- tunnelling whether on-shore or off-shore
- the determination of elevation other natural and artificial features and
- any other projects aimed at harmonious environmental development.

There are several methods of height determination of which levelling is the most common. Levelling is the process of determining the difference in elevation between points with reference to a known datum using a level. The datum usually adopted is the **Geoid**, which in practical terms coincides with the average surface of the ocean and can be referred to as the Mean Sea Level (MSL). The height that is determined with reference to the geoid using the levelling approach and application of Orthometric correction is known as **Orthometric Height**. Methods of determining Orthometric Height can be grouped into direct and indirect methods. The direct method involves practical determination of height using geodetic levelling with application of Orthometric correction. The method is precise but very tedious, time consuming and almost impracticable in some areas such as the rain forested areas and Niger Delta region of Nigeria because of swamp and the nature of the terrain. The indirect method involves observation of ellipsoidal height using Global Navigation Satellite System (GNSS) and a model for the computation of Geoidal Undulation. For the indirect method, two approaches are possible namely;

1) absolute geoid model in which the model used computes absolute values of the Geoidal Undulation.

2) Relative geoid model in which only the Geoidal Undulation relative to existing geoid is computed by the model. The relative model is used in this work. The reference geoid is GEM2008. When the Orthometric Height is computed with Geoidal Undulation determined from relative Global Earth Model (GEM) such as GEM2008, the Orthometric Height so determined is known as **GEM2008 Orthometric Height** in which the geoid is computed with GEM2008 model.

Theoretically, many approaches have been adopted for geoid modelling. They include: gravimetric, astro-gravimetric, astrogeodetic and Geopotential Earth modelled geoid. These are absolute geoid models which determine the magnitude of Geoidal Undulation as a function of positions. Apart from these deterministic methods, there is empirical approach in which height observations are made, processed and used in numerical modelling of the geoid. Examples of the empirical approach are: North Sea Region Model, 4-Parameter Similarity Datum Shift, Zanletnyik Hungarian Model. All the above methods compute absolute geoid. As a result of recent success in the determination of reliable global geoid model for the entire Earth, opportunities now exist for a relative geoid determination. Relative geoid models make use of undulations referred to the established local height datum. This is the approach used in this work.

1.1.1 Geodetic Surfaces

All activities in Surveying are done on three basic surfaces referred to as "geodetic surfaces" namely: the topographic surface, the geoid and the ellipsoid and presented in Figure 1.1.

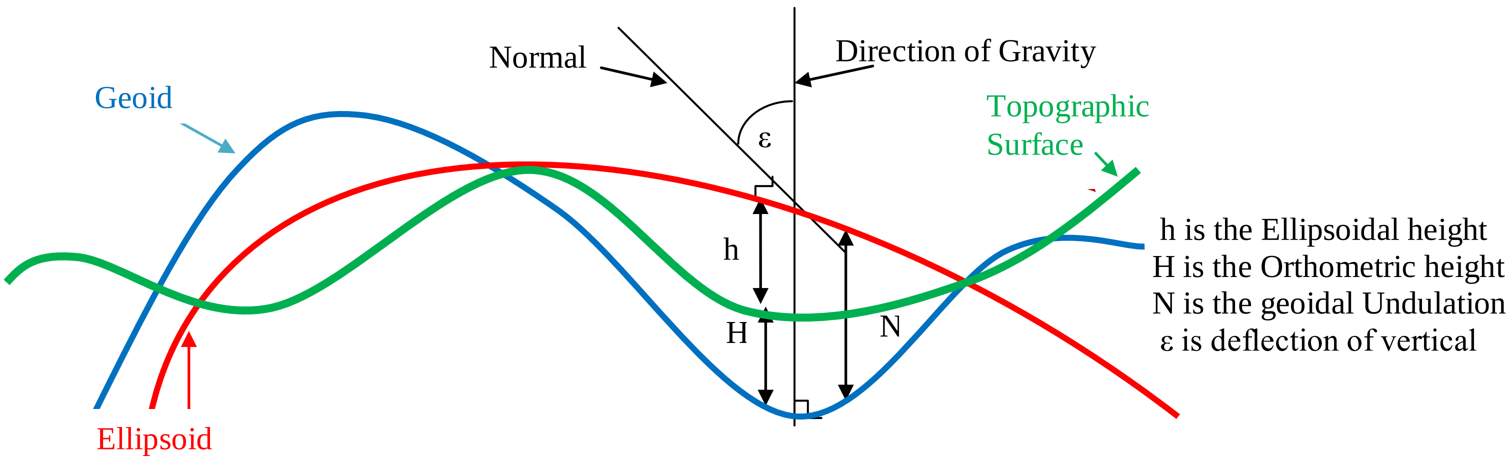


Figure 1.1: The Three Geodetic surfaces (Source: Author, August, 2008)

1.1.2 The Topographical Surface

The topographical surface is generally called the Earth's surface. It is the actual surface of the land and sea. This is the physical surface, where all measurements and observations are done. It is an irregular undulating surface, characterised by mountains, valleys, spurs, dunes and other features. Its undulating and irregular nature is caused by uneven distributions of the Earth masses which make it impossible to describe it with any mathematical relation (Vanicek and Krakiwsky, 1986; Uzodinma and Ezenwere, 1993; Vanicek et all, 2000; Torge, 2000). Hence, geodetic computation cannot be done on this surface. The surface closed to the topographical surface is the **geoid**.

1.1.3 Geoid

The term Geoid comes from the word "geo" which literarily means Earth-shaped. The geoid is an empirical approximation of the figure of the Earth (minus topographic relief). It is defined as the "equipotential surface of the Earth's gravity field which best fits, in the Least Squares sense, the mean sea level" (Deakin, 1996). The geoid can also be defined as the "surface which coincides with that surface to which the oceans would conform over the entire Earth, if free to adjust to the combined effect of the Earth's mass attraction (gravitational force) and the centrifugal force of the Earth's rotation" (Bomford, 1980). Specifically, it is an equipotential surface, meaning that it is a surface on which the gravitational potential energy has the same value everywhere; with respect to gravity. It is more or less corresponding to the Mean Sea Level (MSL) over the oceans. It is the surface of an ideal global ocean in the absence of tides and currents, directed and shaped only by gravity. It is a crucial measuring reference for various phenomena such as sea-level change, ocean circulation and ice dynamics – all affected by climate change. Geoid has a definite physical interpretation, in the sense that it can be fixed by measurements over the ocean with the use of Mean Sea Level.

1.1.4 Mean Sea Level (MSL)

Traditionally, because the sea surface is available worldwide, surveyors, mapmakers and other heights users or professionals have made the task of geoid determination to be simplified by using the average or mean of sea level as the definition of zero elevation. At any point on the geoid the value of the height is zero, while above is positive and below is negative, Figure 1.2 depicts the 3D configuration of the Earth's topography around the geoid, which serves as vertical datum for all Orthometric Heights.

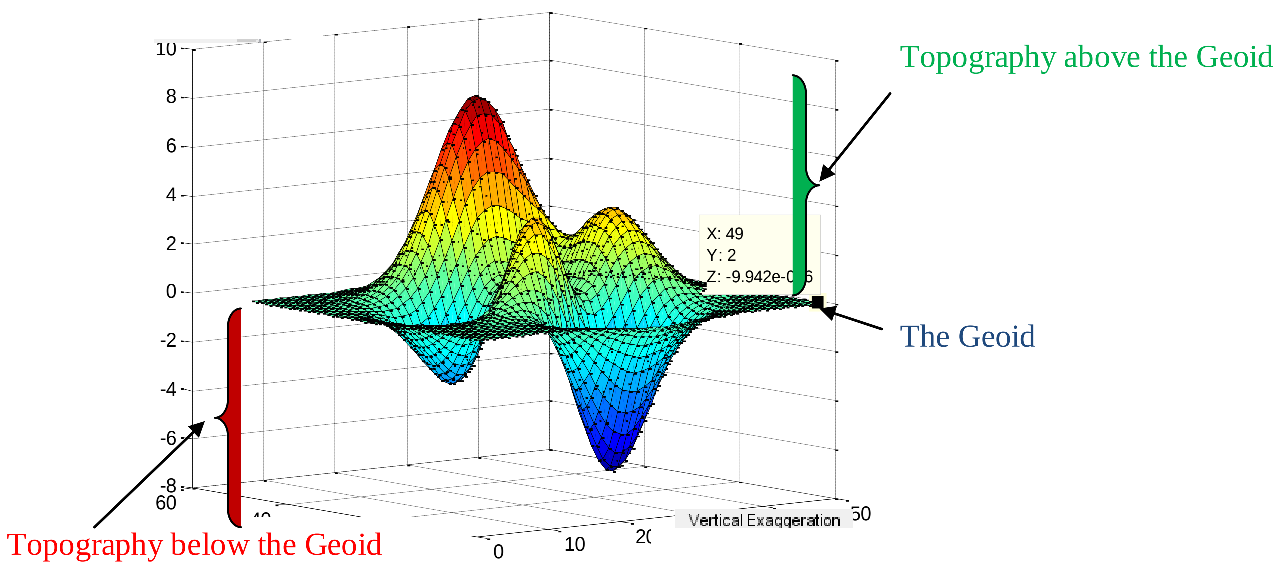


Figure 1.2: 3D Configuration of the view of the Geoid and ocean Topography (Source: Author, October, 2010)

Vertical datum is the surface to which heights of points within a locality are referred. This is always taken to be the MSL for coastal areas. The MSL is determined by continuously measuring the rise and fall of the ocean at "tide gauge stations" on sea coasts for a period of 18.61years (approximately 19 years - this period is described as one cycle of the moon's node). MSL averages out the highs and lows of the tides caused by the changing effects of the gravitational forces from the sun and moon which produce the tides. (DMA, 1996)

The MSL is used by surveyors in the field, when performing temporary adjustment; surveyor levels the instrument with the aid of spirit level, and makes his plumb line perpendicular to the geoid. Therefore, it is a good approximation to say that his spirit level is always parallel to the geoid, even if it is slightly above or below it. MSL can be used to approximate the geoid which can then be fitted to a more regular surface called the ellipsoid (Sideris and Fotopoulos, 2006).

1.1.5 The Ellipsoid

The ellipsoid may be defined as a surface whose plane sections are all ellipses. It is a figure formed when an ellipse is rotated about its minor axis. Ellipse can also be defined as the locus of points such that the sum of the distances from two fixed points (foci) to any point on the ellipse is constant. One particular ellipsoid of revolution, also called the "**Normal Earth**", is the one having the same angular velocity and the same mass as the actual Earth, the potential (U_0) on the ellipsoidal surface equal to the potential (W_0) on the geoid, and the centre coincident with the centre of mass of the Earth (Xiong *et al.,* 2001). The ellipsoid defines a mathematical surface approximating the physical reality of the Earth, while simplifying the geometry. "Ellipsoid is a good approximation to the shape of the Earth but not an exact representation" (Gen, 2003). It is the only regular surface among the three geodetic surfaces (Section 1.1.1); hence, it has a regular shape which makes it possible to be represented mathematically, and therefore enable computations to be done on it. (Rapp, 1981; Vanicek and Krakiwsky, 1986; Petrovskaya and Pishchukhina, 1989; Vanicek, 2001; Gen, 2003; Kaplan and Hegarty, 2006; Moka and Agajelu, 2006; Jokeli, 2006). The ellipsoid serves as a basis for the 3D coordinates of satellite systems such as the Global Navigation Satellite System (GNSS). World Geodetic System 1984 (WGS 84) is the reference ellipsoid of the GNSS.

The reference ellipsoids are always defined by the Semi major axis or the Equatorial radius (a) and flattening (f). Some common reference ellipsoids and their parameters are listed in Table 1.1 below (Dana, 1985; DMA, 1987):

Table 1.1: Reference Ellipsoid (Source: Various)

Ellipsoid	Semi-major axis [m]	1/flattening
Airy 1830	6377563.396	299.324964600
Australian National	6378160.000	298.250000000
Modified Airy	6377340.189	299.324964600
Australian National	6378160.000	298.250000000
Bessel 1841 (Namibia)	6377483.865	299.152812800
Bessel 1841	6377397.155	299.152812800
Clarke 1866	6378206.400	294.978698200
Clarke 1880	6378249.145	293.465000000
Clarke 1880 (Minna-Nigeria)	6378249.145	293.465000000
Everest 1830 (India)	6377276.345	300.801700000
Everest (Sabah Sarawak)	6377298.556	300.801700000
Everest 1956 (India)	6377301.243	300.801700000
Everest 1969 (Malaysia)	6377295.664	300.801700000
Everest (Malaysia and Sing)	6377304.063	300.801700000
Everest (Pakistan)	6377309.613	300.801700000
Fischer 1960 (Mercury)	6378166.000	298.300000000
Fischer 1968	6378150.000	298.300000000
GRS 1965	6378160.000	298.247167427
GRS 1980	6378137.000	298.257222101
Helmet 1906	6378200.000	298.300000000
Hough 1960	6378270.000	297.000000000
Indonesian 1974	6378160.000	298.247000000
International 1924	6378388.000	297.000000000
International Astro. Union 1976	6378140.000	298.257000000
International Earth Rotation Service 1989	6378136.000	298.257000000
Krassovsky 1940	6378245.000	298.300000000
South American 1969	6378160.000	298.250000000
World Geodetic System 1960 (WGS 60)	6378165.000	298.300000000
World Geodetic System 1966 (WGS 66)	6378145.000	298.250000000
World Geodetic System 1972 (WGS 72)	6378135.000	298.260000000
World Geodetic System 1984 (WGS 84)	6378137.000	298.257223563

One of the components of geodetic coordinates of GNSS is ellipsoidal height, which uses ellipsoid as the datum. The geodetic coordinates are related and each of the components of geodetic coordinates have a common origin of the coordinates system, while the geodetic surfaces also related to one another.

1.1.6 Relationship between the Geodetic Surfaces:

Orthometric Heights and ellipsoidal heights are measured with reference to the geoid and the ellipsoid respectively. The relationship between them is Geoidal Undulation. They are also related angularly by the **deflection of vertical (**ε) also called Vertical deflection (VD). It is defined as the angle between the true zenith (plumb line or the direction of gravity) and the normal (that is the line perpendicular to the surface of the ellipsoid chosen to approximate the Earth's sea-level surface). Merry and Vanicek (1974) defined gravimetric deflection as the angle between the actual plumb line and the normal to the geocentric reference ellipsoid, measured at the geoid (Figure 1.3). VDs are caused by mountain and underground geological irregularities. The deflection of vertical has two components. These are the components along the **prime vertical** (North-South component ξ) and along the **meridian** (East-West component η) (Fajemirokun, 1980; 1981 and 1988; Vanicek and Krakiwsky, 1986; Uzodinma and Ezenwere, 1993; Agajelu, 1997 and Vanicek et al., 2000).

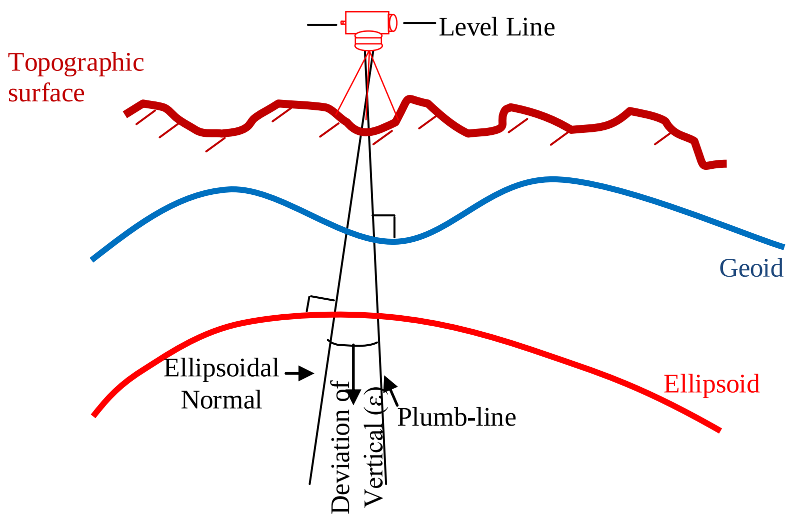


Figure 1.3: Relationship between the Geodetic Reference Surfaces and Deflection of Vertical
(Source: Author, August, 2008)

Deflections of Verticals are usually determined by astronomical observation. VD can be determined by observing the *true zenith* astronomically with respect to the stars, and the *ellipsoidal normal* (theoretical vertical or *ellipsoidal zenith*) by geodetic network computation (Equations 2.18e and 2.18f, Page 36), which always takes place on a reference ellipsoid. Veining Meinesz originally developed the theory of computing the local variations of the VD from gravimetric survey data and

Digital Terrain Modelling (DTM). This deflection of vertical (Figure 1.3) has also been used in astro-gravimetric and astro-geodetic determination of geoid (Sections 2.2.1.3 and 2.2.1.6 respectively). In practice, “the deflections are observed at special points with spacings of 20 to 50 kilometres. The densification is done by a combination of DTM models and a real gravimetry model. Precise VD observations have accuracies of ± 0.2" (on high mountains ± 0.5"), calculated values of about 1–2" (Bomford, 1980 and Torge, 1989).

In physical geodesy, deflection of vertical is defined as the difference in direction between the natural gravity with reference to the geoid and the normal gravity vector with reference to the ellipsoid, while the magnitude is called the **gravity anomaly** (δ)**.** Therefore, deflections of verticals are functions of the gravity gradient and its inhomogeneities because they are always connected with the local and regional undulations of the geoid and also related with gravity anomalies. Therefore, gravity is a vector quantity. It has both magnitude and direction. The direction is the **gravity vector** along the plumb line and its corresponding **normal gravity** along the ellipsoidal normal differed by **gravity disturbance.** This is the difference between the **Normal** (perpendicular to the ellipsoid) and the **plumb line** i.e. direction of gravity (perpendicular to the geoid).

In this work, geoid modelling is based on the three geodetic surfaces (Section 1.1.1) briefly discussed above. The separation between ellipsoidal and geoidal surfaces is known as **Geoidal Undulation (N).** The relationship between the three geodetic surfaces (Figure 1.1) is mathematically represented by Equation 1.1 (Bonford, 1980):

$$N = h - H \cos \varepsilon \tag{1.1}$$

where;

N = Geoidal Undulation

h = Ellipsoidal height

H = Orthometric Height

ε = deflection angle

This deflection angle is usually small; and given the assumption of small and gently rolling geographic area, the angle ε can be taken to be negligible (Bomford, 1980 and Hwang and Hsia, 2003). In Equation 1.1, the cosine function of the angle is required. The value of cosine function of small angle is tending towards unity. The maximum observed deflection of the vertical is approximately 70". Even at a height of 1000 m, neglecting this extreme ε only causes an error of ~0.06 mm (Bomford, 1980). This is supporting the assumption that the deflection angle is usually small and therefore can be neglected.

Neglecting the deviation of the vertical, Equation 1.1 becomes:

$$h = N + H \tag{1.2}$$

Equation 1.2 can equally be written as:

$$N = \mathrm{h} - H \tag{1.3}$$

where;

All the terms are as earlier defined.

The modern method of obtaining Geoidal Undulation is to use data from a satellite positioning method such as GPS and geodetic levelling. This method is sometimes called GPS/levelling geoid. Apart from determination of the geoid, if the transformation parameters are accurately determined, the method serves as important input in the simultaneous determination of control network of any country.

1.1.7 Control Network

Horizontal and vertical geodetic control networks were fully separated from each other due to the different methods of observations. Traversing, triangulation and trilateration are some of the methods adopted for horizontal control network, while spirit and trigonometric levelling are used for vertical control network. (Allan *et al.,* 1968; Davis *et al.,* 1981; Bomford, 1980; Denker *et al.,* 2000 and Fotopoulos, 2003). When there is simultaneous need for vertical and horizontal coordinates, different approaches were used to get each of them before the advent of GPS survey.

GPS survey has solved this problem by providing the three-dimensional (3D) coordinates with reference to World Geodetic System 1984 (WGS 84) reference ellipsoid. The 3D coordinates are the geodetic latitude (φ), geodetic longitude (λ) and ellipsoidal height (*h*), which can be determined accurately. However, *h* is not always used directly as height in normal everyday work because it does not provide elevation above the MSL, which refers to the Earth gravity equipotential surface that is the geoid, the reference surface for Orthometric Heights.

One of the major points of favour of the use of Orthometric Height against the use of ellipsoidal height is its relationship with ocean (water body). However, "the direction of the flow of fluid is not only controlled by height; actually it is the force of gravity that governs fluid flow. Therefore, the selection of a height system that neglects gravity, or does not use it rigorously, allows the possibility of fluids appearing to flow upward" (Featherstone, 2006). A situation like this may occur, when the ellipsoid that approximates the physical and irregular topographical surface falls in an area against the direction of fluid flow. Clearly, such a system is counter-intuitive, the only heights properly related to the Earth's gravity field that is the Orthometric Heights are natural heights and physically meaningful for most applications (Featherstone, 2006 and Isioye et al., 2011). Therefore, Orthometric Height, which may be obtained by spirit levelling, is always preferred.

Spirit levelling is the dominant technique for providing elevation above MSL or geoid. The equipment is inexpensive and the method is highly accurate. However, it is labour intensive over long distance; the field procedures are tedious, and prone to human and other errors. Other problems associated with spirit levelling have been discussed by various authors. (Allan et al., 1968; Bomford, 1980; Fajemirokun, 1980 and1981; Uzodinma and Ezenwere, 1993; Featherstone, 1998; James and Mikhail, 1998; Vanicek, 2001; Fotopoulos, 2003 and Uzodinma, 2005). Spirit (Geodetic) levelling provides Orthometric Height, while ellipsoidal height is easy to obtain with GNSS. The two heights can be used in geoid modelling for any area under study.

1.1.7.2 Height for Control Network

Height is an important component in any control network system. Orthometric Height is natural height and it is preferred by the users. Its determination by direct geodetic levelling, that is, spirit levelling, though, precise and accurate but associated with a lot of problems. However, ellipsoidal height determination using GNSS technology is easier, faster, more economical and user friendly but not as accurate as the direct geodetic levelling method. As a result of the benefits in the use of GNSS technology for ellipsoidal height determination, ellipsoidal height obtained can be applied to the Geoidal Undulation to obtain Orthometric Height, as stated in Equation 1.3. The Geoidal Undulation can equally be obtained from the Global Earth Model (GEM).

The latest GEM available to the public, used in this work is GEM2008 to determine the Global Geoidal Undulation for a number of points in Port-Harcourt in Rivers State and Lagos State of Nigeria. The geoid so determined was compared with other existing methods using data obtained from Differential Global Positioning System (DGPS).

DGPS observation was done to determine the ellipsoidal heights while geodetic levelling was also done to determine the Orthometric Height (but neglecting the Orthometric correction) for the same points. When Orthometric and ellipsoidal heights of a point are available, the local Geoidal Undulation can be computed using Equation 1.3. Geoidal Undulations were determined for points in the study areas to model the geoid. This method shall be referred to as **'Satlevel' Collocation.** Two modelling techniques explored are Spherical 'Satlevel' model and Rectangular 'Satlevel' model. The Geoidal Undulations computed from 'Satlevel' collocation were substituted with the ellipsoidal heights obtained from the DGPS to get the local Orthometric Heights for all the points observed in the two study areas.

1.2 STATEMENT OF PROBLEM

Geodetic levelling is accurate and can be used in the determination of Orthometric Height which is regarded as natural height because of its relation with ocean. This height is needed in many real life situations but its determination by differential levelling is very cumbersome and associated with problems.

The problems associated with height measurements are as follows:

1. Obtaining Orthometric Height by direct method is time consuming, expensive and difficult in some terrains. Data acquisition for Orthometric Height is labour intensive over long distances and the field procedures are tedious and prone to human and other errors. In some areas such as Niger Delta region of Nigeria, it is almost impossible to perform spirit levelling due to weather, terrain conditions and swamps. Despite these problems associated with Orthometric

Heights; it is still the height preferred for engineering works a major area of practice for the survey profession.

2. Nigeria height system was based on the assumption that the ellipsoid and the geoid at the origin (Minna) coincided (Field, 1978; Fajemirokun, 1980; Uzodinma, 2005 and Onyeka, 2006). The values of the geoid in most part of the country negate this assumption. Since there is problem at the origin, the accuracy of the entire Nigerian Heights System is in doubt.

3. In Nigeria, geoid height and geoid model which can aid the adjustment of the Nigerian Geodetic network to be done by the correct projective method rather than the adopted developmental method are not available. Hence, observations were reduced to an irregular surface - the geoid rather than regular mathematical surface - the ellipsoid.

4. At the beginning, astronomical observations were done on four different stations at Kano, Naraguta, Lafia Beriberi and Zaria. The mean of the astronomical observations was compared with another one observed at Minna. Since the difference was not large, Minna geoid was assumed to be equal to the ellipsoid. Consequently, the assumption and geodetic reductions used in analysis have introduced distortions into the Nigerian Geodetic Network. For example, a geoidal profile along the 12th Parallel Traverse (CFL series) shows geoidal height discrepancies of up to 12 metres (Adaminda and Field, 1985; Fajemirokun, 1980; Uzodinma, 2005). Also, Omogunloye (2010) observed a large error in the same CFL series. Furthermore, Agajelu (1985) and Ezeigbo (1985) observed a scale error of 1-3ppm in the north-eastern part of the network, which they attributed to the absence of geoid height model.

5. Another problem associated with the height systems in Nigeria is the lack of a uniform reference datum. In areas that abut the coast; the MSL is often adopted as the basis for determining heights. However, defining the MSL and carrying it to the hinterland have always been problematic resulting in a poor or uncoordinated height system, especially in Port Harcourt where; the surveyors need to establish benchmark each time height data are required.

6. Furthermore, the Nigerian Vertical Control Networks obtained using geodetic spirit levelling are not evenly distributed. Though, covering fairly all parts of the country but most of the work were concentrated on the South-Western and North-Western parts of the country. Unfortunately, like the planimetric data which were computed using developmental method as against the correct projective method, heights data available so far are still provisional heights. They are heights above the geoid rather than geodetic heights.

7. The heights are also based on an arbitrarily chosen datum known as the Lagos Survey Datum. The heights obtained from geodetic levelling were derived after circuit adjustment of the various levelling loops (Fajemirokun, 1980 and Uzodinma, 2005). The accuracy of this height is suspected because it is based on the inaccurate Lagos Vertical Datum (Field, 1978; Ezeigbo and Adisa 1980; Uzodinma, 2005 and Onyeka, 2006).

8. Furthermore, the coplanarity of the ellipsoid with the geoid at the origin is in doubt. In fact, the two surfaces have been determined on different occasions to be inclined at either angle 6″.35 (in 1928) or 1″.6 (in 1968). This inclination is suspected to have resulted in the failure to reduce some of the astronomical observations to the Conventional International Origin (C. I. O). This could also be responsible for the tilt of the geoid from the northwest to the northeast as reported in Ezeigbo and Edoga (1980) and Uzodinma (2005). The Lagos Survey Datum, to which all heights of the benchmarks in the country referred, has been found not to be exactly coinciding with the Mean Sea Level. There were attempts in the past to analyse tidal observations obtained from East Mole (Tide Gauge station near Lagos Port), in order to establish the relationship between the Mean Sea Level and the Lagos Survey Datum. Results of the investigations showed that the Lagos Survey Datum is about 50cm below the Mean Sea Level (Fajemirokun, 1980 and Uzodinma, 2005).

9. There are differences in the values of Geoidal Undulations obtained from different versions of Global geoid. For example, the difference between the Geoidal Undulations obtained from GEM 96 and GEM2008 in Port Harcourt - one of the study areas is about 2m, when compared with the result of GGU (2006). Gravity data used for the global geoid are required all over the world with good spatial distribution, which is difficult to achieve and hence gravity approximation becomes the best alternative. The accuracy of the result varies from

one data point to the other depending on the method of approximation used. Therefore, it is always advisable for the user to test its compatibility in any locality before using the Global geoid.

In summary, there is no geoid model in Nigeria and hence the Nigerian geodetic Control network systems are not uniquely defined.

1.3 AIM AND OBJECTIVES OF THE RESEARCH

The aim of this research is to develop empirical mathematical models for transforming Global Geoidal Undulations (N) to the local equivalents.

The **objectives**:

In order to achieve the stated aim, the following specific objectives are set:

1. To derive optimal empirical Geoidal Undulation-models for transforming Global undulation to local values
2. To compare ellipsoidal height differences with Orthometric Heights differences.
3. To compute the local Orthometric Heights from GNSS ellipsoidal height.
4. To validate the adequacy of the developed models on some data sets.
5. To develop user friendly software for computation of Geoidal Undulation.

1.4 JUSTIFICATION FOR THE STUDY

'Satlevel' collocation technique is used to generate a geoid model for interactive use. It is convenient, saves time and cheaper in geoid determination compare to the classical methods which require data all over the world or astro-geodetic method where data must be concentrated around the computational point, to compute Geoidal Undulation at any location.

The existing models were developed for different countries and are to suite a particular locality while 'Satlevel' collocation can be used in and around the location, where the geoidal coefficients are determined especially in the coastal region.

The latest version of the Global geoid released to the public is GEM2008. It is readily available since it can be found on the internet for any geographical location. The use of this available data from global geoid has not been yielding expected result in terms of accuracy. This research makes use of this opportunity for the Global geoid to be adapted to local geoid in Nigeria.

Since height users prefer the natural Orthometric Height which is difficult to achieve as against the ellipsoidal height which is easy to observe from GNSS. The indirect method will provide a cheaper way to obtain Orthometric Heights. There is a need for the prediction of local Orthometric Heights from GNSS ellipsoidal height and the possibility of using ellipsoidal heights in place of Orthometric Heights for engineering applications and other purposes.

Most of the problems identified with the Nigerian Geodetic Network borders on lack of geoid height. These problems can be solved when there is geoid model and this research provides solution to this problem with use of 'Satlevel" collocation model. It will also demonstrate the possibility to harmonise the irreconcilable different height systems scattered all over the country, especially in Port Harcourt – one of the study areas.

This work is providing the alternative way of testing the compatibility of the Global geoid against the local and transforming the Global geoid to its local equivalent.

1.5. SCOPE AND LIMITATIONS OF THE RESEARCH

1.5.1 Scope:

The scope of the study includes:

- data acquisition using GPS and Geodetic levelling.
- determination of ellipsoidal and Orthometric Heights at discrete points only.
- derivation of empirical Geoid models relative to GEM2008 Global geoid.
- computing the GEM2008 Orthometric Heights by applying Equation 1.3 using the GEM2008 Global Undulation.

- transforming GEM2008 Orthometric Heights to local values using the ‘Satlevel’ Collocation Model.
- statistical testing and validation of the empirical Geoidal Undulation models

1.5.2 Limitations:

The following are the limitations of this research

i. Data inconsistency: During observation, readings were recorded to 3 places of decimal as against 5 places of decimal in some instances for geodetic levelling as observed from Lagos State data, which was acquired from various sources as a result of large extents of area covered.

ii. The models derived were NOT tested in mountainous areas because of lack of data.

iii. Accuracy of the model depends on the accuracy of the data used in the determination of the coefficients, which may be affected by the quality of instruments used for data acquisition, method used in computation and competency of the observer.

1.6 SIGNIFICANCE OF THE STUDY

The uniqueness of this research is the consideration of the relative geoid. This approach uses an existing Geoidal Model (GEM2008) as long wavelength part using a single predictive model for computing the short wavelength component and determines the Geoidal Undulation at another location.

- Global model cannot accurately fit the local environment because the data used in global model are sampled for the locality to provide and good enough for the long wavelength components of the geoid. The data for local geoid are specific and good for both long and short wavelength components of the geoid. This research will overcome this problem with ‘Satlevel’ collocation model use for transforming the Global geoid to its local equivalent.
- ‘Satlevel’ collocation models require data at discriminate points within an area, unlike some of the existing models such as Stoke’s Integral which require data all over the Earth to compute Geoidal Undulation. Also, ‘Satlevel’ collocation models involves no integration, it saves time and cheaper than the classical methods.

- 'Satlevel' collocation models require four coefficients to compute and attain the accuracy comparable to the existing Zanletnyik Hungarian Model (Equation 2.27) which is over – parameterized with 26 coefficients.

The benefits derived from this research include:

- Development of 'Satlevel' collocation geoid models using curvilinear and rectangular coordinates.
- Global geoid provides the long wavelength component of the local geoid.
- Provide empirical evidence that it is possible to replace the Orthometric Heights differences with ellipsoidal heights differences if the project is within a small area.
- Obtaining Orthometric Heights from ellipsoidal heights using 'Satlevel' collocation models and vice versa is easier than that obtained from some of the existing methods.
- The program 'Orthometric Heights on the fly' was developed which will make the determination of Geoidal Undulation and Orthometric Heights easier and more convenient for the users than the existing methods.

1.7 OPERATIONAL DEFINITION OF TERMS:

The following definitions were adopted in the research:

Check levelling: Check levelling is the operation of running levels for the purpose of checking the series of levelling or bench marks, which have been previously fixed.

Differential levelling: Differential levelling is levelling operation which is used to determine the difference in elevations of points some distance a part or to establish bench marks.

Dynamic Height: Dynamic Height is defined as the vertical distance above the geoid of points on the same equipotential surface and measured along the direction of gravity in terms of linear units at given latitude, generally 45°.

Ellipsoid: Ellipsoid may be defined as a surface whose plane sections are all ellipses.

Ellipsoidal height: Ellipsoidal height is the geodetic height determined with reference to the ellipsoid.

GEM2008 Orthometric Height: GEM2008 Orthometric Height is the Orthometric Height determined using the Geopotential Earth Model 2008 as Geoidal Undulation.

Geodetic Levelling: Geodetic Levelling is the determination of elevation or difference in height between two points with reference to a known datum, done with careful measurement and precise equipment to attain high accuracy.

Geoid: The geoid can be defined as the surface which coincides with that surface to which the oceans would conform over the entire Earth, if free to adjust to the combined effect of the Earth's mass attraction (gravitation) and the centrifugal force of the Earth's rotation.

Geoid Modelling: Geoid modelling is a process of developing mathematical algorithms to represent the difference between orthometric and ellipsoidal heights.

Geoidal Undulation: Geoidal Undulation is the vertical distance between the ellipsoid and geoid at a specific point and is also known as the Geoid separation or geoid height.

Geopotential Number: it defined as the numerical value that is assigned to a chosen geopotential surface usually 45 degrees latitude when expressed in geopotential units (1 gpu = 1 m × 1 kilogal). Therefore, Geopotemtial number is a constant of normal gravity (γ_0) for arbitrary standard latitude of 45°.

Global Positioning System (GPS): GPS is a location fixing system initiated by United States (U.S.) Department of Defence (DOD) based on acquiring satellite signals (tracking) with the aid of receiver and processing of data to obtain the three dimensional (3D) coordinates of the receiving station. This positioning system is referenced to a global reference system known as World Geodetic System 1984 (WGS' 84).

Hypsometry Levelling: Hypsometry levelling is the method of levelling that is employed in determination of the heights of mountains by observing the temperature at which water boils.

Levelled Height: Levelled Height is the raw determination of height between points using levelling equipment. It is measurements of distances in a vertical plane like distances in horizontal plane are measured as in chain surveying using distance measuring equipment.

Levelling: Levelling is the process of determining the difference in elevation between points with reference to a known datum using a level.

Mean Sea Level (MSL): MSL is the result of average continuous measurements of the rise and fall of the ocean at "tide gauge stations" on seacoasts

Normal Height: Normal Height is the vertical distance measured along the ellipsoidal normal from topographical surface to the ellipsoid.

Orthometric Height: Orthometric Height is defined as the vertical distance along the curved plumb line from the geoid to the topographic surface.

Outliers: outliers defined as those values in a data set which exceed 3 standard deviations from the mean.

Profile levelling: Profile levelling is the levelling operation in which the object is to determine the elevation of points at known distance apart along a given line, and thus to obtain the accurate outline of the surface of the ground. It is called the longitudinal levelling or sectioning.

Reciprocal levelling: Reciprocal levelling is the method of levelling in which the difference in elevation between two points, accurately determined by two sets of observation when it is not possible to set up the level midway between the two points.

Reference Height Datum: A reference height datum is a smooth surface which is adopted as a basis for heights in a particular locality.

'Satlevel': 'Satlevel' is a method of geoid determination in which the ellipsoidal height from any satellite based system is combined with Orthometric Height to model the geoid.

Trigonometric levelling: Trigonometric levelling is the process of levelling in which the elevations of points are computed from the vertical angles and horizontal distance measured in the field.

CHAPTER TWO
LITERATURE REVIEW

2.1 GEOID DETERMINATION:

C. F. Gauss introduced the “geometric surface of the Earth” in 1828, while Listing designated this level surface as **geoid** in 1873. Geoid determination then was on global bases. However, lack of adequate global data hindered spherical harmonic expansion not to be properly applied to geoid modelling. Therefore, only regional geoid modelling was possible. For regional modelling, the integral formula of Stokes developed in 1849 was used. It was therefore of great significance that F. R. Helmet in 1880/1884 showed, with “astronomical levelling” “how local and continental geoid sections could be computed by path-integration of the deflections of the vertical” as a method of geoid determination (Torge, 1989). From a synthetic evaluation of the influences of continental land-masses, Helmet concluded that the values of Geoidal Undulations were likely to lie within a range of 400m. However, by taking into account plausible isostatic compensation, he found out that the actual geoidal variation lie within ±27m. From later consideration, the value of gravity anomalies has a range of ±50m (Hannover, 1996; Torge, 1989 and 1991). The range of the Geoid Undulation all over the world as well as geoidal surface can also be determined. However, availability of gravity data was a major problem in worldwide determination of geoidal surface using the available method of Stoke’s formula.

International Union of Geodesy and Geophysics (IUGG) attempted to solve this problem by creating the International Bureau of Gravimetric Service (BGI) in 1951. The activities of the Bureau is summarised as follows: “the overall task of BGI is to collect, on a world-wide basis, all measurements and pertinent information about the Earth gravity field, to compile them and store them in a computerized data base in order to redistribute them on request to a large variety of users for scientific purposes”…. “BGI is one of the services of the International Association of Geodesy (IAG). IAG also established International Gravity Field Service (IGFS), which coordinates since 2001, the servicing of the geodetic and geophysical community with gravity field-related data, software and information. IGFS centres are: International Geoid Service (IGeS), International Centre for Global Earth Models (ICGEM), International DEM Service (IDEMS), International Center for Earth Tides (|ICET) and BGI.

BGI is also recognized as one of the services of the Federation of Astronomical and Geophysical Data Analysis Services (FAGS) that operates under the auspices of the International Council for Science (ICSU)" (BGI, 2010). With establishment of BGI and IGFS Global geoid modelling has been improved with different methods as part of efforts in geoid determination.

2.1.1 Efforts in Geoid Determination:

Apart from Global geoid modelling, several other efforts have been made in respect of geoid determination all over the world. A lot of projects are currently being planned, some are ongoing and many have been completed, while methods of geoid determination continue to be improved upon. Some of those efforts are discussed below:

Hirvonen used gravimetric method to carry out the first computation of geoid on a worldwide scale in 1934. He computed the Geoidal Undulation for 62 points distributed in an East –West band encircling the entire Earth surface (Torge, 2001). For the first time, mean free air anomalies were estimated from the available gravity data covering 5˚ by 5˚ blocks. The number of points used for this computation was very small and hence the need for repetition of the exercise for better result.

Also, Tanni was able to use large quantity of gravity data available between 1948 and 1949 to compute the Global geoidal height. He employed the Prat-Hayford system's gravity reduction method and Airy Heiskanen system. He later computed the Global Geoidal Undulation in 5˚ by 5˚ blocks with a more detailed 1˚ by 1˚ blocks geoid of Europe.

Furthermore, since gravity data is the major data required in gravimetric computation, efforts were made to improve on the acquisition of gravity data. By 1957, five times gravity data more than the one used by Tanni was available. Heiskanen used those measurements to compute the gravimetric geoid of Columbus using Free Air anomalies. He used electronic computer for numerical integration of Stoke's formula (Torge, 2001).

There have been several efforts for the definition of Canadian geoid. These efforts continue in 1974, when Department of Energy, Mines and Resources, embarked on the formulation of procedures and techniques necessary for redefinition of geodetic networks in Canada, in which geoid played an important role. Several researches have been done on the use of astro-gravimetric method for geoid determination. This method requires the observation of deflection of vertical. “The result of deflection of the verticals components are correct to within 0.03% for the case where it is sufficient to model the local gravity anomalies by a plane, if a more complex modelling is required, then many additional integrals would have to be evaluated. Several other efforts that were made in Canada include the Canadian geoid ’88. Nagy (1989) observed that the Canadian geoid ’88 differed beyond the expected error bounds, despite the fact that, there has been a number of local geoid determination over Canada. He then focused the attention on the sources of errors, which might have accounted for such discrepancies.

The theory of Stokes-Helmert scheme was developed by Vanicek and Martinec (1994) for the precise determination of geoidal height. Alamdari et al (2005) used this theory for precise determination of the geoid in Iran. In the scheme, the Earth gravity field was first reduced to the so called Helmet gravity field. The topographic and the atmospheric masses above the geoid as a surface material layer with known surface density were both condensed onto the geoid. As a result of condensation, the geoid as an equipotential surface is uplifted to a new position where it is called the **co-geoid**. The co-geoid is determined as a solution to the Geodetic Boundary Value Problem (GBVP) in the Helmert space. In this method of geoid determination, two different kinds of data were used. The purely satellite-derived geo-potential coefficients, already reduced to the Helmet space were used to determine the long wavelengths part up to harmonic degree and order 20 of the co-geoid (spheroid of degree 20). The terrestrial and local gravity anomalies were used into the generalized Stokes Integral employing spheroidal Stokes kernel to determine the remaining short wavelengths part, that is the residual co-geoid. Finally, the co-geoid is transformed to the geoid in the real Earth space by precisely accounting for the indirect effect (uplift). This method is accurate but tedious and requires additional work.

Featherstone et al., (1998) presented the practical approach to gravimetric and geometric geoid height supported by result from three GPS controlled gravity surveys conducted in Western Australia. The methods were combined to accurately recover Orthometric Height from GPS.

There has been several works done in deriving continental-wide geoid models (Roman and Smith, 2000; Featherstone et al., 2001 and Merry, 2003). However, there was no comprehensive work on the continent of Africa until 2001, when the International Association of Geodesy (IAG) initiated the African Geoid Project (AGP), a project for the computation of an African geoid (Merry and Blitzkow, 2001 and Merry, 2003). The status and progress of the AGP was outlined by Merry (2003). He described "how a precise African geoid may be used to convert GPS-derived heights to local vertical reference frames, and how this geoid may be used to establish the relationship between the disparate vertical reference frames in Africa".

Flury and Rummel (2002) reviewed the earlier work on the Boundary Value problems of Physical Geodesy. They used the so-called error constant or error volume as the central element's theory in the "prediction of the accuracy of Geoidal Undulations from error estimates of given mean gravity anomalies or anomaly profiles". They applied this theory to gravity anomalies as they were typically available at this time, both in terms of data density and precision. Based upon a carefully collected and very dense data set in a test area in the Alps, they addressed the following issues

(i) Requirements on DTM's and density models for the reduction of Alpine gravity and height data in order to arrive at a 'flatland' gravity anomalies behaviour,

(ii) Accuracy estimates of height anomalies based upon the results of (i)

(iii) consideration of various gravity functions such as gravity anomalies, gravity disturbances, deflections of the vertical, geoid heights and height anomalies. It could be shown, for example, that in order to arrive at height anomalies with 1cm-precision global gravity measurements are required with a typical data spacing of 5 km and

(iv) study of the representation error of discrete and mean gravity anomaly data down to a typical spacing of 1 km,

In an effort to improve the geoid determination in Greenland, Forsberg and Kort (2002) reported the compilation of gravity data to include the combination of topographic, bathymetric and geological structure. Airborne gravity surveys were done. Free air and Bouguer anomalies data are now available.

The initial problem in the determination of Indonesian geoid or gravity field is accuracy and lack of comprehensive and inadequate land gravity data. Heliani *et al.*, (2004) worked on the determination of Indonesian geoid and proposed the solution to unavailability of data by means of a simulation technique. The simulation was done by combining short wavelength topographic effects from GTOPO30 and long wavelength information from GEM96. The simulated results and the observed gravity data were favourably compared. GEM96 is not as accurate as GEM2008, yet produced a comparable result with observed data. This shows that the approach can equally be used in accurate determination of geoid for any area.

Featherstone (2004) reported that the members of the Western Australian Centre for Geodesy and University of New Brunswick's collaborators in Australia and around the world are active in the determination of future generations of the Australian geoid model and its relation to the Australian Height Datum (AHD). As part of Australian Research Council grant, funds were made to continue improving the theories of geoid determination, techniques and provision of computer software to the National Mapping Division of Geosciences Australia (formerly AUSIG). They released a geoid model in 2004, called AUSGeoid2004. Featherstone (2004) summarized the work on several key aspects to produce a new generation of geoid model for Australia which will better support the direct transformation of GPS-derived heights to the 1971 realization of the AHD.

Mueller (2005) reported the different methods of geoid determination have been applied in part of the North Aegean Trough (NAT), which forms a continuation of the seismically active North Anatolian Fault Zone. Sea Surface Heights (SSH) needed to be determined with high accuracy. The methods to determine highly-precise DV include astro-geodetic observations with the new Zenith Camera DIADEM, as well as GPS boat and buoy measurements to provide accurate values of SSH. The data during the campaign were gathered, computed, stored and compared favourably with the existing local gravimetric and geoid models. Geoid height differences calculated from DV and compared with GPS based SSHs showed a very good agreement. The comparison of these data sets with the gravimetric geoid model HGFFT98 revealed significant disagreements. The comparison with the altimetric geoid model resulted in smaller differences, while it was also found that the altimetric, Deflection of the Vertical and GPS models follow the same variations in the geoid height signal. This showed that any of the methods can produce accurate results when properly apply in any area.

Al Marzooqi et al (2005) while deriving transformation parameters for the Dubai Emirates in United Arab Emirate gave particular attention to the conversion of Orthometric Heights to their corresponding Clark1880 ellipsoidal heights, using Abridged Molodensky formulas and the Dubai precise Geoid model. “The optimal datum transformation parameters between the WGS’84 datum and Clark1880 were determined, which is based on 2966 common points for the mainland and 88 common points in Hatta region with standard deviation of 0.15 cm in planimetry for the mainland and 0.13m for the Hatta region”. The Authors also considered determination of the gravimetric geoid based on the combination of spherical harmonics potential coefficient set with terrestrial gravity data. Since the subject of this discussion is on transformation parameters, much attention was not given to geoid determination.

Zanletnyik et al., (2006) reported the investigations done for the purpose of determination of the separation of the geoid in Hungary. In their work, they reviewed the lithospheric geoid solution as proposed by Rapp and Kalmár (1996), the gravimetric solution to HGR97 as investigated by Kenyeres (1999), HGTUB98 and HGTUB2000 solution reported by Tóth and Rózsa (2000). The authors have done research using sequence of neural networks to approximate the geoid surface in the area of Hungary. The results were analyzed, in which the errors of the estimation were compared with the errors of other approximation methods, such as Zanletnyik Hungarian Polynomial fitting, single Radial Base Function (RBF) and sequence of neural network. In comparing the methods, the sequence of neural networks proved to be better. On the basis of their research, the error of the estimation reduced efficiently using the sequence of neural network. Zanletnyika et al., (2006) also analysed the classical approximation polynomial model fitting for the approximation of the geoid surface, a gravimetric geoid solution was used with 211680 known geoid heights in a regular grid. 8484 points were selected for the teaching set from the whole database, and the approximation method was tested. In accordance with the results, the teaching set can represent quite well the whole database of the known geoid heights. Cutting out an area with unreliable data outside Hungary, the estimation was improved significantly. The standard deviation of the errors of estimation was reduced to 5cm and this accuracy is of the same order as the accuracy of the original data. However, Zanletnyik Hungarian Polynomial fitting is over parameterised with 26 coefficients and requires large computer memory with additional work to determine the order suitable for the area under study.

Benahmed and Fairhead (2007) reported that the Algerian Geodetic Laboratory of National Centre of Space Techniques has focussed a part of the current research on the precise geoid determination using different methods. In 1997, the first Algerian preliminary geoid determination was done in a small zone, especially the Northern parts of Algeria, which was calculated using the Least Squares collocation and the "GRAVSOFT" software package, developed during a number of years at the National Survey and Cadastre (KMS). The Fast Fourier Transformation (FFT) and the Remove-Restore procedure were used to compute the quasi-geoid. Remove-Restore is the operation of subtracting and adding the effect of the systematic parts of the data, before and after the prediction process (Amin et al., 2005). The final estimates were taken as an improved quasi-geoid over the whole of Algeria.

Wang, et al (2012) reported that a new gravimetric geoid model known as United State Gravimetric Geoid 2009 (USGG2009), have been developed for the United States and its territories including the Conterminous US (CONUS), Alaska, Hawaii, Guam, and the Commonwealth of the Northern Mariana Islands, American Samoa, Puerto Rico and the US Virgin Islands. USGG2009 supersedes the previous models USGG2003 and G99SSS. Details of the data were made available online. (NGS, 2009a)

On the 11th of September 2012, the National Geodetic Survey of United States released an updated model for transforming heights between the physical height systems, that relate to water flow (that is the geoid) and the ellipsoidal coordinates. "These models cover regions including the conterminous United States (CONUS), Alaska, Hawaii, Guam and the Commonwealth of the Northern Mariana Islands, and American Samoa. Models for Puerto Rico and the U.S. Virgin Islands (USVI) are being held back pending release of final control data for the USVI but will likely be released later. GEOID09 transforms to NAVD 88 in CONUS and Alaska and to the respective datum for all the other regions (each having its own datum point). The use of Deflection of the Verticals for geoid determination is not common due to the tedious nature of data acquisition. Interestingly, models for the Deflection of the Verticals have been released for these same regions mainly for aid in navigational systems" (NGS, 2009a). The project has solved the problem of geoid and Deflection of the Verticals models in United States.

One of the major problems identified with the Nigeria geodetic network is lack of geoid model. GGU (2006) in their study, have produced the first gravimetric geoid for Nigeria at about 1m accuracy. The study was sponsored by National Space Research and Development Agency (NASRDA). The accuracy obtained is too low for many geodetic networks analysis. Also, the Lagos State Mapping and Geographical Information System concluded the 8 modules of the project. The importance of geoid and GPS are well recognized and hence devoted Module 2 as "Determination of Geoid and Establishment of Active GPS Reference Station" (Nwillo, 2010). The geoid was determined using GPS/levelling and other approaches. The final result was published on the official website of Lagos State government. Also, the proposed and currently ongoing mapping projects of Akwa Ibom, Ogun and Benue States of Nigeria will also compute geoid. With these developments, Nigeria has resulted into piece-meal approach in geoid determination. If the whole country is covered with this approach, a combination of the method will be required so as to have a geoid for the country.

Several other efforts which include development of various methods of geoid determination were made. In this research, 'Satlevel' collocation method combines the accuracy of Orthometric Height and ease of ellipsoidal height in geoid determination. Orthometric Height used the available methods of geodetic levelling and ellipsoidal height from satellite method (Differential Global Positioning System (DGPS)). Existing methods were also used to validate the new models.

2.2 EXISTING METHODS OF GEOID DETERMINATION

The basic task, in any methods of geoid determination, whether the modern or the classical, is to determine the geoidal height or Geoidal Undulation. Several methods of geoid determination exists either as combination of existing methods or new method are developed. These methods can be grouped into two: the classical or deterministic approaches which compute absolute geoid, and the empirical approach in which heights observations are made and used in a numerical modelling of the geoid. This second approach is also referred to as predictive methods. The classical methods are: gravimetric, astro-geodetic, astro-gravimetric, Rudzki geoid, geoid from the Satellite Altimetry and geoid from the New Geopotential Earth Model (GEM2008).

2.2.1 Classical or Deterministic Approaches

The classical methods involved the availability of data beyond the computation points and use of formulae such as Stokes' Formula, Brun's formula and so on to determine the geoid. This requirement for availability of data beyond the computational points made these methods to be tedious, time consuming, expensive and laborious. These existing methods include the following:

2.2.1.1 Gravimetric Geoid: The word gravimetric comes from gravity, which can be defined as the resultant effect of gravitation and centrifugal forces of rotating Earth (Heiskanen and Moritz, 1967; Fubara, 2007). Gravimetry contributes significantly to Geology (and also shared with Geodesy) by "aiding the determination of the mass distribution below the Earth's surface, which has significant implications in terms of prospecting and exploration for hydrocarbons, mineral deposits, water, and so on, as well as to general knowledge of Earth's structure. The interpretation of temporal changes of the gravity field helps in understanding geodynamic phenomena, such as Earthquakes, volcanic and magmatic processes, isostatic rebound, tectonics, other periodic vertical and horizontal land movements etc". Gravity can also be applied in the study of variation in the density of the Earth while micro-gravimetric observations can contribute to archaeology, by detecting caves or cavities (Vajda, 2006). Apart from that, gravity can equally be used in geoid determination and external equipotential. The geoid determined using gravimetric information and the well known Stoke's formula is called '*gravimetric geoid'*.

Gravimetric geoid is the oldest method of geoid determination. The principle of this method requires that the entire Earth's surface be sufficiently and densely covered with gravity observations. Practically, a dense gravity net around the computation point and a reasonably uniform distribution of gravity measurement outside are sufficient. Then, gravity approximation is inevitable; so as to fill the gap with extrapolated values.

Gravimetric geoid solutions can come from various methods such as:

- the addition of local gravity and terrain data,
- satellite-derived global geopotential models,
- combined global geopotential models result from the addition of terrestrial gravity and terrain data

- Satellite-only solution.

However, Satellite-derived global geopotential models are of long wavelength (typically a few hundred kilometres) so they are of less use to the GPS user (Featherstone et al., 1998 and 2005). Depending on area of coverage, gravimetric geoid may be global, regional or local. Regional gravimetric geoid models are the best because they are of high resolution, local gravity and terrain data are often added to the global geopotential model and optimised for the area of interest.

Gravimetric method is the most commonly applied method for geoid determination. The method was applied in various part of the world as discussed by various authors (such as: Field, 1978; Wenzel, 1982; Ezeigbo, 1983; 1985 and 1993; Agajelu, 1990; Ayhan, 1993; Abd-Elmotaal, 1998; Rózsa, 1999; Bajracharya, 2003; Merry, 2003; Fotopoulos, 2003; Alamdari et al., 2005; Osasuwa, 2006; GGU, 2006). Evans and Featherstone (2000) discussed the improvement of convergence rate for the transformation error in gravimetric geoid. However, the application of this technique is mainly dependent on the availability of high-resolution gravity data. The original technique is based on Stoke's Integral formula.

Stokes' Formula: The Geoidal Undulation (N) at any point P (φ, λ) on the Earth's surface can be computed using the evaluation of the Stokes' Integrals, given by Bernhard and Moritz, 2005; GGU, 2006 and Orupabo, 2007 as:

$$N = \frac{R}{4\pi G}\iint_{\sigma} \Delta g \, S(\psi) d\sigma \qquad (2.1)$$

where;

$\iint_{\sigma}$ an integral extended over the whole Earth

$R =$ Mean radius of the Earth.

G = G is the universal gravitational constant:

G = 6.673 × 10^{-11} ms^{-2} (or N m^2 kg^{-2}), which has the same value for all pairs. of particles.

Δg = Gravity anomaly known everywhere; on the Earth

S(ψ) = Stokes' function between the computation and integration points ψ = Spherical distance

dσ = Differential area on the geoid

Stokes's formula, Equation (2.1), given above is often described as classical solution of the geodetic boundary value problem. It computes absolute geoid and requires data all over the Earth to compute Geoidal Undulation. This makes its application to be expensive, tedious and time consuming. The above Stoke's formula also serves as basis for astro-gravimetric method.

2.2.1.3 Astro-gravimetric: Astro-gravimetric geoid is obtained from Stoke's formula. Differentiating Stokes's formula (Equation 2.1) with respect to φ and λ, will result in the corresponding Veining Meinesz expressions given by Torge (1989); Agajelu, (1997); Bernhard and Moritz, (2005); and Orupabo (2007) as:

$$\xi = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g \frac{dS(\psi)}{d\psi} \cos\alpha d\sigma \tag{2.2a}$$

$$\eta = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g \frac{dS(\psi)}{d\psi} \sin\alpha d\sigma \tag{2.2b}$$

where;

ξ = deflection component along the meridian

η = deflection component along the Prime Vertical

γ = mean gravity of the Earth

α = azimuth

$$\frac{ds(\psi)}{d\psi} = -\frac{\cos\frac{\psi}{2}}{2\sin^2(\frac{\psi}{2})} + 8\sin\psi - 6\cos\frac{\psi}{2} - 3(\frac{1-\sin\frac{\psi}{2}}{\sin\psi}) + 3\sin\psi\ln(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}) \tag{2.3}$$

is the derivative of the Stokes function S(ψ)

The above equations (Equations 2.2a and 2.2b) show that deflections of vertical can be computed from gravity anomalies. Therefore, gravimetric technique of geoid computation involves the evaluation of the Stokes' and Veining Meinesz's integrals, given by (Ezeigbo, 1988; 1993; Torge, 1989; GGU, 2006 and Orupabo, 2007):

$$\begin{bmatrix} N \\ \xi \\ \eta \end{bmatrix} = \frac{1}{4\pi\gamma}\begin{bmatrix} R\sum_{i=1}^{n}\Delta\overline{g}_i\iint_{\sigma_i} S(\psi)\,d\sigma \\ \sum_{i=1}^{n}\Delta\overline{g}_i\iint_{\sigma_i}\frac{dS(\psi)}{d\psi}\begin{Bmatrix}\sin\alpha \\ \cos\alpha\end{Bmatrix}d\sigma \end{bmatrix} \tag{2.4}$$

where;

N, ξ and η are the Geoidal Undulation, Component of deflection of the vertical along the meridian direction, and the Component of the deflection of the vertical along the prime vertical direction, respectively.

R and γ are the mean radius and mean normal gravity of the Earth respectively.

$\Delta\overline{g}_i$ is the mean gravity anomaly in the block σ_i of the n blocks in which the gravity anomalies are available.

α is the azimuth of the integration point relative to the computation point. It is given by (GGU, 2006 and Orupabo, 2007):

$$\tan\alpha = \frac{\cos\phi'\sin(\lambda'-\lambda)}{\sin\phi'\cos\phi - \sin\phi\cos\phi'\cos(\lambda'-\lambda)} \tag{2.5}$$

Equation (2.4) is evaluated using geographically defined blocks given by GGU (2006) and Orupabo, (2007) as:

$$d\alpha = \cos\phi' d\phi' d\lambda' \tag{2.6}$$

a numerical evaluation formula of Equation (2.5) is given by (GGU, 2006 and Orupabo, 2007);

$$\begin{bmatrix} N \\ \xi \\ \eta \end{bmatrix} = \frac{1}{4\pi\gamma}\begin{bmatrix} R\sum_{i=1}^{n}\Delta\overline{g}_i \iint_{\sigma_i}\frac{\Delta\alpha}{m}\sum_{j=1}^{m}S\left(\psi_j\right) \\ \sum_{i=1}^{n}\Delta\overline{g}_i \iint_{\sigma_i}\frac{dS\left(\psi_j\right)}{d\psi}\begin{Bmatrix}\sin\alpha \\ \cos\alpha\end{Bmatrix}d\sigma \end{bmatrix} \tag{2.7}$$

where;

$\overline{\Delta}$g is the mean gravity anomaly

$\Delta\sigma$ is the area of each block

m is the number of subdivision of $\Delta\sigma$

$$S\left(\psi\right) = -\frac{1}{\sin(\frac{\psi}{2})} - 6\sin\frac{\psi}{2} + 1 - 5\cos\psi - 3cos\ln(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}) \tag{2.8}$$

called Stoke's function.

Obenson's line Integral Solution transforms the area integral (Equation 2.8) to a line integral given by (Obenson, 1983; Ezeigbo, 1985 and GGU, 2006) as:

$$\begin{bmatrix} N \\ \xi \\ \eta \end{bmatrix} = \frac{1}{4\pi\gamma}\begin{bmatrix} R\sum_{i=1}^{n}\Delta\overline{g}_i C_N \\ \sum_{i=1}^{n}\Delta\overline{g}_i \begin{Bmatrix} C_\xi \\ C_\eta \end{Bmatrix} \end{bmatrix} \tag{2.9}$$

$$\begin{bmatrix} C_N \\ C_\xi \\ C_\eta \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} S\left(\psi_j\right)\Delta\alpha_j \\ \sum_{i=1}^{n}\partial S\left(\psi_j\right)\Delta\beta_j \\ \sum_{i=1}^{n}\partial S\left(\psi_j\right)\Delta\gamma_j \end{bmatrix} \tag{2.10}$$

where;

$$\Delta\alpha_j = \Delta\alpha_{j+1} - \Delta\alpha_j$$
$$\Delta\beta_j = \Delta\beta_{j+1} - \Delta\beta_j$$
$$\Delta\gamma_j = \Delta\gamma_{j+1} - \Delta\gamma_j$$

S(ψ) and đS̅(ψ) given by (Obenson, 1983; Ezeigbo, 1993):

$$S(\psi) = 1.75\cos^2\psi - \cos\psi + \sin\frac{\psi}{2}(3\cos\psi + 1) - 1.5\sin^2\psi \ln(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}) \tag{2.11a}$$

$$\begin{aligned}\partial\bar{S}(\psi) = {} & \frac{\psi}{4} + \cos\frac{\psi}{2} - 6.5\sin\frac{\psi}{2}\cos\frac{\psi}{2} - 6\sin^2\frac{\psi}{2}\cos\frac{\psi}{2} + 13\sin^2\frac{\psi}{2}\cos\frac{\psi}{2} - 2\ln\tan\frac{\psi}{2} + \\ & \left(1.54 - 3\sin\frac{\psi}{2}\cos\frac{\psi}{2} + 6\sin^2\frac{\psi}{2}\cos\frac{\psi}{2}\right)\ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right) - 3\frac{\psi^2}{4} + \frac{1}{6}\psi^3 - \frac{3}{128}\psi^4 + \\ & \frac{1}{150}\psi^5 - \frac{1}{3072}\psi^6 + \frac{1}{2205}\psi^7 - \frac{61}{491520}\psi^8 + \ldots.\end{aligned} \tag{2.11b}$$

The intention is to get the exact integral of the above Equation (2.7), which can be evaluated using Ezeigbo's Analytical Formulas for Stokes' and Veining Meinesz's Integrals in any geographically defined blocks (Ezeigbo, 2005). This method will transform the geographical grid block method to the template method. Hence, the exact integral solution to the Equation (2.7) is obtained using the relevant equations as given by GGU (2006) and Orupabo (2007):

$$\begin{bmatrix} N \\ \xi \\ \eta \end{bmatrix} = \frac{1}{4\pi\gamma}\begin{bmatrix} R\sum_{i=1}^{n}\Delta\overline{g}_i \int_{\psi_i}^{\psi_{i+1}}\int_{\alpha_i}^{\alpha_{i+1}} S(\psi)\sin\psi\, d\psi\, d\alpha \\ \sum_{i=1}^{n}\Delta\overline{g}_i \int_{\psi_j}^{\psi_{j+1}}\int_{\alpha_j}^{\alpha_{j+1}} \frac{\partial S(\psi_j)}{d\psi}\begin{Bmatrix}\sin\alpha \\ \cos\alpha\end{Bmatrix}\sin\psi\, d\psi\, d\alpha \end{bmatrix} \tag{2.12a}$$

$$\begin{bmatrix} N \\ \xi \\ \eta \end{bmatrix} = \frac{1}{4\pi\gamma}\begin{bmatrix} R\sum_{i=1}^{n}\Delta\overline{g}_i \sum_{j=1}^{m}\left(\vec{S}(\psi_{j+1}) - \bar{S}(\psi)\right)(\alpha_{j+1})(\alpha_j) \\ \sum_{i=1}^{n}\Delta\overline{g}_i \sum_{j=1}^{n}\left(\partial\bar{S}(\psi_{j+1}) - \partial\bar{S}(\psi_j)\right)\begin{Bmatrix}\sin\alpha_{j+1} - \sin\alpha_j \\ \cos\alpha_{j+1} - \cos\alpha_j\end{Bmatrix}\sin\psi\, d\psi\, d\alpha \end{bmatrix} \tag{2.12b}$$

where;

All the terms are as earlier defined.

The above procedures were used by the GGU (2006) in the computation of optimum geoid for Nigeria. Optimum geoid is the geoid computed from all possible geoidal quantities that could be obtained, based on a given set of data. The computation of geoid using the above procedure is tedious, time consuming and required gravity data and large computer memory. Other formulae that also relate Geoidal Undulation to certain quantities includes;

2.2.1.4 Brun's formula: Brun's formula relates the Geoidal Undulation (N) to the disturbing potential (T) and normal gravity (γ). The equation is given by Heiskanen and Moritz (1967) as:

$$N = \frac{T}{\gamma} \tag{2.13}$$

Computation of anomalous potential T is a boundary value problem. The fundamental equation used to solve boundary condition from Third Boundary Value Problem is given by (Moritz, 1980) as:

$$\Delta g = \frac{\partial T}{\partial \eta} + \frac{1}{\gamma}\frac{\partial \gamma}{\partial \eta} T \tag{2.14}$$

Brun's Equation (2.13) can be used to compute the Geoidal Undulation, if the boundary condition is satisfied on the geoid, with the use of fundamental equation in Physical Geodesy, the geodetic boundary value problem can be solved for the precise determination of the geoid (Najafi- Alandani, 2006). The condition is that gravity anomaly (Δg) must be known at every point on the geoid for a linear combination of T and $\frac{\partial T}{\partial \eta}$ to be given upon the surface.

T can equally be computed using fully normalised spherical harmonics. In this approach, Moritz (1980) assumed that T(θ, λ) contains no spherical harmonic of degrees zero and one. Thus, the spherical harmonic expansion of T has the form:

$$T(\phi,\lambda) = \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left[a_{nm}\bar{R}(\phi,\lambda) + b_{nm}\bar{S}_{nm}(\phi,\lambda)\right] \tag{2.15}$$

The spherical harmonic expansion of the function Equation (2.15) can be written as:

$$K(\psi) = \sum_{n=2}^{\infty} k_n p_n(\cos\psi) \tag{2.16}$$

where; P_n (cos ψ) are the (usual or "conventional") Legendre polynomials. The k_n can be expressed in terms of a_{nm} and b_{nm} by:

$$k_n = \sum_{n=2}^{\infty}\left(\bar{a}_{nm}^{2} + \bar{b}_{nm}^{2}\right) \tag{2.17}$$

k_n refers to conventional harmonic, where a_{nm} and b_{nm} are coefficients of fully normalised harmonics.

2.2.1.5 The Rudzki geoid: Bajracharya (2003) investigated the effect of various methods of gravity reduction on Helmert geoid. Each of the reduced gravity was used in geoid determination. He concluded that the Rudzki geoid which had never been used in the past for geoid determination proves to be as good as the Helmert and Residual Terrain Model (RTM) geoids, and better than the Airy-Heiskanen (AH) and Pratt-Hayford (PH) geoids, but compared to GPS-levelling after fit. Also, he noted that Rudzki geoid has the smallest bias among all other reduction schemes. The main advantage of using this method is that one does not have to compute the indirect effect on the geoid required for all other reduction schemes. Therefore, it can become an alternative tool for gravimetric geoid determination in the future.

2.2.1.6 Astro-geodetic Geoid:

This is often referred to as Astronomic levelling. The method of geoid determination is based on the assertion of F. R. Helmert. The observation of transit time of stars yields astronomic coordinates while GPS gives ellipsoidal coordinates. The angle between the plumb-line and ellipsoidal normal represent the deflection of the vertical (Section 1.1.6). Integrating these values along a profile will lead to differences in Geoidal Undulation.

Geoidal Undulation can be computed from known deflection of vertical using the expression for Helmet's formula as follows:

$$dN = -\varepsilon dS \tag{2.18a}$$

$$N_B - N_A = -\int_A^B \varepsilon dS \tag{2.18b}$$

$$N_B = N_A - \int_A^B \varepsilon dS \tag{2.18c}$$

where;

$$\varepsilon = \xi \cos\alpha + \eta \sin\alpha \tag{2.18d}$$

$$\xi = \Phi - \phi \tag{2.18e}$$

$$\eta = A - \lambda \cos\phi \tag{2.18f}$$

ε = deflection of vertical components

ξ = deflection of vertical component along the meridian

η = deflection of vertical component along the prime vertical

Φ = astronomic latitude

Λ = astronomic longitude

ϕ = geodetic latitude

λ = geodetic longitude

dS = element of distance

When deflection of vertical components are obtained by comparing astronomic and geodetic coordinates of the same point to determine the geoid, then the method is referred to as **astro-geodetic determination of geoid.** In this method, the integration is performed along the profile, hence deflection of vertical should be known along the profile which is a limited area. However, the points on which deflection of verticals are known should be closed to one another. Thus, a profile for ε can be constructed by interpolation, so that the integration can be performed numerically or digitally. In practice, ε = ξ and ε = η are often used for North-South and East – West profiles respectively.

$$dN = \varepsilon dS \tag{2.18g}$$

$$\varepsilon = \frac{dN}{dS} \tag{2.18h}$$

North – South direction ε = ξ and

$$dS_\phi = Rd\phi$$

East – West direction $\varepsilon = \eta$ and

$$dS_{\lambda} = R\cos\phi d\lambda$$

$$\therefore dS^2 = R^2 d\phi^2 + R^2\cos^2\phi d\lambda^2 \quad (2.19)$$

$$\xi = \frac{dN}{dS_{\phi}} = \frac{1}{R}\frac{dN}{d\phi} \quad (2.20a)$$

$$\eta = \frac{dN}{dS_{\lambda}} = \frac{1}{R\cos\phi}\frac{dN}{d\lambda} \quad (2.20b)$$

Astro geodetic method can give a better accuracy in geoid determination but can only be implemented, if two neighbouring Astro geodetic stations are closed to each other, so that the geoidal profile between them can be approximated by the arc of a circle. Then the formula can be written as:

$$N_B - N_A = \frac{\varepsilon_A - \varepsilon_B}{2} \quad (2.21)$$

'in this way the interpolation can be avoided; but this is only apparent, since the assumption that the geoid between A and B form a circular arc itself equivalent to an interpolation and not necessarily the best one' (Agajelu, 1997).

In moderate or flat terrain, a distance of 25km between the stations is satisfactory, while in the mountainous area 10km may not be sufficient. Hence interpolation in such an area is inevitable. Interpolation between Astro-geodetic stations can be used for the following measurements:

i. measurement of zenith distance
ii. astro-gravimetric levelling
iii. use of topographic – isostatic deflection.

2.2.1.7 The Geoid from the Geopotential Earth Model: The geoid models that are defined by a set of coefficients of spherical harmonic expansion for the entire Earth is called **Geopotential Earth Model** (**GEM**) Or **Earth Gravitational Model (EGM)** or **Geopotential Gravitational Earth Model.** GEM is a global and hence generalise data in a particular locality. They are of different versions as a result of availability of data to improve the result. The year of publication is always used as the version number. These includes: GEM 96, GGM01S, PGM 2000A, EIGEN-CGO1C and

EIGEN-GRACE2S. These models cannot accurately model the local variation. As earlier observed, GEM '96 differed from the observed values by 2m in Nigeria. However, the latest edition released to the public named GEM2008 is said to provide sub-meter accuracy at every point on the Earth.

Geopotential Earth Model 2008 (GEM2008): GEM2008 is the new Global geoid model made available to the public and published by the National Geospatial-Intelligence Agency (NGA). It replaced the GEM96 model which had been the default Global geoid since its publication in 1996. GEM2008 was developed "to degree and order of 2160 with the availability of improved versions of worldwide 5′×5′ gravity databases and GRACE-derived satellite solutions. The accurate 5′×5′ global gravity anomaly database that takes advantage of all the latest data and modelling for both land and marine areas worldwide were used to achieve a geoid accuracy of 15 centimetres Root Mean Square (RMS) worldwide. This was possible with an improved long wavelength model from GRACE, improved terrain and altimetry data, and the very best surface gravity database that were compiled from available data all over the world. The Shuttle Radar Topographic Mission (SRTM) data has been used with other elevation sources (GTOPO30, ICE Sat, and others.) to develop a worldwide 30 sec by 30 sec topographic database that is being used for terrain corrections and Residual Terrain Modelling (RTM) of all the surface gravity data. ICE Sat has been used over Antarctica and other polar regions above the Shuttle Radar Topography Mission (SRTM) coverage along with other available altimetry sources (Kenyon et al., 2007). The development of a Mean Sea Surface (MSS) over the oceans and associated Dynamic Ocean Topography (DOT) has been one of the key components and major improvement on the new GEM2008.

GEM2008 is provided in terms of spherical harmonic coefficients which generally need to be converted into a grid of geoid undulations before they can be used. To compute GEM2008 undulation, Geopotential coefficients is available on the INTERNET and plugged into the Geopotential model. Program for the conversion have been developed by several authors. Some of them are available on the NGA website and Alltrans calculator on the softpedia website

In case of the lack of proper gravity data, the geoid could be modelled with different geometric methods such as astro-geodetic method or geoid height from GPS in conjunction with spirit levelling (Kuhar *et al.,* 2001 and Mustafa *et al.,* 2007). Featherstone, (2004) and Soltanpour (2006) called these methods geoid-type surface. The approaches of using GNSS/GPS and spirit levelling was referred to as GPS-Levelling geoid (Véronneau, 2002; Johnston and Luton, 2001; Fotopoulos, 2003 and Soltanpour et al., 2006). The method is a predictive approach to geoid determination.

2.2.2 Empirical or Predictive Approaches

The alternatives to gravimetric methods are the predictive methods which require data at discriminate points within the study area. In empirical approach, the heights measurements are taken and used in numerical modelling of the geoid. The empirical method has the advantage of simplicity, ease of use and provides sufficient accuracy but requires extensive data collection for meaningful result over large area. Such methods may include: First degree Harmonic, North Sea Region Model, 4-Parameter similarity datum shift, 5-Parameter similarity datum shift, 7-Parameter similarity datum shift and Zanletnyik Hungarian Polynomial model fitting (Engelis et al., 1984; Haagmans *et al*., 1998; Featherstone *et al.*, 1998; Fotopoulos 2003; Danila, 2006; Zanletnyik *et al.,* 2006). In this approach, different types of functions are used. They are:

2.2.2.1. Single Function: Depending on the area of coverage and availability of data, single function such as linear, trigonometric, harmonic, Fourier series, splines, wavelets and combination of two or more functions may be used. These were used in some researches such as Featherstone (2000). The author has investigated the use of continuous curvature splines in parts of Australia (Featherstone, 2000).

2.2.2.2 Polynomials Functions: Some classic orthogonal polynomials include: Legendre, Tschebyscheff of first and second kind, Jacobi, Laguerre and Hermite. If the application of these models is not suitable or too complex for practical use, then one can also apply orthogonalization or orthonormalization procedures to decorrelate the existing base functions. A common orthonormalization procedure that is relatively simple to implement in practice is the Gram-Schmidt orthonormalization method (Fotopoulos, 2003). Fotopoulos (2003) investigated the use of orthogonalization procedures and submitted that it can be used to decorrelate parameters of any parametric model; however the results cannot be applied for prediction of height values at new points (GNSS-levelling) due to the lack of an analytical form for the 'orthogonalized' model.

2.2.2.3 First Degree Harmonic of Geoidal Height: Another astro-geodetic method of computing the geoid is the use of first degree harmonic of geoidal height. Heiskanen and Moritz (1967) gave the expression as:

$$N_i(\phi,\lambda) = \xi \sin\phi \cos\lambda + \eta \sin\phi \sin\lambda + \zeta \cos\phi \tag{2.22}$$

where;

ξ, η *and* ζ are real coordinates of centre of gravity, the origin being the centre of the ellipsoid. All other terms are as earlier defined.

2.2.2.4 North Sea Region Model: In this model, two or more different types of base functions were combined. Haagmans et al. (1998) developed the recent North Sea region model where; the selected models can be represented by the following equations (Haagmans et' al, 1998 and Fotopoulos 2003):

$$a + b\lambda + c\phi + d\phi\lambda \tag{2.23}$$

where;

ϕ = geodetic latitude

λ = geodetic longitude

a, b, c and d are the coefficients which may be estimated using Least Squares adjustment procedure.

The model Equation (2.23) above is described as a bilinear trend function. The implementation of the model is combination of trigonometric function based on Fourier analysis, which was used in different parts of the North Sea region to model the long-wavelengths. The model gave a comparative accuracy with other existing models. However, the use of trigonometric function based on Fourier analysis is time consuming and not popular among the practitioners and hence not convenient.

2.2.2.5 The 7-Parameter Similarity Datum Shift: Another family of North Sea region model which is closely related is based on the general 7-Parameter Similarity Datum Shift Transformation. This model is simplified with classic 4-Parameter Similarity Datum Shift model given by Fotopoulos (2003) as:

$$ax = x_1 + x_2 \cos\phi \cos\lambda + x_3 \cos\phi \sin\lambda + \mathrm{x}_4 \sin\phi \qquad (2.24)$$

The model was extended to include the fifth parameters as follows:

$$ax = x_1 + x_2 \cos\phi \cos\lambda + x_3 \cos\phi \sin\lambda + \mathrm{x}_4 \sin\phi + \mathrm{x}_5 \sin^2\phi \qquad (2.25)$$

Fotopoulos (2003) reported that Kotsakis et al., (2001) developed a more complicated form of the differential similarity model. The model was tested in the Canadian region and is given by Fotopoulos, (2003) as:

$$\begin{aligned} ax = x_1 + x_2 \cos\phi \cos\lambda + x_3 \cos\phi \sin\lambda + \mathrm{x}_4 \left(\frac{\sin\phi_i \cos\phi_i \sin\lambda_i}{W} \right) + \\ \mathrm{x}_5 \left(\frac{\sin\phi_i \cos\phi_i \cos \mathrm{s}\, \lambda_i}{W} \right) + x_6 \left(\frac{1 - f^2 \sin^2\phi_i}{W} \right) + x_7 \left(\frac{\sin^2\phi_i}{W} \right) \end{aligned} \qquad (2.26)$$

where;

$W = \left(1\text{-}\ e^2 \sin^2 f\right)^2$

e is the eccentricity

f is the flattening of the ellipsoid

All other terms are as earlier defined.

Equation 2.26 is similar to datum shift transformation model given by Heiskanen and Moritz (1967). However, Fotopoulos (2003) reviewed the model and observed that: “the parameters from such a 'datum shift transformation' do not represent the *true* datum shift parameters (translations, rotations and scale) because other long-wavelength errors inherent in the data (such as those in the geoid heights) will be interpreted as tilts and be absorbed by the parameters to some degree". The model (Equation 2.26) considered the Earth as both ellipsoid and sphere. The assumption of sphere and ellipsoid for the shape of earth at the same time is not a common practice.

2.2.2.6 Classical Approximation Polynomial Model Fitting: Instead of the application of this huge geoid database for practical purposes, Zanletnyik et al., (2006) found a simple mathematical formula (an equation of surface of geoid forms in Hungary). Using this mathematical formula to compute geoid heights in arbitrary points in Hungary would be simpler than interpolating the geoid heights between known points, especially if it should be implemented in a computational procedure.

Zanletnyik et al., (2006) developed the classical approximation model polynomial fitting to approximate the geoid heights as a function of geographic coordinates (φ, λ). The formula for the 6th order fitting of the Classical Approximation Polynomial is called Zanletnyik Hungarian Polynomial Model, which is given as follows:

$$\begin{aligned} N = {} & a_0 + a_1 \cdot \phi + a_2 \cdot \lambda + a_3 \cdot \phi^2 + a_4 \cdot \phi \cdot \lambda + a_5 \cdot \lambda^2 + a_6 \cdot \phi^3 + a_7 \cdot \phi^2 \cdot \lambda \; + \\ & a_8 \cdot \phi \cdot \lambda^2 + a_9 \cdot \lambda^3 + a_{10} \cdot \phi^4 + a_{11} \cdot \phi^3 \cdot \lambda + a_{12} \cdot \phi^2 \cdot \lambda^2 + a_{13} \cdot \phi \cdot \lambda^3 \; + \\ & a_{14} \cdot \lambda^4 + a_{15} \cdot \phi^5 + a_{16} \cdot \phi^4 \cdot \lambda + a_{17} \cdot \phi^3 \cdot \lambda^2 + a_{18} \cdot \phi^2 \cdot \lambda^3 + a_{19} \cdot \phi \cdot \lambda^4 \; + \\ & a_{20} \cdot \lambda^5 + a_{21} \cdot \phi^6 + a_{22} \cdot \phi^5 \cdot \lambda + a_{23} \cdot \phi^4 \cdot \lambda^2 + a_{24} \cdot \phi^3 \cdot \lambda^3 + a_{25} \cdot \phi^2 \cdot \lambda^4 \; + \\ & a_{26} \phi \cdot \lambda^5 + a_{27} \cdot \lambda^6 + \ldots\ldots\ldots \end{aligned} \tag{2.27}$$

where;

a_i = coefficients of the Zanletnyik Hungarian Polynomial

N = geoid height

Φ, λ = geodetic latitude, longitude.

The authors (Zanletnyik et al., 2006) submitted that, differences between known geoid heights and approximated values are characteristic of accuracy of geoid heights computed by Zanletnyik

Hungarian Polynomials model. Increasing the degree of polynomials, first accuracy will increase, and then may decrease above the sixth degree, because of the deterioration of conditions of equations. The inconsistency in accuracy of this model becomes a serious issue which call for concern and may need further investigation because accuracy and precision are parts of properties of geodetic measurements, observations and computations.

Using this model, Zanletnyik et al., (2006) have the best result at sixth order with 8484 dataset. Second degree gave a good result for the 88 data used in Port Harcourt Nigeria (See section 3.3.5). This shows that, the model cannot be solely relied on because the order of polynomial at which the model satisfied a given set of data needs to be determined for better result.

2.2.2.7 Geometric Methods: Different geometric methods can be used to compute the geoid. Featherstone et al (1998) uses linear interpolation which is sufficient over a small area using Equation 2.28. The model is given as:

$$h - H = N = N_0 + N_1 e + N_2 n \tag{2.28}$$

where;

N_o = bia**s**

N_1 *and* N_2 = tilt of the geoid with respect to the ellipsoid

h = ellipsoidal height

H = Orthometric Height

e and n = easting and northing in plane coordinates system.

Featherstone et al (1998) submitted that geometric determination of the Orthometric Height is trivial for a short profile, where; GPS surveys are rarely conducted. (Equation 2.28) above is modified to include the tilt of the geoid and the use of rectangular coordinates.

2.2.2.8 Sequence of Neural Networks Method: Zanletnyik et al., (2006) done a research in which a sequence of neural networks was applied to approximate the geoid surface in the area of Hungary. They estimated the geoid, using RBF (Radial Basis Function) neural network with 35 neurons having

Gaussian activation functions. They submitted that, the radial basis type activation function proved to be the most efficient in case of function approximation problems. They applied RBF network with input geodetic latitude, longitude (φ, λ) and output N (geoid height). The RBF network consists of one hidden layer of activation functions, or neurons. The method also yields good result in Hungary.

2.2.2.9 The Coefficient of Representativity (CR): Paláncz et al (2006) discussed the extensive use of machine learning algorithms, such as artificial neural networks (ANN) and support vectors machines (SVM) with their wide range of applications. The applications include classification, regression, feature extraction, data prediction and spatial data analysis. The Authors proposed a simple method based on 'the Coefficient of Representativity (CR)' for extracting representative learning set from measured geospatial data. In this method, sample points having low CR value from the dataset were eliminated successively. They illustrated its application in data preparation for the correction and used it to model the Hungarian gravimetrical geoid based on the available GPS measurements. The results were analysed and found to be reliable.

2.2.2.10 Ellipsoidal Approximation in Geometry and Gravity Space: Ardalan and Grafarend (2007) developed and tested new methods for high-resolution regional geoid and quasi-geoid determination based on ellipsoidal approximation in geometry and gravity space.

2.2.2.11 Combination of Wavelet and Fast Fourier Transformation (FFT): A computational scheme using a combination of wavelet and FFT transforms has been developed for local geoid approximation. Wavelet multi-resolution analysis, FFT, and the combined algorithm were introduced for the solution of the Stokes problem. The wavelet algorithm was built based on using an orthogonal wavelet base function. Different thresholding and filtering techniques are used in the case of the wavelet only solution. Different mother wavelets are tested for both the wavelet only and the combined FFT Wavelet solution. The combined scheme showed an indication to the existence of a shift invariant wavelet solution. The direct proof and numerical results were given for the combined algorithm. The combined algorithm has overcome the problem of FFT when dealing with non-stationary signal and kernel. The comparison between FFT, wavelet transform, and combined FFT and wavelet transform was done through the solution of both stationary and non-stationary cases (El-Habiby and Sideris, 2006).

Along the same direction, Spherical Fast Fourier Transformation Method was used in some studies done by Tóth and Rózsa, 2000; and Rózsa 2003). In both studies, the gravimetric geoid was computed with 1D Spherical Fast Fourier Transformation method. Their solutions were based on terrestrial gravity data, height data and the GEM96 geopotential model. Rózsa (2003) included Digital Terrain Model (DTM) data, and GPS/Levelling data, which lead to the improve accuracy of the result.

2.2.2.12 A Non-Conventional Interpretation: Petrovskaya, and Pishchukhina (1989) observed complexity of the integral kernel (the Stokes' function) when implementing the Stokes integral formula which is commonly used for evaluating the geoid heights. The complexity of the integral kernel results in a bulky set of formulae for determining the remote zone influence. Therefore, a non-conventional interpretation method was developed by Petrovskaya, and Pishchukhina (1989) which allows derivation of very simple formulae to evaluate the contributions of both close and remote zone components of the geoid heights. The methods used were classified into different techniques. This eases the problem of geoid computation and produced accurate result.

2.2.2.13 Application of Fuzzy Logic: Fuzzy logic is a mathematical logic that attempts to solve problems by assigning values to an imprecise spectrum of data in order to arrive at possible conclusion. It is an approach to computing based on degrees of truth rather than the usual true or false or 1 or 0. It has been extended to handle the concept of partial truth, where the truth value may range between completely true and completely false. The method of fuzzy logic has been extended to geoid determination as Mustafa et al (2007) reported the application of fuzzy logic approach in geoid surface approximation in Istanbul and Sakarya, a town about 150 km east of Istanbul in Turkey. The topographic structures of these regions have different characteristics. For these two regions, geoid heights have been determined through fuzzy logic approach and the obtained results are interpreted and found to be reliable.

2.2.3 Interpolation Methods

There are several interpolation methods that can be adopted in geoid determination. For example SURFER a contouring and 3D surface mapping software package from Golden Software Incorporation can transform random surveying data, using interpolation, into continuous curved face contours. It is provided with eighteen different gridding/trend surface algorithms. The methods were grouped into two as smoothing and exact interpolators. Smoothing interpolators are: Inverse Distance to a Power, Kriging , Polynomial Regression, Radial Basis Function, Modified Shepard's Method, Local Polynomial, Moving Average; while the exact interpolators are: Inverse Distance to a Power, Kriging, Nearest Neighbor, Radial Basis Function, Modified Shepard's Method, Triangulation with Linear Interpolation, and Natural Neighbor. Any of these interpolation methods can be used to approximate the geoid in an area if adequate data are available. The software is originally meant for interpolation of height but can be adopted for geoidal Undulation by using the Undulation value in place of height as data input.

2.2.4 Geoid from the Satellite Altimetry

Satellite altimetry is based on satellite-borne altimeter which transmits approximately 13.5GHz frequency radar pulses in the vertical direction to the Earth’s surface. The height of satellite above the Earth’s surface is received by measurement of the time of propagation of the reflected pulse. Satellite altimetry measurements lead to geoid heights or with inverse Stoke’s formula to gravity anomalies. The mean sea surface height sensed by the altimeter is corrected for sea surface topography to yield geoid heights (Featherstone, 2003). Apart from that, satellite altimetry may also be used for the following (Sideris and Fotopoulos, 2006):

- determination of MSL,
- determination and removal of Sea Surface Height (SSH) and Sea Surface Temperature (SST) from tide gauge and sea level data,
- development of precise global geopotential models,
- determination of marine geoid,
- gravity models for the solution of the geodetic Boundary value problem,
- improved geopotential models,
- Improving the determination of bathymetric data, which in turn leads to better models of the marine geoid and gravity.

Kenyon et al (2007) highlighted the efforts undertaken by the Danish National Space Centre to produce a new MSS utilizing data from altimetry satellites GEOSAT, TOPEX/Poseidon, ENVISAT, JASON-1, ERS-1/2, GFO, and ICE Sat. Satellite altimetry method is used for geoid determination on the sea, where measurement generally suffered poor accuracy because of unfavourable condition. The recent advances in orbit modelling and it's corrections have been a major factor leading to the improvements to the altimetry data along with re-tracking waveforms, particularly with the ERS and GEOSAT Geodetic mission data (Kenyon et al, 2007).

2.2.5 Integration of the Models

Most of the types of models for geoid determination discussed above are based on the use of a single model to represent the study area irrespective of the extent. This approach assumes that a homogeneous set of discrepancies exist over an entire region, regardless of its coverage and data distribution. This is not always the case and hence sometimes limits the application of the method. If the area is large, the accuracy will be low. For example, consider an instance of the task of selecting a single model to adequately model all of the discrepancies across large regions such as Africa, Canada, Europe and Australia, where comparatively sparsely distributed sets of GPS-levelling control points are available (Véronneau, 2002; Johnston and Luton, 2001; Fotopoulos, 2003). An additional limitation of this approach is that it relies on a single model to deal with both long and short wavelength discrepancies. Fotopoulos (2003) suggested one way to deal with this situation, that is to divide the region into a number of smaller sub-regions and fit the appropriate model to that region using, for example, any of the aforementioned models. The type of model or extent of the model (e.g. order of Zanletnyik Hungarian Polynomial) may vary for each sub-region. The author also noted new problems that may be associated with implementing this approach to include:

(i) How to divide the region,

(ii) How to connect across adjacent sub-regions and

(iii) The type of parametric model suited for a particular set of control points may be completely incompatible for a different region.

Therefore, the importance of empirical tests with real data cannot be over emphasised. These models can now be integrated to produce the undulation of the whole region. Fotopoulos (2003) called the procedure **mosaic of parametric models.** The model is of the form (Fotopoulos, 2003):

$$\Delta N = \Delta N_0 + a\left(\phi - \phi_0\right) + b\left(\lambda - \lambda_0\right)$$
$$\Delta N_{ij} = a\left(\phi_j - \phi_i\right) + b\left(\lambda_j - \lambda_i\right) \quad (2.29)$$

where;

ΔN is the observed Geoidal Undulation as given with respect to the geoid

φ, λ and ΔN are the mean values of latitude, longitude and Geoidal Undulation, respectively, in the sub-region

ΔN_{ij} is the difference of Geoidal Undulation between points i and j, and the coefficients to be determined from the adjustment of the common points are denoted by a and b.

The model (Equation 2.29) gives a comparative accuracy over a large area with the other existing models. However, the area has to be divided in to compartments with common points between the compartments. The common points may eventually have slight different values as a result of prediction from different compartments.

The present application in various part of the world is to model the geoid using the empirical models. The empirical models determined by different Authors were also used to validate the GEM2008 which was used to compute the Global Geoidal Undulation for the study areas (See Tables 4.4a 4.4b for Port Harcourt and Lagos State respectively).

Most of the scholars have worked on geoid but no attempt has been made to consider the relative. In this research, efforts are made to compute GEM2008 Geoidal Undulation and use it as long wavelength part in determination of relative geoid to compute the Orthometric Height.

2.3 METHODS OF ORTHOMETRIC HEIGHT DETERMINATION

Orthometric Height is natural "height above sea level" measured along the plumb line from the foot point on the geoid to the point on the surface gravity value. Orthometric Height belongs to the group of physical/natural height systems that are fundamentally related to the Earth's gravity field. Therefore, gravity is a major factor in determination of Orthometric Height. Other height systems are: levelled height, geopotential numbers, dynamic heights and normal heights (Bomford, 1980; Davis et al., 1981; Ebong, 1981, Fajemirokun, 1981; Fotopoulos, 2003; Ceylan, 2005 and Robert, 2011).

Methods of determining Orthometric Height can be grouped into two namely, direct and indirect methods. The direct method involves practical determination of Orthometric Height using level, with application of Orthometric correction where necessary, while the indirect method involves observation of ellipsoidal height using Global Navigation Satellites System (GNSS) and computation of Geoidal Undulation using any geoid modelling technique (Section 2.2). When the Geoidal Undulation is determined from global model such as GEM2008, the Orthometric Height so determined is GEM2008 Orthometric Height. This is applicable to any other global model. The direct methods include the following:

2.3.1 Spirit (Geodetic) levelling:

Levelling is the determination of height or difference in elevation between two points with reference to a known datum. Geodetic Levelling is levelling operation done with refined measurement and precised equipment to attain high accuracy and precision. As earlier discussed, geodetic levelling is very precise in procedure, accurate and self checking but tedious, time consuming and can be affected by terrain, swamp and climate. These difficulties have forced researchers to examine alternative methods of height determination. As a result of modern high-tech instrument developments, research has again been focused on precision trigonometric levelling.

2.3.2 Trigonometric levelling/heighten:

Trigonometric levelling is the process of levelling in which the elevations of points are computed from the observed vertical angles and measured horizontal distance. Recent technological developments allow the use of total station to observe vertical angles and measure horizontal distances for high accurate levelling over long distances. The accuracy compared favourably with that of geometric levelling. Trigonometric levelling can provide centimetre accuracy in height differences and better, depending on the accuracy and precision of equipment used. (Obong, 1985; SURCON. 2003; Davis, 1981; Mikhail et al., 1981)

2.3.3 EDM-Height-Traversing: "Leap-Frog":

EDM-Height-Traversing is also called Leap-frog. In this method a target is set and remaining at a particular change point for both fore and back sights. The target is not moved between the two sightings in order to avoid the possibility of the target being placed on a different point. This is similar to three tripod system in theodolite traversing. “Two targets/reflectors are employed (on reflector rods with struts). As in spirit levelling, it is imperative that the electronic tacheometer (total station) is set up in the middle between the two reflectors. The height differences (between the instrument's trunion axis and the reflector) are observed, recorded and computed by the electronic tacheometers. Consequently, the ambient temperature and pressure are input into the instrument since the slope distances must be corrected for temperature and pressure" (AusAID, 2007). Also, Ceyla and Baykal (2006) analyzed the result of leap-frog trigonometric levelling for the sight of distance S = 150 m which resulted in a standard deviation of ±1.87 mm/$\sqrt{km}$ and a production speed of 5.6 km/day. The Total Station levelling technique has a number of benefits over normal spirit levelling. The elimination of collimation errors, Staff calibration errors and the minimization of refraction errors make the technique attractive to those undertaking Class A ("First Order") levelling. The use of significantly longer sight lengths makes it attractive to everyone else. The technique does require slightly longer observation periods per standpoint; however this is offset by fewer instrument standpoints for the same length of run. (Ceyla and Baykal, 2006; AusAID, 2007; Davis, 1981**;** Mikhail et al., 1981 and Robert, 2011)

2.3.4 Stadia (Tacheometric) Levelling:

Tacheometry or tachymetry or telemetry is a swift method of surveying in which both the horizontal and vertical distances of points are obtained by optical mean relative to one another. Here, elevations of points are computed from the vertical angles and horizontal distance measured in the field using trigonometric principle (Davis et al., 1981). The main objective is production of topographical or contour map. Heights determined by tacheometric means are only use for mapping and lower order jobs but not accurate for geodetic purposes**.**

2.3.5 Barometric Levelling (Altimetric heighten):

The method is based on the use of atmospheric pressure to determine the elevation of points. This method is less precise and not used for any geodetic exercise.

2.3.6 Hydrostatic Levelling:

This method uses long tubes on the seabed. The tube is filled with water. It also required special instruments to define level of water on both sides of the tube. The idea looked simple but not practicable. In longer tubes air bubbles can make measuring very difficult and unstable. Likewise, installation of long tubes is very expensive in practice (Davis et al., 1981). This method is less precise and not use for geodetic exercise.

2.3.7 Sea-transition Levelling:

Sea transition levelling is a method of height transfer over water and valley (for example from mainland to an island). Transfer of height over water is special and may be tasking because of refraction error which is always difficult to calculate over the surface of open water. The method of levelling was developed particularly for transferring height across large volume of water, by the Zeiss/Oberkochen Company. The method was originally developed for height determination on islands (Ilija et al., 2008). This method required two targets and a level. The only condition is the visual contact (intervisibility) between the two stations. Ilija et al (2008) reported that, this method was used for transmission of height to major benchmarks to Rab Island in the Republic of Croatian. Accuracy of 1cm was obtained in this project.

2.3.8 Hypsometry:

it is the method of levelling in which the heights of mountains are determined by observing the temperature at which water boils. The method cannot be used for accurate work in Geodesy.

All these methods have their advantages and disadvantages. Geodetic levelling is the most precised and accurate method, and therefore adopted in this research for height determination, from where difference in elevation can be obtained, which required the application of Orthometric correction to get the Orthometric Height.

2.3.9 Orthometric (Height) Correction

Orthometric Height Correction is the small correction needed to be applied to the observed elevation difference along a given line of précised levelling so as to get the Orthometric Height. Orthometric Height correction is a function of gravity and related to latitude, longitude and height of any chosen point. Hence, it should be computed and applied to get the geopotential number that is equal in a particular locality. The popular notion is that Orthometric correction is very small especially in low elevation area and it is always neglected. There are different methods of computing Orthometric correction which may yield difference results and differences can reach several centimetres. "This implies that OHs from levelling may mismatch the true Orthometric Height by several centimetres, if the OC computation is not sufficiently accurate" (Hwang and Hsiao, 2003). One of the formulae for computing Orthometric Correction is given as (Heiskanen and Moritz 1967):

$$OC_{HB}^{HM} = \sum_{i=1}^{k} \frac{g_i - \gamma_0}{\gamma_0} \delta n_i + \frac{\bar{g}_A - \gamma_0}{\gamma_0} H_A - \frac{\bar{g}_A - \gamma_0}{\gamma_0} H_B \tag{2.30}$$

where;

OC is the Orthometric correction

γ_0 is normal gravity at some latitude (usually 45°N or 45°S).

H is the elevation height

g is the mean gravity along the plumb line between the surfaces

A and B are the two benchmarks A and B.

Another simplified formula for computing Orthometric Correction was developed by Hwang and Hsiao (2003) as:

$$OC_{AB} = \frac{1}{g_B}\left(\frac{\bar{g}_A - g_B}{2} - g_B\right)\Delta n_{AB} + H_A\left(\frac{\bar{g}_A}{g_B} - 1\right) \tag{2.31}$$

where;

all the terms are as earlier defined.

When Orthometric Height correction is applied to difference in elevation, it will result in the Orthometric Height of the point.

Most of the scholars have worked on Orthometric Height and different height systems but no attempt has been made to compute Global Orthometric Height and adaptation of the global geoid to its local equivalent. In this research, efforts are made to compute GEM2008 Orthometric Height and to adapt the global Orthometric Height to local Orthometric Height. Other information required to implement Equation 1.3 is the ellipsoidal height.

2.4 METHODS OF ELLIPSOIDAL HEIGHT DETERMINATION

Ellipsoidal height is the determined with reference to the ellipsoid and can be obtained mainly from satellite methods.

2.4.1 Satellite Methods

Heights determined from satellite methods are always ellipsoidal height. Using today's available technology and techniques, ellipsoidal heights can be obtained from a number of difference systems, such as (Engeliset al., 1984; King et al., 1985; Adhikery, 2001; El-Rabbany, 2002; Fotopoulos, 2003; Uzodinma, 2005 and Fubara, 2007):

i. Very Long Baseline Interferometry (VLBI),
ii. Satellite Laser Ranging (SLR),
Other navigation based systems such as:
iii. Doppler Orbitography by Radio-positioning Integrated on Satellite (DORIS)

The families of Global Navigation Satellites System such as:

iv. Global Positioning System (GPS) of USA
v. Global Navigation Satellite System (GLONASS) of Russia
vi. GALELIO of Europe
vii. Compass Navigation Satellite System (CNSS) of China
viii. Indian Regional Navigation Satellite System (IRNSS)
ix. Beidouin of China
x. Quasi Zenith Navigation Satellite System (QZSS) of Japan and
xi. Satellite altimetry.

2.4.1.1 Global Positioning System (GPS): GPS is a location fixing system initiated by the United States (U.S.) Department of Defence (DoD) based on acquiring satellite signals (tracking) with the aid of receiver and processing of data to obtain the three dimensional (3D) coordinates of the receiving station. GPS is fully functional Global Navigation Satellite System (GNSS). At present, it utilizes a constellation of about 31 medium Earth orbiting satellites. These satellites transmit precise microwave signals and enable the GPS receiver to determine its location, time and speed (if the antenna is moving). Various Authors have discussed the system segments, configuration, policies, implementation and applications (King et al., 1985; Grenoble and Mark, 1995; Leick, 1995; Gregory, 1996; Featherstone, 1996; Agajelu, 1997; Franke, 1999; Higgins, 2000; Adhikery, 2001; Vanicek, 2001; El-Rabbany, 2002; Martti, 2002; Seeber G. 2003; NIS, 2004; Moka and Okeke, 2005; Uzodinma, 2005; Kaplan and Hegarty, 2006; Fajemirokun, and Nwillo, 2007; Ogundare, 2007a; Olopa, 2007 and Sarumi, 2007). Apart from GPS, there are other systems, which serve the same function like GPS but belong to other nations. They are discussed below:

2.4.1.2 GLONASS: The former Soviet Union and now Russia developed 'GLObal'naya NAvigatsionnaya Sputnikovaya Sistema' meaning GLObal NAvigation Satellite System (GLONASS). The GLONASS constellation also reached its full operational capability of 24 satellites in 1996. Currently, only twenty satellites are in operation with two active spares four are under maintenance. The average lifetime of satellite which was about 4.5 years was improved. Russia has announced publicly its intention to restore the GLONASS constellation to full health status, through the deployment of longer life satellites. The fully operational capability expected in 2010 was achieved on the 5th of March, 2013, with the assistance of India that is currently participating in the restoration project.

With 24 satellites, Russia successfully developed its own analogue of the American GPS, named GLONASS. It is providing now a complete global coverage, a Russian daily reported Dr Andrei Ionin, who works for the operators of GLONASS explained that with 18 satellites, GLONASS was able to provide precise navigation across Russia. With all 24 GLONASS satellites in orbit, GLONASS receivers can pick signal from the quartets that is necessary for precise positioning anywhere in the world.

2.4.1.2 BeiDou Satellite Navigation Experimental System: The BeiDou system was developed by the People Republic of China. The system was officially called *BeiDou* Satellite Navigation Experimental System. The system started in the year 2000 and consists of 3 satellites called **BeiDou-1**, but has limited coverage and applications mainly for customers in China and from neighboring regions.

2.4.1.3 Compass Navigation Satellite System (CNSS) of China: The second generation of the BeiDou Satellite Navigation Experimental System is known as *Compass* or *BeiDou-2*. China has indicated her interest to have a global navigation system similar to GPS. It became operational with coverage of China in December 2011 with 10 satellites in use. It is expected to be in full operation by 2020 with 35 satellites.

2.4.1.4 Indian Regional Navigation Satellites System (IRNSS): IRNSS is being developed by Indian Space Research Organisation. The government approved the project in May 2006. It is expected to be in operation by 2014. The system is envisaged to establish a constellation of seven satellites made up of a combination of Geostationary Earth Orbit (GEO) and Geosynchronous Orbit (GSO) spacecraft over the Indian region. The seven satellites in the IRNSS constellation will consist of—three in GEO orbit (at 34º E, 83º E and 131.5º E) and four in GSO orbit inclined at 29 degrees to the equatorial plane with their longitude crossings at 55º E and 111.5º E (two in each plane). All the satellites will be continuously visible in the Indian region for 24 hours a day.

2.4.1.5 Quasi-Zenith Satellite System (QZSS): QZSS is owned and managed by Japan Aerospace Exploration Agency (JAXA). The first QZSS satellite called 'Michibiki' was launched on 11th of September 2010. Other relevant information available online on JAXA website. Interestingly, JAXA has adopted a data interface based on Receiver Internet Exchange "RINEX 3.01" format in "MGM-Net" which includes the participating ground stations. The idea is to know the availability, capabilities evaluation of multipath and Radio Frequency Interference (RFI) environment of the GNSS for the future QZSS satellites to be launched. Full operational status was expected by 2013.

2.4.1.6 Galileo Positioning System: Galileo is currently being built by European Union and the European Space Agency. The first satellite was launched in 2005 and second in 2008. Full operational capability of 30 satellites is expected by 2019. Europe’s Galileo system (a navigation satellite system) has passed its latest milestone, transmitting its very first test navigation signal back to the Earth. According to European Space Agency (ESA) press statement, the different Galileo signals are being activated and tested one by one. Soon after the payload power amplifiers were switched on and ‘outgassed’– warmed up to release vapours that might otherwise interfere with operations – the first test signal was captured at Redu. The test signal was transmitted in the ‘E1’ band, which will be used for Galileo’s Open Service once the system begins initial operations in 2014. The result of the Galileo is also 3D coordinates. The first two Galileo operational satellites were launched at an altitude of 23.600km on 21st of October 2011. The launch of the Galileo satellites will lead to the provision of initial satellite navigation services in 2014. Successive launches are expected to complete the constellation by 2019.

The development in GNSS application is to integrate the system with other tools for various applications. Some of these integrations are as discussed below:

2.4.1.7 Integration of GNSS and Other Tools: GNSS has been integrated with other methods of data acquisition in other to improve the quality of data for various applications. These have helped in solving a lot of problems where individual method failed. Such integration includes: GNSS and Geodetic levelling, GNSS and GIS, GNSS and Inertial Navigation System (INS), Satellite – to - Satellite Tracking.

a. GNSS and Geodetic Levelling: All GNSS measure height with reference to the ellipsoid while geodetic levelling heights measurements are reduced to the geoid. The difference between the two is Geoidal Undulation. This application is the focus of this research.

b. GNSS and Remote Sensing: Remote Sensing and GNSS have a common origin in the use of satellites as the basic source of data. This shows that there is a closed link between the two systems. Therefore, integrating the system has improved the accuracy. Remote Sensing is capable of revealing a lot of information that may be hidden by other methods and it is used to monitor the environment while GNSS will give the position anywhere on the globe. The positions are accurate and the problems of image distortion in Remote sensing method are solved with the integration. GNSS coordinates are equally used in geo-referencing the Remote sensing image for processing in any application.

c. GNSS and GIS Integration: GNSS and Geographic Information System (GIS) have been combined together and used in various applications. According to Olaleye *et al* (1999) "two technologies on their own show different areas of use but the integration of the two, open a new world of application". This means fundamentally, that one can locate the position of any feature on the Earth surface (GPS) and plot this position in relation to a bigger spatial representation such as map on digital environment (GIS)." Presently, there are software in the market which are capable of interfacing GNSS with GIS. Such may include: IDRIS, ARC/INFor, ERDARS, TNTLite and so on.

d. GNSS and Inertial Navigation System (INS) Integration: GNSS has been integrated with Inertial Navigation System (INS). With this integration, INS has been updated with velocity or position to refine the navigation and measurement of gravity especially deflection of vertical. The complementary characteristics of the two systems made the integration of GNSS and INS viable and widely used for a variety of positioning, navigation, and geo-referencing applications. Depending on type of applications and other factors, GNSS /INS integration can be developed in three modes, viz.: loose, tight, and ultra-tight integrations. The integration Kalmar filter is at the heart of integrated GPS/INS systems. The widely used integration Kalmar filter is based on the INS error dynamic model, including both navigation states and sensor error states. Precise GPS measurements are used to estimate the INS errors and thus the calibrated INS can provide precise position, velocity and attitude information for the user platform.

Apart from the above, this integration has been extensively applied in the mapping of gravity field. The integration also made it possible to precisely determine the following:

i. Velocities
ii. Gravity anomalies
iii. Deflection of vertical.

In this integration it is possible to included deflection of vertical in the post mission analysis and also feasibility of conventional gravimetry from airplane and other vehicles. In order to enhance the capabilities of this integration, University of New South Wales developed both commercial system and in-house software packages for operations of integrated GPS/INS systems through the real data analysis.

e. Satellite – to - Satellite Tracking: The orbit of low orbiting satellite is much affected by gravitational pull, air drag and other effects. Low orbiting satellite may be integrated with GNSS (a high orbiting) satellite for satellite –to- satellite tracking method in gravity determination. The low orbiting satellite will have the capability of taking gravimetric data from the Earth surface which can be sent to GNSS satellite of high altitude, which then determines the position accurately. The two data can be supplied simultaneously to the users.

The theory of satellite –to- satellite tracking was applied in Gravity Recovery and Climate Experiment (GRACE) and Gravity Field and Ocean Circulation Experiment (GOCE). Presently, the system is called Global Earth Observation System of Systems.

Global Earth Observation System of Systems (GEOSS): GEOSS is an international effort to build a public infrastructure which interconnects a diverse and growing array of instruments and systems for monitoring and predicting changes in the global environment (Ezeigbo, 2010). Among the major instruments, which belong to the GEOSS system are the Global Navigation Satellite System (GNSS), Very Long Baseline Interferometer (VLBI), Interferometric Synthetic Aperture Radar (InSAR), Challenging Mini-Satellite Pay-load (CHAMP), Gravity Recovery and Climate Experiment (GRACE), Gravity Field and Steady-State Ocean Circulation Explorer (GOCE).

i. **Gravity Recovery and Climate Experiment (GRACE):** On the 17th of March 2002, GRACE was launched under the Earth System Science Pathfinder Program (ESSP) by the National Administration of Space Agency (NASA). The GRACE mission has 2 identical space crafts (the twin GRACEs) at 220km apart in a polar orbit at an altitude of 500km above the Earth surface. It does consist of satellite range rate measurements, accelerometer GPS and altitude measurement from each satellite. This program enables the accurate mapping of the

Earth’s gravity every 30 days over its five years lifespan with spatial resolution of 400km (half wavelength).

The results of the gravity mapping have been an unprecedented view of the local gravity conditions. Another area of gravity use is detection of groundwater. Water has value of mass, and “GRACE can detect differences in groundwater with outstanding accuracy, along with improvements in the precision of the geoid (a model of the Earth's gravity field) of between 10- to 100-fold. Measurements of ocean bottom pressure obtained from GRACE are of high accuracy, which surprised oceanographers, and GRACE even profiles the global water vapor content of the Earth's atmosphere”. The GRACE satellites have changed the way people look at water. It shows the changes in the Earth's atmosphere, and provides different data on melting rates of the world's ice. For example, it was GRACE that determined that ice loss from the high Asian mountain ranges which was only 4 billion tons a year, compared to the 50 billion tons of ground-based estimates. GRACE pegs global ice loss over the period from 2003 to 2010 at about 4.3 trillion tons, adding about 0.5 inches to the global sea level in eight years.

ii. **Gravity Field and Steady-State Ocean Circulation Explorer (GOCE):** In March 17, 2009, the European Space Agency launched satellite based gravity mission called GOCE into orbit. This mission carries a 3 axis gradiometer and a GPS/GLONASS receiver. The reference orbit is down dust, sun synchronous at an altitude of 250/270km above the Earth’s surface. The combination of high and low altitude satellites that is satellite –to- satellite track and a gradiometer enable an excellent mapping of the Earth gravitation field (Ezeigbo, 2005).

The mission of GOCE by European Space Agency is mapping of the Earth’s gravity field with the same level of accuracy as GRACE and a higher spatial resolution. “GRACE and GOCE are complementary in terms of spectral sensitivity. A series of GOCE and GRACE and GOCE-based global gravity models have been released since 2010. Assessment of these models is commonly based on comparisons with other independent data that are direct and indirect observations of the Earth's gravity field including geoid heights from GPS and spirit levelled heights, airborne and surface gravity measurements, marine geoid heights from mean oceanographic sea surface topography models, altimetry observations, orbits from other geodetic and altimetry satellites. In response to the call of having an independent, coordinated and inclusive team for the assessment of the new GOCE models, a Joint Working Group (JWG) was approved by IGFS and the IAG Commission 2 during IUGG 2011 in Melbourne, Australia. Its objectives are to develop new standard validation/calibration procedures and to perform the quality assessment of GOCE- GRACE and GOCE-based satellite-only and combined solutions for the static Earth's gravity field".

GOCE was able to gather enough data to map Earth's gravity just after two years in orbit. By 31 March 2011, this satellite was able to produce with unrivalled precision the most accurate model of the 'geoid', while on the 12th March 2012; the first global high-resolution map of the boundary between Earth's crust and mantle – the Moho – was produced based on data from GOCE gravity satellite. The most accurate gravity map of Earth has already been delivered by ESA's GOCE gravity satellite on the 16th November 2012. In order to obtain even better results, the orbit of the satellite is being lowered. The incredibly low orbit of the satellite kept less than 260km was maintained and responsible by GOCE's innovative ion engine, together with its accelerometer measurements. GOCE was able to provide new insight into air density and wind speeds in the upper atmosphere. It was also planned that GOCE will give dynamic topography and circulation patterns of the oceans with unprecedented quality and resolution in the near future.

Unfortunately, the plan was dusted on the 21st of October, 2013 when the mission came to a natural end as it ran out of fuel and the satellite gradually descended, with most of the 1,100kg satellite disintegrated in the atmosphere, an estimated 25% reached Earth's surface on Monday 11th November 2013. ESA's GOCE satellite re-entered Earth's atmosphere on a descending orbit pass that extended across Siberia, the western Pacific Ocean, the eastern Indian Ocean and Antarctica. Fortunately, there was no damage to property.

iii**. Multi-GNSS Monitoring Network (MGM-Net**): MGM-Net is a multi-constellation GNSS augmentation and assistance systems which include a plurality of reference stations across the world. Each of the reference stations may be adapted to receive navigation data from a plurality of different GNSS and to monitor integrity and performance data for each of the GNSS. An operation center may receive the integrity and performance data transmitted from each of the plurality of all the reference stations in the network. The Japan Aerospace Exploration Agency (JAXA) has established Multi-Global Navigation Satellites System Monitoring Network under international collaboration as part of "Multi-GNSS Demonstration Campaign". The receiver used in this system can track any GNSS satellites for various applications.

iv. Augmentation Systems: These are the navigational aid developed for different functions in order to improve the accuracy, integrity, and availability of satellites. There are several of such systems worldwide, some are satellites based while other are ground based:

Ground Based Augmentation Systems (GBAS): GBAS is a satellite-based precision approach established at an airport, aimed to provide accurate landing system to the airplane. GBAS provides aircraft with very precise positioning guidance, both horizontal and vertical, which is especially critical during the approach and landing phase of flight. This allows for a safer, more efficient and descent landing operation.

Satellite Based Augmentation Systems (SBAS): Satellite Based Augmentation Systems deliver error corrections, extra ranging signals (from the geostationary satellite) and integrity information for each GPS satellite being monitored. Augmentation Systems includes: US WAAS **Wide Area Augmentation System** (**WAAS**) a navigational aid designed to enable aircraft to rely on GPS for taken off, enrouting, landing operations and any other phases of flight, including precision approaches to all airports within its coverage area. Examples of WAAS are the: European EGNOS Japan’s MSAS, India’s GAGAN, Russia’s SDCM and China’s COMPASS

Research are still continuing on the applications of Global Navigation Satellites System and integrating with other methods of positioning in order to solve geo-spatial problems.

2.4.1.8 Satellite Altimetry: Satellite altimetry measurements are used to obtain ellipsoidal heights over the oceans, which cover more than 70% of the Earth's surface. Fubara and Mourad (1974) considered the use of altimeter data for the determination of the geoid in ocean areas and discussed the analytical data handling formulations. The overall objective of the investigation was a demonstration of the feasibility of the use of altimeter data for the determination of the geoid in ocean areas. The analytical data handling formulations were equally discussed.

The present satellite altimetry mission is the TOPEX/POSEIDON measurement system. If the satellite is accurately positioned, then the orbital height of the space craft minus the altimeter RADAR ranging to the sea surface corrected for path delays and environmental corrections yields the sea surface height as demonstrated in Figure 2.2 and Equation (2.32):

$$h = N + \xi + \varepsilon \tag{2.32}$$

where;

ξ is the ocean topography

ε is the error

other terms as defined earlier

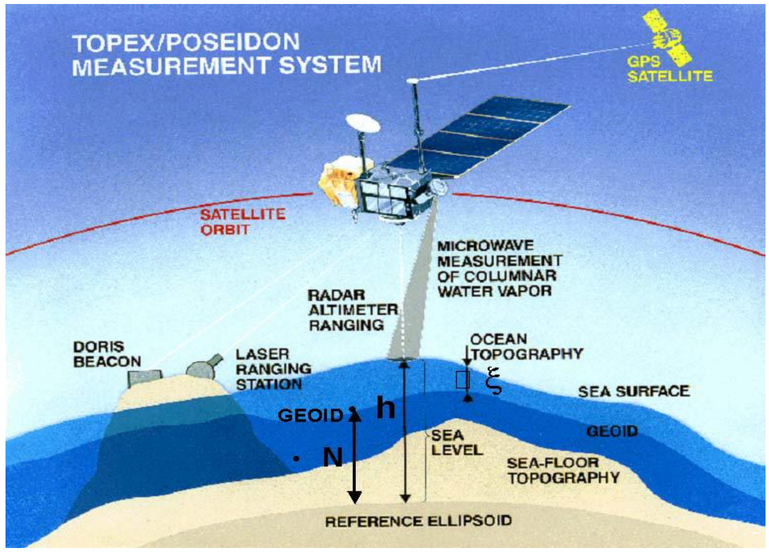


Figure 2.1: Demonstration of Sea Surface Topography for Geoid Determination (Source: JPL, 2012)

The ocean topography is related to deflection of vertical and gravity anomaly. This is useful because the gravity anomalies are more easily interpreted and correlated with seafloor structure, and also because they can be checked against independent measurements made by ships carrying gravimeters (Sandwell and Smith, 2004). The satellites measure ellipsoidal height and the ocean topography which are related to Orthometric Height, Equation 1.3 can then be applied to get the Geoidal Undulation.

The accuracy of GPS and other satellites based height measurements depend on several factors but the most crucial one is the "imperfection" of the Earth's shape. Height can be measured in different ways. The traditional, Orthometric Height (H) is the height above an imaginary surface called the geoid, which is determined by the Earth's gravity and approximated by MSL. The GPS uses ellipsoidal height (h) above the reference ellipsoid. This ellipsoid approximates the Earth's surface to give a definite mathematical shape. (Leick, 1995; Featherstone *et al.*, 1998; El-Habiby and Sideris, 2006; Christopher, 2008; Hofmann-Wellenhof and Moritz, 2005). All these satellite’s systems give the 3D coordinates of the receiving station as the final results.

GPS is the most popular method among the systems discussed above. It is one of the GNSS that presently has complete constellation with spares in space, which has made it universally accepted and hence the chosen method adopted in this work to acquire data for ellipsoidal height. The alternatives methods of obtaining ellipsoidal heights are set to broaden the applications of ‘Satlevel’ collocation in the near future.

2.5 ADAPTATION OF REGIONAL TO GLOBAL GEOID

More meaningful applications of data on a global basis may be enhanced with adaptation from regional data to global data. This was realised in Republic of Croatia, where the geographical shape has unfavourable condition and the Adriatic coast is also unhelpful for geoid modelling on state borders. Adriatic coast with high mountains provide large vertical gradients on geoid surface. Therefore, the need for adaptation of Croatian territory to Global geoid became a necessity. Different methods were used by Ilijah (2008) to get detailed analysis of available gravity data and the new Croatian geoid HRG2000 was calculated and connected to old height system with origin in Trieste (Bašiæ 2001and Ilija et al., 2008).

Furthermore, Al Marzooqi et al., 2005 used the Abridged Molodensky transformation of height to compute the height differences between the local geodetic system ellipsoid and the WGS 84 ellipsoid in Dubai Emirate. Since ellipsoidal height is involved, the need for Geoidal Undulation becomes necessary. Ellipsoidal height was transformed using the Equation of the form (Al Marzooqi et al., 2005):

$$\Delta h = \Delta X \cos\phi_i \cos\lambda_i + \Delta Y \cos\phi_i \sin\lambda_i + \Delta Z \sin\phi_i + \left(a\Delta f + f\,\Delta a\right)\sin^2\phi_i - \Delta a \qquad (2.86)$$

where;

$\Delta X, \Delta Y, \Delta Z$ = corrections to transform local datum co-ordinates to WGS84 X, Y, Z ;

$\Delta a, \Delta f$ = (WGS84 minus local) semi-major axis and flattening respectively;

a = semi-major axis of the local geodetic system ellipsoid;

f = flattening of the local geodetic system ellipsoid.

The above equation assisted in converting the grid coordinates into the geocentric coordinates. However, the computation of the geoid heights (N_{WGS84}) on WGS 84 system above WGS 84 ellipsoid, computation of Cartesian coordinates of the point in the two system, their differences and the associated constants with the Abridged Molodensky model will be additional work before the adaptation. The authors have transformed the heights on WGS84 system to CLARKE 1880 system reference ellipsoid.

All these scholars have worked on geoid and different height system but no attempt has been made to compute Global Orthometric Height and adaptation of the global geoid to its local equivalent. In this research, concerted efforts are made to compute GEM2008 Orthometric Height and to adapt the global Orthometric Height to local Orthometric Height, so as to make it more useful to the surveyor and other height users who preferred a natural height system. The adaptation is based on developed ‘Satlevel’ collocation models as described in the following chapter.

CHAPTER THREE

METHODOLOGY

3.1 THE THEORETICAL CONCEPT IN ‘SATLEVEL’ COLLOCATION

Any curve fitting model based on ellipsoidal and Orthometric Heights can be used in ‘Satlevel’ collocation, depending on the area/region, nature of topography and the assessment of the model performance. The type of base functions may however vary. One possibility is a polynomial (of various orders) represented by the Multiple Regression Equation (MRE) can also be used (Fotopoulos, 2003). Other types of base functions include trigonometric, harmonic, Fourier series, splines and wavelets. In this research, regression methods were used to fit the geoid in two study areas.

The following theoretical concepts are used in this work

1. **Geoidal Undulation:** With GNSS, ellipsoidal height can be obtained, while the Orthometric Heights can be determined by geodetic levelling with application of Orthometric correction, where applicable. The difference between ellipsoidal and Orthometric Heights is called **Geoidal Undulation** or **geoid separation** (N) (See Equation 1.3). Hence, the work is based on geodetic surfaces (Section 1.1) and GPS satellites

   The concept of GPS can be summarised as Wright (1990) “Whilst attempting to track the position the position of satellites from known position on the surface of the Earth, it was realised in USA that if the orbit of the satellite was known accurately, then the position of a receiver could be determined using satellite’s position”. Theoretically, the principle is based on the original idea of using the known positions of the satellites in space to get the position(s) of receiver(s) on the surface of the earth. This can be compared to the resection in traditional (conventional) positioning technique. As earlier discussed, the final result of GNSS is the 3D coordinates.

   Figure (3.1) shows geodetic surfaces and GPS Satellites.

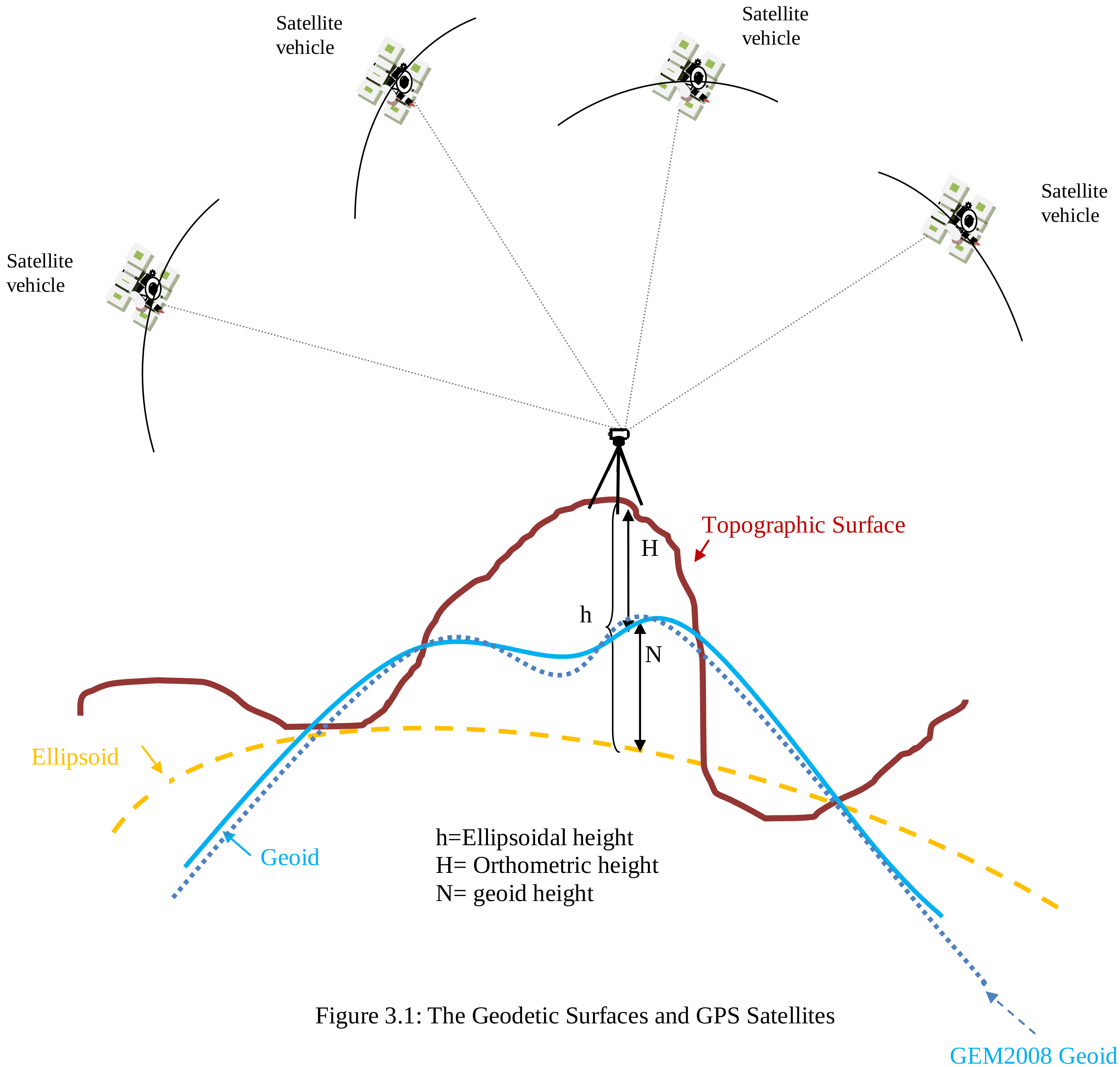


Figure 3.1: The Geodetic Surfaces and GPS Satellites

2. Absolute geoids are computed using Equations 2.23 to 2.27 to obtain global undulations.

GEM2008 undulation was determined using Alltrans calculator (an online program) downloaded from softpedia website. (Softpedia, 2009)

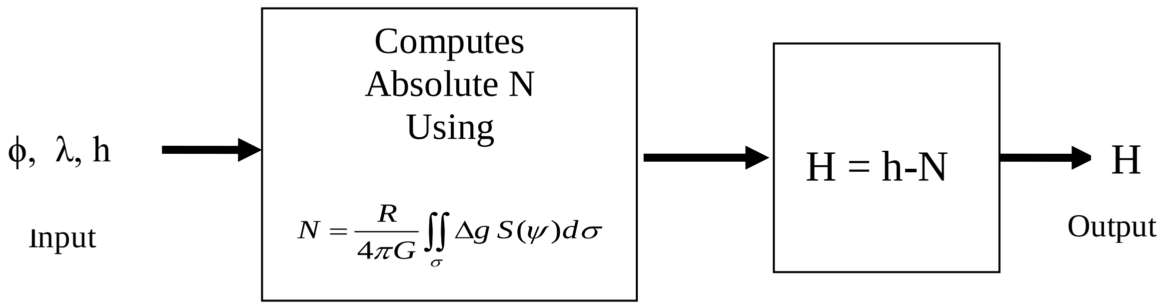


Figure 3.2a Input / Output Procedure for Computation of Absolute Geoidal Undulation

(Source: Author: October, 2009)

3. The residuals between the GEM2008 and Geoidal Undulations (local undulations) were computed using Equation 3.77

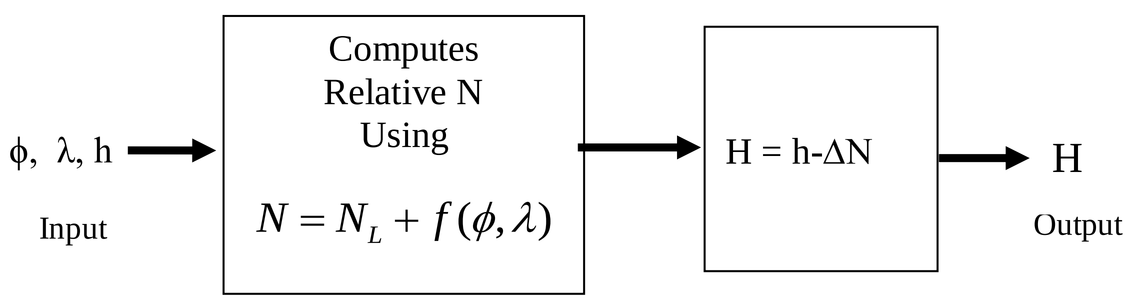


Figure 3.2b Input / Output Procedure for Computation of Relative Geoidal Undulation

(Source: Author: October, 2009)

where;

N_L = the long wavelength component of the Geoidal Undulation

ϕ, λ, h = Geodetic latitude, longitude and ellipsoidal height

$\iint_{\sigma}$ = an integral extended over the whole Earth

R = Mean radius of the Earth

Δg = gravity anomaly known everywhere; on the Earth surface

$S(\psi)$ = Stokes' function between the computation and integration points

ψ = Spherical distance

$d\sigma$ = Differential area on the geoid.

4. 'Satlevel' model was derived using regression analysis

3.1.2 Regression Analysis

This is a technique commonly applied to measure the relationship which can be used to estimate between one dependent variable and one or more independent/causal variables. Different regression analysis includes: linear, quadratic, cubic, compound, logarithmic, inverse, power, growth, logistic and exponential regression analysis. A combination or a series of each of the above will result in polynomial or multiple regression analysis. Multiple regression analysis is used to examine the influence of two or more independent variables on the dependent variable. This analysis utilizes the Least Squares method to fit a general linear model to a set of data along with the estimation and test procedures associated with it (Belsley et al., 1980; Sincich, 1986; and Nicholson, 1986). In this work, Least Squares adjustment was used to estimate the geoidal coefficients.

3.1.3 Least Squares Adjustment

Least Squares adjustment was first introduced by C. F. Gauss, a German mathematician and a geodesist in 1794. It was published by A. M**.** Legendre, a French mathematician in 1806. The "principle requires the minimization of sum of squares of residuals" (Leick, 1980; Mikhail and Gracie, 1981; Young, 1985; Moka, 1990; Moka, 1999 and Ayeni, 2001). The residual is the difference between the observed and computed values.

The observation should be over-determined to provide redundancy. Generally, a geodetic network is observed with more observations than the minimum observations necessary, so as to give redundant observations. These redundant observations give the possibility to adjust the network with the following advantages (Strang Van Hees, 1984; Iyalla 1988):

(i) increasing the accuracy of the computed unknowns,

(ii) estimating the standard deviation of the observations and the unknowns,

(iii) testing the functional and stochastic model,

(iv) finding gross-errors in the observations.

With modern instruments in use today, the precision of observations is not the reason for measuring redundant observations. The most important purpose for redundant observations is to detect gross errors or blunders and ensure adjustement of network using Least Square techniques. The principle is based on assumptions.

3.1.3.1 Assumption in Least Squares Adjustment

The theory of Least Squares is based on the following assumptions:

1. Observations are normally distributed
2. Observations have mean and variance
3. Residual is assumed to be random
4. Expected value of residual and the errors are zero
5. Weight and relative weight are known

Unfortunately, the nature of observation in surveying is not always linear. As a result, observation equations should be written for all observation. The observation equations should be linearized to form the normal equations, so as to be able to impose the Least Squares condition. Any series expansion formula to linearize non linear equation can be adopted. In this work, Taylor's series expansion was used for linearization.

As earlier discussed, the observations are over-determined. Since more observations than the minimum observations necessary are made, then mean and variance can be computed. Therefore, the observations have mean and variance.

A residual is the difference between an observed value and the most probable value of the same quantities. Squaring and adding the residuals will be minimum when the partial differential coefficient with respect to each of the parameters is zero. In Least Squares adjustment, the sum of squares of the residuals is minimum, when those residuals are calculated from the most probable value (Ayeni, 2001). These assumptions are applied in any method of least squares adjustment.

3.1.3.2 Methods of Least Squares Adjustment

Depending on the problems at hand any of the following methods of adjustment can be adopted.

i. Observation equation: The method involves an iterative solution for the differential displacements of the parameters, when equation is written for every observation.

ii. Condition equation: The generalized Least Squares method and the mixed model can be related to condition equations if certain physical assumptions are made.

iii. Mixed model: Observations, condition, parameter and / or constraint may be added to observation and / or condition equation to form a different model.

   Other techniques which are more suitable for handling larger networks and dependant on the model formulation of (i) and (ii) above. These include; Phase and Sequential adjustments.

   Phase adjustment: Phased adjustment is used for networks where inverting large matrices becomes too large and time consuming. Phased adjustment allows the network to be broken into smaller networks which are adjusted independently, and then the results of each smaller network are combined by treating the already estimated parameters and corrected observations of a previous phase as quasi-observations in the subsequent phase.

   Sequential adjustment: This involves the same set of unknowns, updated with observation taken at different time or epochs. Updating the parameter estimates sequentially yields the same result as adjusting all observations in a single observation (Jones, 1999).

The least square adjustment yields the following results:

i. Most Probable value of the observations
ii. Most Probable value of adjusted parameters
iii. Statistical analysis to determine the precision and reliability of the observation

Since observations were made, observation equation method is most suitable method of Least Squares adjustment and was therefore adopted in this work. The observation equation model and its derivation are discussed below.

3.1.3.3 Least Squares Adjustment Using Observation Equations Method

Observation equation method, often referred to as parametric method involves an iterative solution for the differential displacements of the parameters (Allman, 1974; Leick, 1980). The observation equation model can thus be expressed as (Allman, 1974; Ayeni, 1982; Leick, 1980; Ezeigbo, 1988; Ndukwe, 1991 and 1997; Ayeni, 2001 and Moka et al., 2006):

$$L^a = F\left(X^a\right) \tag{3.1}$$

where;

$$L^a = L^b + V \tag{3.2}$$

$$V = L^a - L^b \tag{3.3a}$$

$$X^a = X^o + X \tag{3.3b}$$

L^a is the adjusted vector of observations (coordinates)

L^b is Vector of observations (given coordinates in local and geocentric datum), that is, the actual observed values

X^a is the adjusted vector of parameters

X^o is the approximate parameters (usually the first approximation is taken $X^0 = 0$; for linear equation only)

X is the vector of the (unknown) corrections to the approximate parameters $\left(X^o\right)$

V is the vector of the residuals of the observations

Substituting equations (3.3a) and (3.3b) in (3.1) we have:

$$L^b + V = F\left(X^0 + X\right) \tag{3.4}$$

The functional model of equations (3.1) and (3.2) are not linear and therefore Least Squares cannot be imposed. It can however be linearized using Taylor's series (Leick, 1980; Moritz, 1972, 1976 and 1980; Ayeni, 1982; Ezeigbo, 1988; Sevilla et al., 1989; Ayeni, 2001; Ayeni *et al.*, 2006; Nwilo *et al.*, 2006; Oyewusi, 2008 and Isioye, 2008).

Equation 3.1 can be linearized as follows;

$L = L^0 - L^b$ (The vector of absolute or constant terms) (3.5)

$F\left(X^0\right) = L^0$ (Observed values for the first iteration) (3.6)

$V = AX + L$ (3.7)

$A = \dfrac{\partial F\left(X^a\right)}{\partial X^a} X$ [The differential expression (for different observation)]

where;

A is the design matrix of unknown parameter or coefficient matrix which is the partial of condition equations with respect to adjusted parameters

L is the vector of the misclosures (which is the vector of absolute or constant terms)

L^o observables

Equation (3.7) is the linearized mathematical model for equation (3.1) representing equations (3.3a) and (3.3b).

Minimizing the sum of squares of residual $\hat{V}^T P \hat{V}$ subject to equation (3.7) using Lagrange multiplier method, (Leick, 1980; Anderson, 1982; Ayeni, 1982; Ezeigbo, 1988; Simon, 1991; Mikhail and Anderson, 1998; Ayeni, 2001; Nwilo *et al.*, 2006) gives:

$$\phi = \hat{V}^T P \hat{V} - 2\hat{K}^T\left(A\hat{X} + L - \hat{V}\right)$$

$$\phi = \hat{V}^T P \hat{V} - 2\hat{K}^T A\hat{X} - 2\hat{K}^T L + -2\hat{K}^T \hat{V} \qquad (3.8)$$

where,

P is the weight matrix of the observations

$\hat{K}$ is the Lagrange Multiplier

Differentiating equation (3.8) with respect to $\hat{V}, \hat{K}$ and $\hat{X}$ to derive normal equation, the expression becomes:

$$\frac{\partial \phi}{\partial \hat{V}} = 2\hat{V}^T P + 2\hat{K}^T = 0 \tag{3.9a}$$

$$\frac{\partial \phi}{\partial \hat{V}} = \hat{V}^T P + \hat{K}^T = 0 \tag{3.9b}$$

$$P\hat{V} + \hat{K} = 0 \tag{3.9c}$$

$$\hat{K} = -P\hat{V} \tag{3.9d}$$

$$\hat{V} = -P^{-1}\hat{K} \tag{3.9e}$$

$$\frac{\partial \phi}{\partial \hat{K}} = -2A\hat{X} - 2L + 2\hat{V} = 0$$

$$\frac{\partial \phi}{\partial \hat{K}} = -A\hat{X} - L + \hat{V} = 0$$

$$\hat{V} = A\hat{X} + L \tag{3.10}$$

$$\frac{\partial \phi}{\partial \hat{X}} = -2\hat{K}^T A = 0$$

$$\frac{\partial \phi}{\partial \hat{X}} = -\hat{K}^T A = 0$$

$$-A^T\hat{K} = 0 \tag{3.11}$$

Substituting equation (3.9e) into equation (3.7) the expression becomes:

$$A\hat{X} + L = -P^{-1}\hat{K}$$

$$\hat{K} = -P\left(A\hat{X} + L\right)$$

$$\hat{K} = -PA\hat{X} - PL \tag{3.12}$$

Substitute equation (3.12) into equation (3.11) the expression becomes:

$$-A^T\left(-PA\hat{X}-PL\right)=0$$

$$A^TPA\hat{X}=-A^TPL \tag{3.13}$$

Equation (3.13) is called Reduced Normal Equation or simply Normal Equation (Leick, 1980; Mikhail and Anderson, 1998 and Ayeni, 2001).

The Least Squares solution of equation (3.13), which is the estimate of the correction to approximate parameter vector, X is given by (Hirvonen, 1971; Leick, 1980; Ayeni, 1980 and 1982; Ezeigbo, 1988 and 1990c, Mikhail, *et al.,* 1981; Mikhail and Anderson, 1998; Ayeni, 2001, and Nwilo *et al.,* 2006):

$$\hat{X}=-\left(A^TPA\right)^{-1}A^TPL \tag{3.14}$$

Putting the dimension of each matrix we have:

$${}_m\hat{X}_1=-\left({}_mA_n^T\,{}_nP_n\,{}_nA_m\right)^{-1}{}_mA_n^T\,{}_nP_n\,{}_nL_1$$

where;

n	no of observation equations
m	no of parameters to be determined
A^TPA	is a non-singular matrix called Normal Equation Coefficient Matrix N.
A^TPL	is the Normal Equations Constant (or absolute) Term Vector U.

Therefore, equation (3.14) can be written as:

$${}_m\hat{X}_1=-\left({}_mN_m\right)^{-1}{}_mU_1 \tag{3.15}$$

Equation (3.15), the estimate of the correction to the approximate parameter $\hat{X}$ represent estimate of adjusted parameters, it is known as solution vector to the Normal Equation.

Iterate equation (3.9b) using current X^a as new X^o until $\hat{X} = 0$ (Ayeni, 1980 and 2001). Given the variance of unit weight for weighted observation (A-posteriori) as (Ayeni, 1980, 1982 and 2001 and Leick, 1980);

$$\hat{\sigma}_0^2 = \frac{\hat{V}^T P \hat{V}}{n-m} \qquad (3.16)$$

where;

$$P = \frac{\hat{\sigma}_0^2}{\sum_{L^b}}$$

$$\Sigma_{L^b} = \hat{\sigma}_0^2 P^{-1}$$

$n-m$ is the degree of freedom

$\hat{\sigma}_0^2$ is a -posteriori variance of unit weight (estimate of σ_o^2)

σ_o^2 is the a-priori variance of unit weight

$\sum_{L^b}$ is the variance matrix of observation

The standard deviation of unit weight for weighted observation is given as (Leick, 1980; Elujobade, 1987);

$$\sqrt{\hat{\sigma}_0^2} = \sqrt{\frac{\hat{V}^T P \hat{V}}{n-m}} \qquad (3.17)$$

In order to know the variance (which is a measure of accuracy of a quantity) associated with the parameters, we need to derive the expression for the covariance matrix of the estimated parameters $\hat{X}$.

$$\sum_{\hat{X}^a} = \hat{\sigma}_0^2 \left(A^T P A\right)^{-1} \qquad (3.18)$$

This is the variance-covariance matrix of adjusted parameter (Uotila, 1974; Ayeni, 1980 and 2001), which is the measure of accuracy of the estimated vector of the parameters $\hat{X}$ (Ezeigbo, 1990; Nwilo *et al.*, 2006 and Oyewusi, 2008);

Also, the covariance of adjusted observation is given by (Ayeni, 1980 and 2001);

$$\sum_{L^a} = A\sum_{\hat{X}^a} A^T \tag{3.19}$$

where;

L^b = observations

L^a = adjusted observations

$\hat{V}$ = vector of residuals (v = estimate of V)

$\hat{X}^a$ = vector of adjusted parameters

X^0 = approximate values of parameters

$\widehat{X}_1^a$ = vector of estimate of one set of adjusted parameters (set 1); (estimate of X_1^a)

$\widehat{X}_2^a$ = estimate of second set of adjusted parameters (set 2); (estimate of X_2^a)

A = partial derivatives of condition equations with respect to adjusted parameters

B = partial derivatives of condition equations with respect to adjusted observations

P = weight matrix of observations

$\sum_{\hat{X}^a}$ = estimate of the covariance matrix of adjusted parameters (estimate of $\sum_{X^a}$)

$\sum_{L^a}$ = estimate of the covariance matrix of adjusted observations (estimate of $\sum_{L^a}$)

$\sum_{\hat{V}}$ = estimate of the covariance matrix of residuals (estimate of $\sum_v$)

m = number of parameters

n = number of observations

L_1^a = adjusted observation for set 1

L_2^a = adjusted observation for set 2

σ_0^2 = a-priori variance of unit weight

$\widehat{\sigma}_0^2$ = a-posteriori variance of unit weight (estimate of σ_0^2)

$\Delta\hat{X}$ = influence of addition (subtraction of observations)

$\hat{k}\,\hat{k}_1, \hat{k}_2 ... \hat{k}_n$ = estimates of vectors of Langranges multipliers

$\sum_w$ = estimate of the covariance matrix of the misclosures

r = number of condition equations

r_1, r_2 = number of condition equations for set 1 and set 2

m_1 = number of parameters for set 1

m_2 = number of parameters for set 2

n_c = number of equations from functional constraints.

The observation equation method yields the results which provide some statistical analysis.

3.1.4. STATISTICAL ANALYSIS

Statistical analysis were done in order to ascertain the margin of error that is to determine how the most probable values differed from the observed values because of the omitted predictor, random variation and the inaccuracy of the form of the model. The accuracies of the models were tested and the following statistical quantities were computed namely; Model Parameter Estimators, Model Covariance Estimators and Model Validation

3.1.4.1 Model Parameter Estimators

The first objective is to estimate the 4 unknown parameters. These parameters were obtained as solution vector (x_0, x_1, x_2, x_4.**)** and represented by a column vector x. The random errors r and the response variables y are represented by (m) vectors, denoted by $\boldsymbol{r}$ and $\boldsymbol{y}$ respectively. The base functions $\mathbf{A}_{ij}$ are contained in the (m x 4) matrix **A** (designed matrix). The designed matrices were estimated depending on the base function. For example, the base function used for Spherical 'Satlevel' is given as:

$$A = \begin{bmatrix} 1 & P_1 & Q_1 & R_1 \\ 1 & P_2 & Q_2 & R_2 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 1 & P_m, & Q_m & R_m \end{bmatrix} \tag{3.20}$$

where;

$$P_1 = \cos^3\phi_1 \cos\lambda_1 + \sin^2\phi_1 \cos\phi_1 \cos\lambda_1 + \cos^3\lambda_1 \cos\phi_1 + \sin^2\lambda_1 \cos\phi_1 \cos\lambda_1$$
$$P_2 = \cos^3\phi_2 \cos\lambda_2 + \sin^2\phi_2 \cos\phi_2 \cos\lambda_2 + \cos^3\lambda_2 \cos\phi_2 + \sin^2\lambda_2 \cos\phi_2 \cos\lambda_2$$
$$P_m = \cos^3\phi_m \cos\lambda_m + \sin^2\phi_m \cos\phi_m \cos\lambda_m + \cos^3\lambda_m \cos\phi_m + \sin^2\lambda_m \cos\phi_m \cos\lambda_m$$

$$Q_1 = \cos^3\phi_1 \sin\lambda_1 + \sin^2\phi_1 \cos\phi_1 \sin\lambda_1 + \cos^2\lambda_1 \cos\phi_1 \sin\lambda_1 + \sin^3\lambda_1 \cos\phi_1$$
$$Q_2 = \cos^3\phi_2 \sin\lambda_2 + \sin^2\phi_2 \cos\phi_2 \sin\lambda_2 + \cos^2\lambda_2 \cos\phi_2 \sin\lambda_2 + \sin^3\lambda_2 \cos\phi_2$$
$$Q_m = \cos^3\phi_m \sin\lambda_m + \sin^2\phi_m \cos\phi_m \sin\lambda_m + \cos^2\lambda_m \cos\phi_m \sin\lambda_m + \sin^3\lambda_m \cos\phi_m$$

$$R_1 = \cos^2\phi_1 \sin\phi_1 + \sin^3\phi_1 + \cos^2\lambda_1 \sin\phi_1 + \sin^2\lambda_1 \sin\phi_1$$
$$R_2 = \cos^2\phi_2 \sin\phi_2 + \sin^3\phi_2 + \cos^2\lambda_2 \sin\phi_2 + \sin^2\lambda_2 \sin\phi_2$$
$$R_m = \cos^2\phi_m \sin\phi_m + \sin^3\phi_m + \cos^2\lambda_m \sin\phi_m + \sin^2\lambda_m \sin\phi_m$$

The Rectangular ‘Satlevel’ (Equation 3.79) and the ‘Satlevel’ equation for fitting the local Orthometric Height to GEM2008 Orthometric Heights (Equation 3.86) have their designed matrices depend on their base functions.

The design matrix ***A*** can be called the carrier matrix because it includes the 3 explanatory variables and, according to the assumption made about the composition of the geoidal variations, it also has a column of 1’s to cater to the constant x_0. or N_L in case of Spherical ‘Satlevel’ Model.

Thus, the postulated geoidal model can be written as:

$$\mathbf{y} = Ax + \mathbf{r}. \tag{3.21}$$

where;

y = (N_i, i = 1, 2, …, m) vector of the observed undulations and ***r*** is the vector of residuals.

The vector of mean values $E[\mathbf{y}]$ of ***y*** is obtained by taken the expected values of Equation 2.50 as:

$$E[\mathbf{y}] = Ax \tag{3.22}$$

For estimation purposes, suppose we have a set of m Geoidal Undulation observed at m known geographic locations $(N_i, (\phi, \lambda)_i), i = 1, 2, ..., m,$ (in order not to confuse N used for Geoidal

Undulation in Geodesy with N used for Normal equation in Least Squares Adjustment, y will be used for Geoidal Undulation in the Statistical analysis) the Least Squares solution is obtained by minimizing the L_2-norm of the residual errors $\|\mathbf{y} - Ax\|_2$ with respect to the unknown parameters ***x***, that is***:***

$$S = \mathbf{r}^T \mathbf{r} = (\mathbf{y} - Ax)^T (\mathbf{y} - Ax) \tag{3.23}$$

Differentiation of Equation (3.23) with respect to ***x*** gives the following linear equations:

$$2A^T (\mathbf{y} - Ax) = \mathbf{0} \rightarrow A^T A\hat{x} = A^T \mathbf{y} \tag{3.24}$$

$$N\hat{x} = U$$

Equation (3.24) is called the *normal equations* and provides the 4 estimators of the model parameters provided the (4 x 4) matrix $A^T A$, the normal matrix which is symmetric can be inverted. The Least Squares solution to the unknown parameters is:

$$\hat{x} = -(A^T A)^{-1} A^T y \tag{3.25}$$

$When\ A^T A = N\ and\ A^T y = U,\ then\ Equation\ 3.25\ becomes:$

$$\hat{x} = -N^{-1} U$$

where; $\hat{x} = (\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)$ or A_1 A_2 A_3 in case of “Satlevel" spherical Model is the vector of estimated model parameters.

Thus the vector of estimated mean values of **y** is given by:

$$\hat{\mathbf{y}} = A\hat{x} \tag{3.26}$$

And the vector of estimated residuals is:

$$\hat{\mathbf{r}} = \mathbf{y} - A\hat{x} \tag{3.27}$$

this is the estimate of the original errors r

$$\mathbf{r} = \mathbf{y} - A\hat{x} \tag{3.28}$$

and is used in the assessment of the model.

The Least Squares estimators $\hat{x} = (\hat{x}_0, \hat{x}_1, \hat{x}_2, \hat{x}_3)$ can be shown to be unbiased estimators of the postulated model parameters under the assumption that the errors are independent of the explanatory variables. From Equation. (3.28), $E[\hat{x}] = E[(A^T A)^{-1} A^T \mathbf{y}]$, taking the ***y*** values as the random variables and the ***A*** values as known or fixed, the equation becomes:

$$E[x] = [(A^T A)^{-1} A^T E[\mathbf{y}]. \tag{3.29}$$

Hence, by using Equation. (3.22) in Equation. (3.29), the equation becomes:

$$E[\hat{x}] = [(A^T A)^{-1} A^T Ax]. \tag{3.30}$$

And since $[(A^T A)^{-1}(A^T A) = \mathbf{I}$, which is a (4 x 4) identity matrix, then,

$$E[x] = x. \tag{3.31}$$

3.4.1.2 Model Covariance Estimators

The covariance of the Least Squares estimators can be expressed as the elements of a matrix **C** as follows:

$$\mathbf{C} = E[(\hat{x} - x)(\hat{x} - x)^T. \tag{3.32}$$

Using Equation (3.25) for $\hat{x}$, and putting Equation (3.29) into Equation (3.31) for ***x***, and because $(A^T A)^{-1}$ is symmetric then Equation (3.32) becomes:

$$\mathbf{C} = E[(A^T A)^{-1} A^T (\mathbf{y} - E[\mathbf{y}])(\mathbf{y} - E[\mathbf{y}])^T A(A^T A)]. \quad (3.33)$$

The errors represented by $\boldsymbol{r} = \boldsymbol{y} - E[\boldsymbol{y}]$ are assumed to have a zero expectation and a common variance σ^2 normality assumption (Hamilton, 1964 and Ayeni, 2001). Also, because the errors are mutually independent, covariance between pairs are zero. Thus:

$$E[(\mathbf{y} - E[\mathbf{y}])(\mathbf{y} - E[\mathbf{y}])^T] = \sigma^2 \mathbf{I}. \quad (3.34)$$

which is (m x m) matrix with the diagonal elements equal to σ^2 and the off-diagonal elements equal to zero. It follows that:

$$\mathbf{C} = \sigma^2 (A^{\mathbf{T}} A)^{-1} A^{\mathbf{T}} A (A^{\mathbf{T}} A)^{-1} = \sigma^2 (A^{\mathbf{T}} A)^{-1} \quad (3.35)$$

The quantities $\sigma^2 c_{ii}^{'}$; where; $c_{ii}^{'}$, $i = 0, 1, ..., n-1$ are the diagonal elements of the $\sigma^2 (A^{\mathbf{T}} A)^{-1}$ matrix and are the variances of the estimators of the model parameters often referred to as Variance-Covariance matrix. They are used in making inferences about the parameters and for setting confidence limits on the parameters. However, the value of σ^2 is not usually known. An estimate can be computed for it from the estimated residual errors.

3.1.4.3 The Error Variance Estimation (σ^2.)

The error variance σ^2 is unknown. As a result, the residuals are used for its estimation. The residual sum of squares is estimated as (Olaleye, 1992):

$$\begin{aligned} SS_E &= (\mathbf{y} - A\hat{x})^T (\mathbf{y} - A\hat{x}) \\ &= \mathbf{y}^T \mathbf{y} - \hat{x}^T A^T \mathbf{y} - \mathbf{y}^T A\hat{x} + \hat{x}^T A^T A\hat{x}. \end{aligned} \quad (3.36)$$

From the normal equations, it is noted that $A^T A\hat{x} = A^T \mathbf{y}$. Furthermore, the scalar quantity $\mathbf{y}^T A\hat{x}$ is equivalent to its transpose, $x^T A^T \mathbf{y}$. Therefore, the residual sum of squares is given as Equation (3.37);

$$SS_E = \mathbf{y}^T \mathbf{y} - \hat{x}^T A^T \mathbf{y} \qquad (3.37)$$

Because n parameters need to be estimated, an unbiased estimator of σ^2 is:

$$\hat{\sigma}^2 = \frac{SS_E}{m-n} = \frac{\mathbf{y}^T \mathbf{y} - \hat{x} A^T \mathbf{y}}{m-n} \qquad (3.38)$$

Confidence limits can be obtained on σ^2 because the variable $(m-n)\hat{\sigma}^2 / \sigma^2$ is A^2_{m-n} distributed on consideration of the assumptions of independence and normality. **Error variance** and the residual sum of squares are:

$$SS_E = \mathbf{y}^T \mathbf{y} - \hat{x}^T A^T \mathbf{y} \qquad (3.39)$$

After estimating n parameters from a set of m observations, an unbiased estimator of σ^2 is:

$$\hat{\sigma}^2 = \frac{\mathbf{y}^{\mathbf{T}} \mathbf{y} - \hat{x}^T A^{\mathbf{T}} \mathbf{y}}{m-n} \qquad (3.40)$$

3.1.4.4 Model Validation

Empirical modelling is an iterative procedure. One starts with a chosen set of explanatory variables arranged (subjectively) in a decreasing order of physical importance. Then, if the researcher follows the commonly used backward elimination procedure, the test of significance of these variables starting with the last, and making changes where necessary, considering the results of the tests. The changes involve, for instance, the exclusion of some variables and the inclusion of others.

Significance tests applied to the model selection process range from the F Distribution with a chosen number (p – 1) of explanatory variables to individual tests on the model parameters. Assumptions are usually made concerning the errors represented by the term r. It is assumed that the errors are mutually independent with a common distribution and also that the errors are independent of the explanatory variables. It is part of the test procedure to verify the assumptions made.

3.1.4.4.1 Initial Significance Tests on the Model Parameters: After estimating the parameters of the model, it is necessary to find the evidence of a linear relationship between the response and a subset of the explanatory variables, as already mentioned, which can consequently be used in forecasting. For the initial significance test, the hypotheses are;

Null Hypothesis $H_0: \; x_i = 0$ for all $i, i = 1, 2, \ldots, n-1$

and

Alternate Hypothesis $H_0: \; x_i \neq 0$ for one or more i , $i = 1, 2, \ldots$ n – 1.

The total sum of squares of the errors of observations of the response variable is the sum of squares deviations from the mean:

$$S_{yy} = \mathbf{y}^{\mathbf{T}}\mathbf{y} - \frac{(\mathbf{y}^{T}\mathbf{1})^{2}}{m} \tag{3.41}$$

Note that **1** in this equation is a vector of ones that is; $\mathbf{1} = (1_1, 1_{2.} \; \ldots, 1_m)^T$

This total error can be separated into two parts, $S_{yy} = SS_R + SS_{E,}$ which are respectively the sum of squares due to the regression and the sum of squares due to the errors. From Equation (3.41),

$$SS_E = \mathbf{y}^T\mathbf{y} - \hat{x}^T A^T \mathbf{y}$$

Therefore,

$$SS_R = S_{yy} - SSE = \mathbf{y}^T\mathbf{y} - \frac{(\mathbf{y}^T\mathbf{1})^2}{m} - \mathbf{y}^T\mathbf{y} + \hat{x}^T A^T \mathbf{y} \tag{3.42}$$

$$SS_R = \hat{x}^T A^T \mathbf{y} - \frac{(\mathbf{y}^T \mathbf{1})^2}{m}$$

Under the Null Hypothesis, $SS_R / \sigma^2 \sim X^2_{n-1}$, where; σ^2 is the common variance of the errors and n – 1 is the number of explanatory variables (that is, there are n parameters including x_0); also from the F distribution based on the assumption that the **y** and **A** have a multivariate normal distribution:

$$\frac{SS_R / n-1}{SS_E /(m-n)} \sim F_{n-1,\ m-n}. \tag{3.43}$$

The expression on the left denoted as F is called the *ratio of the means of the two respective sums of squares*

$$\frac{MS_R}{MS_E} \sim F_{n-1,\ m-n}.$$

The Null Hypothesis is rejected if F > $F_{n-1,\ m-n,\ \alpha}$ for a level of significance α. A summary of the procedure is given in Table 3.1 below;

Sums of squares and ANOVA. The total sum of squares from m observations is

$$S_{yy} = \mathbf{y}^{\mathbf{T}} \mathbf{y} - \frac{(\mathbf{y}^T \mathbf{1})^2}{m} \tag{3.44}$$

ANOVA for testing significance in multiple linear regressions with n parameters including x_0 in vector x using m observations are shown in Table 3.1:

Table 3.1: ANOVA for Testing Significance in Multiple Linear Regressions

Source of variation	Degrees of freedom	Sum of squares	Mean square	F value
Model	$n-1$	$SS_R = \hat{x}^T A^T \mathbf{y} - \frac{(\mathbf{y}^T \mathbf{1})^2}{m}$	$MS_R = \frac{SS_R}{n-1}$	$F = \frac{MS_R}{MS_E}$
Residual	$m-n$	$SS_E = \mathbf{y}^T \mathbf{y} - \hat{x}^T A^T \mathbf{y}$	$MS_E = \frac{SS_E}{m-n}$	
Total	$n-1$	$S_{yy} = \mathbf{y}^{\mathbf{T}} \mathbf{y} - \frac{(\mathbf{y}^T \mathbf{1})^2}{m}$		

The estimated regression and error sums of squares are respectively

$$SS_R = \hat{x}^{\mathbf{T}} A^{\mathbf{T}} \mathbf{y} - \frac{(\sum_{i=1}^{n} y_i)^2}{n} \qquad \text{and} \qquad SS_E = \mathbf{y}^T \mathbf{y} - \hat{x}^T A^T \mathbf{y}$$

With ratio of means

$$\frac{SS_R / n-1}{SS_E /(m-n)} \sim F_{n-1,\ m-n}. \tag{3.45}$$

where; n – *1* is the number of explanatory variables.

3.1.4.4.2 Significance Tests on a Set of Parameters: A significance test on each of the parameters is an approximate procedure. An alternative is to use a statistic that has the F distribution, as in the case of testing a set of parameters. This is based on assumptions such as the multivariate normal distribution of the variables. In the modification, the denominator remains the same. The numerator in the F ratio is, however, changed so that it represents the difference between

- the sum of squares due to the regression when a full set of variables is included and
- the sum of squares when a chosen partial set of variables is eliminated from the regression.

Let the original model contain n – 1 explanatory variables (that is, there are n parameters, including β_0) arranged in descending order of importance. As stated, the choice was made through physical considerations. Suppose the test that the last p variables do not make a significant contribution to the model is to be conducted. Then, the two hypothesis are;

Null Hypothesis $H_0 : x_{n-p} = x_{n-p+1} = \cdots = x_{n-1} = 0$

Alternative Hypothesis $H_1 : x_i \neq 0$ for at least one i, $i = n-p, n-p+1, \ldots n-1$

Also, let

$SS_{R,\, n-1}$ be the sum of squares due to the model usually all n – 1 explanatory variables,

$SS_{R,\, n\text{-}p-1}$ be the sum of squares due to the model using the first n-p–1 explanatory variables, and

$SS_{E,\, n-1}$ be the sum of squared residuals using all n – 1 explanatory variables, with m-n degrees of freedom.

Then

$$\frac{(SS_{R,\, n-1} - SS_{R,n-p-1})/p}{SS_{E,n-1}/(m-n)} \sim F_{p,m-n.} \qquad (3.46)$$

F Distribution on a set of regression parameters: The test statistic is;

$$\frac{(SS_{R,\, n-1} - SS_{R,n-p-1})/p}{SS_{E,n-1}/(m-n)} \sim F_{p,m-n.} \qquad (3.47)$$

Here, $SS_{R,\, n-1}$ and $SS_{R,\, n\text{-}p-1}$ are the sums of squares due to the regression using all n – 1 and the first n-p explanatory variables respectively. Also, $SS_{E,\, n-1}$ is the sum of squared residuals using all n – 1 explanatory variables, with m-n degrees of freedom.

3.1.4.5 Model Adequacy

3.1.4.5.1 Coefficient of Determination: From the sums of squares defined in ANOVA, Table 3.1, one can define a measure of model adequacy by the statistic;

$$R^2 = \frac{SS_R}{SS_{yy}} \tag{3.48}$$

This is the ratio of the sum of squares due to regression model to the total sum of squares; it is sometimes called the ***coefficient of Correlation***, or simply, R^2. It gives the proportion (or fraction) of the variability of the response variable, that is accounted for by the explanatory variables. Tests of hypotheses, however, were used to determine the explanatory variables to be included in the model. The higher the value of R^2 the better the fitting of the model, although this can be misleading if one does a comparison for different transformations.

3.1.5 Detection of Outliers

Outliers are data points which lie outside the general linear pattern of the entire population. Outliers may be misleading or deceptive indicative of data points that belong to a different population than the rest of the sample set. They can occur by chance in any distribution or sample population, but they are often indicative either of measurement errors or statistically, that the population has a heavy tailed-distribution.

Unexpectedly high or low values in the dataset (outliers) can unduly influence the estimation of the parameters of an empirical probability model unless one identifies and deals appropriately with them. In model analysis, there can be some observations that can have excessive influence on the estimates of the parameters and the tests of hypothesis. The researcher need to examine whether they can be classed as influential or not. If the causes are as a result of errors in observation, they should be discarded. Otherwise, the issue to be considered is how to cope with outliers? The outliers may not need to be discarded in any set of observations because the more the observations, the more accurate the description of the relationship.

There are different ways of detecting outliers. Some of which are discussed below:

Outliers can be detected for a given population using the inner quartile range (IQR) criterion, that is, 1.5*IQR. This is given as Mathforum (2009) :

$$IQR = UQ - LQ. \qquad (3.49)$$

Sometimes other criterion such as: Upper quartile range that is, 3*IQR below the L.Q. or 3*IQR above the U.Q. to determine "highly suspected" outliers are also used. (Mathforum, 2009)

where;

IQR = the inner quartile range

UQ = Upper quartile range

LQ = Lower quartile range

Isioye (2008) presented the following methods of detecting outliers.

1. **The Global test:** The Global test of the variance factor is done basically to test the hypothesis that the initial observational standard errors (and therefore weights) are consistent with the magnitude of residuals generated in the Least Squares adjustment.
2. **The F – Distribution:** This is the statistical testing of the a posteriori variance factor against the adopted a priori variance factor.
3. **The W – Test (Data Snooping Test):** Data snooping was suggested by Baarda (1968) to test and to assess the reliability of Geodetic networks. Baarda's data snooping technique was presented in matrix notation in Strang Van Hees (1984b).
4. **Tau Test:** If the a priori variance is not known or a value cannot be assigned to it before adjustment, the a posteriori variance ($\hat{\sigma}^2$) calculated at the end of adjustment is used for outlier detection.
5. **The t-Test:** If an observation includes an error, detection of outlier using the a-posteriori variance obtained from the invalid adjustment model is not appropriate. In this situation better accurate approach is to compute the a-posteriori variance value from the residuals that are free from the model errors (Gokalp and Boz, 2005).

3.1.5.1 Treatment of Outliers: Hwang *et al* (2003) used an iterative method to remove outliers in along-track altimeter data. The largest difference that also exceeds three times of the standard deviation is considered an outlier and the corresponding data value is removed from the time series. The cleaned time series is filtered again and the new differences are examined against the new standard deviation to remove a possible outlier. This process stops when no outlier is found.

The outlier should be included because it may explain an unusual occurrence and its removal from the data set under analysis can at times dramatically affect the performance of a regression model. However, deletion of outlier data is a controversial practice frowned at by many scientists and science instructors; while mathematical criteria provide an objective and quantitative method for data rejection, they do not make the practice more scientifically or methodologically sound, especially in small sets or where a normal distribution cannot be assumed. Rejection of outliers is more acceptable in areas of practice where the underlying model of the process being measured and the usual distribution of measurement error are confidently known. An outlier resulting from an instrument reading error may be excluded but it is desirable that the reading is at least verified. (Wikipedia, 2009)

Alternatively, outliers can be treated using the statistic obtained from Least Squares adjustment by multiplying the residual by the square root of the input weight (the inverse of the square of the standard error).

Ferland and Piraswewishi (2006) developed a computer package, using draw methods and double buffering, which displays a screen shot of the picture. It involved listing which can display a simple radar graph that plots a collection of values in the range of 0-100units onto a polar coordinates system designed to easily show outliers. It is possible to use this kind of graph to monitor some sort of resource allocation metrics, and a quick glance at the graph can tell the researcher, when conditions are good (within some accepted tolerance level), or approaching critical levels (total resource consumption).

Other diagnoses can be based on what is known as the leverage matrix, the use of standardized residuals, and a measure of influence called *Cook's distance.*

3.1.5.2 The Leverage Matrix: For a set of n observations, the n x n leverage matrix **H** defined by substituting the solution of the vector of estimated parameters in the vector of estimated expected values of the response variable. That is:

$$\hat{\mathbf{y}} = A\hat{x} = A(A^T A)^{-1} A^T \mathbf{y} = \mathbf{H}\mathbf{y}. \tag{3.50}$$

Sometimes **H** is called the hat *matrix* because it puts a "hat" (circumflex) on **y.** This is formed solely by the **X** values. Thus, when pre-multiplying the vector of observed y values by the leverage matrix **H,** the vector of fitted values of Y estimated by the Least Squares method can be obtained.

From above the residuals $\hat{\mathbf{r}}_i$ are related to **H** as follows:

$$\hat{\mathbf{r}} = (\mathbf{I} - \mathbf{H})\mathbf{y} \tag{3.51}$$

Where; I is an m x m identity matrix and the leverage matrix H and the residuals matrix I – H are symmetrical and idempotent an idempotent matrix is a matrix which, when multiplied by itself, yields itself. That is, the matrix H is idempotent if and only if HH = H. For this product to be defined H must necessarily be a square matrix. Viewed this way, idempotent matrices are idempotent elements of matrix rings, that is, $\mathbf{H}^2 = \mathbf{H}$**,** the following relationships hold:

$$\hat{\sigma}^2 = \frac{\mathbf{r}^\mathbf{T}\mathbf{r}}{m-n} = \frac{\mathbf{y}^T(\mathbf{I}-\mathbf{H})\mathbf{y}}{m-n} \tag{3.52}$$

$$\mathrm{Var}[\hat{\mathbf{y}}] = \hat{\sigma}^2\mathbf{H},$$

$$\mathrm{Var}[\hat{r}] = \sigma^2(\mathbf{I}-\mathbf{H}),$$

and

$$\mathrm{Cov}[\hat{r}] = \sigma^2\mathbf{H}(\mathbf{I}-\mathbf{H}) = 0. \tag{3.53}$$

Belsley Kuh, and Welsch (1980) define the leverage (h_i) of the ith observation as:

$$h_i = \frac{1}{n} + \frac{\left(x_i - \overline{x}\right)^2}{\left(n-1\right)S_x^2}$$

3.1.5.3 Partial Regression Plots: It is also called **partial regression leverage plots** or **added variable plots** are other ways of detecting influential sets of cases of outliers. Partial regression plots are a series of bivariate regression plots of the dependent variable with each of the independent variables in turn. The plots show cases by number or label instead of dots. One looks for cases which are outliers on all or many of the plots. (Sonona, 2009)

3.1.5.4 Standardized Residuals: For comparative purposes in the assessment of the magnitudes of residuals, it is useful to compute the standardized residuals:

$$v_i = \frac{\hat{r}_i}{\sqrt{\hat{\sigma}_i^2(1-h_i)}} \tag{3.54}$$

$i = 1, 2, ..., m,$ obtained by dividing the residuals $\hat{r}_i$ by the square root of the estimated variance computed from above. The v_i are also called the *studentized residuals*, or *internally studentised residuals* (where; the alternate term *external* denotes residuals obtained by using the error variance $\hat{\sigma}_{(i)}^2$ computed after deleting the ith row of observations). The standardized residuals v_i can be advantageous and enable influential observations, in particular, to be more easily seen.

Significance tests on a set of Parameters, model validation, the error Variance Estimation, Model Covariance Estimators and Model Parameter Estimators were used to tests the explanatory variable used in the model before arriving at the conclusion for the 'Satlevel Collocation models.

3.2 'SATLEVEL' COLLOCATION METHODS OF GEOID DETERMINATION

The 'Satlevel' collocation methods of geoid determination involve the use of both ellipsoidal and Orthometric Heights to model the geoid. The methodology involves acquisition of data relating to ellipsoidal Height from GNSS and Orthometric Heights with application of Orthometric correction, formulating the problems to develop the models and use of the data to drive the model. GEM2008 provided the long wavelength component while the observed GNSS coordinates and observed Orthometric Height at discrete points was used to model the short wavelength component in adapting the global Orthometric Height to its local equivalent. The data from selected two study areas were used for the research.

3.3 FIELD OPERATIONS

The field operations were done for the purpose of acquiring ellipsoidal heights using Differential Global Positioning System (DGPS) and to obtain the Orthometric Height through the spirit levelling for a number of uniformly distributed points in the study area.

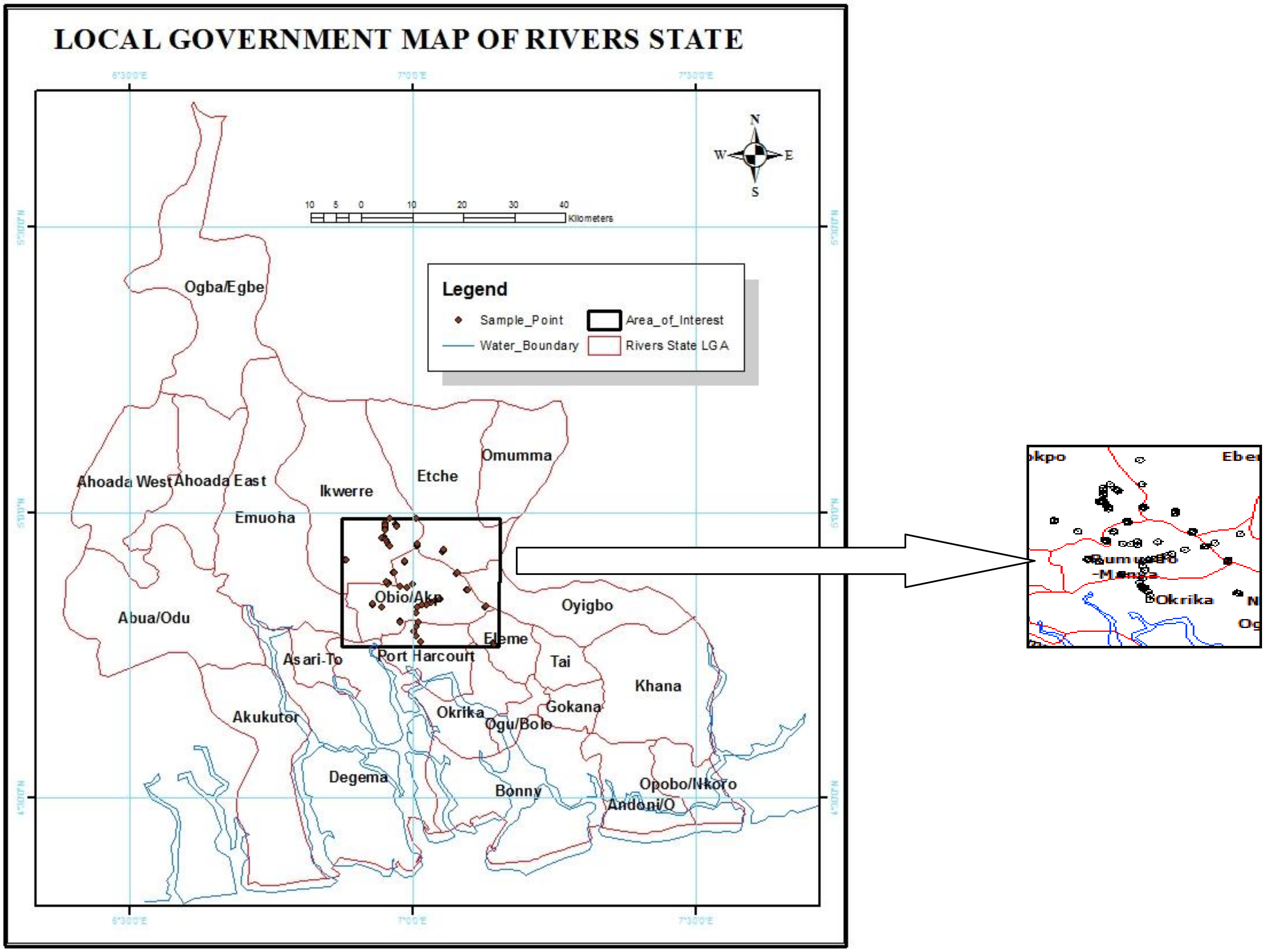
3.3.1 Study Areas:

Two areas were used for this research, namely, Port Harcourt and Lagos State.

1. Port Harcourt lies within Latitudes: 4°45'N and 5°02'N and Longitudes: 6°52'E and 7°09'E along the Bonny River. It is the seat of Rivers State Government in oil rich Niger delta region of Nigeria. Many companies, business organizations and government agencies locate and operate their corporate offices in Port Harcourt. Many of these organisations have used the services of surveyors for projects that needed height information. The surveyors, unable to get a bench mark to connect, will simply establish a local datum to do the work. This practice has created a situation where many different height values which are irreconcilable, exist in the area. Therefore, there is the need for simple method of obtaining the correct values for the benchmarks. The points used for the study were plotted on the local government map of Rivers State to show the distribution of points (Figure 3.3).
See Page 94

The new ‘Satlevel’ collocation and existing models were also tested using data from Lagos State.

2. **Lagos State:** Lagos lies approximately between latitudes 6°22' and 6°52’ North of the Equator and longitudes 2°42' and 3°42' East of the Greenwich Meridian. Ogun State formed the boundary of Lagos State in the Northern and Eastern parts, while the 180km long Atlantic coastline forms the southern boundary and the Republic of Benin borders it on the western side (Figures 3.4 and 3.5).



the Distribution of Points Used for
odified by the author)

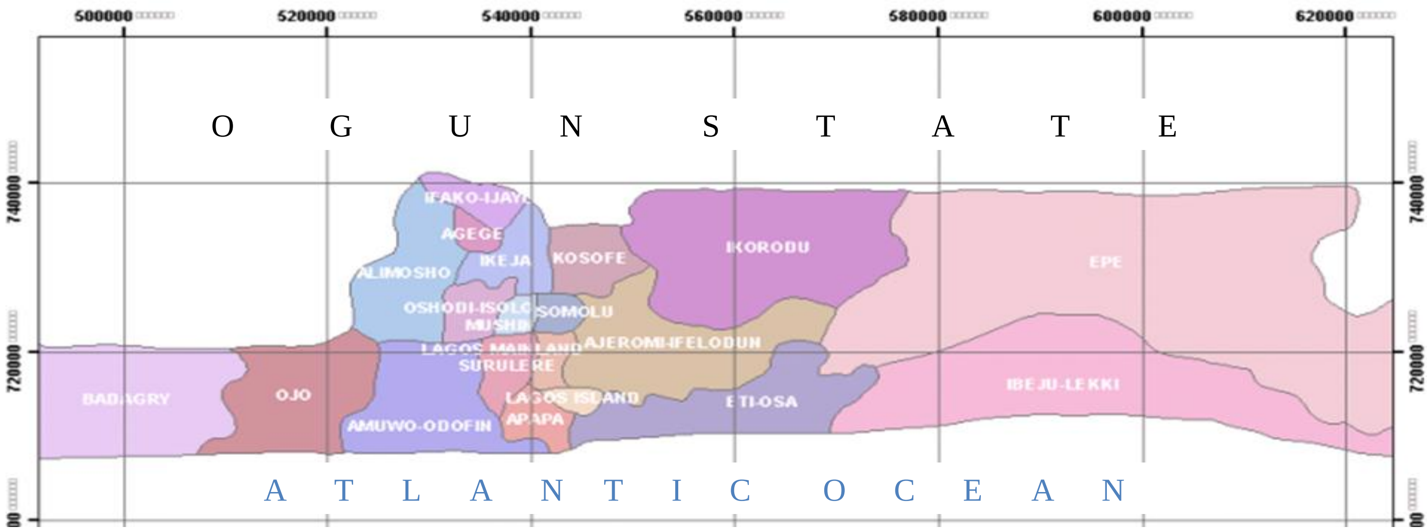


Figure 3.4: The Administrative Map of Lagos State (Source: Map digitized and modified by the Author)

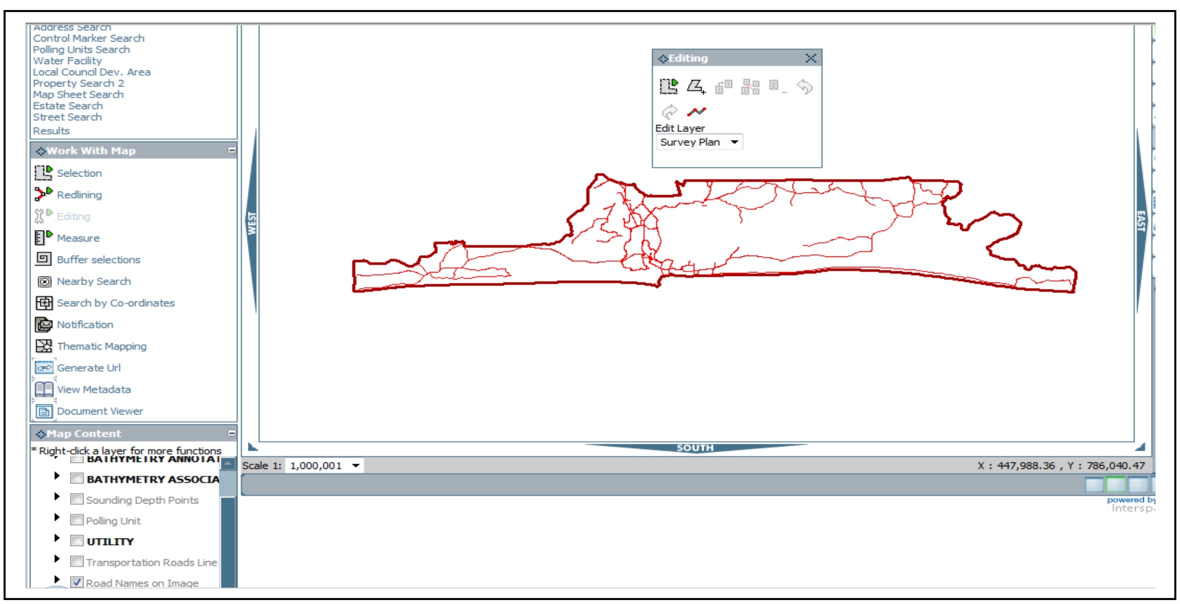


Figure 3.5: Map of Lagos used for Cadastre Enterprise Geographic Information System (Source: Lagos State Office of Surveyor General)

3.3.2 Spirit Levelling

Every survey job must be planned to attain certain accuracy. In this research, first order accuracy was planned and achieved in Port Harcourt, while the Lagos State data is available on Lagos State website (Lagos State Government, 2009) Levelling runs were made along selected routes and locations around the study area. Guidelines and specifications for control of Geodetic Surveys in Nigeria were followed strictly to ensure that the levelling operation was consistent with geodetic standards (Davis et al., 1981; SURCON, 2003). The MSL benchmark established by the Nigerian Ports Authority in 1923 located in Port Harcourt was checked to be in-situ and therefore was adopted as the datum.

The spirit levelling was done to obtain the height differences between the points. The height differences between the first point and the benchmark was added to the Orthometric Height value of the benchmark to get the reduced level of the following point. The procedure was repeated for all the points used for the study. The two study areas are of low topography and closed to the sea-coast, the Orthometric correction was therefore neglected and the reduced level of the point is assumed to be Orthometric Heights, since the job was connected to benchmark with Orthometric Heights values. The diagrammatic sketch for acquiring spirit levelling data for Orthometric Height is as shown in Figure 3.6:

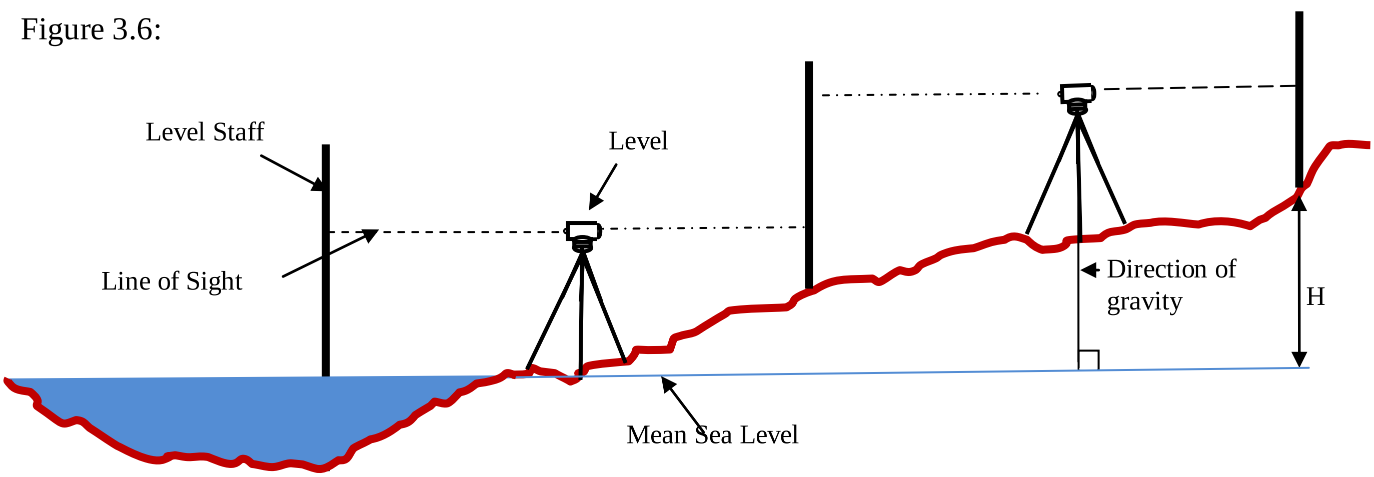


Figure 3.6: Levelling Procedure for acquiring Orthometric Height Data.
(Source: Author, August, 2008)

After acquiring the data for Orthometric Heights, GPS observation was done to obtain data for the ellipsoidal heights of the all the points used in Port Harcourt.

3.3.3 GPS Observation

The methodology of DGPS as given by Trimble (2007) was adopted. DGPS observations were made on the same point along the levelling routes using Trimble 4700 dual frequency GPS receiver. The points were coordinated to geodetic accuracy. Also, GPS observations were made on some of the existing points, particularly the Federal Surveys and Shell Petroleum Development Company (SPDC) points found within the project area. From the GPS observations, ellipsoidal heights were derived, while Orthometric Heights were derived from the data acquired by geodetic levelling (Section 3.3.1). Data used for Lagos State were extracted from the Lagos State Office of the Surveyor General.

3.3.4 Data Quality Validation: Verification of data quality is an important part of any geodetic and other scientific researches, as it helps to ensure that the data used in the models are accurate enough to satisfy the requirement of the application at hand. Data validation assisted in identification of suspicious and invalid cases such as outliers, variables, and suspicious data values in the active data set. Geodetic levelling and DGPS data acquisitions were done in Port Harcourt. Levelling operations were carried in loops and according to specifications for first order geodetic levelling (SURCON, 2003). The data were checked to be precise and the mean of height differences were taken as the most probable value of measurements. Therefore, the good quality of this data from Port Harcourt is guaranteed.

The existing data for some of the stations found on the field were used to check and validate the results of new job in Port Harcourt metropolis.

Table 3.3: Result of Data Quality Validation

Stations	Latitude	Longitude	Existing Data	Mean of New Data
RPCS 209p	4.771628736	7.013283025	29.885	29.885
HS 8	4.755137533	7.016561928	26.028	26.028
RPCS 146p	4.872683436	7.028375606	35.644	35.644
XSV 662	4.873506919	6.99841315	27.603	27.603
ZVS 3003	4.847971022	7.047811589	32.308	32.308
RHS 8A	4.755136992	7.016562314	23.529	23.529

The variance of unit weight for weighted observation (A-posterior) was computed using Equation (3.18). A value of 1.959E-06 was obtained. The standard deviation of unit weight for weighted observation was computed to be 0.0013998m. The diagonal elements of the variance-covariance matrix were small as shown in Page 217. The Covariance matrixes of adjusted observation were computed using Equation 3.19 as shown in Page 217. These show that the data are of good quality.

The data used for Lagos State were obtained from Lagos State Office of the Surveyor General. The data were acquired from several sources, which form part of the limitation of this research. It was based on the assumption and trust that Lagos State government has high reputation.

3.3.5 Data Processing: The data acquired from the field were processed to get the Orthometric Heights from the geodetic levelling (Section 3.3.1) and ellipsoidal height from GPS observation (Section 3.3.2). The ellipsoidal and Orthometric Heights were substituted into Equation 1.2 to obtain the values of the Geoidal Undulation as shown in Tables 3.1a and 3.1b for Port Harcourt and Lagos State respectively.

Table 3.2a: Local Geoidal Undulations in Port Harcourt. (Field work as reported by Akom, 2008)

Stations	**Latitude** [^{o}]	**Longitudes** [^{o}]	**Ellipsoidal Heights (h) [m]**	**Orthometric Heights (H) [m]**	**Local Geoid Undulation (N) [m]**
PT.4 EMMA	4.798391819	7.005574083	30.6930	11.6910	19.0020
PT.5 EMMA	4.806938314	7.009407025	29.3740	10.3800	18.9940
GPS 03	4.981133603	6.949840522	40.0650	21.240	18.8250
GPS 04	4.972244803	6.951180808	38.7710	19.9380	18.8330
GPS 13	4.975173192	6.971955836	40.5890	21.7280	18.8610
GPS 26	4.832460906	6.945637275	20.1800	01.2500	18.9300
GPS 45	4.833776561	7.127300578	33.4320	14.3110	19.1210
GPS 50	4.912119492	6.985296881	35.1170	16.1990	18.9180
GPS 56	4.781655028	7.006075439	28.0330	09.0150	19.0180
GPS 58	4.783296731	7.005240433	27.4410	08.4250	19.0160
GPS 59	4.916896858	6.880102978	20.4940	01.7030	18.7910
GPS 60	4.916108350	6.881154569	20.9820	02.1890	18.7930
XSV 662	4.873506919	6.998413150	27.6030	08.6480	18.9550
ZVS 3003	4.847971022	7.047811589	32.3080	13.2820	19.0260

The full data set for this table is as shown in Appendix C1.

Table 3.2b: Local Geoidal Undulation in Lagos State (Lagos State, SG Office 2010)

STATIONS	**Latitude** [º]	**Longitude** [º]	**Ellipsoidal Height (h) [m]**	**Orthometric Height (H) [m]**	**Geoidal Undulations (N) [m]**
XST 237	6.454802139	3.470396222	25.8360	3.2720	22.5640
YTT78A	6.470008869	3.646457902	27.3350	4.8610	22.4740
FGPLA-Y-003	6.427041234	2.890722633	27.0450	4.2620	22.7830
CFPA21	6.440896094	2.919119213	30.9400	8.1120	22.8280
XST 55	6.37965975	2.706952389	30.0470	7.3470	22.7000
YTT1703A	6.419998574	2.712921902	25.0470	2.1350	22.9120
LWBC5-61P	6.504592611	2.926533297	26.0300	2.8440	23.1860
YTT19-54	6.510901227	2.954208526	37.7640	14.5740	23.1900
YTT2-66A	6.441722983	3.84345449	26.8840	4.6140	22.2700
MCS1174S-A	6.665027289	3.323236155	73.1510	49.5700	23.5810
YTT2-48A	6.429279172	3.718083886	26.6820	4.4510	22.2310
FGPLA-Y-008	6.441898015	2.948674497	30.5720	7.7810	22.7910
MCS1178T-A	6.474988831	3.56779892	25.5580	3.0310	22.5270
ZTT34-34	6.644054924	4.036229785	30.9890	7.8590	23.1300
MCS1188T-A	6.493459685	3.582388693	25.3970	2.7750	22.6220

The full data set for this table is as shown in Appendix C2

3.4. COMPARISON OF ELLIPSOIDAL AND ORTHOMETRIC HEIGHT DIFFERENCES

Differences in elevation between successive heights for each of the points in Port Harcourt and Lagos State were computed and compared. Equation (1.3) was adopted for computing ellipsoidal and Orthometric Height differences and rewritten for any two points as:

$$(h_i - h_{i+1}) - (H_i + H_{i+1}) = (N_i - N_{i+1})$$
$$(h_i - h_{i+1}) - (H_i + H_{i+1}) = (N_i - N_{i+1}) \qquad (3.56)$$

Equation 1.3 can also be written as;

$$H_1 = h_1 - N_1$$
$$H_2 = h_2 - N_2$$

$$\Delta H = H_1 - H_2$$
$$\Delta h = h_1 - h_2$$
$$\Delta N = N_1 - N_2$$

$$\Delta H = \Delta h - \Delta N$$

If ΔN is small, which is always the case within a particular locality, especially the coastal area; then

$$\Delta H \doteq \Delta h$$

where;

h_i is the ellipsoidal height of station *i*

h_{i+1} is the ellipsoidal height of points preceding station *i*

H_i is the Orthometric Height of station *i*

H_{i+1} is the Orthometric Height of points preceding station *i*

N_i is the Geoidal Undulation of station *i*

N_{i+1} is the Geoidal Undulation of points proceeding station *i*

The results are as shown in Tables 3.1a and Table 3.1b for Port Harcourt and Lagos State respectively.

3.4.1 Elevation Differences from Ellipsoidal and Orthometric Heights: The differences in elevation as determined by both ellipsoidal and Orthometric Heights were compared. The results were shown in Table 4.1a and Table 4.1b (See Pages 123 and 124 respectively) for Port Harcourt and Lagos State respectively. Tables 4.1a and 4.1b were plotted as shown in Figures 4.1a and 4.1b (See Pages 123 and 124 respectively). These results show that there is no significant difference between the successive elevation computed from both ellipsoidal and Orthometric Heights. The gap between the two is the magnitude of the Geoidal Undulations for each of the plots.

3.5 Existing Geoidal Undulation

Geoidal Undulation of the study areas were computed using the existing models such as; North Sea Region Model (Equation 2.23), The 4-Parameter Similarity Datum shift (Equation 2.24), 5-Parameter Similarity Datum Shift (Equation 2.25), 7-Parameter Similarity Datum Shift (Equation 2.26), Zanletnyik Hungarian Polynomial Model (Equation 2.27), Mosaic of parametric model (Equation 2.29) and Geopotential Earth Model 2008 using the Alltrans EGM2008 calculator. In some instance, different computational procedures from the original approaches were used to compute the geoidal undulation. For instance;

3.5.1. North sea Region Model (Equation 2.24) was originally based on the use of trigonometric function based on Fourier analysis. This is time consuming and requires additional computational efforts. This model was implemented by using simple Least Squares observation equation method in the two study areas.

3.5.2 The 7-Parameter Similarity Datum Shift Model: The model deviates from the observed Geoidal Undulation and those of other existing models. Close investigation revealed that the adition of terms involving flattening $\left(\frac{1-f^2\sin^2\phi_i}{W}\right)$ affected the result and else did not fit the the two study areas in Nigeria. However, this can be corrected by adition of another term involving flattening and eccentricity as $\left(\frac{1-f^2\sin\phi_i}{W}\right)$.

3.5.3 Zanletnyik Hungarian Polynomial Model: This was originally developed for Hungarian geoid as reviewed in Section 2.2.2.6. The research investigated the deterioration of conditions of equations as observed by Zanletnyik et al., (2006). Observation equation method of Least Squares Adjustment was applied to determine their Geoidal Coefficients (Tables 4.2a and 4.2b; See Pages.126). The Geoidal undulation was computed for each point using each degree of polynomial; the result is tabulated in Table 4.3 (See Pages.126) for Port Harcourt metropolis.

The behaviour of this model for any single points is plotted as shown in Figure 3.7 below:

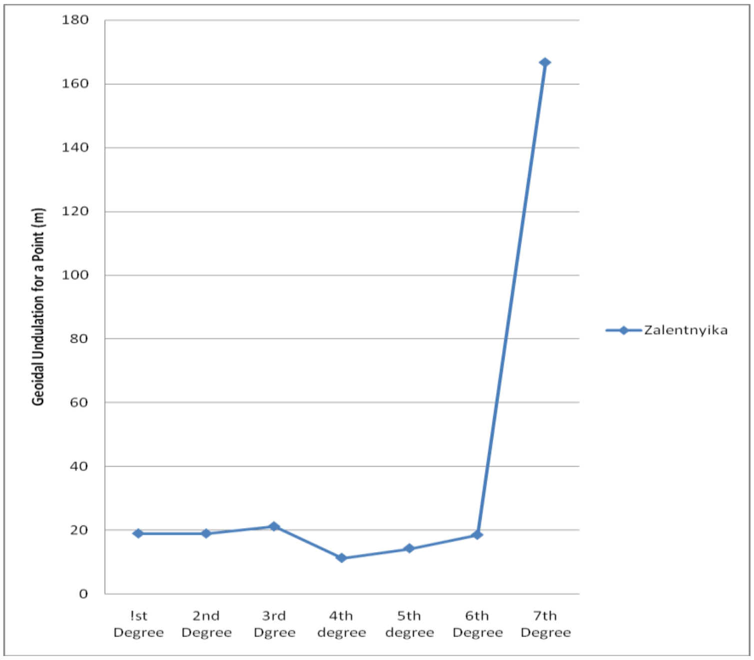


Figure 3.7: The Curve for the Solutions of Zanletnyik Hungarian Polynomial Model
(Source: Author, October, 2009)

The Zanletnyik Hungarian Polynomial Model deviated from the observed Geoidal Undulation and after second degree. Therefore, second order of the model satisfied the dataset in the two study areas.

3.6 MATHEMATICAL MODELS FOR 'SATLEVEL' COLLOCATION

The establishment of an empirical geoid model is premised on the assumption that the total geoidal variation at a geographic location is partly constant, and partly varies with location. The constant part of the geoidal variation is easy to understand and establish the component that varies with position is a function of many complex spatial phenomena that are not simple to describe in precise mathematical terms. A number of computation algorithms have been used by different authors to developed models. However, these models are either not easy to apply or not readily accessible to local surveyors, who are daily faced with the need for Geoidal Undulation values even in real time applications. It is therefore useful to establish an empirical model to represent the relationship between the observed undulation values and the changes in geographic coordinates over the area.

Establishment of an empirical model that will represent the relationship between the observed undulation values and the changes in geographic coordinates over the area is usually of a very complex nature regarding the exact relationship between variables in the model. Our preference is for a model that is linear and easy to use, thus this research tries to find a simplified but best possible solution on the basis of certain assumptions.

Physical evidence of the views of the surface of the Earth supports the hypothesis that the totality of Geoidal Undulation at a geographic location composed of two parts. These two parts are:

1) the constant (long wavelength) part throughout the study area that is $N_L = X_0$ (independent of position) and

2) the changing (short wavelength) part that is $N_S = f(\varphi, \lambda)$ which depends on changes in geographic location within the study area. The statistical significance of these relationships was considered in this work. The following models were developed;

i. Spherical 'Satlevel' Geoid Model

ii. Rectangular 'Satlevel' Geoid Model

3.6.1. Spherical 'Satlevel' Geoid Model

The method assumed that the Geoidal Undulation was a function of geographical location. Here the Earth is assumed to be a sphere. From Equation 1.3, N can be represented functionally as:

$$N = h - H = N_L + f(\phi, \lambda) \tag{3.57}$$

The challenge in Equation 3.57 is to find an explicit expression for $f(\phi, \lambda)$. Assuming a geographic area of interest to be located in a right-handed 3D Cartesian coordinate system (Figure 3.8), the position vector ***p*** for a point ***P*** has a unit vector $\bar{p}$ which can be written as (Olaleye, 1992):

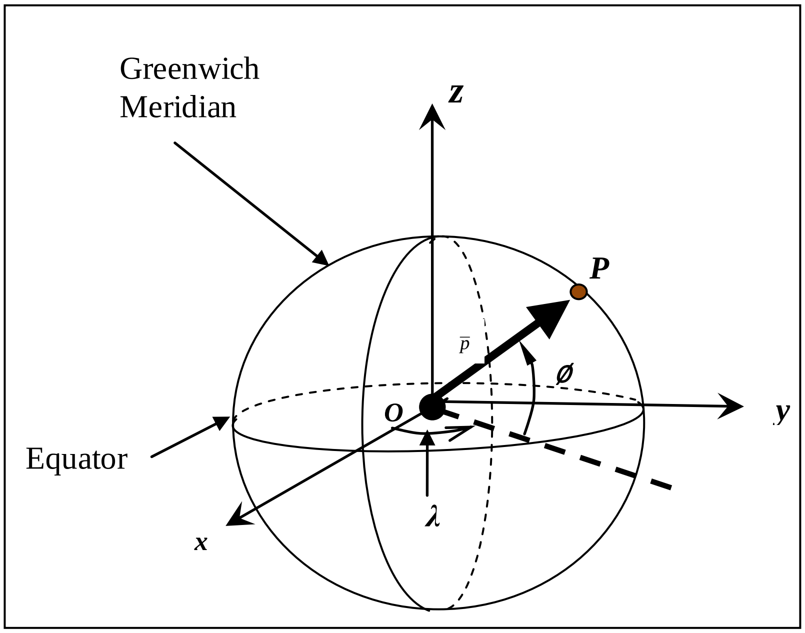


Figure 3.8: A 3D Spherical Coordinates System (Olaleye, 1992 and Agajelu, 1997)

$$\bar{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\phi\cos\lambda \\ \cos\phi\sin\lambda \\ \sin\phi \end{bmatrix} \tag{3.58}$$

The components of this unit vector can serve as signal carriers in the three dimensions of the coordinate system. Thus, any variability of Geoidal Undulation as a result of changes in location within the geographic area can be represented as multiples of components of the unit vector. They are spatial base-functions in terms of the latitude and longitude of a point in a geographic area. This is a vector that has both magnitude and direction. The direction is from the centre of the Earth to the point located on the sphere. Since the area under consideration is small, the sphere was assumed as the shape of the Earth. Hence, the direction cosine was used, and the magnitude of the vector neglected.

The set of base functions involved in Equation (3.58) can be represented as:

$$\begin{bmatrix} 1 & \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix} \tag{3.59}$$

Or symbolically as:

$$\begin{bmatrix} p_0 & p_1 & p_2 & p_3 \end{bmatrix} \tag{3.60}$$

where;

$$\begin{aligned} &p_0 = 1, \\ &p_1 = \cos\phi\cos\lambda, \\ &p_2 = \cos\phi\sin\lambda, \\ &p_3 = \sin\phi \end{aligned}$$

If we represent these base functions by p_0, p_1, p_2, p_3 respectively, Equation (3.60) may be written as a linear combination of these base functions to provide an expression for the Geoidal Undulation as given below;

$$N = h - H = \beta_0 p_0 + \beta_1 p_1 + \beta_2\, p_2 + \beta_3\, p_3 \tag{3.61}$$

where;

β_0 is the coefficient of the predictor variable for the constant part of N L $\beta_1,\ \beta_2, \beta_3$. are the coefficients of the explanatory variables which model the changing part of N

$p_0 = 1,\ p_1 = \cos\phi\cos\lambda, p_2 = \cos\phi\sin\lambda,\ p_3 = \sin\phi$

It is apparent that this collection of base functions meets our hypothesis of a mixture of constancy and variability of the Geoidal Undulation at a point.

Note that both h and H are assumed to be observable in the model, thus these observed values provide the observed values of the Geoidal Undulation (N). Thus, at a point i, where; h and H are observed, the observed undulation at a point is represented by a response variable, an equation of type can be written as Equation (3.62):

$$N_i = (\beta_0 p_0)_i + (\beta_1 p_1)_i + (\beta_2\, p_2)_i + (\beta_3\, p_3)_i \tag{3.62}$$

It is noted here that the linearity of the model is defined with respect to the coefficients (β_o, β_1, β_2, β_3) and not the base functions. Furthermore, we assume that the geographic coordinates implicit in the base functions are known and are error-free but the response variable β is observed with possible sampling errors. Obviously, Equation (3. 62) is never satisfied due to random errors in the measured heights and datum inconsistencies. Thus, the Geoidal Undulation model at a single point *i* then take the form:

$$N_i = h_i - H_i = (\beta_0 p_0)_i + (\beta_1 p_1)_i + (\beta_2\ p_2)_i + (\beta_3\ p_3)_i + r_i \tag{3.63}$$

where; N_i is the response variable and $r_{\mathbf{i}}$ is residual at an observation point i

The residual r_i, also known as error is assumed to be independently and identically distributed with mean 0 and variance σ^2. For hypothesis testing and the setting of confidence limits, we also assume that *r* is normally distributed. The model is thus represented in the 4-dimensional hyperspace of the base functions.

$$\begin{bmatrix}1 & \cos\phi\cos\lambda & \sin\phi\cos\lambda & \sin\phi\end{bmatrix} \tag{3.64}$$

It is apparent that they meet the hypothesis of a mixture of constancy and variability of the Geoidal Undulation at a point. A linear combination of this base functions will provide an expression for the function $f_i(\phi,\lambda)$ in Equation (3.65),that is;

$$f_i(\phi_i,\lambda_i) = A_1\cos\phi_i\cos\lambda_i + A_2\cos\phi_i\sin\lambda_i + A_3\sin\phi_i + r_i \tag{3.65}$$

Thus, at a point *i* where *h* and *H* are observed, with addition of the long wavelength component, type of Equation (3.65) can be written as;

$$h_i - H_i = N_L + A_1\cos\phi_i\cos\lambda_i + A_2\cos\phi_i\sin\lambda_i + A_3\sin\phi_i + r_i \tag{3.66}$$

From Equation 1.3, N= h-H, then Equation 3.66 becomes:

$$N_i = N_L + A_1\cos\phi_i\cos\lambda_i + A_2\cos\phi_i\sin\lambda_i + A_3\sin\phi_i + r_i \tag{3.67}$$

3.6.1.1 Modelling the Short Wavelength: The short wavelength component of the Geoidal Undulation can be modelled using the Pythagoras trigonometric expression for an angle. This expression was used by Rapp (1980) when modelling the expression for prime vertical and meridional radii of curvature. It is also used in spherical triangle. The expression is:

$$\cos^2\phi_i + \sin^2\phi_i = 1 \tag{3.68}$$

This is as shown in Figure 3.9a.

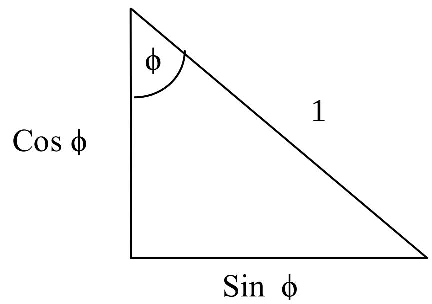


Figure 3.9a: Pythagoras Theorem for Latitude

(Source: Author, June, 2011)

3.6.1.2 Modelling the Short Wavelength along the Direction of Latitude: The short wavelength variation is modelled along the latitude, by multiplying both sides of Equation (3.67) by $\cos^2\phi_i + \sin^2\phi_i$

$$\begin{aligned} N_i\left(\cos^2\phi_i + \sin^2\phi_i\right) = N_L\left(\cos^2\phi_i + \sin^2\phi_i\right) + \\ A_1\left(\cos^2\phi_i + \sin^2\phi_i\right)\cos\phi_i\cos\lambda_i + \\ A_2\left(\cos^2\phi_i + \sin^2\phi_i\right)\cos\phi_i\sin\lambda + \\ A_3\left(\cos^2\phi_i + \sin^2\phi_i\right)\sin\phi_i + r_i \end{aligned} \tag{3.69}$$

$$\begin{aligned} N_i\left(\cos^2\phi_i + \sin^2\phi_i\right) = N_L\left(\cos^2\phi_i + \sin^2\phi_i\right) + \\ A_1\left(\cos^3\phi_i\cos\lambda_i + \sin^2\phi_i\cos\phi_i\cos\lambda_i\right) + \\ A_2\left(\cos^3\phi_i\sin\lambda_i + \sin^2\phi_i\cos\phi_i\sin\lambda_i\right) + \\ A_3\left(\cos^2\phi_i\sin\phi_i + \sin^3\phi_i\right) + r_i \end{aligned} \tag{3.70}$$

$$But\ \cos^2\phi_i + \sin^2\phi_i = 1$$

Then Equation 3.70 becomes:

$$\begin{aligned} N_i = N_L + A_1\left(\cos^3\phi_i \cos\lambda_i + \sin^2\phi_i \cos\phi_i \cos\lambda_i\right) + \\ A_2\left(\cos^3\phi_i \sin\lambda_i + \sin^2\phi_i \cos\phi_i \sin\lambda_i\right) + \\ A_3\left(\cos^2\phi_i \sin\phi_i + \sin^3\phi_i\right) + r_i \end{aligned} \qquad (3.71)$$

3.6.1.3 Modelling the Short Wavelength along the Direction of Longitude: Since there are two components of two dimensional geodetic coordinates (the geodetic latitude and geodetic longitude), the short wavelength is also modelled along the longitude by multiplying both sides of Equation (3.67) by: $\cos^2\lambda_i + \sin^2\lambda_i$

This is as shown in Figure 3.9b.

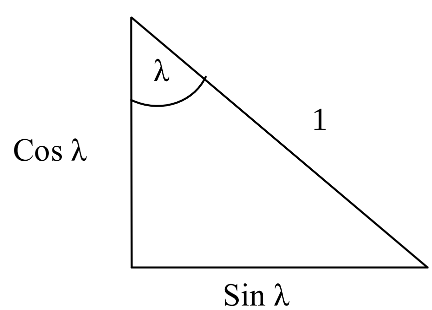


Figure 3.9b: Pythagoras Theorem Longitude (Source: Author, June, 2011)

$$\begin{aligned} N_i\left(\cos^2\lambda_i + \sin^2\lambda_i\right) = N_L\left(\cos^2\lambda_i + \sin^2\lambda_i\right) + \\ A_1\left(\cos^2\lambda_i + \sin^2\lambda_i\right)\cos\phi_i \cos\lambda_i + \\ A_2\left(\cos^2\lambda_i + \sin^2\lambda_i\right)\cos\phi_i \sin\lambda_i + \\ A_3\left(\cos^2\lambda_i + \sin^2\lambda_i\right)\sin\phi_i + r_i \end{aligned} \qquad (3.72)$$

$$N_i\left(\cos^2\lambda_i+\sin^2\lambda_i\right)=N_L\left(\cos^2\lambda_i+\sin^2\lambda_i\right)+$$
$$A_1\left(\cos^3\lambda_i\cos\phi_i+\sin^2\lambda_i\cos\phi_i\cos\lambda\right)+$$
$$A_2\left(\cos^2\lambda_i\cos\phi_i\sin\lambda_i+\sin^3\lambda_i\cos\phi_i\right)+ \quad (3.73)$$
$$A_3\left(\cos^2\lambda_i\sin\phi_i+\sin^2\lambda_i\sin\phi_i\right)+r_i$$

But $\cos^2\lambda_i+\sin^2\lambda_i=1$

Therefore, Equation 3.73 becomes:

$$N_i=N_L+A_1\left(\cos^3\lambda_i\cos\phi_i+\sin^2\lambda_i\cos\phi_i\cos\lambda\right)+$$
$$A_2\left(\cos^2\lambda_i\cos\phi_i\sin\lambda_i+\sin^3\lambda_i\cos\phi_i\right)+ \quad (3.74)$$
$$A_3\left(\cos^2\lambda_i\sin\phi_i+\sin^2\lambda_i\sin\phi_i\right)+r_i$$

Adding Equations (3.71) and (3.74) together:

$$2N_i=2N_L+2A_1\left(\cos^3\phi_i\cos\lambda_i+\sin^2\phi_i\cos\phi_i\cos\lambda_i+\cos^3\lambda_i\cos\phi_i+\sin^2\lambda_i\cos\phi_i\cos\lambda\right)+$$
$$2A_2\left(\cos^3\phi_i\sin\lambda_i+\sin^2\phi_i\cos\phi_i\sin\lambda_i+\cos^2\lambda_i\cos\phi_i\sin\lambda_i+\sin^3\lambda_i\cos\phi_i\right)+$$
$$2A_3\left(\cos^2\phi_i\sin\phi_i+\sin^3\phi_i+\cos^2\lambda_i\sin\phi_i+\sin^2\lambda_i\sin\phi_i\right)+2r_i$$

(3.75)

Divide both sides of Equation (3.75) by 2

$$N_i=N_L+A_1\left(\cos^3\phi_i\cos\lambda_i+\sin^2\phi_i\cos\phi_i\cos\lambda_i+\cos^3\lambda_i\cos\phi_i+\sin^2\lambda_i\cos\phi_i\cos\lambda\right)+$$
$$A_2\left(\cos^3\phi_i\sin\lambda_i+\sin^2\phi_i\cos\phi_i\sin\lambda_i+\cos^2\lambda_i\cos\phi_i\sin\lambda_i+\sin^3\lambda_i\cos\phi_i\right)+$$
$$A_3\left(\cos^2\phi_i\sin\phi_i+\sin^3\phi_i+\cos^2\lambda_i\sin\phi_i+\sin^2\lambda_i\sin\phi_i\right)+r_i$$

(3.76)

The model is of the form:

$$N_i=N_L+A_1\left(\cos^3\phi_i\cos\lambda_i+\sin^2\phi_i\cos\phi_i\cos\lambda_i+\cos^3\lambda_i\cos\phi_i+\sin^2\lambda_i\cos\phi_i\cos\lambda\right)+$$
$$A_2\left(\cos^3\phi_i\sin\lambda_i+\sin^2\phi_i\cos\phi_i\sin\lambda_i+\cos^2\lambda_i\cos\phi_i\sin\lambda_i+\sin^3\lambda_i\cos\phi_i\right)+$$
$$A_3\left(\cos^2\phi_i\sin\phi_i+\sin^3\phi_i+\cos^2\lambda_i\sin\phi_i+\sin^2\lambda_i\sin\phi_i\right)+r_i \quad (3.77)$$

where;

N is the geoid undulation,

A_1, A_2 and A_3 are the trend coefficients which are unknown coefficients to be determined.

ϕ and λ are the WGS 84 geodetic coordinates (Latitudes and Longitudes)

r_i **is** residual at an observation point.

The unknown parameters in Equation (3.77) were estimated by Least Squares adjustment, since sufficient observation points were available.

3.6.2 Rectangular 'Satlevel' Geoid Model:

Equation (1.3) can be represented functionally as:

$$N = h - H = N_m + f(X,Y,Z) \qquad (3.78)$$

where;

X, Y, Z = the 3D Space Rectangular Coordinates

All the terms are as earlier defined.

The challenge in Equation (3.78) is to find an explicit expression for $f(X,Y,Z)$. Assuming a geographical area of interest is located in a right-handed 3D Cartesian coordinates system, with the origin at Earth centre (Figure 3.8). The same explanations for spherical 'Satlevel' model still hold here. The model is of the form;

$$N = B_0 + B_1 X_i + B_2 Y_i + B_3 Z_i + r_i \qquad (3.79)$$

The geodetic coordinates were converted to rectangular using the algorithm below (Bomford, 1980; Rapp, 1980; Uzodinma and Ezenwere, 1993 and Jokeli, 2006);

$$X = (v+h)Cos\phi Cos\lambda \qquad (3.80a)$$

$$Y = (v+h)Cos\phi Sin\lambda \qquad (3.80b)$$

$$Z = \left[v\left(1-e^2\right)+h \right] Sin\phi \qquad (3.80c)$$

where;

v is the radius of curvature in the prime vertical direction at the point of projection of P on the ellipsoid.

$$v = \frac{a}{W} = \frac{a}{\sqrt{1-e^2 \sin^2 \phi}} \tag{3.81}$$

a = Ellipsoid equatorial radius

e^2 = Squared of eccentricity of the reference ellipsoid that is used for the definition of the geodetic coordinates (ϕ, λ, h)

$$e^2 = 2f - f^2 \tag{3.82a}$$

f = flattening

$$f = \frac{a-b}{a} \tag{3.82b}$$

b = Ellipsoid polar radius.

The above algorithm was used to develop a user-friendly program in FORTRAN Programming Language (See Appendix B2).

The set of base functions involved in Equation (3.78) can be represented as;

$$\begin{bmatrix} 1 & \frac{X_i}{r} & \frac{Y_i}{r} & \frac{Z_i}{r} \end{bmatrix} \tag{3.83}$$

where;

$$r = \sqrt{X^2 + Y^2 + Z^2}$$

All other terms are as earlier defined

It is apparent that they met our hypothesis of a mixture of constancy and variability of the Geoidal Undulation at a point. The same explanations still hold as in Spherical 'Satlevel' model.

In summary, for all the methods (Sections 3.6.1 and 3.6.2) the following models have been developed for computing local Geoidal Undulations:

1.
$$N_i = N_L + A_1\left(\cos^3\phi_i \cos\lambda_i + \sin^2\phi_i \cos\phi_i \cos\lambda_i + \cos^3\lambda_i \cos\phi_i + \sin^2\lambda_i \cos\phi_i \cos\lambda\right) + A_2\left(\cos^3\phi_i \sin\lambda_i + \sin^2\phi_i \cos\phi_i \sin\lambda_i + \cos^2\lambda_i \cos\phi_i \sin\lambda_i + \sin^3\lambda_i \cos\phi_i\right) + A_3\left(\cos^2\phi_i \sin\phi_i + \sin^3\phi_i + \cos^2\lambda_i \sin\phi_i + \sin^2\lambda_i \sin\phi_i\right) + r_i \tag{3.77}$$

2. .
$$N = B_0 + B_1 X_i + B_2 Y_i + B_3 Z_i + r_i \tag{3.79}$$

3.7 ADAPTATION OF GLOBAL GEOID MODEL TO LOCAL GEOID MODEL

Spherical 'Satlevel' Model used the data format (geodetic coordinates) commonly available on most GNSS devices unlike Rectangular 'Satlevel' Model which used rectangular coordinates. Rectangular 'Satlevel' Model required additional effort of coordinate's conversion. Also, Spherical 'Satlevel' Model because it is based on the assumption that the Earth is spherical. The model clearly depicts the variation in Geoidal Undulation. Spherical 'Satlevel' Model was therefore used to compute the variation between the Global geoid and the local datum for proper fixing and adaptation.

Recall Equation (3.66);

$$h_i - H_i = N_L + A_1 \cos\phi_i \cos\lambda_i + A_2 \cos\phi_i \sin\lambda_i + A_3 \sin\phi_i + r_i \tag{3.66}$$

Making the Orthometric Height (H_i) the subject of the formula, Equation (3.66) can be written as:

$$H_i = h_i - \left[N_L + A_1 \cos\phi_i \cos\lambda_i + A_2 \cos\phi_i \sin\lambda_i + A_3 \sin\phi_i + r_i\right] \tag{3.84}$$

Using GEM2008 as the long wavelength component of Equation 3.84, then N_L is substituted by N_{GEM08}, the Equation 3.84 becomes:

$$H_i = h_i - \left[N_{GEM08} + A_1 \cos\phi_i \cos\lambda_i + A_2 \cos\phi_i \sin\lambda_i + A_3 \sin\phi_i + r_i \right] \tag{3.85}$$

Since the coefficients will be different, Equation (3.85) can be written as;

$$H_i = h_i - \left[N_{GEM08} + D_1 \cos\phi_i \cos\lambda_i + D_2 \cos\phi_i \sin\lambda_i + D_3 \sin\phi_i + r_i \right] \tag{3.86}$$

As discussed in Section 3.6, N_{GEM08} is the long wavelength component (constant part) while $D_1\cos\phi_1 \cos\lambda_1 + D_2 \cos\phi_i \sin\lambda_i + D_3 \sin\phi_i + r_i$ are the short wavelength component (variable part). To implement Equation (3.86), the mean of residuals between Geoidal Undulation computed from local geoid and that of Global (GEM2008) will be deducted from each of the residuals. These are the mean corrected global residuals, which were used in the Least Squares adjustment to compute the coefficients. The coefficients computed are for: N_L, A_1, A_2, A_3, B_1, B_2, B_3, D_1, D_2 and D_3 in Equations (3.77), (3.79) and (3.86).

The Geoidal Undulations adapted from GEM2008 to its local equivalent called local Geoidal Undulations. Geoidal Undulation obtained from Equation 3.86 was subtituted in Equation 1.3 with Ellipsoidal height to obtain GEM2008 Orthometric Height and are tabulated in Tables 4.13a and 4.13b and plotted in form of charts (Figures 4.10a and 4.10b) for Port Harcourt and Lagos State respectively.

3.8 'SATLEVEL' COLLOCATION MODEL PARAMETER ESTIMATION

For estimation purposes, suppose a set of *m* Geoidal Undulation observed with equal reliability at *m* known geographic locations $(N_i, (\phi, \lambda)_i)$, $i = 1, 2, ..., m$, then the base functions p_0, p_1, p_2, p_3 become base vectors $\mathbf{P}_0$, $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$ which form the coordinates (spanning) axes of the hyperspace in which the undulation measurements are made (Figure 3.10).

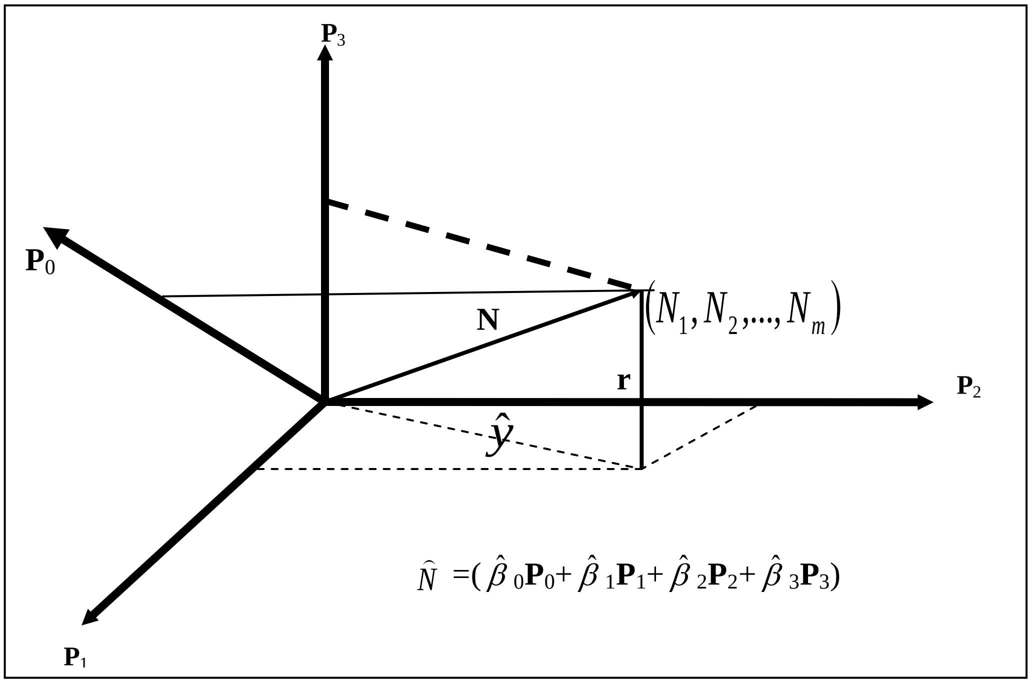


Figure 3.10: The Four Dimensional Observation Space Spanned by $\mathbf{P}_0$, $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$

Each base vector has *m* elements corresponding to the number of points at which undulation values are observed. The spanning vectors, as they are often called, may be represented as;

$$\mathbf{P}_0 = \begin{bmatrix} 1 \\ 1 \\ . \\ . \\ 1 \end{bmatrix}, \quad \mathbf{P}_1 = \begin{bmatrix} p_1 \\ p_2 \\ . \\ . \\ p_m \end{bmatrix}, \quad \mathbf{P}_2 = \begin{bmatrix} Q_1 \\ Q_2 \\ . \\ . \\ Q_m \end{bmatrix}, \quad \mathbf{P}_3 = \begin{bmatrix} R_1 \\ R_2 \\ . \\ . \\ R_m \end{bmatrix} \tag{3.88}$$

Where;

$$P_1 = \cos^3\phi_1 \cos\lambda_1 + \sin^2\phi_1 \cos\phi_1 \cos\lambda_1 + \cos^3\lambda_1 \cos\phi_1 + \sin^2\lambda_1 \cos\phi_1 \cos\lambda_1$$
$$P_2 = \cos^3\phi_2 \cos\lambda_2 + \sin^2\phi_2 \cos\phi_2 \cos\lambda_2 + \cos^3\lambda_2 \cos\phi_2 + \sin^2\lambda_2 \cos\phi_2 \cos\lambda_2$$
$$P_m = \cos^3\phi_m \cos\lambda_m + \sin^2\phi_m \cos\phi_m \cos\lambda_m + \cos^3\lambda_m \cos\phi_m + \sin^2\lambda_m \cos\phi_m \cos\lambda_m$$

$$Q_1 = \cos^3\phi_1 \sin\lambda_1 + \sin^2\phi_1 \cos\phi_1 \sin\lambda_1 + \cos^2\lambda_1 \cos\phi_1 \sin\lambda_1 + \sin^3\lambda_1 \cos\phi_1$$
$$Q_2 = \cos^3\phi_2 \sin\lambda_2 + \sin^2\phi_2 \cos\phi_2 \sin\lambda_2 + \cos^2\lambda_2 \cos\phi_2 \sin\lambda_2 + \sin^3\lambda_2 \cos\phi_2$$
$$Q_m = \cos^3\phi_m \sin\lambda_m + \sin^2\phi_m \cos\phi_m \sin\lambda_m + \cos^2\lambda_m \cos\phi_m \sin\lambda_m + \sin^3\lambda_m \cos\phi_m$$

$$R_1 = \cos^2\phi_1 \sin\phi_1 + \sin^3\phi_1 + \cos^2\lambda_1 \sin\phi_1 + \sin^2\lambda_1 \sin\phi_1$$
$$R_2 = \cos^2\phi_2 \sin\phi_2 + \sin^3\phi_2 + \cos^2\lambda_2 \sin\phi_2 + \sin^2\lambda_2 \sin\phi_2$$
$$R_m = \cos^2\phi_m \sin\phi_m + \sin^3\phi_m + \cos^2\lambda_m \sin\phi_m + \sin^2\lambda_m \sin\phi_m$$

The vector form of the undulation model then becomes:

$$N = \beta_0 \begin{bmatrix} 1 \\ 1 \\ . \\ . \\ 1 \end{bmatrix} + \beta_1 \begin{bmatrix} P_1 \\ P_2 \\ . \\ . \\ P_m \end{bmatrix} + \beta_2 \begin{bmatrix} Q_1 \\ Q_2 \\ . \\ . \\ Q_m \end{bmatrix} + \beta_3 \begin{bmatrix} R_1 \\ R_2 \\ . \\ . \\ R_m \end{bmatrix} + \mathbf{r} \tag{3.89}$$

Where; $\boldsymbol{N}$ = (N_{i}, i = 1, 2, …, m) is the vector of the observed undulations and $\boldsymbol{r}$ is the vector of residuals. All other terms as earlier defined.

When sufficient observations of undulation values are made within a geographic area, the postulated geoidal model can be written in vector form as;

$$N = \beta_0 \mathbf{P}_0 + \beta_1 \mathbf{P}_1 + \beta_2 \mathbf{P}_2 + \beta_3 \mathbf{P}_3 + \mathbf{r} \tag{3.90}$$

Or in terms of residuals as;

$$\mathbf{r} = N - (\beta_0 \mathbf{P}_0 + \beta_1 \mathbf{P}_1 + \beta_2 \mathbf{P}_2 + \beta_3 \mathbf{P}_3) \tag{3.91}$$

It follows that the observed vector of undulations is a linear combination of the base vectors which define the observation space.

The Least Squares solution is obtained by minimizing the L_2-norm of the residual errors $\|N - \mathbf{X}\boldsymbol{\beta}\|_2$. The 4-unknown parameters $\beta_0,\ \beta_1,\ \beta_2, \beta_3.$ are represented by a column vector $\boldsymbol{\beta}$.

The vector $\hat{y}$ (in Figure 3.10) is inclined to each of the spanning base vectors of the data space so that it has an image (an approximation) $\hat{\mathbf{y}}$ in the data space. If $\hat{y}$ is orthogonal to any base vector, it does not have any component along such axis; and when it is orthogonal to all of the base vectors, it has no representation or image in the data space. The data space is spanned by the base vectors (coordinate axes) $\mathbf{P}_0$, $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$

Also, the base vectors form the axes of a multi-dimensional space in which **y** are observed. Geometrically, vector of residuals (**r**) has the minimum length only when it is perpendicular or normal (normality of the normal equations) to each of the axes vectors $[\mathbf{P}_0 \quad \mathbf{P}_1 \quad \mathbf{P}_2 \quad \mathbf{P}_3]$. This implies that at the minimum value of **r**, its inner product with each of the axes vectors should be zero that is perpendicular to **P.**

$$\langle [N-(\beta_0\mathbf{P}_0+\beta_1\mathbf{P}_1+\beta_2\,\mathbf{P}_2+\beta_3\,\mathbf{P}_3)],\mathbf{P}_i\rangle = 0,\ \text{i=1, 2, 3, 4} \tag{3.92}$$

Thus, the normal equations can be arranged in matrix-vector form as:

$$\begin{bmatrix} \langle \mathbf{p}_0,\mathbf{p}_0\rangle & \langle \mathbf{p}_1,\mathbf{p}_0\rangle & \langle \mathbf{p}_2,\mathbf{p}_0\rangle & \langle \mathbf{p}_3,\mathbf{p}_0\rangle \\ \langle \mathbf{p}_0,\mathbf{p}_1\rangle & \langle \mathbf{p}_1,\mathbf{p}_1\rangle & \langle \mathbf{p}_2,\mathbf{p}_1\rangle & \langle \mathbf{p}_3,\mathbf{p}_1\rangle \\ \langle \mathbf{p}_0,\mathbf{p}_2\rangle & \langle \mathbf{p}_1,\mathbf{p}_2\rangle & \langle \mathbf{p}_2,\mathbf{p}_2\rangle & \langle \mathbf{p}_3,\mathbf{p}_2\rangle \\ \langle \mathbf{p}_0,\mathbf{p}_3\rangle & \langle \mathbf{p}_1,\mathbf{p}_3\rangle & \langle \mathbf{p}_2,\mathbf{p}_3\rangle & \langle \mathbf{p}_3,\mathbf{p}_3\rangle \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} \langle y,\mathbf{p}_0\rangle \\ \langle y,\mathbf{p}_1\rangle \\ \langle y,\mathbf{p}_2\rangle \\ \langle y,\mathbf{p}_3\rangle \end{bmatrix}. \tag{3.93}$$

These are the normal equations which result from the perpendicularity requirement between **r** and the base vectors for minimum residual **r**. $\mathbf{r} \perp \mathbf{P}_i$. The normal equations can be put in vector-matrix form as Equation (3.93), which can be solved using Least Squares Adjustment. Least Square Adjustment provides the values of the coefficient called the "**Geoidal Coefficients**". The geoidal coefficients were used to estimate the absolute values of the undulations.

As a way of checking for arithmetic errors or blunders, the values of the coefficients were substituted into the original model (Equations 3.77 and 3.79 for Spherical and Rectangular 'Satlevel' respectively) and both equations must check. Problems were experienced with regard to the number of decimal places causing rounding errors. Computer program was used and data stored in the computer memory to eliminate the copying error. The models were validated after the estimate.

3.9 'SATLEVEL' COLLOCATION MODEL VALIDATION

After estimating the parameters of the model, it is important in empirical modelling to find evidence of a linear relationship between the response and a subset of the explanatory variables to justify the model. The test will assure the significance or otherwise of the selected base functions. Significance tests applied to the model selection process are in two parts:

1) F Distribution for the significance of the three explanatory base functions in the model and
2) Model Validation

3.9.1 Significance Test of the 'Satlevel' Collocation Model Parameters: The hypotheses formulated are as follows;

$$\text{Null Hypothesis } H_0 : x_1, x_2, x_3 = 0 \tag{3.94a}$$

$$\text{Alternative Hypothesis } H_1 : x_1,\ x_2,\ x_3 \neq 0 \tag{3.94b}$$

F Distribution was used for this test. The values of the quantities computed include: residual sum of squares, sum of square total and sum of square regression. The results were as presented in the Table 4.15a and 4.15b for both Port Harcourt and Lagos State respectively.

Decision Rule: - H_o may be rejected at significance level $\alpha < 0.05$
if $F > F_{3,71,\alpha=0.05} = 8.565011359$ and $F > F_{3,110,\alpha=0.05} = 8.551420939$ were obtained as F value from the table using Microsoft excel for both Port Harcourt and Lagos State respectively. (See Section 4.1.13 for the result)

3.9.1.1 Assessing the Parametric of the 'Satlevel' Collocation Model Performance

In general, the process of selecting the best parametric model in a particular region suffers from a high degree of arbitrariness in both choice of model type and assessing model performance. This is always based on hypotheses testing. In this research, the models so derived satisfied our hypothesis.

Nevertheless, the performances of the models need to be tested. The tests used to assess the performance of parametric models includes: classical empirical approach, assessing the goodness of fit, model validation and the significance test of the model parameters.

2) '**Satlevel' Collocation Model Validation:** Five points which were not part of the initial data used to derive the models were randomly selected as checks for model validation.. These checked points were used to compute the geoidal coefficients, which were later used to compute the datum for the selected points. The results of which and the mean square errors were computed and shown in Tables 4.12a and 4.12b for Spherical 'Satlevel', and Tables 4.12c and 4.12d for Rectangular 'Satlevel' for the two study areas.

The other data were used to compute the coefficients of each of the models. The checked points were also used to compute the geoidal coefficients, which were later used to compute the datum for the points. The mean square errors were computed for each of the models and are also tabulated in Tables 4.12a, 4.12b, 4.12c and 4.12d for the two study areas.

3.9.1.2. 'Satlevel' Collocation Classical Empirical Approach: "The most common method used in practice to assess the performance of the selected parametric model(s) is to compute the statistics for the adjusted residuals after the Least Squares fit" (Fotopoulos, 2003). The residuals were computed for the existing models as shown in Tables 4.5a and 4.5b for both Port Harcourt and Lagos State respectively. Residuals from the New 'Satlevel' Collocation Geoid Model for both Port Harcourt and Lagos State are shown in Tables 4.9a and 4.9b. These residuals compared favourably with those of existing model in Tables 4.5a and 4.5b. The residuals for the Existing and new 'Satlevel' Collocation models were also summarised in Table 4.11a and 4.11b.

3.9.1.3. Ellipsoidal and Orthometric Heights: The local Geoidal Undulations were computed using Equation (1.3). Since the data used were observed quantities, therefore, local undulations were the also observed undulations. The adjusted local undulations were adopted as 'gold' standard for bases of comparison with both existing and the new 'Satlevel' Collocation models. The results of the local/observed undulations are tabulated in Tables 3.2a and 3.2b and plotted into charts Figure 4.1a and Figure 4.1b for Port Harcourt and Lagos State respectively.

3.9.1.4 Differences Between the Local (Observed) Undulations and New 'Satlevel' Collocation Models: The results of the existing model such as: North Sea Region Model (Equation 2.23), The 4-Parameter Similarity Datum shift (Equation 2.24), 5-Parameter Similarity Datum Shift (Equation 2.25), 7-Parameter Similarity Datum Shift (Equation 2.26), Zanletnyik Hungarian Polynomial Model (Equation 2.27), Mosaic of parametric model (Equation 2.29) along with the Geopotential Earth Model 2008 which was calculated using the Alltrans EGM2008 calculator were presented as shown in Tables 4.3a and 4.3b and plotted in charts (Figure 4.3a and 4.3b) for Port Harcourt and Lagos State respectively.

3.10 'SATLEVEL' COLLOCATION MODEL ADEQUACY

A statistical measure of the goodness of fit for a discrete set of points is denoted by R^2. In the extreme case where; the parametric model fit is perfect, R^2 equals one. The other extreme occurs, if one considers the variation from the residuals to be nearly as large as the variation about the mean of the observations resulting in the fractional part. The closer the value is to one, the smaller the residuals and hence the better the fit.

3.10.1 'Satlevel' Collocation Model adequacy test using the coefficient of determination $\mathbf{R^2}$: The two tests are based on the assumptions that the residual errors ***r*** is independent of errors in the base functions and are normally distributed with zero mean and common variance.

1) For the significance test on the base functions, the hypothesis are:

   Null Hypothesis $H_0: \ \beta_i = 0$ for all *i*, *i* = 1, 2, 3

   and

   Alternate Hypothesis $H_0: \ \beta_i \neq 0$ for one or more *i* , *i* = 1, 2, 3

The significance of the explanatory variables in the model can be established by testing the ratio *of the means of the two sums of squares* (SS_R) and ($SS_{E,}$) which follows an F-distribution*:*

$$\frac{SS_R/4}{SS_E/88} = \frac{(\hat{\boldsymbol{\beta}}\mathbf{X}^T\mathbf{y} - \frac{(\mathbf{y}^T\mathbf{P}_0)^2}{m})/4}{(\mathbf{y}^\mathbf{T}\mathbf{y} - \hat{\boldsymbol{\beta}}\mathbf{X}^T\mathbf{y})/88} \sim F_{4,\,88,\,\alpha} \tag{3.94}$$

The Null Hypothesis is rejected if $F >\ F_{4,\,88,\,\alpha}$ for a level of significance α .

Model adequacy was checked by computing the coefficient of determination. This is the ratio of the sum of squares due to model to the total sum of squares; it is sometimes called the *coefficient of correlation,* or simply, R^2.

$$R^2 = \frac{SS_E}{SS_{yy}} \tag{3.95}$$

where;

SS_E is the sum of square of residuals

SS_{yy} is the sum of square Total

It gives the proportion (or fraction) of the variability of the response variable, that is accounted for by the model variables. The higher the value of R^2 the better the fitting of the model. This also enables the computation of variation of Geoidal Undulations not accounted by the models.

The coefficients of determination for the models were as shown in Tables 4.15a and 4.15b for Port Harcourt and Lagos State respectively. The coefficient of determination for fitting the local geoid into GEM2008 in Lagos State gave the same results with the actual predicted values. The variation of Geoidal Undulations not accounted by each of the Geoid models were computed for each of the new models and equally shown in Tables 4.15a and 4.15b for Port Harcourt and Lagos State respectively. The results satisfied 95% significant level in Port Harcourt. This shows the reliability of the New ‘Satlevel’ Collocation models when points are evenly spaced and well distributed. However, the result is a little bit less in Lagos because the data used in Lagos State are too small compared to area of coverage. From Table 4.6b, the spacing between points was several kilometres apart.

3.10.3. Orthometric Heights The GEM2008 Orthometric Heights computed using Equation (3.86) and Local Orthometric computed from Equation (1.3) are tabulated in Tables 4.13a and 4.13b and plotted inform of charts (Figures 4.10a and 4.10b) for Port Harcourt and Lagos State respectively. The differences obtained from Tables 4.13a and 4.13b and that of Tables 4.14a and 4.14b were of the same magnitude. Though, this is expected and shows the reliability of the results.

3.10.4 Geoidal Map and Surface Modelling of the Study Area: The geoidal maps and 3D surface modelling of the study areas were produced for each of the models using SURFER software. Figures 4.12a through 4.12j show the Geoidal Undulation and 3-D Models. Some of the existing and the new 'Satlevel' collocation models were also used to produce the geoidal map of Port Harcourt, which was overlaid on the Local Government map of Rivers State (Figure 4.12k). The geoidal map of Port Harcourt was overlaid on the full Local Government map of Rivers State as the final product (Figure 4.13). The geoid slopes towards the ocean. This is equally expected but an indication of reliability of the results. GEM2008 fit perfectly in the Coastal areas of Nigeria and therefore adapted for Orthometric Height with the use of 'Satlevel' Collocation Models developed in this research. The usage is better enhanced with interactive software designed in this research called "Orthometric Height on Fly".

3.11 COMPUTER PROGRAMMING

All the computations were done using spread sheet (Microsoft Excel). A sample of the computations is as shown in Appendix A. Determination of initial coefficients for the area under study is required. This can be done using any convenient methods. A program for computation of coefficients which is required at first instance using Least Squares Adjustment observation equation method is designed in MATLAB. The program listing is as contained in Appendix B1.

Also, a user-interactive program called "Orthometric Height on the Fly" was designed using FORTRAN PowerStation. The flowchart (Figure. 3.11) for the program gives detailed procedure of its usage. Orthometric Height on the fly was designed using Microsoft FORTRAN PowerStation. The program listing is shown in Appendix B3.

D.

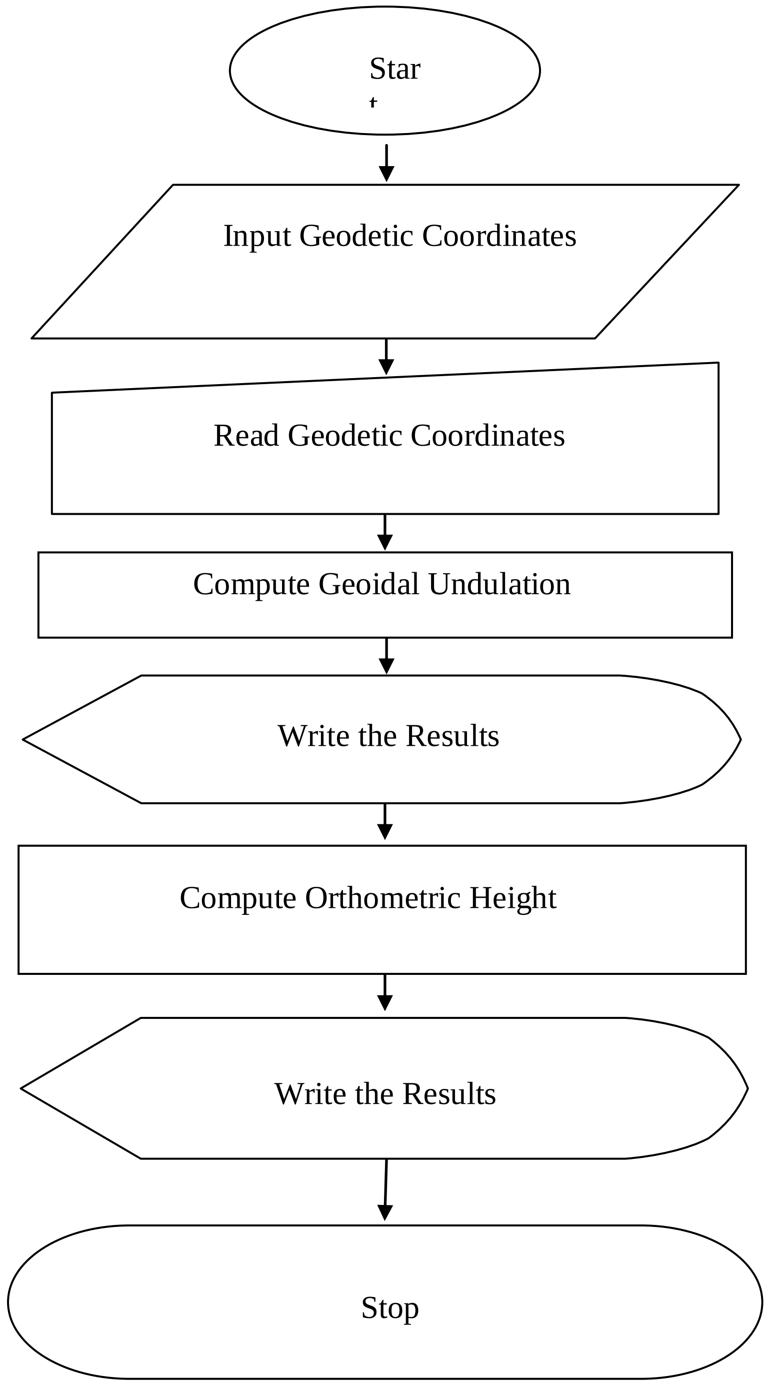


Figure 3.11: Flowchart for “Orthometric Height on the Fly” Programs

This user-friendly interactive software computes Orthometric Height of any point from the given geodetic coordinates using the New ‘Satlevel Collocation models. The program was tested using some data from acquired data and results show true resemblance with manual computation, thereby confirming the capability of the program. The sample of the displayed result is attached in Appendix D.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 THE RESULTS:

The GNSS data acquired from the field were processed and the ellipsoidal heights were extracted from the processed result. Similarly, the results of the geodetic levelling operation that was done were reduced and adjusted to give the Orthometric Heights.

4.1.1 Local Geoidal Undulation: The results of the Orthometric Heights acquired from the geodetic levelling (Section 3.6.1) and ellipsoidal heights from GPS observation (Section 3.6.2) were substituted into Equation 1.3 to obtain the values of the Geoidal Undulation for both Port Harcourt and Lagos State as shown in Tables 3.2a and 3.2b respectively.

Ellipsoidal and Orthometric Heights were plotted (Figures 4.1a and 4.1b) for both Port Harcourt and Lagos State respectively, to see the relationship between them as discussed in Section 3.9.1.3. (See Page 118)

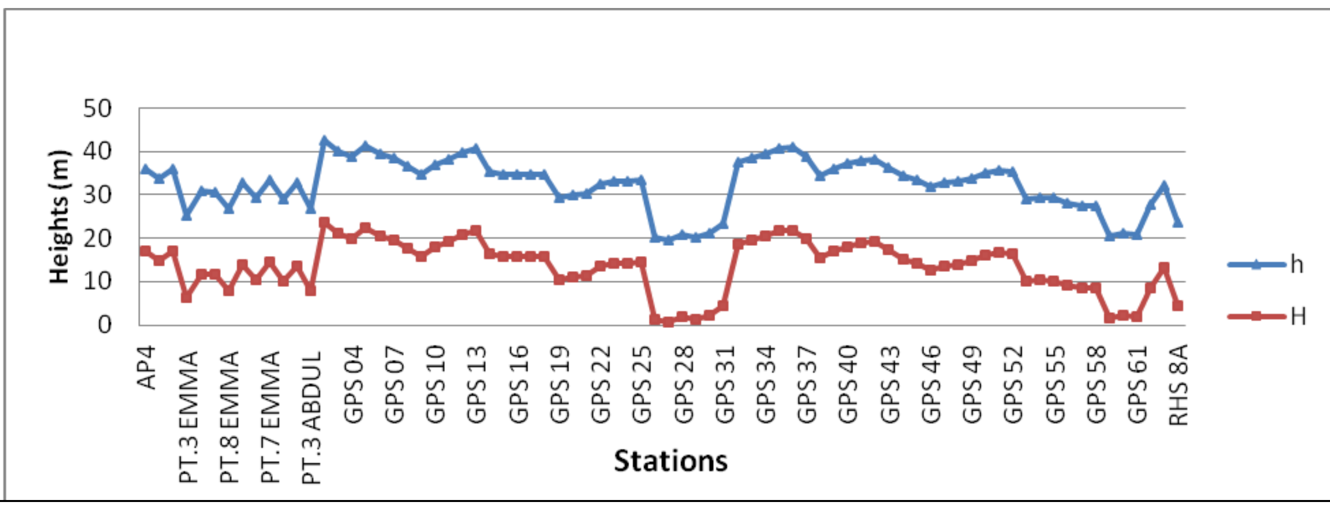


Figure 4.1a: Chart showing the relationship between Ellipsoidal and Orthometric Heights in Port Harcourt

Ellipsoidal and Orthometric Heights follow the same pattern in each of the two charts, (Figures 4.1a and 4.2), which portray that; the data were true reflection of the same terrain.

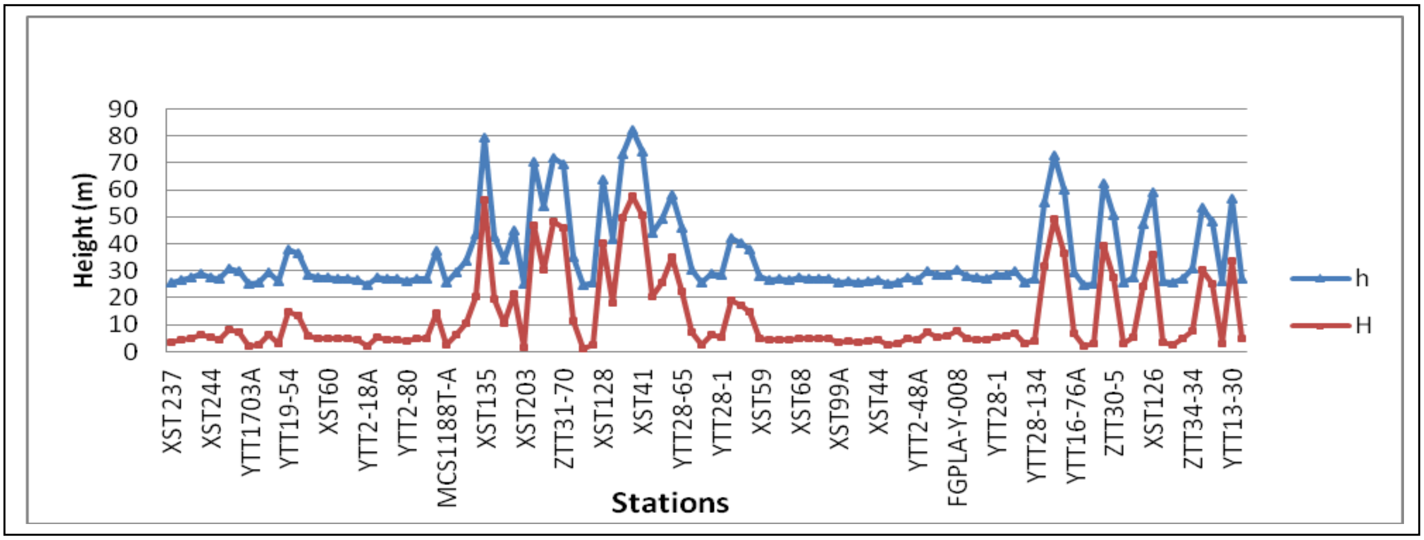


Figure 4.1b: Chart showing the Relationship between Ellipsoidal and Orthometric Heights in Lagos State

4.1.3 Comparison of Height Difference

The Orthometric and ellipsoidal height differences using Equation (3.1) were computed and compared as shown in Table 4.1a and Table 4.1b and discussed earlier in Section 3.4.2 (See Page 99)

Table 4.1a: Comparison between the Differences in Elevation of Ellipsoidal and Orthometric Heights in Port Harcourt: (Source: Author, October, 2009)

Stations	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Changes in Elevation of Ellipsoidal Height (Dh) [m]	Changes in Elevation of Orthometric Height (DH) [m]	Difference Between Changes in Elevations (Diff) [m]	Mean Square Error (MSE) [m]
PT.4 EMMA	30.6930	11.6910	00.1030	00.1070	-0.0040	3.38724E-05
PT.5 EMMA	29.3740	10.3800	03.4680	03.4590	0.0089	5.65504E-05
GPS 04	38.7710	19.9380	01.2940	01.3020	-0.0080	8.79844E-05
GPS 13	40.5890	21.7280	-00.9280	-00.9240	-0.0040	2.89444E-05
GPS 26	20.1800	01.2500	13.3520	13.3800	-0.0280	0.000863184
GPS 30	20.9840	02.0720	-00.7450	-00.7460	0.0010	1.444E-07
GPS 45	33.4320	14.3110	00.9790	00.9790	0.0000	1.9044E-06
GPS 54	29.3360	10.3600	-00.2580	-00.2580	0.0000	1.9044E-06
GPS 55	29.1730	10.1970	00.1630	00.1630	0.0000	1.9044E-06
GPS 56	28.0330	09.0150	01.1400	01.1820	-0.0420	0.001881824
GPS 57	27.5360	08.5190	00.4970	00.4960	0.0010	1.444E-07
GPS 58	27.4410	08.4250	00.0950	00.0940	0.0010	1.444E-07
GPS 61	20.6720	01.8770	00.3100	00.3120	-0.0020	1.14244E-05
XSV 662	27.6030	08.6480	-06.9310	-06.7710	-0.160	0.026043504
ZVS 3003	32.3080	13.2820	-04.7050	-04.6340	-0.0710	0.005238864

The full data set for this table is as shown in Appendix C3

The difference in both Ellipsoidal and Orthometric Heights in Port Harcourt (Table 4.3a) are plotted in form of chart (Figure 4.2a)

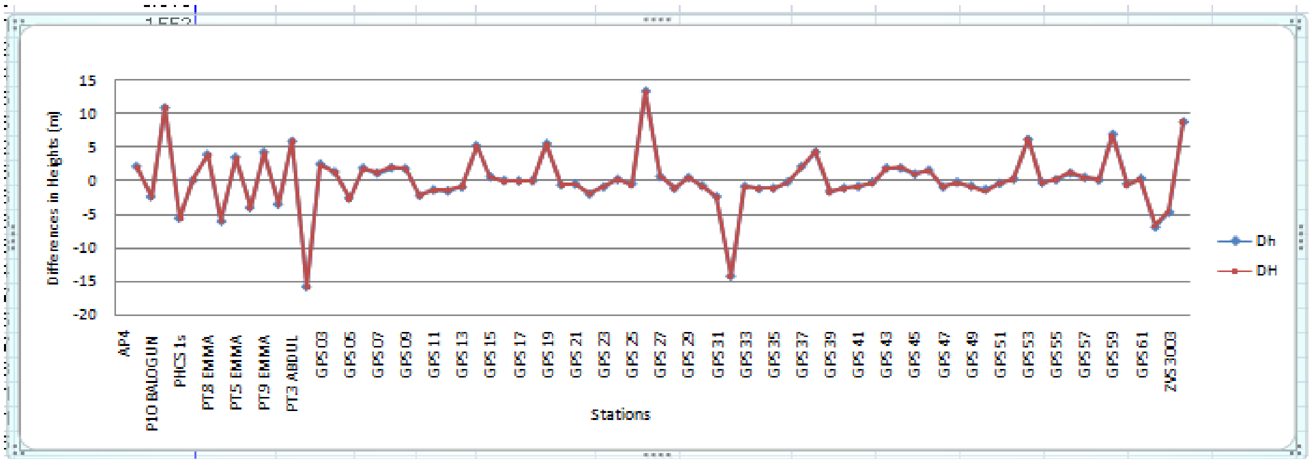


Figure 4.2a: Chart showing the differences in Heights for both Ellipsoidal and Orthometric heights in Port Harcourt.

Table 4.1b: Comparison between the Differences in Elevation of Ellipsoidal and Orthometric Heights in Lagos State (Source: Author, October, 2009)

Stations	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Changes in Elevation of Ellipsoidal Height (Dh) [m]	Changes in Elevation of Orthometric Height (DH) [m]	Difference Between Changes in Elevations (Diff) [m]	Mean Square Error (MSE) [m]
XST 237	25.8360	3.2720				
FGPLA-Y-003	27.0450	4.2620	0.4270	0.9860	-0.5590	0.316009012
CFPA21	30.9400	8.1120	-3.8950	-3.8500	-0.0450	0.002318113
YTT1703A	25.0470	2.1350	50.0000	5.2120	-0.2120	0.046288141
LWBC5-61P	26.0300	2.8440	3.1560	3.4620	-0.3060	0.095571737
YTT19-54	37.7640	14.5740	-11.730	-11.7300	-0.0040	5.10766E-05
CFPA40	28.3150	5.6600	8.1280	7.7600	0.3680	0.133117866
ZTT2-57A	26.8840	4.6100	0.7790	0.8360	-0.0570	0.003617636
MCS1188T-A	25.3970	2.7750	11.9120	11.5500	0.3610	0.128058921
MCS1174S-A	73.1510	49.570	-31.7100	-31.6000	-0.1090	0.012678037
YTT13-30	56.5500	33.5130	-30.4900	-30.5700	0.0830	0.006376535
XST204	27.1270	4.9060	29.4230	28.6100	0.8160	0.660730343

The full data set for this table is as shown in Appendix C4

where;

h is the Ellipsoidal Height

H is the Orthometric Height

Dh is the Changes in Elevation of Ellipsoidal Height

DH is the Changes in Elevation of Orthometric Height

Diff is the Difference between Changes in the Elevations

The difference in both Ellipsoidal and Orthometric Heights in Lagos State (Table 4.3b) are plotted in form of chart (Figure 4.2b)

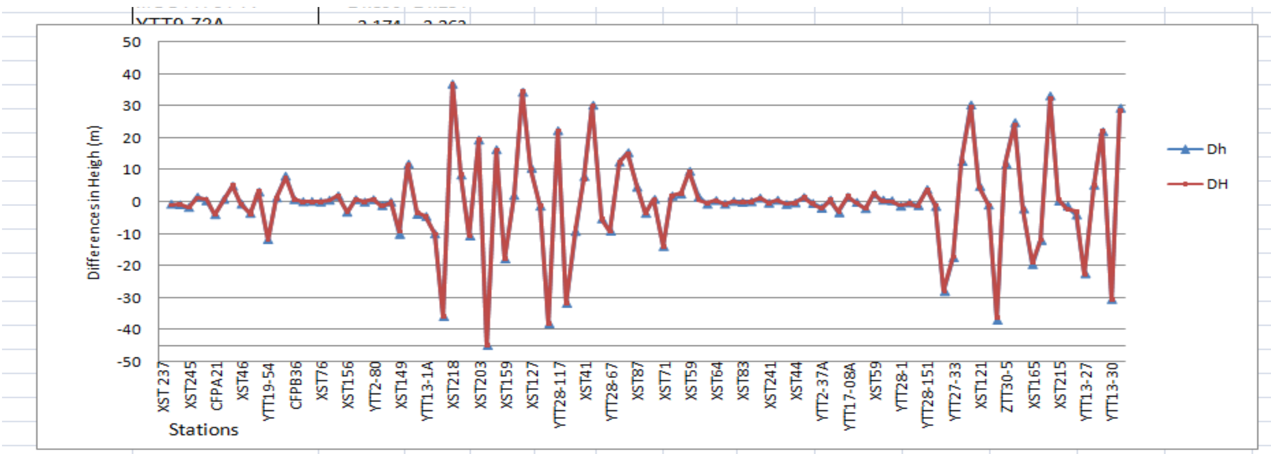


Figure 4.2b: Chart showing the differences in Heights for both Ellipsoidal and Orthometric heights in Lagos State

4.1.4 The Geoidal Coeeficients for the Existing Models

Least Squares adjustment was used to estimate the parameters as discussed in section 3.6. The Least Square solution results in Geoidal Coefficients. However, Least Squares Adjustment was not applied to some of the existing models such as Local Undulation (Equation 1.3), Mosaic of parametric model and Geopotential Earth Model (GEM2008). Microsoft Excel was used to compute Geoidal Undulations for Local Undulation and Mosaic of parametric model while EGM2008 Calculator was used to compute the undulations and hence geoidal coefficients were not required. The geoidal coefficients (Table 4.2a and 4.2b) were computed for Port Harcourt and Lagos State respectively.

Table 4.2a: The Geoidal Coefficients for Port Harcourt (Source: Author, June, 2011)

	North Sea Region Model	**4-Parameters Similarity Datum Shift**	**5-Parameters Similarity Datum Shift**	**7-Parameters Similarity Datum Shift**	**Zanletnyik Hungarian Polynomial**
N_L	1345.20654	1345.20654	-1936.76337	15281.00928	319.0917454
A_1	114.5869999	1335.467728	1897.508453	-14327.0999	-2576.306183
A_2	164.3047874	163.3021734	299.036705	-3163.15067	-3164.1421
A_3	13.99514285	114.4484052	874.5935555	16977.31518	2747.18713
A_4			-4357.56714	-2573.64957	17026.0236
A_5				-637.826782	7286.43756
A_6				-4671.14542	

Table 4.2b: The Geoidal Coefficients for Lagos State (Source: Author, June, 2011)

	North Sea Region Model	**4-Parameters Similarity Datum Shift**	**5-Parameters Similarity Datum Shift**	**7-Parameters Similarity Datum Shift**	**Zanletnyik Hungarian Polynomial**
N_L	-27.62028811	-7.794061756	2001.758099	1517.191162	32.14482264
A_1	462.6794576	0.546303484	-2033.68359	-3259.33982	-687.325703
A_2	294.883289	-33.5917543	-155.658038	479.1723868	466.352393
A_3	-2904.7024	282.9507267	776.2075272	-6315.33471	5953.5467
A_4			-3171.38221	-674.170835	-6177.1055
A_5				1771.392578	1661.00804
A_6				4228.470734	

4.1.5 Results of the Existing Empirical Geoid Models

Based on the empirical models reviewed in section 2.1.2, the observed field data (Tables 3.2a and 3.2b) were used to compute the Global Geoidal Undulation for the two study areas (Tables 4.3a and 4.3b respectively). The result of investigation done on the deterioration of conditions of equations of the Zanletnyik Hungarian Polynomial Model as discussed in Section 3.5.3 (See Page 100) is tabulated in Table 4.3. The best data closest to the observed values and those of other existing Geoidal Undulation is the one computed for the polynomial of second degree and therefore adopted for this model and presented with other existing models as shown in Tables 4.4a and 4.4b for both port Harcourt and Lagos State respectively.

Table 4.3: Geoidal Undulations Computed Using each Degree of the Zanletnyik Hungarian Polynomial Model (Source: Author, October, 2009)

Stations	1st Degree [m]	2nd Degree [m]	3rd Degree [m]	4th Degree [m]	5th Degree [m]	6th Degree [m]	7th Degree [m]	8th Degree [m]
AP4	18.9443	18.9408	21.1712	11.2761	14.2913	18.5150	166.8048	402.7213
PHCS 1s	19.0223	19.0170	21.1653	10.6370	14.0435	18.2426	168.7446	405.1322
PT.4 EMMA	18.9994	18.9960	21.1624	10.8043	14.0536	18.3238	168.3134	404.5598
PT.3 ABDUL	19.0054	18.9990	21.2038	11.0665	15.4369	18.5404	166.4281	400.7091
GPS 02	18.8972	18.9120	21.2469	12.1190	16.2482	18.7604	162.1648	396.4643
GPS 13	18.8668	18.8620	21.1888	12.0289	14.8696	18.6684	163.7662	399.7721
GPS 25	18.8984	18.8990	21.1336	11.3494	13.0813	18.4499	167.6288	405.1482
GPS 39	19.0287	19.0470	21.3098	11.4209	17.7147	18.8343	163.3975	394.5971
GPS 40	19.0281	19.0480	21.3127	11.4363	17.7631	18.8416	163.3033	394.4462
GPS 55	18.9631	18.9700	21.1448	10.8829	13.1728	18.2903	168.8586	406.2539
GPS 60	18.7970	18.8000	21.0548	11.6352	10.9978	18.3925	168.5052	408.6847
XSV 662	18.9510	18.9470	21.1836	11.3079	14.6514	18.5509	166.3992	401.8501
ZVS 3003	19.0200	19.0150	21.2291	11.1090	16.1105	18.6122	165.7315	399.0682

The full data set for this table is as shown in Appendix C5

The Zanletnyik Hungarian Polynomial Model deviated from the observed Geoidal Undulation and after second degree. Therefore, model developed and fit in a particular locality may not necessarily fit in another place.

Table 4.4:The Local, Existing Geoid Models Equations and Model Numbers

Observed Undulation [m]	North Sea Region Model [m]	4-Parameters Similarity Datum Shift [m]	5-Parameters Similarity Datum Shift [m]	7-Parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM 2008 [m]
Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
Eqn. 1.1	Eqn.2.23	Eqn. 2.24	Eqn. 2.25	Eqn. 2.26	Eqn. 2.27	Eqn. 2..29	GEM

Table 4.4a: Summary of the Results from the Local and Existing Geoid Models for Port Harcourt

STATIONS	Model 1 [m]	Model 2 [m]	Model 3 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 8 [m]
PT.4 EMMA	19.0024	19.0014	19.0044	18.9910	19.5819	18.9958	19.0139	19.0080
PT.5 EMMA	18.9939	19.0009	19.0054	18.9929	19.5839	18.9938	18.9944	19.0060
GPS 03	18.8250	18.8270	18.8361	18.8234	19.4134	18.8257	18.8010	18.8250
GPS 04	18.8330	18.8349	18.8440	18.8325	19.4227	18.8329	18.8795	18.8320
GPS 13	18.8610	18.8634	18.8674	18.8542	19.4444	18.8619	18.8778	18.8590
GPS 26	18.9300	18.9210	18.9179	18.9107	19.5015	18.9281	18.9065	18.9320
GPS 30	18.9120	18.8993	18.8939	18.8886	19.4792	18.9126	18.9044	18.9140
GPS 45	19.1210	19.1127	19.1163	19.1135	19.7037	19.1234	19.0627	19.1210
GPS 50	18.9180	18.9183	18.9244	18.9162	19.5073	18.9119	19.0819	18.9150
XSV 662	18.9550	18.9553	18.9614	18.9534	19.5447	18.9473	18.7952	18.9530
ZVS 3003	19.0260	19.0230	19.0296	19.0211	19.6123	19.0150	18.9625	19.0200

The full data set for this table is as shown in Appendix C6

The results of the existing models using data acquired in Port Harcourt (Table 4.4a) are plotted inform of chart (Figure 4.3a)

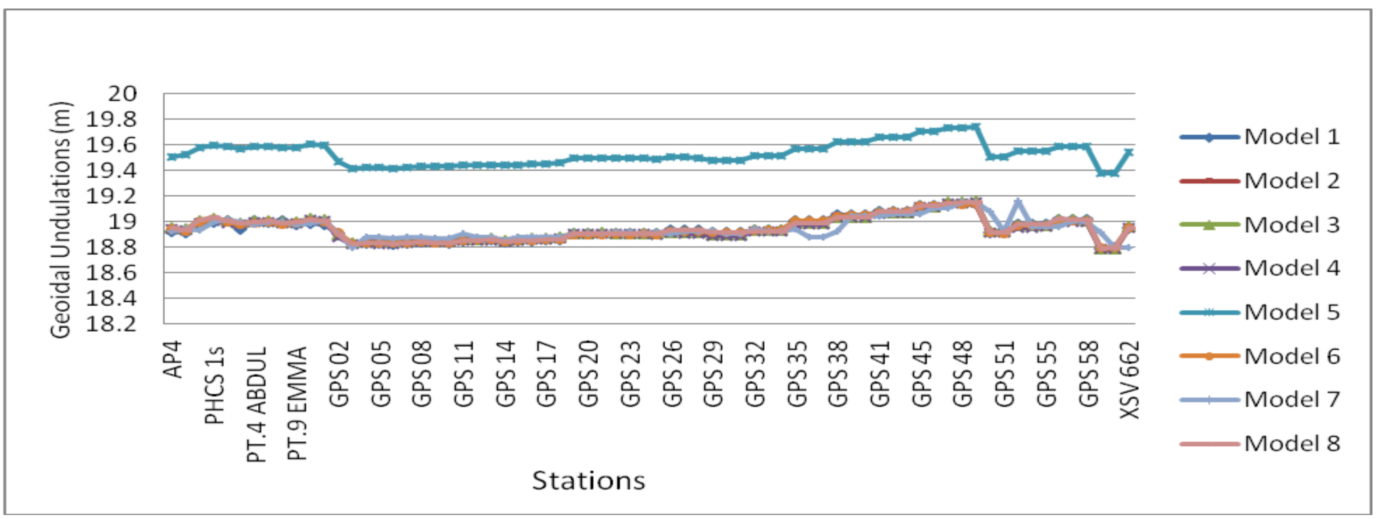


Figure 4.3a: The Relationship between the Local Undulation and Existing Model for Port Harcourt

Table 4.4b: Summary of the Results from the Local and Existing Geoid Models for Lagos State

Stations	Model 1 [m]	Model 2 [m]	Model 3 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 8 [m]
XST44	22.2540	22.3805	22.3754	22.3222	20.1455	22.30554	22.6056	22.0660
YTT78A	22.4740	22.5187	22.5086	22.4782	20.3016	22.4617	22.8674	22.3580
XST245	22.4910	22.3672	22.3564	22.3141	20.1505	22.3105	22.6140	22.1350
XST244	22.2240	22.3137	22.3015	22.2606	20.1065	22.2665	22.7004	22.0970
FGPLA-Y-003	22.7830	22.7186	22.7376	22.7760	20.5955	22.7555	23.1494	22.4460
CFPA21	22.8280	22.7793	22.7891	22.8211	20.6477	22.8077	22.7958	22.4870
YTT1703A	22.9120	22.7746	22.8066	22.9127	20.7507	22.9108	22.9243	22.6340
XST50	22.8800	22.7744	22.7936	22.8554	20.6862	22.8462	22.6300	22.54700
LWBC5-61P	23.1860	23.1247	23.0975	23.1376	21.0207	23.1807	23.0213	22.8570
YTT19-54	23.1900	23.1423	23.1123	23.1448	21.0278	23.1878	22.7630	22.8690
XST75	23.0230	23.0105	22.9896	22.9918	20.8408	23.0008	22.6386	22.7120
CFPA40	22.6550	22.5418	22.5951	22.6634	20.4619	22.6219	22.3994	22.3700
CFPB36	22.6490	22.5506	22.5968	22.6498	20.4491	22.6092	22.7510	22.3450
XST72	22.3960	22.4826	22.5075	22.4929	20.2893	22.4493	22.6995	22.1630
XST76	22.3650	22.4677	22.4893	22.4657	20.2630	22.4230	22.7332	22.1160
XST44	22.2540	22.3716	22.3657	22.3130	20.1385	22.2985	22.6346	22.0630
YTT2-18A	22.2580	22.3572	22.3487	22.2996	20.1316	22.2916	22.7340	22.0800
XST156	22.2170	22.2923	22.2782	22.2439	20.0983	22.2583	22.6852	22.1230
ZTT2-57A	22.2740	22.2915	22.2752	22.2615	20.1276	22.2876	22.7461	22.2800
YTT2-66A	22.2700	22.2726	22.2551	22.2572	20.1349	22.2949	22.7311	22.3420

The full data set for this table is as shown in Appendix C7

The results of the existing models using data acquired in Lagos State (Table 4.4b) are plotted inform of chart (Figure 4.3b)

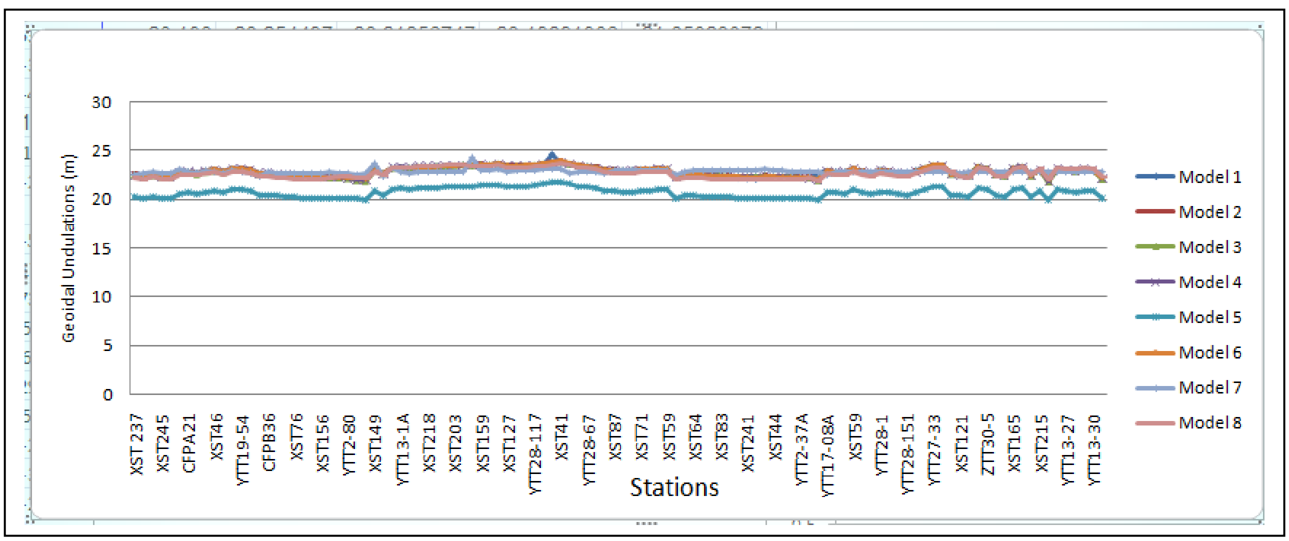


Figure 4.3b: The Relationship between the Undulations of the Existing Models for Lagos State

The results of the Geoidal Undulation computed for Port Harcourt metropolis gave averages of 18.9465 and 18.9482 for spherical ‘Satlevel’ and rectangular ‘Satlevel’ respectively, while Lagos State gave averages of 22.854m and 22.857m for Spherical ‘Satlevel’ and Rectangular ‘Satlevel’ respectively. The mean of residuals for Spherical ‘Satlevel’are 0.0033mm and 6.151mm, while Rectangular ‘Satlevel’ are 1.728mm and. 0.00032mm for Port Harcourt and Lagos State respectively.

The charts shows that there is deviation in some of the model especially 7 – Parameters Similarity Datum Shift that deviate for more than 2m from others (Section 2.2.2.5). Improved result can be obtained with addtion of another term as observed in (See Section 3.5.1). The difference in Geoidal Undulation for every single point computed for each degree of Zanletnyik Hungarian Polynomial Model as shown in Table 4.3 is an indication of the deviation of the existing models and hence the need for a new model like ‘Satlevel’ collocation.

The field data in Table 3.2a were used to compute the difference between the local undulation and existing models for Port Harcourt (Table 4.4a)

Table 4.5a: Residuals for the Existing Geoid Models for Port Harcourt

STATIONS	Model 2 [m]	Model 3 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 8 [m]
AP1	-0.0217	-0.0275	-0.0200	-0.6113	-0.0159	-0.0325	-0.0221
PHCS 1s	-0.0234	-0.0252	-0.0080	-0.5985	-0.0191	0.0044	-0.0350
GPS 02	-0.002o	0.0066	0.0228	-0.5671	-0.0077	0.0000	0.0040
GPS 13	-0.0024	-0.0064	0.0068	-0.5834	-0.0009	-0.0168	0.0020
GPS 19	0.0004	-0.0066	8.6E-05	-0.5910	0.0050	0.0000	0.0000
GPS 29	0.0133	0.0189	0.0243	-0.5663	-0.0003	-0.0129	-0.0020
GPS 33	0.0014	0.0019	0.0126	-0.5782	0.0036	0.0054	0.0100
GPS 41	0.0065	0.0066	0.0116	-0.5791	0.0022	0.0342	0.0010
GPS 42	0.0066	0.0071	0.0120	-0.5788	0.0016	0.0188	0.0010
GPS 43	0.0066	0.0075	0.0122	-0.5785	0.0008	0.0191	0.0010
GPS 53	0.0074	0.0088	0.0202	-0.5706	0.0071	-0.0020	-0.1801
GPS 54	0.0079	0.0095	0.0211	-0.5698	0.0072	-0.0020	0.0178
GPS 55	0.0068	0.0083	0.0199	-0.5709	0.0064	-0.0030	0.0184
GPS 59	0.0121	0.0036	0.0020	-0.5880	-0.0078	-0.1284	0.0050
GPS 60	0.0121	0.0037	0.0022	-0.5878	-0.0075	-0.0004	-0.0020
XSV 662	-0.0003	-0.0064	0.0016	-0.5897	0.0077	0.1598	0.0020
ZVS 3003	0.0030	-0.0036	0.0049	-0.5863	0.011	0.0635	0.0060
Mean	1E-07	-0.0037	0.00529	-0.585	1E-05	-0.0002	-0.0019

The full data set for this table is as shown in Appendix C8

The results of the residuals in Port Harcourt (Table 4.5a) are plotted in form of chart (Figure 4.4a)

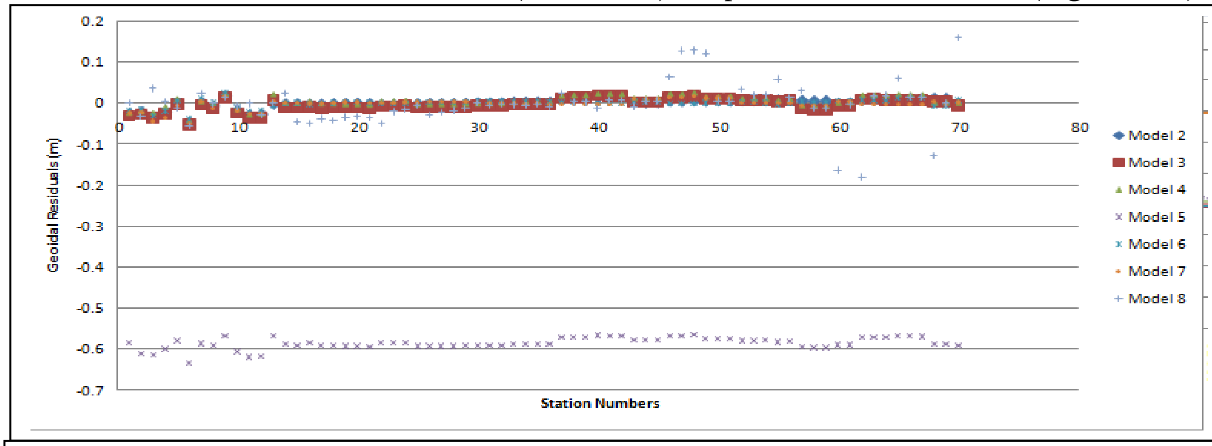


Figure 4.4a: The Relationship between the Residuals of the Existing Models for Port Harcourt

The field data in Table 3.2b were used to compute the difference between the local undulation and existing models (Residuals) for Lagos State (Table 4.5b)

Table 4.5b: Residuals for the Existing Geoid Models for Lagos State

Observed Undulation [m]	North Sea Region Model [m]	4-Parameters Similarity Datum Shift [m]	5-Parameters Similarity Datum Shift [m]	7-Parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM 2008 [m]
YTT2-66A	-0.0026	0.0149	0.0128	2.1351	-0.0250	-0.4800	-0.0720
YTT2-80	0.0267	0.0474	0.0215	2.1192	-0.0410	-0.5120	-0.1320
XST42	0.0325	-0.0440	-0.1400	2.1108	-0.0490	0.3857	-0.0550
XST209	0.0321	-0.0150	-0.0390	2.1841	0.0241	0.6105	-0.0670
XST201	0.0327	-0.0060	-0.0180	2.1957	0.0357	0.6631	-0.0560
XST203	-0.0110	-0.0330	-0.0260	2.1672	0.0072	0.6614	-0.0490
XST177	0.0122	-0.0040	0.0112	2.1907	0.0307	0.7293	-0.2110
YTT28-67	-0.0051	0.0424	0.0641	2.1705	0.0105	0.1168	0.1961
YTT28-65	0.0099	0.0509	0.0700	2.1952	0.0352	-0.2790	0.2401
XST87	-0.0251	-0.0020	0.0153	2.1767	0.0167	-0.4490	0.2480
YTT28-30	-0.0121	0.0078	0.0259	2.1917	0.0317	-0.2320	0.2686
YTT28-1	-0.0009	0.0184	0.0273	2.1878	0.0278	-0.2050	0.2871
CFPA18	0.0195	0.0198	-0.0030	2.1635	0.0035	0.0763	0.3245
XST69	-0.0343	-0.0430	-0.0420	2.1454	-0.0150	-0.1470	0.3090

The full data set for this table is as shown in Appendix C9

The results of the residuals (Table 4.5b) are plotted inform of chart (Figure 4.4b)

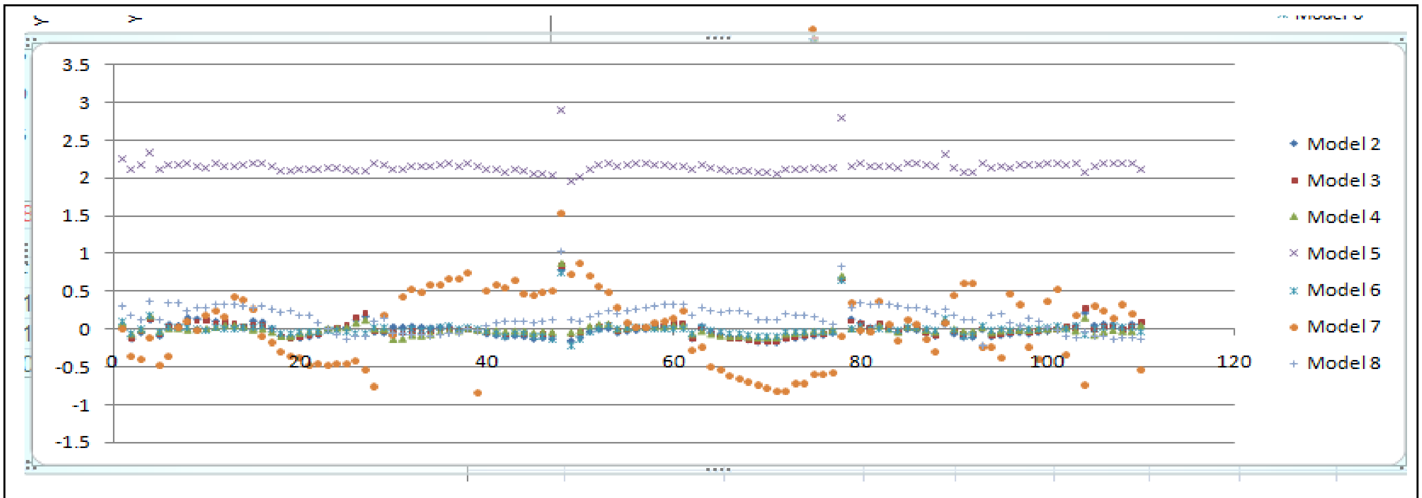


Figure 4.4b: The Relationship between the Residuals of the Existing Models for Lagos State

Residuals: Tables 4.5a and 4.5b show the residuals for the existing models in both Port Harcourt and Lagos State respectively. The data were equally presented in form of charts (Figures 4.4a 4.4b). The residuals for the new 'Satlevel' collocation models are tabulated in Tables 4.9a and 4.9b, and plotted in form of charts (Figures 4.6a and 4.6b). The residuals for the existing and new 'Satlevel' Collocation models are tabulated in Tables 4.11a and 4.11b and plotted in charts (Figures 4.8a and 4.8b) for Port Harcourt and Lagos State respectively. The deviation in the 7- Parameter similarity datum shift as discussed in Sections 2.2.2.5 and 4.2.2.3 is still observed.

Roman (2009) observed that the slight change in GEM2008 is mainly due to shift in reference model GEM96 => GEM08 (GRACE). Significant changes included surface gravity data that are already in the mountains. Roman (2009) concluded that GEOID09 for United States better reflects the true geophysics and current ellipsoidal and Orthometric Heights. In another study here in Nigeria, GEM96 differed by about 2m from the GEM2008 geoid. It should be noted that GEOID09 for the United States of America with high accuracy was produced from the GEM2008 Global geoid. This is the latest model released to the public.

4.1.6 Results of 'Satlevel' Collocation Geoid Models

Spherical 'Satlevel' model was computed using Equation (3.77), while Equation (3.79) was used to compute the Geoidal Undulations for the Rectangular 'Satlevel'. Meanwhile, the coordinates of all stations were converted from geodetic coordinates to rectangular coordinates using Equation (3.80a), (3.80b) and (3.80c). The results are tabulated in Table 4.6a for Port Harcourt

Table 4.6a: Curvilinear and Space Rectangular coordinates of the points used for Port Harcourt

Station Name	Latitude [°]	Longitude [°]	X [m]	Y [m]	Z [m]	Distance [m]
RPCS 209p	4°46'17.86345"	7°00'47.8189"	6308650.6760	776089.5453	527024.4120	11061.080
PHCS 1s	4°46'20.60153"	7°00'48.69008"	6308641.3540	776115.4473	527108.3030	2333.4640
PT.3 ABDUL	4°50'26.70761"	7°01'52.74514"	6307767.3550	777996.5102	534641.1130	992.1820
UNIPORT GATE	4°53'37.49584"	6°54'52.00249"	6308850.5060	765068.6973	540480.7560	14226.840
GPS 09	4°57'17.82054"	6°56'49.49213"	6307841.8780	768592.4569	547224.0230	284.1821
GPS 10	4°57'13.61218"	6°56'39.42241"	6307892.6430	768286.1249	547095.4250	336.0861
GPS 29	4°50'11.32868"	6°55'41.77726"	6309189.4970	766654.7432	534169.8470	1817.1900
GPS 30	4°50'14.59804"	6°55'42.51984"	6309179.0680	766676.5252	534269.9780	103.0024
GPS 49	4°46'06.13308"	7°08'34.02402"	6306914.0480	790350.7421	526665.6610	213.3693
GPS 50	4°54'43.63017"	6°59'07.06877"	6307732.6120	772849.1508	542505.26600	23619.250
GPS 61	4°54'50.33509"	6°52'51.17259"	6309098.7740	761348.8490	542709.2300	237.0870
XSV 662	4°52'24.62491"	6°59'54.28734"	6307909.5620	774336.5702	538250.2940	13783.220
ZVS 3003	4°50'52.69568"	7°02'52.12172"	6307481.7270	779804.6764	535437.0170	6164.2320
RHS 8A	4°45'18.49317"	7°00'59.62433"	6308750.2650	776468.3413	525206.4860	10835.320

The full data set for this table is as shown in Appendix C10

The same procedures were done for Lagos Sate as shown in Table 4.6b

Table 4.6b: Curvilinear and Space Rectangular coordinates of the point used in Lagos State

Station Name	Latitude [°]	Longitude [°]	X [m]	Y [m]	Z [m]	Distance [m]
XST 237	6.45480214	3.470396222	6326376.79	383656.9222	712259.3733	
YTT78A	6.47000887	3.646457902	6324980.599	403083.1823	713930.5512	19857.8072
FGPLA-Y-003	6.42704123	2.890722633	6330279.633	319650.5076	709208.8429	81898.4157
CFPA21	6.44089609	2.919119213	6329952.831	322779.2959	710731.8211	3495.07918
LWBC5-61P	6.50459261	2.926533297	6329113.107	323557.5998	717730.4949	13700.1800
CFPB36	6.39047864	2.824224997	6331097.606	312325.6154	705190.7575	4804.90637
ZTT2-57A	6.43808236	3.778118170	6324433.208	417642.4318	710422.1614	11087.1112
MCS1174S-A	6.66502729	3.323236155	6324736.803	367255.5981	735361.2880	2956.09324
YTT28-96	6.68580244	3.288081883	6324703.436	363360.1436	737644.3198	4515.29773
YTT16-76A	6.50349199	3.719303861	6324047.590	411097.4419	717609.9399	42103.7497
XST149	6.56550677	3.588484489	6324198.692	396608.7811	724424.4592	16011.9267
MCS1188T-A	6.49345969	3.582388693	6325133.279	395991.8080	716507.1560	7996.11148
YTT2-11A	6.42250489	3.513237463	6326488.494	388411.7561	708710.2553	10958.3510
XST225	6.42348209	3.531541184	6326352.054	390432.0548	708817.6473	5423.2625

The full data set for this table is as shown in Appendix C11

4.1.7.1 'Satlevel' Collocation Geoidal Coefficients for Port Harcourt

Least Squares Adjustment was applied to Equations (3.77) and (3.79) using the field data in Table 3.2a which were used to derive the Geoidal coefficients for the New 'Satlevel Collocation models for Port Harcourt (Table 4.7a).

Table 4.7a: Geoidal Coefficients for Port Harcourt

Geoidal Coefficients	Spherical 'Satlevel'	Rectangular 'Satlevel'
N_L	12559.38861	-5703.882111
A_1	-6305.379486	5654.355621
A_2	402.0375862	761.384052
A_3	236.0263758	452.612663

4.1.7.2 'Satlevel' Collocation Geoidal Coefficients for Lagos State

Least Squares Adjustment was aso applied to Equations (3.77 and 3.79) using the field data in Table 3.2b which were also used to derive the geoidal coefficients for the New 'Satlevel Collocation models for Lagos State (Table 4.7b).

Table 4.7b: Geoidal Coefficients for Lagos State

Geoidal Coefficients	Spherical 'Satlevel'	Rectangular 'Satlevel'
N_L	-4176.787667	1717.275164
A_1	2092.822366	-0.00026828
A_2	-77.5007162	-2.148E-05
A_3	27.30095914	1.5023E-05

Results of Geoidal Undulation from 'Satlevel' Collocation models were computed using the following equations as given in Table 4.8

Table 4.8: New 'Satlevel' Collocation Geoid Models, Equations and Model Number

Actual Name of the Geoid Models	Spherical 'Satlevel' Model [m]	Rectangular 'Satlevel' Model [m]
Model Numbers	SATLEVEL 1	SATLEVEL 2
Equation number	3.77	3.79

The field data in Table 3.2a were used in Equations 3.77 and 3.79 along with the Geoidal coefficients (Table 4.7a) to compute the local undulations for the New ‘Satlevel Collocation models for Port Harcourt (Table 4.8a).

Table 4.8a: Local Geoid and New ‘Satlevel’ Collocation Geoid Models for Port Harcourt

Stations	Local [m]	Spherical ‘Satlevel’ [m]	Rectangular ‘Satlevel’ [m]
Model Number	Model 1	SATLEVEL 1	SATLEVEL 2
Equation Number	Equation 1.2	Equation 3.22	3.24
AP4	18.9229	18.9476	18.9497
PHCS 1s	18.9980	19.0164	19.0232
PT.4 EMMA	19.0024	18.9977	19.0018
PT.3 ABDUL	18.9803	19.0084	19.0058
GPS 02	18.9040	18.8910	18.8953
GPS 13	18.8610	18.8605	18.8657
GPS 29	18.9130	18.8875	18.8885
GPS 30	18.9120	18.8873	18.8887
GPS 49	19.1420	19.1488	19.1601
GPS 50	18.9180	18.9177	18.9184
GPS 51	18.9170	18.9162	18.9171
GPS 53	18.9760	18.9607	18.9644
GPS 54	18.9760	18.9599	18.9639
GPS 55	18.9760	18.9611	18.9650
GPS 56	19.0180	19.0047	19.0092
GPS 57	19.0170	19.0037	19.0079
GPS 60	18.7930	18.7817	18.7865
XSV 662	18.9550	18.9548	18.9521
ZVS 3003	19.0260	19.0229	19.0224
RHS 8A	19.0296	19.0997	19.0260

The full data set for this table is as shown in Appendix C12

The results of the Geoidal Undulations computed from Local Geoidal Undulation and the New ‘Satlevel’ Collocation Models for Port Harcourt (Table 4.8a) are plotted in form of chart (Figure 4.5a)

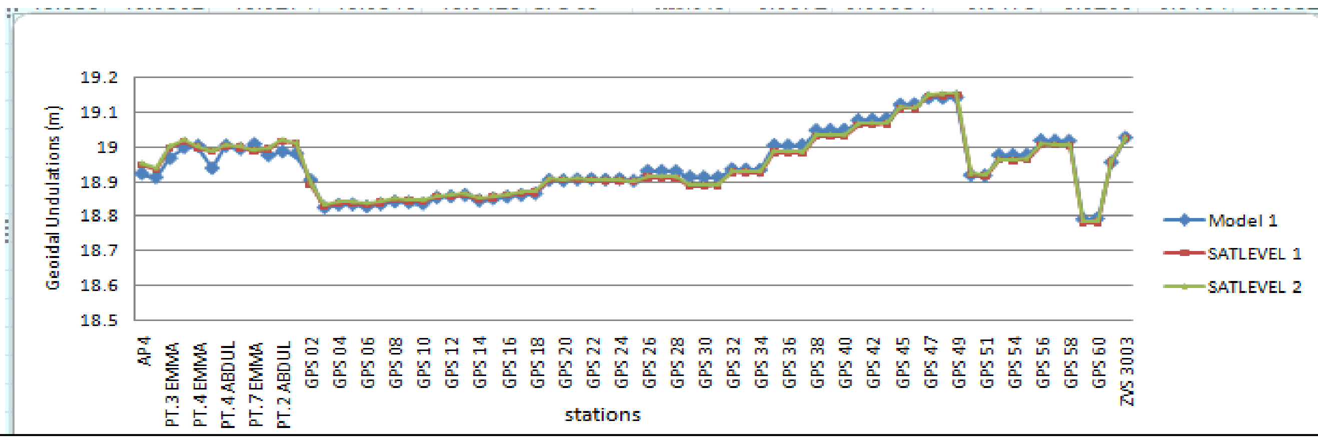


Figure 4.5a: Chart showing the Relationship between the Local Geoidal Undulations and the Geoidal Undulations of the New "Satlevel" Collocation Models for Port Harcourt

The results of the Spherical 'Satlevel' and Rectangular 'Satlevel' were computed and presented in Tables 4.8a and 4.8b and plotted into charts (Figures 4.5a and 4.5b) for Port Harcourt and Lagos State respectively. Tables 4.10a and 4.10b and charts (Figures 4.7a and 4.7b) for Port Harcourt and Lagos State respectively summarised the results of the local, existing and new 'Satlevel' collocation models. The matching of the two quantities as observed in Figure 4.7a and 4.7b shows that both Spherical and Rectangular 'Satlevel' models agrees with each other. 'Satlevel' Collocation model can produce predicted geoid to 95% significant level as shown in Tables 4.15a for Port Harcourt. Also, mean of residuals for Spherical 'Satlevel' were computed to be 0.006151 and 0.00003252 for Port Harcourt and Lagos State respectively. Rectangular 'Satlevel' were computed to be 0.00172811 and 0.0000031968 Port Harcourt and Lagos respectively. The root mean square errors were also computed. Therefore, It was observed that there is no significant difference between the observed Geoidal Undulations and the undulations computed from 'Satlevel' collocation models as shown by the residuals tabulated in Tables 4.11a and 4.11b.

The field data in Table 3.2b were used in Equations (3.77 and 3.79) along with the Geoidal coefficients (Table 4.7b) to compute the local undulations for the New 'Satlevel Collocation models for Lagos State (Table 4.8b).

Table 4.8b: Local Geoidal Undulation and each of the New 'Satlevel' Collocation Geoid Models for Lagos State

Stations	Local [m]	Spherical 'Satlevel' [m]	Rectangular 'Satlevel' [m]
STATION	Local	SATLEVEL 1	SATLEVEL 2
Equation Number	Equation 1.2	Equation 3.22	Equation3.24
XST 237	22.5640	22.4859	22.4944
YTT78A	22.4740	22.4719	22.4768
FGPLA-Y-003	22.7830	22.7769	22.7766
CFPA21	22.8280	22.8196	22.8199
YTT1703A	22.9120	22.9149	22.9061
LWBC5-61P	23.1860	23.1291	23.1336
CFPB36	22.6490	22.6590	22.6541
ZTT2-57A	22.2740	22.2599	22.2581
MCS1188T-A	22.6220	22.6185	22.6268
CFPA31	22.5800	22.6163	22.6145
XST99A	22.2150	22.3396	22.3468
XST241	22.1750	22.2997	22.3065
XST114	22.2850	22.3562	22.3635
XST44	22.2540	22.3150	22.3213
YTT2-14A	22.2480	22.2971	22.3031
FGPLA-Y-008	22.7910	22.7982	22.7997

The full data set for this table is as shown in Appendix C13

The results of the Geoidal Undulations computed from Local Geoidal Undulation and the New 'Satlevel' Collocation Models for Lagos State (Table 4.7b) are plotted in form of chart (Figure 4.5b)

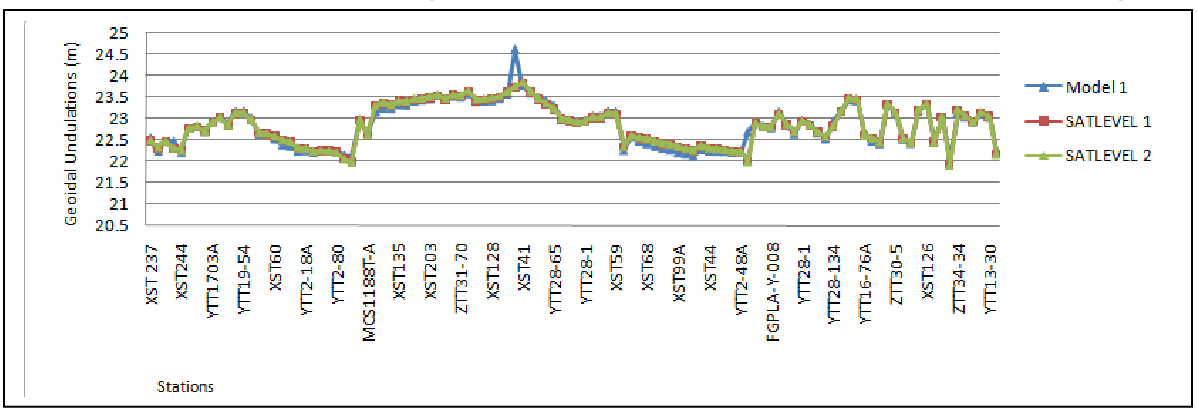


Figure 4.5b: Chart showing the Relationship between the Geoidal Undulations of the New "Satlevel" Collocation Models for Lagos State

The field data in Table 3.2a were used to compute the difference between the local undulation and the 'Satlevel' Collocation models for Port Harcourt (Table 4.9a).

Table 4.9a: Computed Residuals from the New 'Satlevel' Collocation Geoid Model for Port Harcourt

Stations	Spherical 'Satlevel' [m]	Rectangular 'Satlevel' [m]
AP1	-0.0208	-0.0226
PT.3 EMMA	-0.0304	-0.0326
PHCS 1s	-0.0184	-0.0252
PT.3 ABDUL	-0.0280	-0.0255
GPS 02	0.0130	0.0087
GPS 19	0.0002	0.0005
GPS 20	0.0001	0.0002
GPS 39	0.0173	0.0185
GPS 49	-0.0068	-0.0181
GPS 59	0.0112	0.0065
GPS 60	0.0113	0.0065
XSV 662	0.0002	0.0030
ZVS 3003	0.0031	0.0036

The full data set for this table is as shown in Appendix C14

Equations (3.77 and 3.79) are adopted for 'Satlevel' Collocation and are referred to as SATLEVEL 1 and SATLEVEL 2 respectively.

The Geoidal Residuals of the New 'Satlevel' Collocation Models for Port Harcourt (Table 4.8a) are plotted inform of chart (Figure 4.6a)

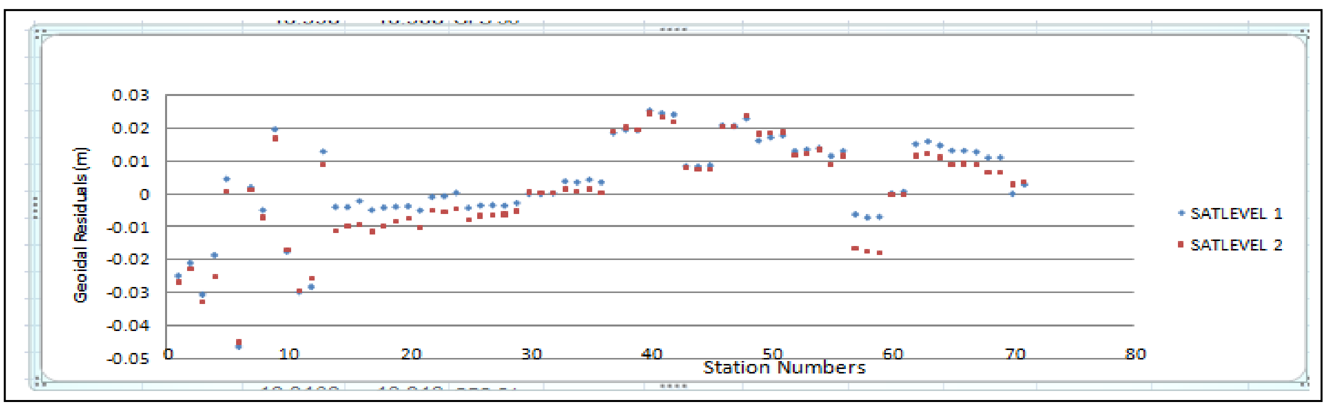


Figure 4.6a: The Relationship between the Residuals of the "Satlevel' Collocation Models for Port Harcourt

The field data in Table 3.2a were used to compute the difference between the local undulation and the 'Satlevel' Collocation models for Lagos State (Table 4.9b)

Table 4.9b: Computed Residuals from the New 'Satlevel' Collocation Geoid Models for Lagos State

Stations	Spherical 'Satlevel' [m]	Rectangular 'Satlevel' [m]
XST 237	0.0781	0.0696
YTT78A	0.0021	-0.0028
FGPLA-Y-003	0.0061	0.0065
CFPA21	0.0084	0.0081
XST 55	-0.0220	-0.0102
YTT1703A	-0.0030	0.0059
XST46	0.0075	0.0156
XST50	0.0244	0.0279
YTT1703A	-0.0030	0.0059
LWBC5-61P	0.0569	0.0524
CFPA40	-0.0190	-0.0116
CFPB36	-0.0100	-0.0051
MCS1188T-A	0.0035	-0.0048
YTT2-48A	-0.0090	-0.0092
YTT17-08A	0.0006	0.0097
CFPA18	0.0010	-0.0024
XST69	-0.0420	-0.0464
ZTT45-200	0.0247	0.0159
MCS1144S-A	-0.0140	-0.0221
XST165	0.0054	-0.0012
XST126	0.0107	0.0076
YTT9-29A	0.0248	0.0286
XST215	-0.0170	-0.0172
XST165	0.0054	-0.0012
ZTT35-26	0.1427	0.1743
ZTT34-34	-0.0810	-0.0753
YTT13-27	-0.0300	-0.0198
XT161	-0.0070	0.0012
XST202	-0.0270	-0.0303
YTT13-30	-0.0260	-0.0151

The full data set for this table is as shown in Appendix C15

The results of the residuals (Table 4.8b) for spherical 'Satlevel' SATLEVEL 1 and rectangular satlevel" SATLEVEL 2 are plotted inform of chart (Figure 4.6b):

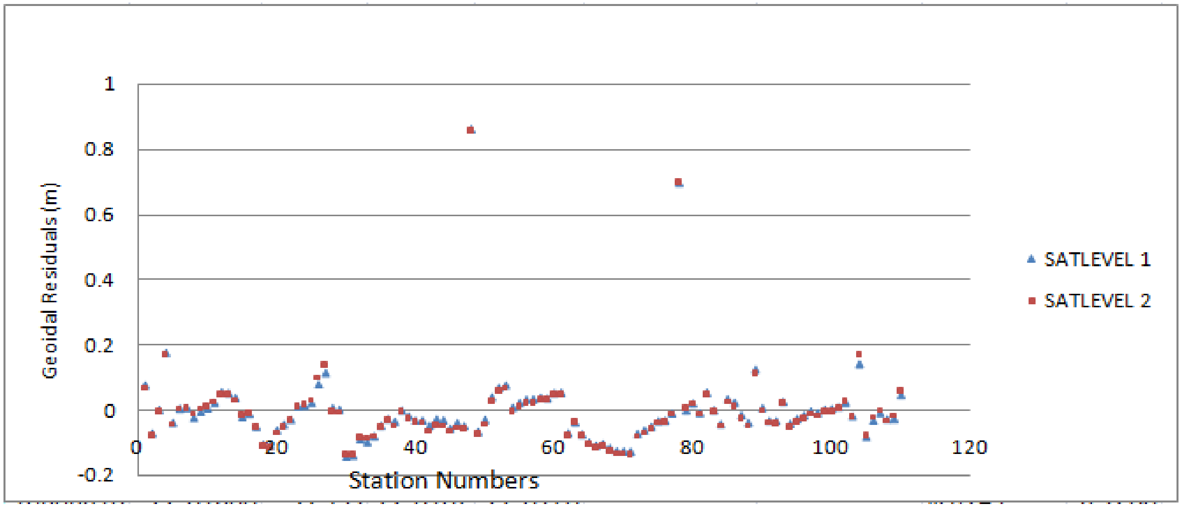


Figure 4.6b: The Relationship between the Residuals of the "Satlevel' Collocation Models for Lagos State

The results shown in Table 4.8 are too closed on each of the points, which is an indication that the two 'Satlevel' Collocation models agree with each other in terms accuracy and precision.

4.1.8 Result of Local, Existing Geoid and New 'Satlevel' Collocation Models:

The field data in Table 3.2a were used to compute the local undulations using Equation 1.3, the Existing Geoidal Undulations using Equations 2.23, 2.24. 2.25, 2.26, 2.27, 2.29 and Altrans EGM 2008 Calculator, along with the New 'Satlevel Collocation models using Equations 3.77 and 3.79 for Port Harcourt and Lagos State (Table 4.10a and 4.10b respectively)

Table 4.10: Local, Existing Geoid, New 'Satlevel' Collocation Models, Equations and model numbers

Actual Name of the Geoid Models	Local Undulation [m]	North Sea Region Model [m]	4-Parameters Similarity Datum Shift [m]	5-Parameters Similarity Datum Shift [m]	7-Parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM2008 [m]	'Satlevel' Spherical Model [m]	'Satlevel' Rectangular [m]
Model Number	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	SATLEVEL 1	SATLEVEL 2
Equation Number	Eqn. 1.1	Eqn.2.23	Eqn. 2.24	Eqn. 2.25	Eqn. 2.26	Eqn. 2.27	Eqn. 2..29	GEM	Eqn. 3.22	Eqn. 3.24

Table 4.10a: Summary of the Results from the Local, Existing Geoid and New ‘Satlevel’ Collocation Models for Port Harcourt.

Stations	Model 1 [m]	Model 2 [m]	Model 4 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 8 [m]	SATLEVEL 1 [m]	SATLEVEL 2 [m]
AP4	18.9229	18.9482	18.9542	18.9463	19.5078	18.9408	18.9229	18.9470	18.9476	18.9522
PHCS 1s	18.9980	19.0214	19.0232	19.0060	19.5965	19.0171	18.9935	19.0330	19.0164	19.0214
PHCS 1s	18.9980	19.0214	19.0232	19.0060	19.5965	19.0171	18.9935	19.0330	19.0164	19.0214
PT.4 EMMA	19.0024	19.0014	19.0044	18.9910	19.5819	18.9958	19.0139	19.0080	18.9977	19.0025
PT.4 ABDUL	19.0028	19.0004	19.0073	18.9976	19.5889	18.9910	18.9788	19.0000	19.0006	19.0053
PT.3 ABDUL	18.9803	19.0080	19.0151	19.0057	19.5969	18.9986	19.0093	19.0060	19.0084	19.0131
GPS 02	18.9040	18.9060	18.8974	18.8812	19.4711	18.9117	18.9040	18.9000	18.8910	18.8955
GPS 03	18.8250	18.8270	18.8361	18.8234	19.4134	18.8257	18.8010	18.8250	18.8288	18.8342
GPS 13	18.8610	18.8634	18.8674	18.8542	19.4444	18.8619	18.8778	18.8590	18.8605	18.8655
GPS 14	18.8450	18.8464	18.8562	18.8469	19.4374	18.8435	18.8517	18.8430	18.8491	18.8542
GPS 19	18.9040	18.9036	18.9106	18.9039	19.4950	18.8990	18.9040	18.9040	18.9038	18.9086
GPS 29	18.9130	18.8997	18.8941	18.8887	19.4793	18.9133	18.9259	18.9150	18.8875	18.8921
GPS 30	18.9120	18.8993	18.8939	18.8887	19.4792	18.9126	18.9044	18.9140	18.8873	18.8919
GPS 48	19.1400	19.1345	19.1551	19.1463	19.7358	19.1374	19.1527	19.1470	19.1470	19.1532
GPS 49	19.1420	19.1360	19.1570	19.1482	19.7376	19.1393	19.1539	19.1490	19.1488	19.1550
GPS 54	18.9760	18.9681	18.9665	18.9549	19.5458	18.9688	18.9582	18.9780	18.9599	18.9645
GPS 55	18.9760	18.9692	18.9677	18.9561	19.5469	18.9696	18.9576	18.9790	18.9611	18.9657
GPS 59	18.7910	18.7789	18.7874	18.7890	19.3790	18.7988	18.9195	18.7860	18.7798	18.7854
GPS 60	18.7930	18.7809	18.7893	18.7908	19.3808	18.8005	18.7934	18.7950	18.7817	18.7873
XSV 662	18.9550	18.9553	18.9614	18.9534	19.5447	18.9473	18.7952	18.9530	18.9548	18.9594
ZVS 3003	19.0260	19.0230	19.0296	19.0211	19.6123	19.0150	18.9625	19.0200	19.0229	19.0276

The full data set for this table is as shown in Appendix C16

The results of the ‘Satlevel’ Collocation Geoidal Undulation (Table 4.9a) are plotted inform of chart (Figure 4.7a)

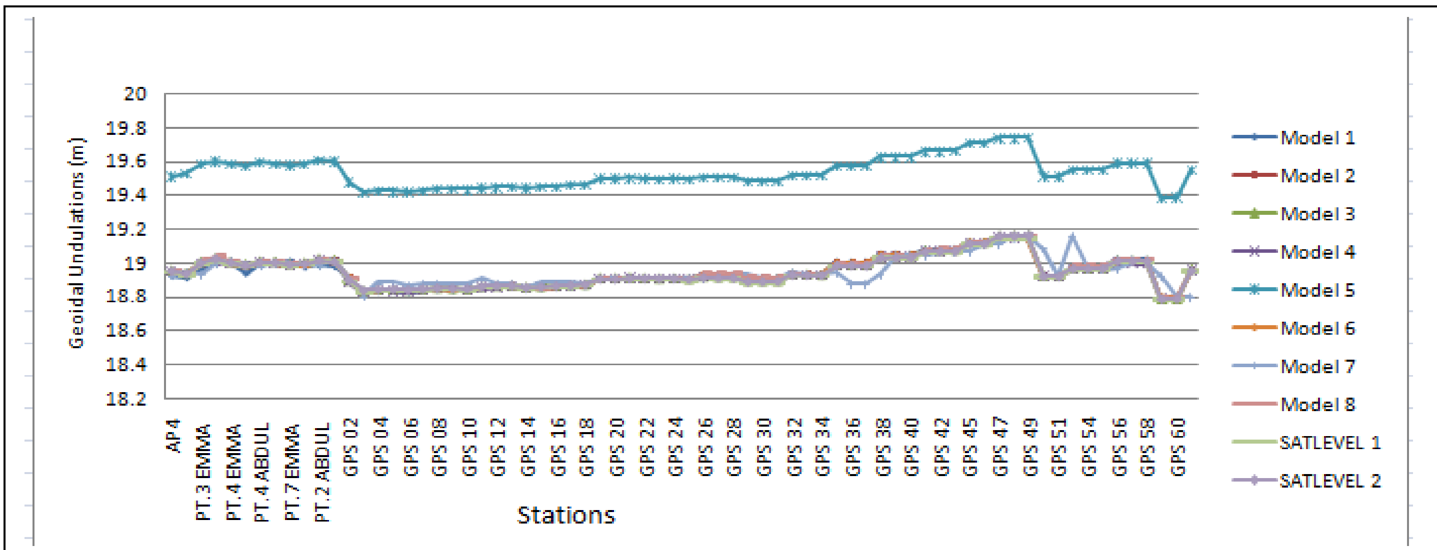


Figure 4.7a: Chart Showing the Relationship between the Local Geoidal Undulations of the Existing and the Geoidal Undulations of the New “Satlevel” Collocation Models for Port Harcourt

The field data in Table 3.2b were used to compute the local undulations for the existing and the new 'Satlevel' Collocation models for Lagos State (Table 4.10b)

Table 4.10b: Summary of the Results from the Local, Existing Geoid and New 'Satlevel' Collocation Models for Lagos State

Stations	Model 1 [m]	Model 2 [m]	Model34 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 8 [m]	SATLEVEL 1 [m]	SATLEVEL 2 [m]
XST 237	22.5640	22.5444	22.5389	22.4898	20.3034	22.4634	22.2640	22.4859	22.4944
YTT78A	22.4740	22.5187	22.5086	22.4782	20.3016	22.4617	22.3580	22.4719	22.4768
FGPLA-Y-003	22.7830	22.7186	22.7376	22.776	20.5955	22.7555	22.4460	22.7769	22.7766
CFPA21	22.8280	22.7793	22.7891	22.8211	20.6477	22.8077	22.4870	22.8196	22.8199
LWBC5-61P	23.1860	23.1247	23.0975	23.1376	21.0207	23.1807	22.8570	23.1291	23.1336
CFPA40	22.6550	22.5418	22.5951	22.6634	20.4619	22.6219	22.3700	22.6740	22.6666
CFPB36	22.6490	22.5506	22.5968	22.6498	20.4491	22.6092	22.3450	22.6590	22.6541
ZTT35-14	22.1190	21.9463	21.9019	21.9989	20.0245	22.1846	22.2090	22.0036	21.9775
CFPA31	22.5800	22.5386	22.5775	22.6081	20.4059	22.566	22.2900	22.6163	22.6145
XST55	22.7000	22.0505	22.0297	21.9881	19.8913	22.0513	21.8810	22.0009	21.9977
YTT17-08A	22.9050	22.7689	22.8004	22.9022	20.7385	22.8986	22.6190	22.9044	22.8953
FGPLA-Y-008	22.7910	22.7685	22.7768	22.7999	20.6237	22.7837	22.4610	22.7982	22.7997
YTT28-200	22.4260	22.5100	22.5028	22.4547	20.2697	22.4297	22.2210	22.4520	22.4603
MCS1178T-A	22.5270	22.5887	22.5788	22.5409	20.3556	22.5157	22.3760	22.5341	22.5418
YTT9-73A	22.4380	22.4790	22.4683	22.4404	20.2693	22.4294	22.3370	22.4348	22.4384

The full data set for this table is as shown in Appendix C17

The results of the Geoidal Undulation computed from the Local, Existing and New 'Satlevel' Collocation Models (Table 4.11b) are plotted inform of chart (Figure 4.7b)

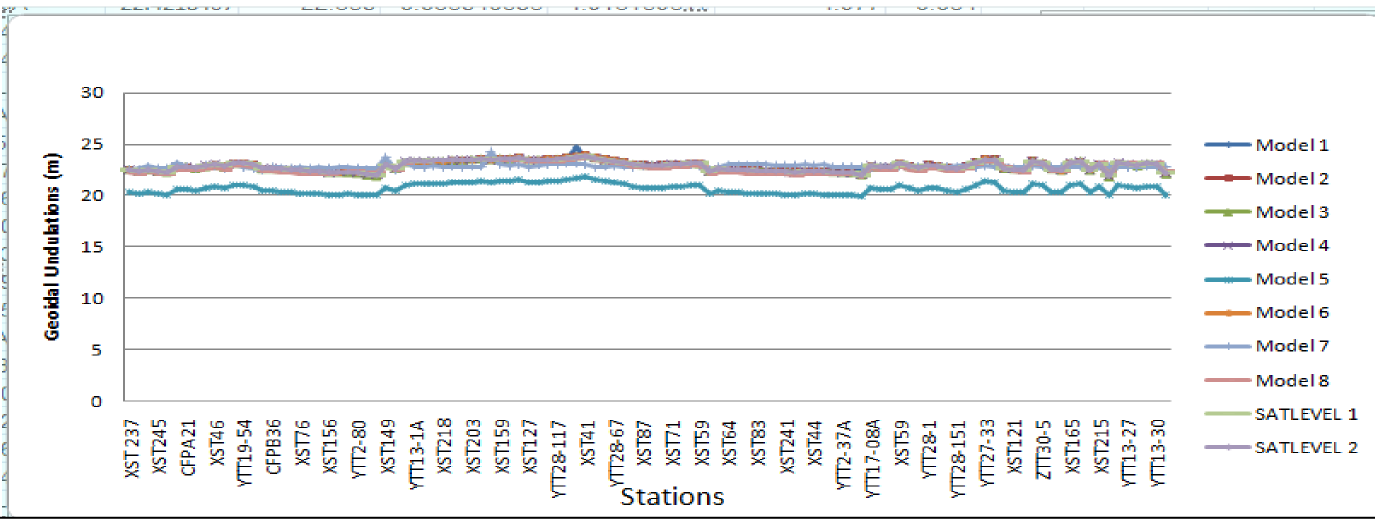


Figure 4.7b: Chart showing the Relationship between the Local, Geoidal Undulations of the Existing and the Geoidal Undulations of the New "Satlevel" Collocation Models for Lagos State

The field data in Table 3.2a were used to compute the differences between the local undulation of the Exisiting and the New ‘Satlevel’ Collocation models for Port Harcourt (Table 4.11a)

Table 4.11a: Residuals obtained from the Existing and New ‘Satlevel’ Collocation Models for Port Harcourt

Stations	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	SATLEVEL 1	SATLEVEL 2
AP4	-0.0253	-0.0313	-0.0234	-0.5850	-0.0179	0.0000	-0.0241	-0.0300	-0.0293
PHCS 1s	-0.0234	-0.0252	-0.0080	-0.5985	-0.0191	0.0044	-0.0350	-0.0247	-0.0234
PT.9 EMMA	-0.0176	-0.0241	-0.0143	-0.6056	-0.0086	-0.0072	-0.0180	-0.0229	-0.0221
PT.2 ABDUL	-0.0293	-0.0363	-0.0273	-0.6186	-0.0203	-0.0004	-0.0269	-0.0351	-0.0343
GPS 09	-0.0005	-0.0108	-0.0016	-0.5920	0.0021	-0.0324	0.0020	-0.0098	-0.0088
GPS 10	-0.0013	-0.0120	-0.0032	-0.5937	0.0010	-0.0357	0.0010	-0.0110	-0.0101
GPS 29	0.0133	0.0189	0.0243	-0.5663	-0.0003	-0.0129	-0.0020	0.0202	0.0209
GPS 30	0.0127	0.0181	0.0234	-0.5672	-0.0006	0.0077	-0.0020	0.0193	0.0201
GPS 49	0.0060	-0.0150	-0.0062	-0.5956	0.0027	-0.0119	-0.0070	-0.0143	-0.0130
GPS 50	-0.0003	-0.0064	0.0018	-0.5893	0.0061	-0.1639	0.0030	-0.0051	-0.0044
GPS 60	0.0121	0.0037	0.0022	-0.5878	-0.0075	-0.0004	-0.0020	0.0050	0.0057
XSV 662	-0.0003	-0.0064	0.0016	-0.5897	0.0077	0.1598	0.0020	-0.0051	-0.0044
ZVS 3003	0.0030	-0.0036	0.0049	-0.5863	0.0110	0.0635	0.0060	-0.0023	-0.0016

The full data set for this table is as shown in Appendix C18

The results of the Residuals computed from the Local, Existing and New ‘Satlevel’ Collocation Models for Port Harcourt (Table 4.11b) are plotted inform of chart (Figure 4.8)

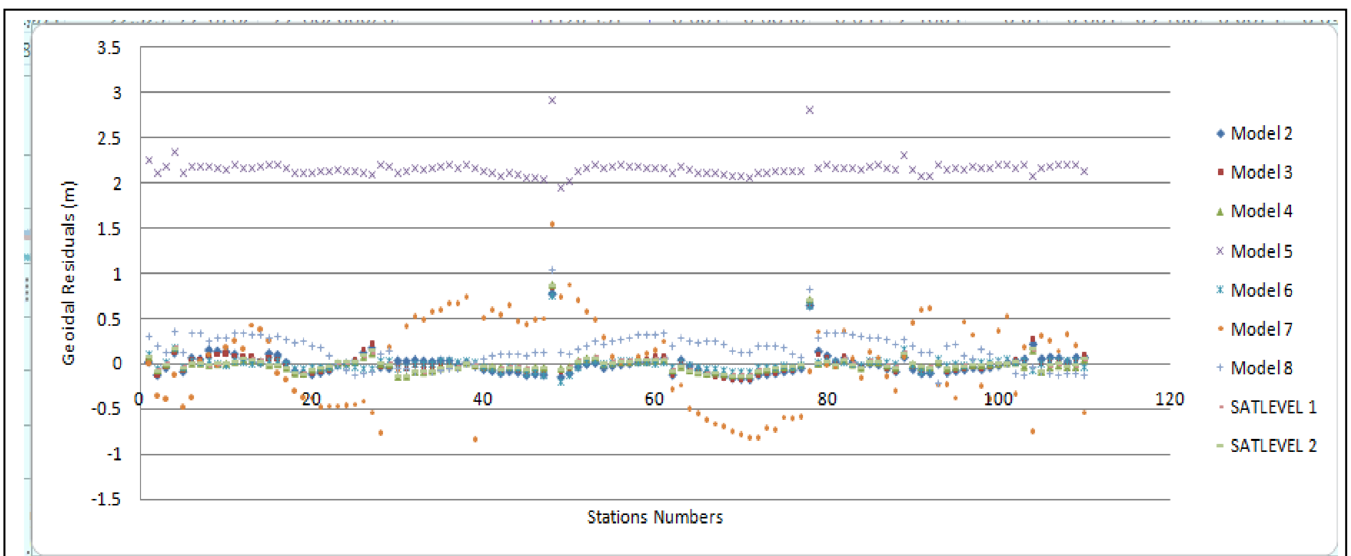


Figure 4.8a: Chart showing the Residuals Computed from the Local, Existing and New “Satlevel” Collocation Models for Port Harcourt.

The field data in Table 3.2b were used to compute the difference between the local undulation of the Exisiting and the New 'Satlevel' Collocation models in Lagos State (Table 4.11b)

Table 4.11b: Residuals obtained from the Existing and New 'Satlevel' Collocation Models for Lagos State

STATIONS	North Sea Region Model [m]	4-parameters Similarity Datum Shift [m]	5-parameters Similarity Datum Shift [m]	7-parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM2008 [m]
XST 237	0.0196	0.0251	0.0742	2.2606	0.1006	-0.0770	0.3000
XST44	-0.1265	-0.1210	-0.0680	2.1085	-0.0520	-0.3980	0.1880
YTT78A	-0.0447	-0.0350	-0.0040	2.1724	0.0123	-0.1970	0.1160
XST 55	0.1513	0.0880	-0.0100	2.1892	0.0291	-0.1550	0.2460
YTT1703A	0.1374	0.1054	-7E-04	2.1613	0.0012	0.1128	0.2780
XST46	0.1319	0.1181	0.0053	2.1401	-0.0200	0.1570	0.2770
XST50	0.1056	0.0864	0.0246	2.1938	0.0338	0.0722	0.3330
LWBC5-61P	0.0613	0.0885	0.0484	2.1653	0.0053	0.4497	0.3290
ZTT2-57A	-0.0175	-0.0010	0.0125	2.1464	-0.0140	-0.4090	-0.0060
MCS1188T-A	-0.0488	-0.0390	-0.0050	2.1843	0.0242	-0.1720	0.1380
XST42	0.0325	-0.0440	-0.1400	2.1108	-0.0490	0.3857	-0.0550
XST128	-0.1277	-0.0880	-0.0540	2.0565	-0.1030	0.1140	0.0930
YTT28-117	-0.114	-0.0680	-0.0360	2.0606	-0.0990	-0.0060	0.1135
MCS1174S-A	-0.126	-0.0730	-0.0410	2.0284	-0.1320	0.2858	0.1170
XST165	-0.0208	-0.0190	0.0004	2.1929	0.0328	0.3378	0.0170
XST126	0.0102	-2E-04	0.0116	2.2061	0.0460	0.4359	-0.0070
YTT9-29A	0.0286	0.0357	0.0161	2.1699	0.0098	-0.2090	-0.1060
XST215	0.0472	0.0165	-0.0250	2.1965	0.0365	0.3614	-0.1240
ZTT35-26	0.2175	0.2718	0.1501	2.0775	-0.0830	-0.6110	-0.0410
XST59	0.0609	0.0867	0.0469	2.1662	0.0062	0.4077	0.3290
CFPA18	0.0195	0.0198	-0.0030	2.1635	0.0035	0.0763	0.3245
XST202	0.0239	-0.0050	-0.0320	2.1901	0.0300	0.4158	-0.1160
YTT13-30	0.0677	0.0275	-0.0330	2.1922	0.0321	0.3137	-0.1120
XST204	0.0659	0.0906	0.0469	2.1202	-0.0400	-0.5100	-0.1310

The full data set for this table is as shown in Appendix C19

The results of the results of the Geoidal Undulation computed from the Local, Existing and New 'Satlevel' Collocation Models (Table 4.10b) are plotted in form of chart (Figure 4.8b)

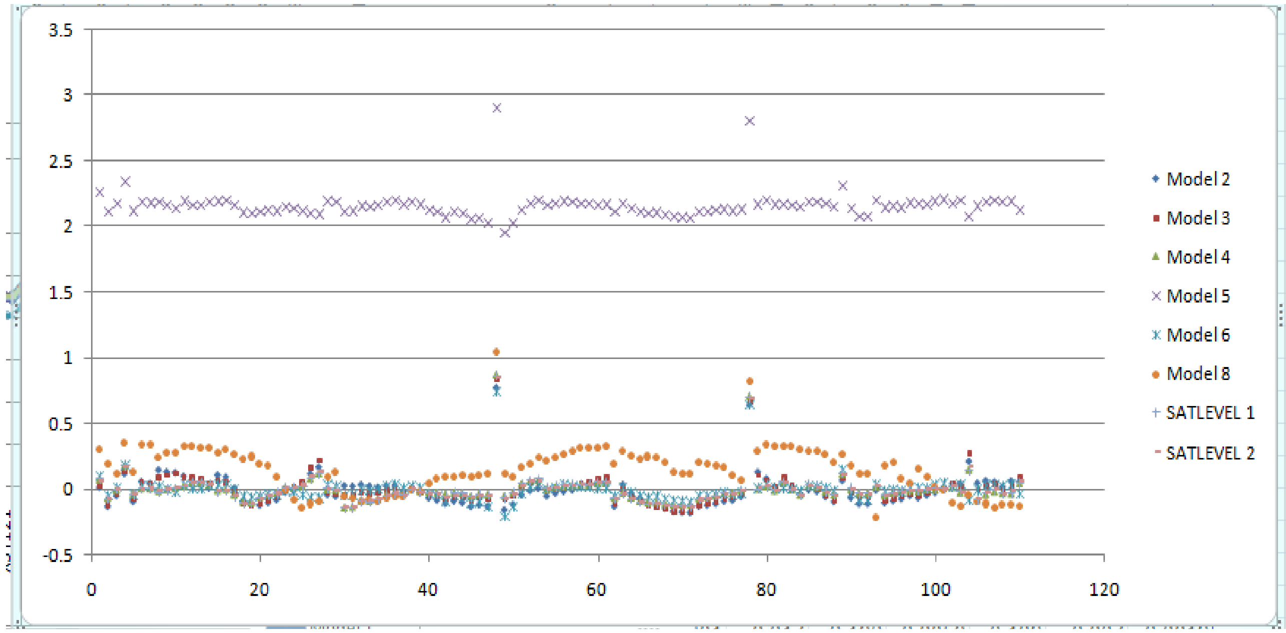


Figure 4.8b: Chart showing the residuals computed from the Local, Existing and New "Satlevel" Collocation Models for Lagos State

4.1.9 Results of Validation of 'Satlevel' Collocation Geoid Models

Based on the methodology adopted as discussed in Section 3.8, the results of the points used as checks for model validation are tabulated in Tables 4.12a, 4.12b, 4.12c and 4.12d

Table 4.12a: Results of Validation of Spherical 'Satlevel' Models for Port Harcourt

Stations	Latitude [^{o}]	Longitude [^{o}]	Observed Ellipsoidal Height (h) [m]	Observed Orthometric Height (H) [m]	Observed Undulation (N) [m]	Computed Undulation (N) [m]	Difference Between the Observed and Computed Undulation [m]
UNIPORT GATE	4.893749	6.914445	29.7120	10.8670	18.8450	18.84664	-0.0016
GPS 44	4.832048	7.126734	34.4110	15.2900	19.1210	19.1170	0.0040
GPS 52	4.915312	6.983789	35.2540	16.3390	18.9150	18.9125	0.0025
GPS 61	4.913982	6.880881	20.6720	1.8770	18.7950	18.7982	-0.0032
RHS 8A	4.755137	7.016562	23.5290	4.4860	19.0430	19.0354	0.0076

Table 4.12b: Results of Validation of Spherical 'Satlevel' Models for Lagos State

Stations	Latitude [^{o}]	Longitude [^{o}]	Observed Ellipsoidal Height (h) [m]	Observed Orthometric Height (H) [m]	Observed Undulation (N) [m]	Computed Undulation (N) [m]	Difference Between the Observed and Computed Undulation [m]
YTT2-11A	6.422504894	3.513237463	26.4160	4.3660	22.0500	22.48514	-0.06914
XST126	6.623861573	3.528768937	24.6990	2.2640	22.4350	22.57468	-0.1397
XST136	6.468232669	3.56529207	30.2510	7.7600	22.4910	22.61483	-0.1238
XST137	6.426358515	3.580480429	27.0900	4.8320	22.2580	22.37867	-0.1207
XST225	6.423482091	3.531541184	26.4470	4.1980	22.2490	22.47998	-0.2310

Table 4.12c: Results of Validation of 'Satlevel' Rectangular Model for Port Harcourt

Stations	Latitude [^{o}]	Longitude [^{o}]	Observed Ellipsoidal Height (h) [m]	Observed Orthometric Height (H) [m]	Observed Undulation (N) [m]	Computed Undulation (N) [m]	Difference Between the Observed and Computed Undulation [m]
P10 BALOGUN	4.866626	6.999611	36.0840	17.18136	18.90264	18.96148	-0.05883667
GPS 44	4.832048	7.126734	34.4110	15.2900	19.1210	19.11247	0.008532236
GPS 52	4.915312	6.983789	35.2540	16.3390	18.9150	18.90981	0.005191819
GPS 61	4.913982	6.880881	20.6720	1.8770	18.7950	18.79232	0.002678822
RHS 8A	4.755137	7.016562	23.5290	4.4860	19.0430	19.03440	0.008598406
			Mean				0.01676759
			Mean Square Error				0.000001

Table 4.12d: Results of Validation of 'Satlevel' Rectangular Model for Lagos State

Stations	Latitude [^{o}]	Longitude [^{o}]	Observed Ellipsoidal Height (h) [m]	Observed Orthometric Height (H) [m]	Observed Undulation (N) [m]	Computed Undulation (N) [m]	Difference Between the Observed and Computed Undulation [m]
YTT2-11A	6.422504894	3.513237463	26.4160	4.3660	22.0500	22.30913097	0.106869031
XST126	6.623861573	3.528768937	24.6990	2.2640	22.4359	23.31351975	-0.878519746
XST136	6.468232669	3.56529207	30.2510	7.7600	22.4910	22.50887863	-0.017878631
XST137	6.426358515	3.580480429	27.0900	4.8320	22.2580	22.29144198	-0.033441979
XST225	6.423482091	3.531541184	26.4470	4.1980	22.2490	22.30352167	-0.054521669

4.1.10 Results of Fitting the Global (GEM2008) Geoid Model to Local Geoid

The result of the geodetic levelling observation was processed to observed local Orthometric Heights while GEM2008 Geoidal Undulation was calculated and applied in Equation (1.3) to get GEM2008 Orthometric Heights. Equation (3.86) was used to compute the fitted local Orthometric Heights (Table 4.12a)

In section 3.8, Local Geoid was fitted to Global (GEM2008); Equation (3.86) was used for the adaptation. The results were tabulated in Tables 4.13a and 4.13b for both Port Harcourt and Lagos State respectively.

Table 4.13a: Results of Fitting the Local Geoid to GEM2008 Model for Port Harcourt

Stations	Adjusted Local Geoidal Undulation [m]	GEM2008 Geoidal Undulations [m]	Differences [m]
Models	Equation (3.31)	Alltrans EGM Calculator	
AP4	18.9552	18.9470	0.0082
PHCS 1s	19.0327	19.0330	-0.0003
PT.9 EMMA	19.0022	18.9930	0.0092
PT.3 ABDUL	19.0170	19.0060	0.0110
GPS 02	18.8858	18.9000	-0.0142
GPS 10	18.8425	18.8350	0.0075
GPS 29	18.9009	18.9150	-0.0141
GPS 30	18.9006	18.9140	-0.0134
GPS 40	19.0309	19.0360	-0.0051
GPS 41	19.0664	19.0750	-0.0086
GPS 42	19.0678	19.0770	-0.0092
GPS 43	19.0693	19.0790	-0.0097
GPS 45	19.1147	19.1210	-0.0063
GPS 46	19.1139	19.1210	-0.0071
GPS 47	19.1581	19.1460	0.0121
GPS 48	19.1591	19.1470	0.0121
GPS 40	19.0309	19.0360	-0.0051
GPS 59	18.7882	18.7860	0.0022
GPS 60	18.7902	18.7950	-0.0048
XSV 662	18.9616	18.9530	0.0086
ZVS 3003	19.0301	19.0200	0.0101

The full data set for this table is as shown in Appendix C20

The results of the Local and GEM2008 Geoidal Undulation (Table 4.12a) are plotted in form of chart (Figure 4.9a)

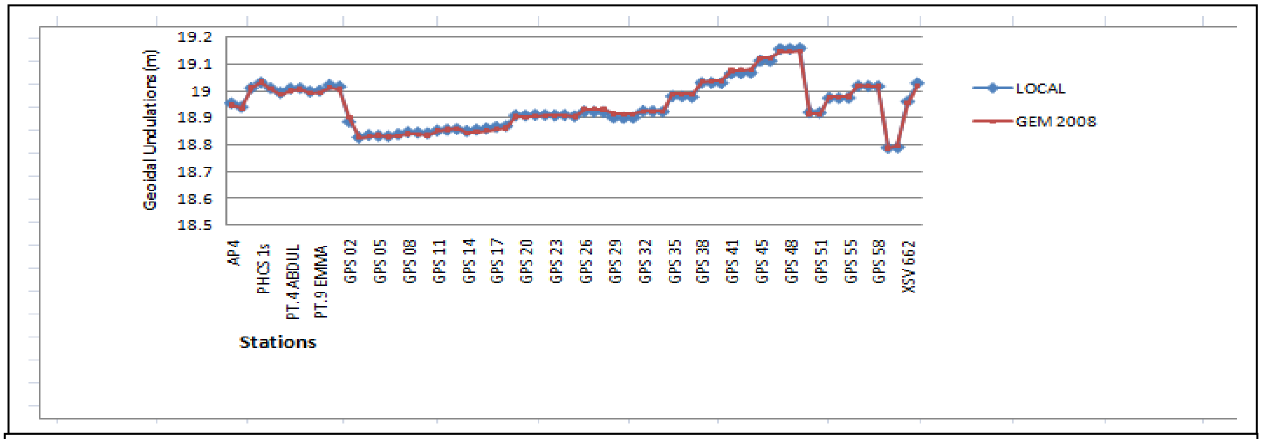


Figure 4.9a: Chart showing the Relationship between the local and GEM2008 Geoidal Undulations for Port Harcourt

Table 4.13b: Results of Fitting the Local Geoid to GEM2008 Model for Lagos State

Stations	Adjusted Local Geoidal Undulations [m]	GEM2008 Geoidal Undulations [m]	Difference [m]
Models	Equation (3.31)	Alltrans EGM Calculator	
XST 237	22.3910	22.2640	0.1270
FGPLA-Y-003	22.4057	22.4460	-0.0403
CFPA21	22.4686	22.4870	-0.0184
LWBC5-61P	22.7911	22.8570	-0.0659
CFPB36	22.2376	22.3450	-0.1074
YTT28-200	22.3552	22.2210	0.1342
MCS1178T-A	22.4680	22.3760	0.0920
YTT9-73A	22.3883	22.3370	0.0513
XST165	23.1602	23.1840	-0.0238
XST126	23.3256	23.3660	-0.0404
YTT9-29A	22.4351	22.5860	-0.1509
XST215	23.0412	23.1540	-0.1128
ZTT35-26	21.8902	22.1250	-0.2348
YTT13-27	23.0677	23.1490	-0.0813
XT161	22.9273	23.0590	-0.1317
XST202	23.1401	23.2360	-0.0959
YTT13-30	23.0582	23.1490	-0.0908
XST204	22.1465	22.3520	-0.2055

The full data set for this table is as shown in Appendix C21

The results of the Local and GEM2008 Geoidal Undulation (Table 4.12b) are plotted in form of chart (Figure 4.9b)

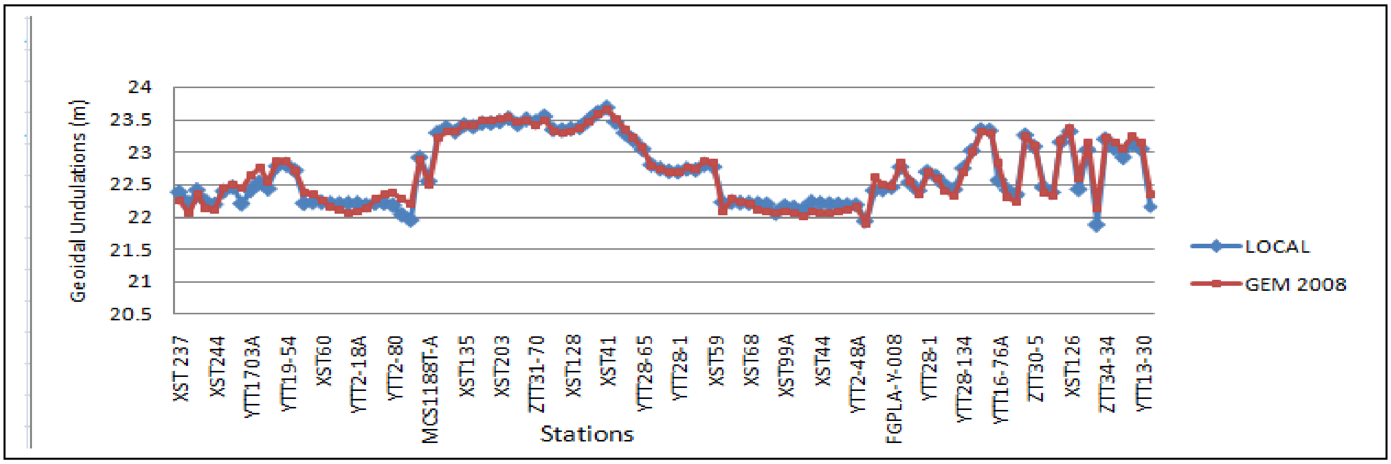


Figure 4.9b: Chart showing the Relationship between the local and GEM2008 Geoidal Undulations for Lagos State

4.1.11 GEM2008 Orthometric Height and Local Equivalent:

The result of the geodetic levelling observation was processed to obtain local Orthometric Heights while GEM2008 Geoidal Undulation was applied in Equation (1.3) to get GEM2008 Orthometric Heights. Equation (3.86) was used to compute the local Orthometric Heights and GEM2008 Orthometric as tabulated in Table 4.14a for Port Harcourt

Table 4.14a: Summary of the Result of GEM2008 Orthometric Height Computed from New 'Satlevel' Collocation for Port Harcourt

Stations	GEM2008	Local	Differences
Models	Alltrans EGM Calculator	Equation (3.86)	
AP4	16.9020	16.8938	0.0083
PHCS 1s	11.7630	11.7633	-0.0003
PT.9 EMMA	10.1480	10.1388	0.0092
PT.3 ABDUL	7.7440	7.7330	0.0110
GPS 02	23.6420	23.6562	-0.0142
GPS 19	10.3620	10.3557	0.0063
GPS 20	10.9670	10.9607	0.0063
GPS 39	17.0070	17.0112	-0.0042
GPS 40	18.0920	18.0971	-0.0051
GPS 59	1.7080	1.7058	0.0022
GPS 60	2.1870	2.19182	-0.0048
XSV 662	8.6500	8.64139	0.0086
ZVS 3003	13.2880	13.2779	0.0101

The full data set for this table is as shown in Appendix C22

The results of the Local and GEM2008 Orthometric Heights (Table 4.14a) are plotted in form of chart (Figure 4.10a)

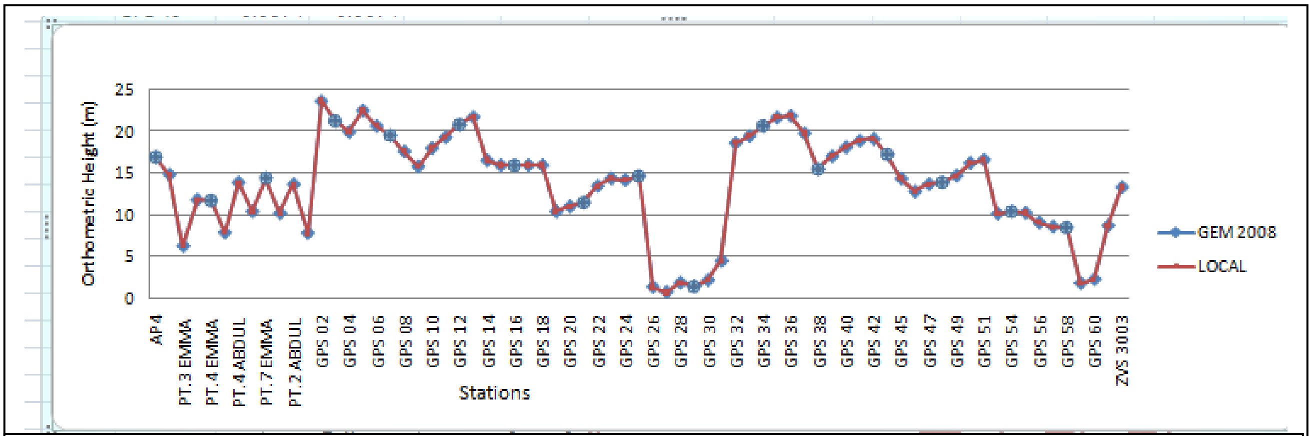


Figure 4.10a: Chart showing the Local and GEM2008 Orthometric Heights in Port Harcourt

The result of the geodetic levelling observation was processed to obtain observed local Orthometric Heights while GEM2008 Geoidal Undulation was applied in Equation (1.3) to get GEM2008 Orthometric Heights. Equation (3.86) was used to compute the local Orthometric Heights and GEM2008 Orthometric as tabulated in Table 4.14b for Lagos State.

Table 4.14b: Summary of the Result of GEM2008 Orthometric Height Computed from New 'Satlevel' Collocation for Lagos State

Stations	GEM2008	Local	Differences
Models	Alltrans EGM Calculator	Equation (3.86)	
FGPLA-Y-003	4.5990	4.6393	-0.0403
CFPA21	8.4530	8.4714	-0.0184
LWBC5-61P	3.1730	3.2389	-0.0659
YTT19-54	14.8950	14.948	-0.0533
XST75	13.7310	13.718	0.0128
MCS1174S-A	49.6870	49.6500	0.0374
CFPA31	4.8940	4.9445	-0.0505
YTT2-48A	4.5200	4.4865	0.0335
FGPLA-Y-008	8.1110	8.1064	0.0046
MCS1144S-A	7.2660	7.1791	0.0869
YTT28-151	3.4297	3.3233	0.1064
MCS1178T-A	3.1820	3.0900	0.0920
XST204	4.7750	4.9805	-0.2055
YTT19-54	14.8950	14.948	-0.0533

The full data set for this table is as shown in Appendix C23

The results of the Local and GEM2008 Orthometric Heights (Table 4.13b) are plotted inform of chart (Figure 4.10b)

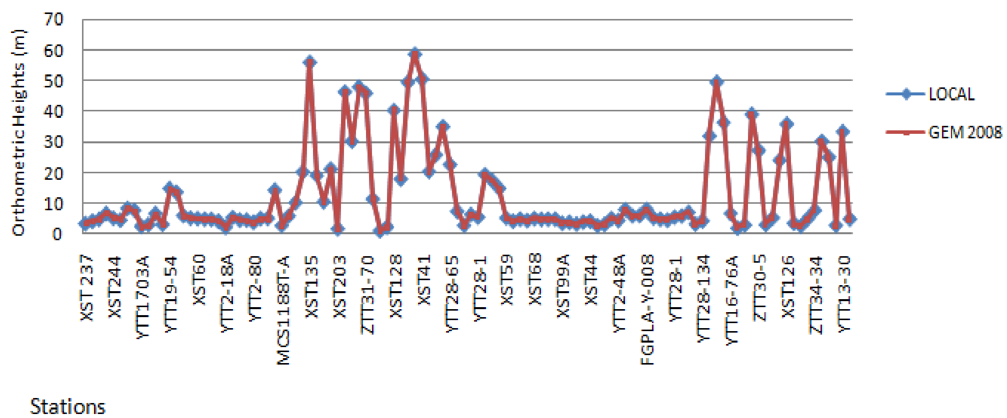


Figure 4.10b: Chart showing the Local and GEM2008 Orthometric Heights in Lagos State

GEM2008 Orthometric Height superimposed on the local Orthometric Height is an indication that GEM2008 fit perfectly in the study area after adaptation.

4.1.12 Comparing the Difference between the Local Geoidal Undulations and GEM2008 Orthometric Heights

The difference between the Local Geoidal Undulation and GEM2008 Geoidal Undulation were calculated in Table 4.10a for Port Harcourt. The difference beween the local and GEM2008 Orthometric Heights were also calculated in Table 4.13a. The two differences were compared as shown in Figure.4.10a for Port Harcourt.

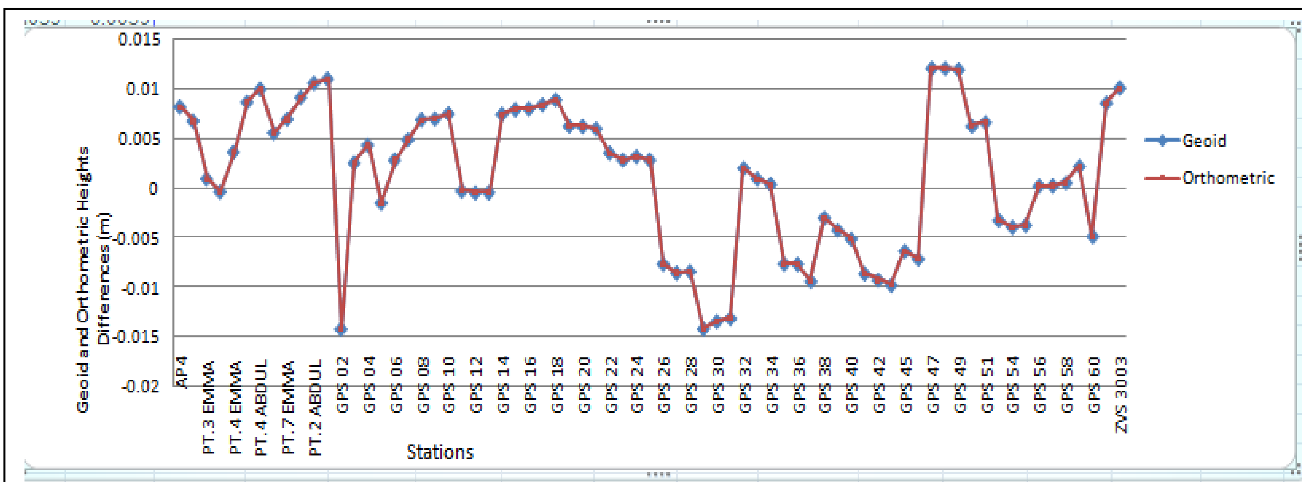


Figure 4.11a: Geoidal and Orthometric Heights Differences between the Local and GEM2008 values in Port Harcourt

The difference between the Local Geoidal Undulation and GEM2008 Geoidal Undulation were calculated in Table 4.12b. The difference beween the local and GEM2008 Orthometric Heights were also calculated in Table 4.13b. The two differences were compared as shown in Figure.4.9b for Lagos State.

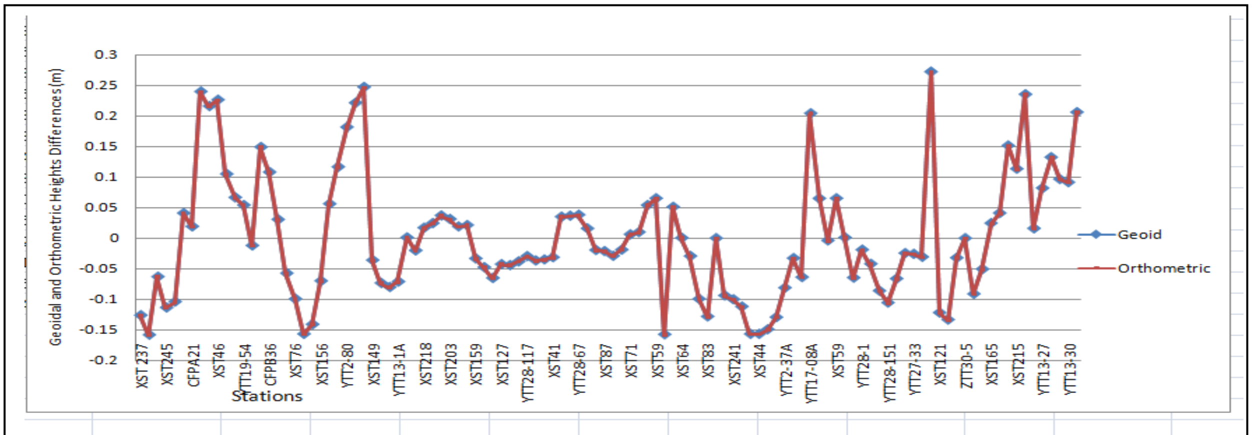


Figure 4.11b: Geoidal and Orthometric Heights Differences between the Local and GEM2008 values In Lagos State

4.1.13 Statistical Quantities for the New Models

The results of the computed statistical quantities for the New Models using data set from Port Harcourt and Lagos State were tabulated in Tables 4.15a and 4.15b respectively.

Table 4.15a: Computed Statistical Quantities for the New Models using Data from Port Harcourt

Quantities	Spherical 'Satlevel' Model	Rectangular 'Satlevel'
The Residual Sum of Squares	0.0152	0.0151
Sum of Squares Total	0.5469	0.5462
Sum of Squares Regression	0.5316	0.5313
The Coefficient of Determination R^2 for each of the method	0.9721	0.9728
The Variation not accounted for by each of the Model.	3%	3%
The Corresponding Product-Moment Correlation Coefficient	0.9859	0.9863
F Computed	34.8935	33.3000
F Table	8.56501136	8.56501136

The F computed as shown above is greater than F from the table. Therefore, the Null Hypothesis that the explanatory variables were equal to zero is rejected. Therefore, it is concluded that, explanatory variable made significant contributions to the variability of Geoidal Undulation in Port Harcourt.

Table 4.15b: Computed Statistical Quantities for the New Models using Data from Lagos State

Quantities	'Satlevel' Spherical Model	'Satlevel' Rectangular
The Residual Sum of Squares	1.6292	1.8916
Sum of Squares Total	26.1565	26.6893
Sum of Squares Regression	24.5274	24.7977
The Coefficient of Determination R^2 for each of the method	0.9377	0.9291
The Variation not accounted for by each of the Model.	6%	7%
The Corresponding Product-Moment Correlation Coefficient	0.9684	0.9639
F Computed	15.0552	15.9268

The F computed as shown above is greater than F from the table. Therefore, the Null Hypothesis that the explanatory variables were not equal to zero is rejected. Therefore, explanatory variable made significant contributions to the variability of Geoidal Undulation in Lagos State.

4.1.14 Geoidal Map and 3-Dimensional Surface Modelling

Geoidal Map and Three Dimensional Surface Models of the area were produced using SURFER software. The geoidal map was overlaid on the Local Government map of the Rivers State using ArcGIS software Figures 4.12a through 4.12j.

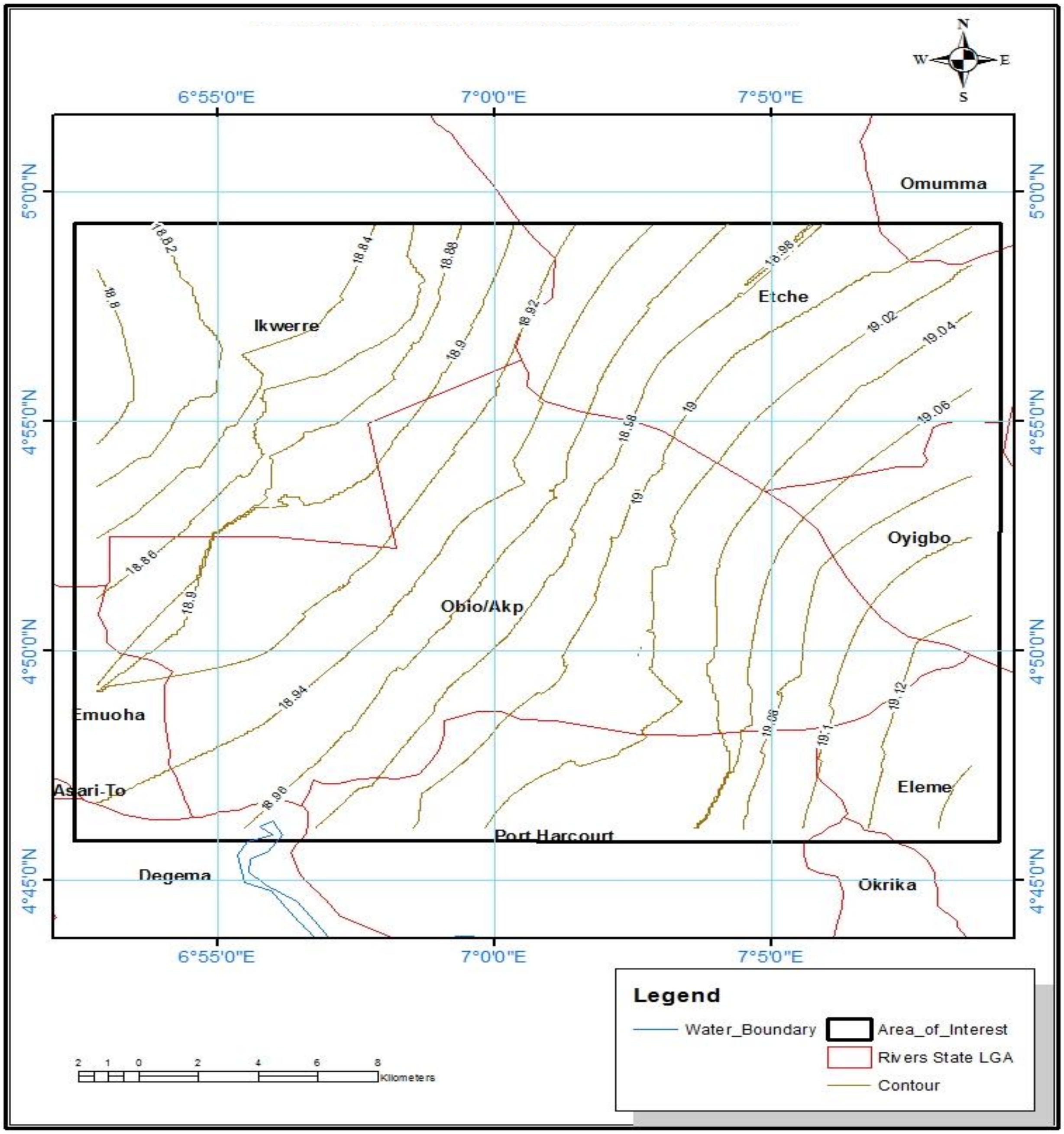


Figure 4.12a: The Geoidal Map Plotted From GEM2008 Model for Port Harcourt (Source: Author, October, 2009)

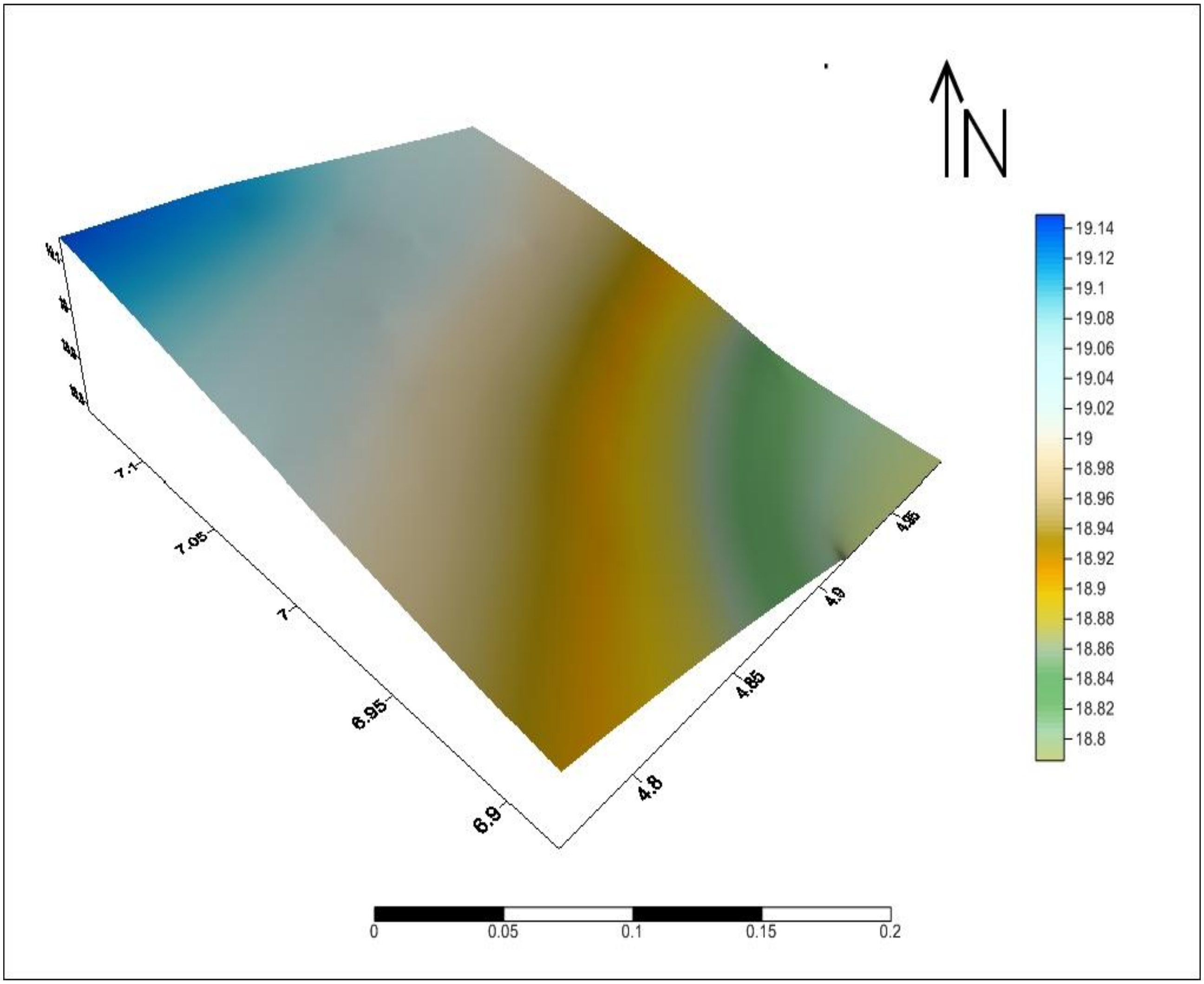


Figure 4.12b: Three Dimensional Surface Modelling obtained Using GEM2008 Model for Port Harcourt(Source: Author, October, 2009)

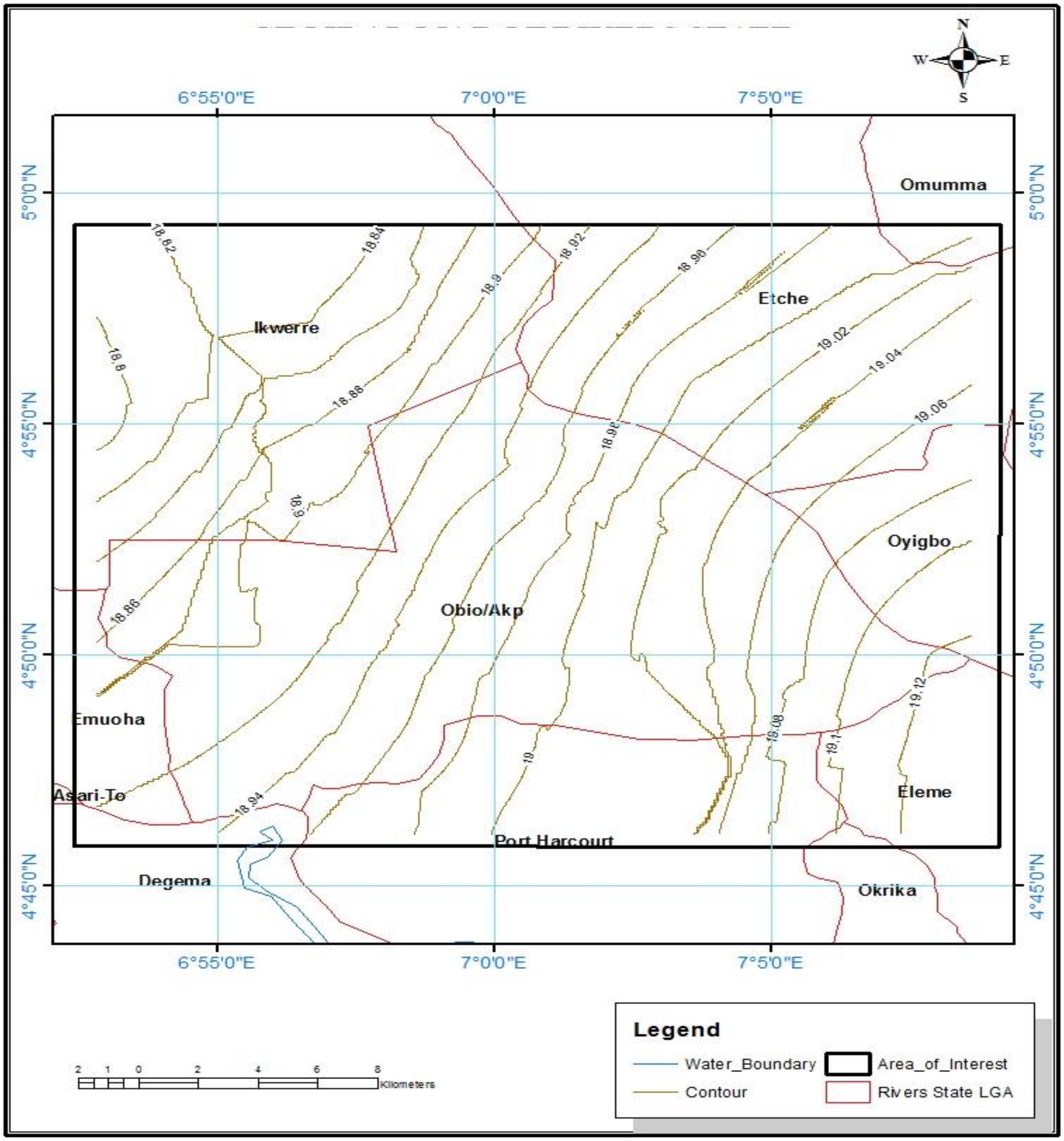


Figure 4.12c: Geoidal Map Plotted From Spherical **'Satlevel'** Model for Port Harcourt (Source: Author, October, 2009)

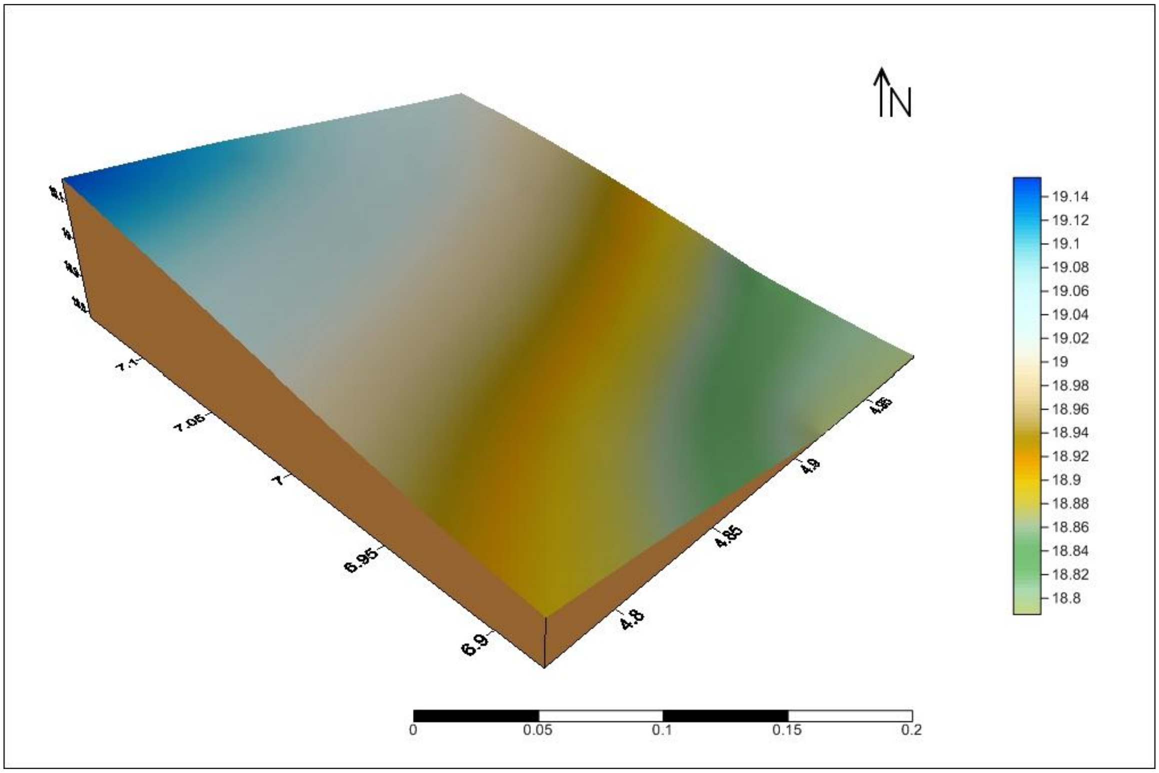


Figure 4.12d: Three Dimensional Surface Modelling Plotted From Model Spherical **'Satlevel'** Model for Port Harcourt

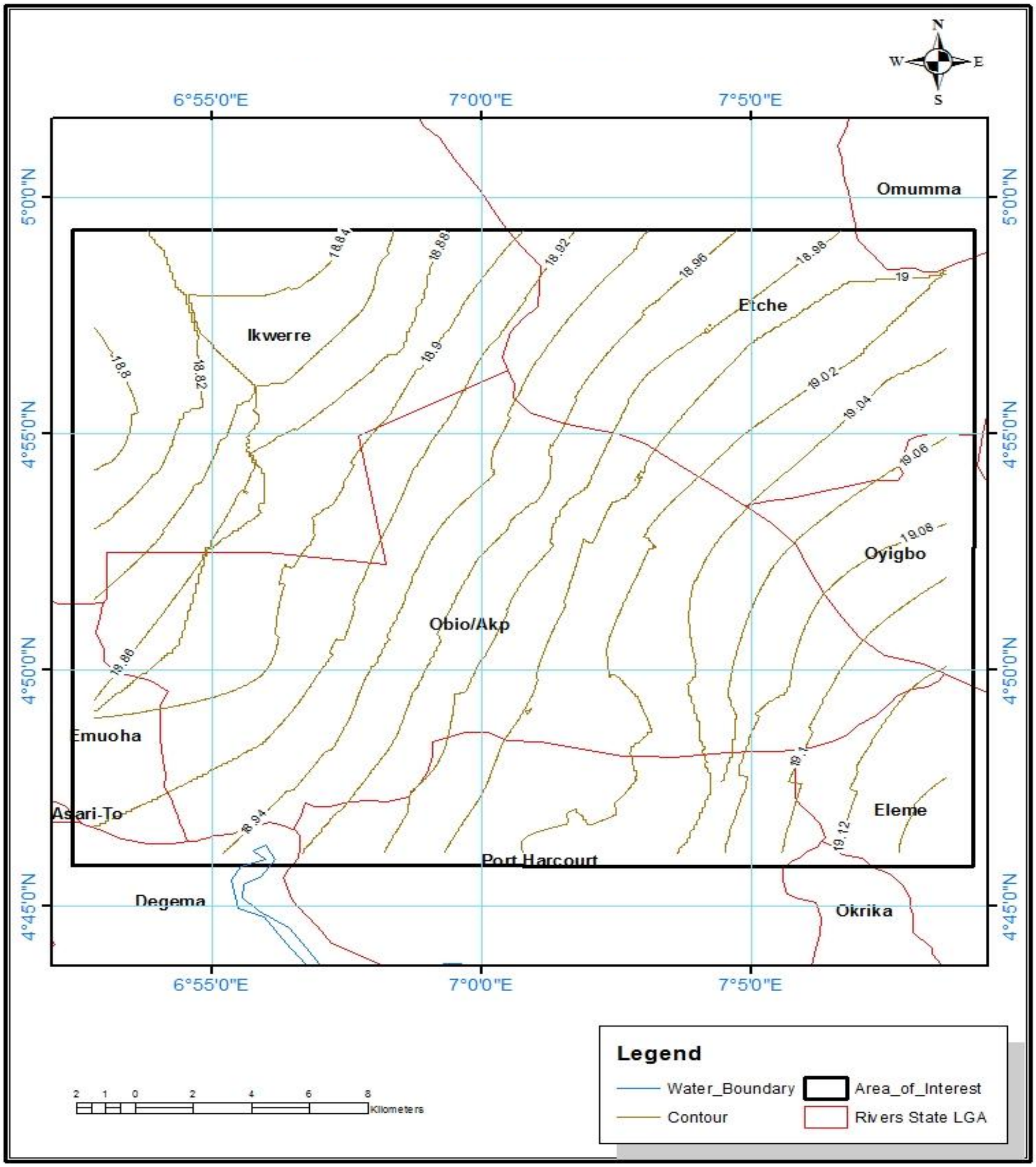


Figure 4.12e: The Geoidal Map Plotted From Port Harcourt Rectangular 'Satlevel' Model for Port Harcourt (Source: Author, October, 2009)

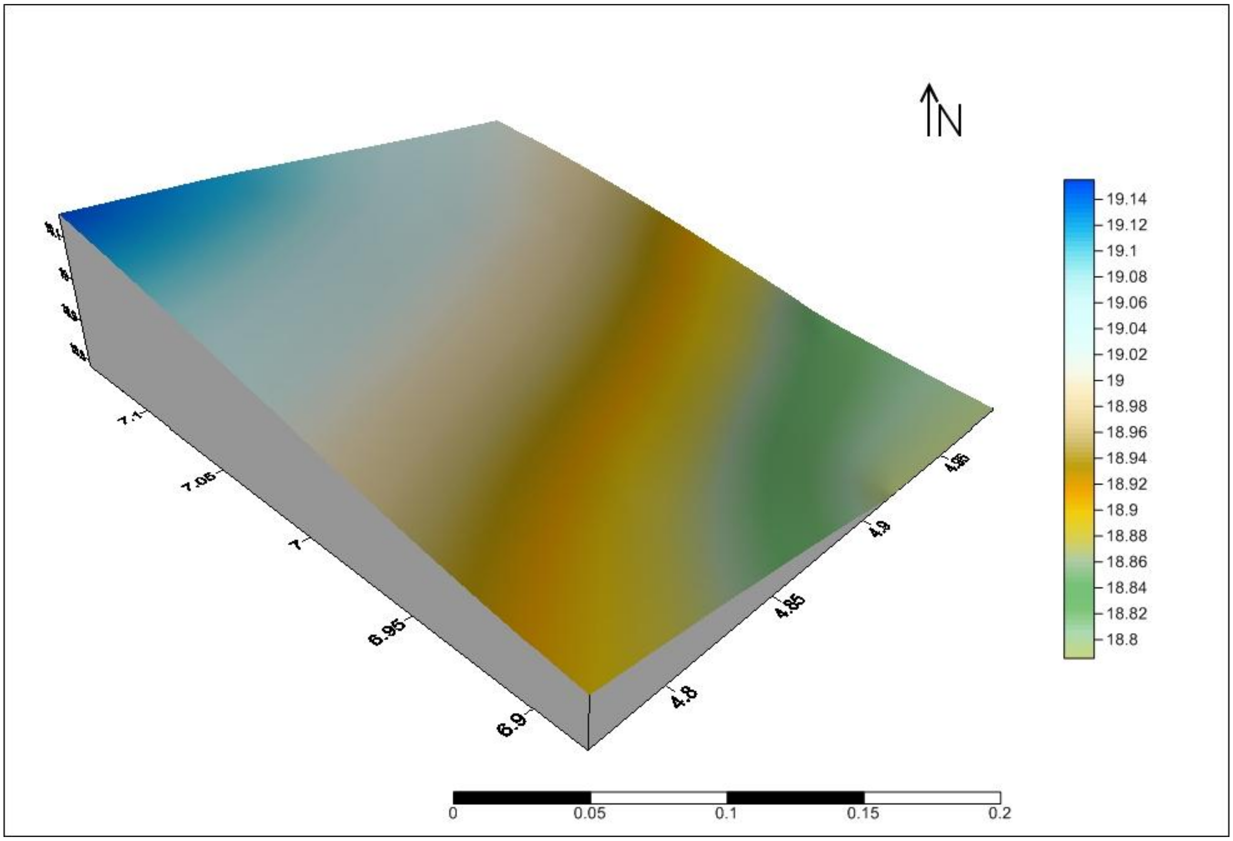


Figure 4.12f: Three Dimensional Surface Modelling Plotted From Rectangular 'Satlevel' for Port Harcourt (Source: Author, October, 2009)

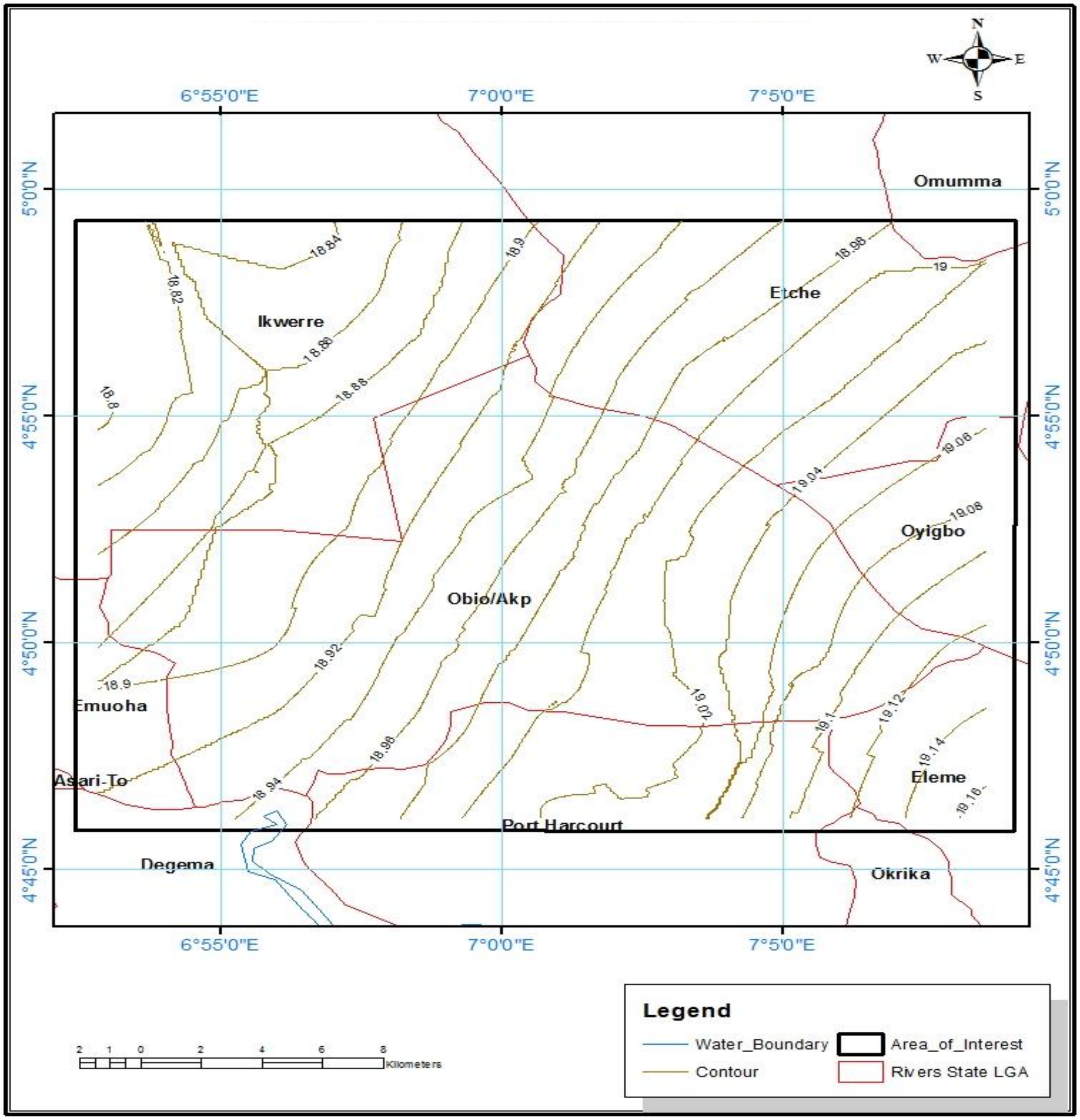


Figure 4.12g: Geoidal Map Plotted From Zanletnyik Hungarian Polynomial Fitting Model for Port Harcourt (Source: Author, October, 2009)

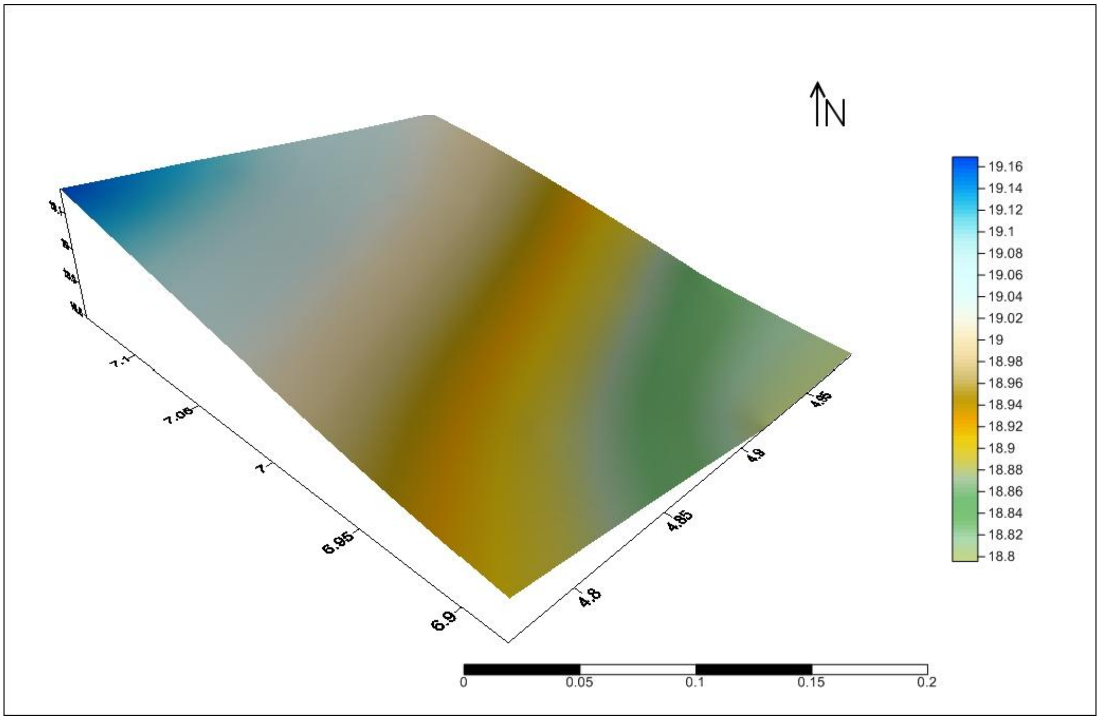


Figure 4.12h: Three Dimensional Surface Modelling Plotted From Zanletnyik Hungarian Polynomial Model for Port Harcourt (Source: Author, October, 2009)

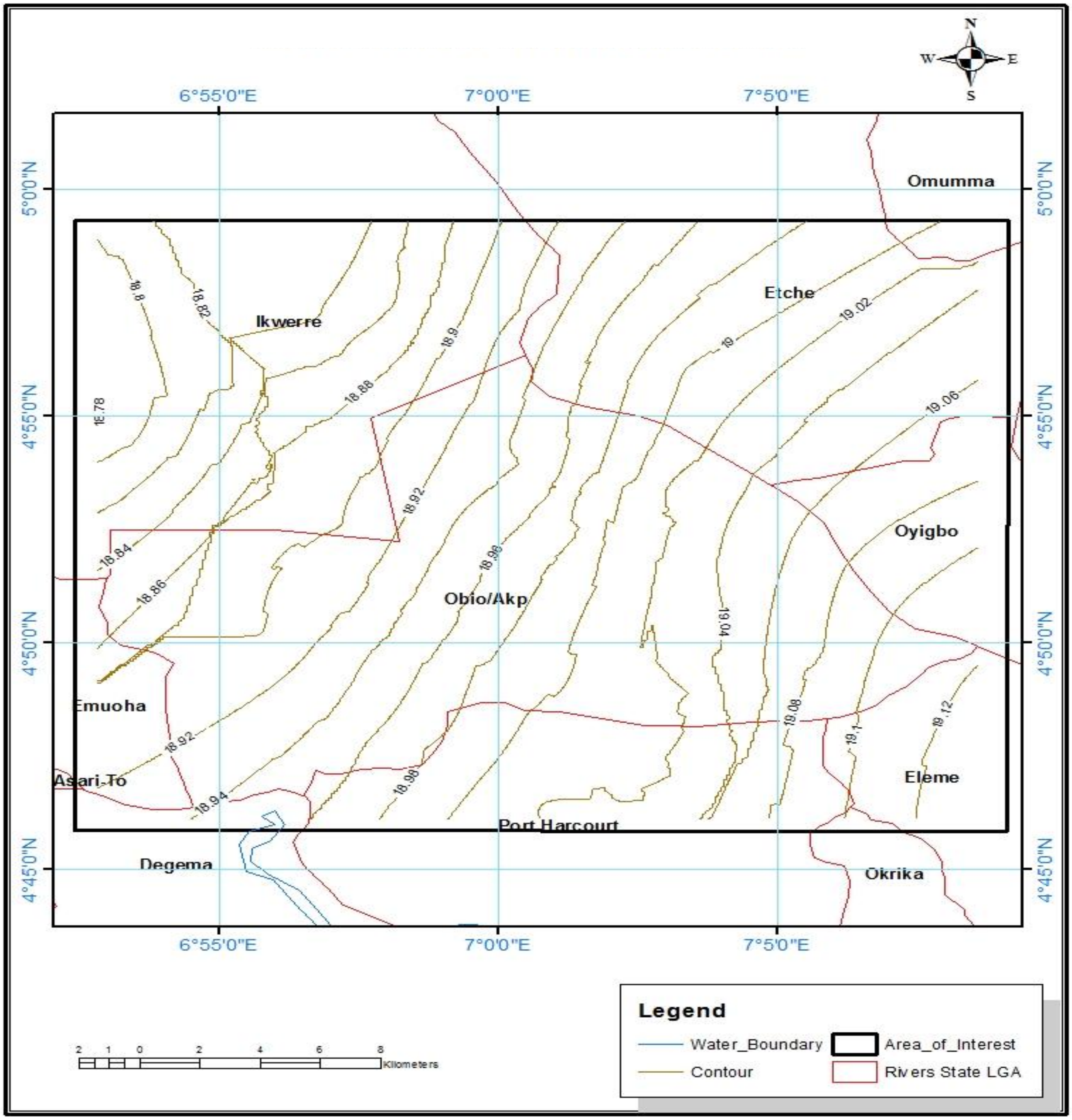


Figure 4.12i: Geoidal Map Plotted From North Sea Region Model for Port Harcourt

(Source: Author, October, 2009)

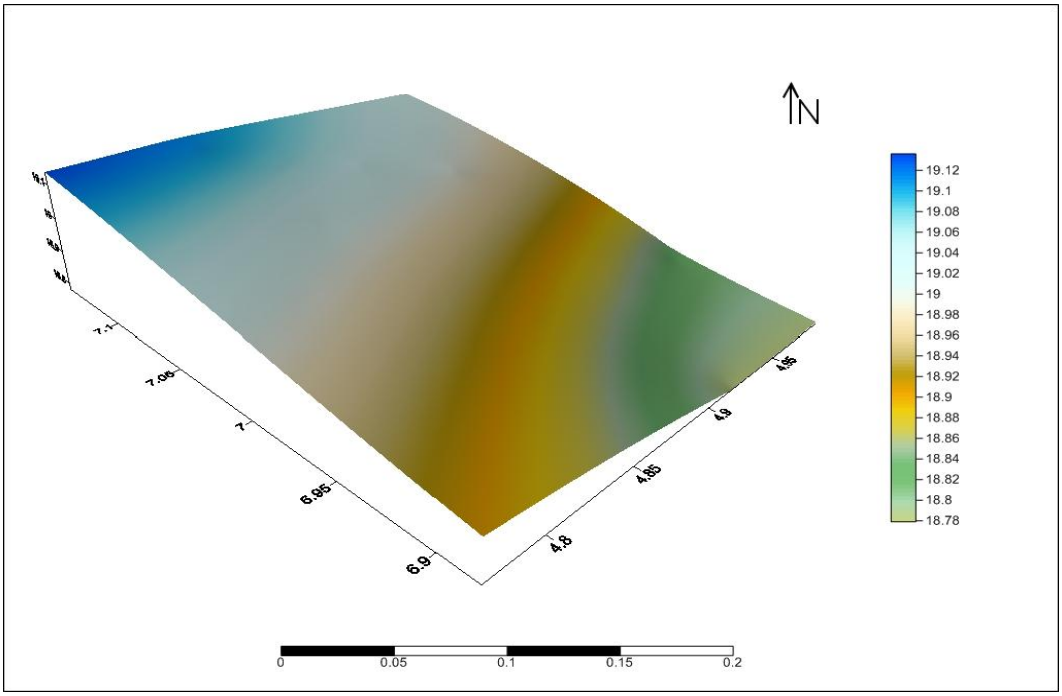


Figure 4.12j: Three Dimensional Surface Modelling Plotted From North Sea Region Model for Port Harcourt (Source: Author, October, 2009)

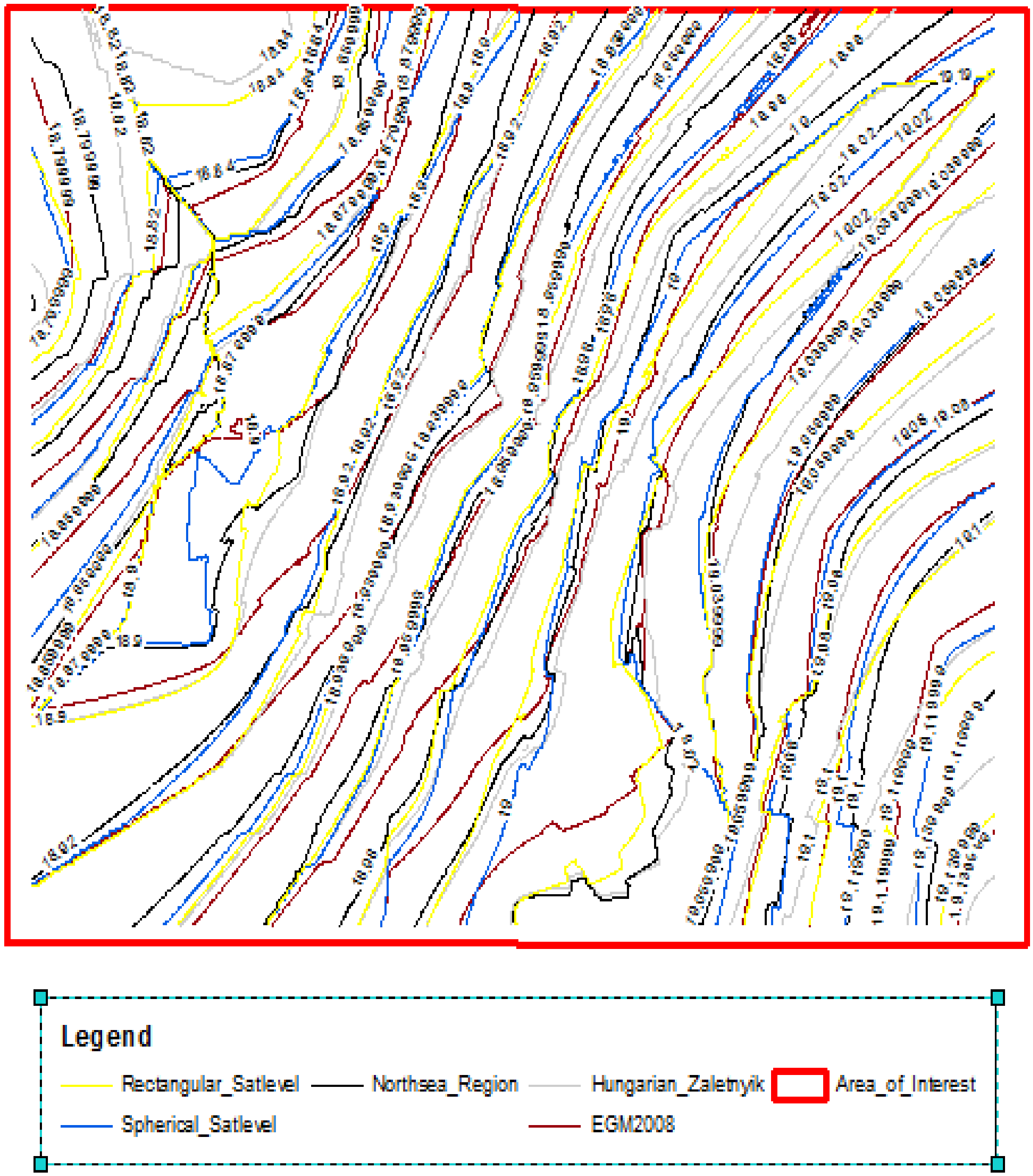


Figure 4.12k: Geoidal Map Plotted From Some of the Existing and 'Satlevel' Collocation Geoid Models for Port Harcourt (Source: Author, October, 2009)

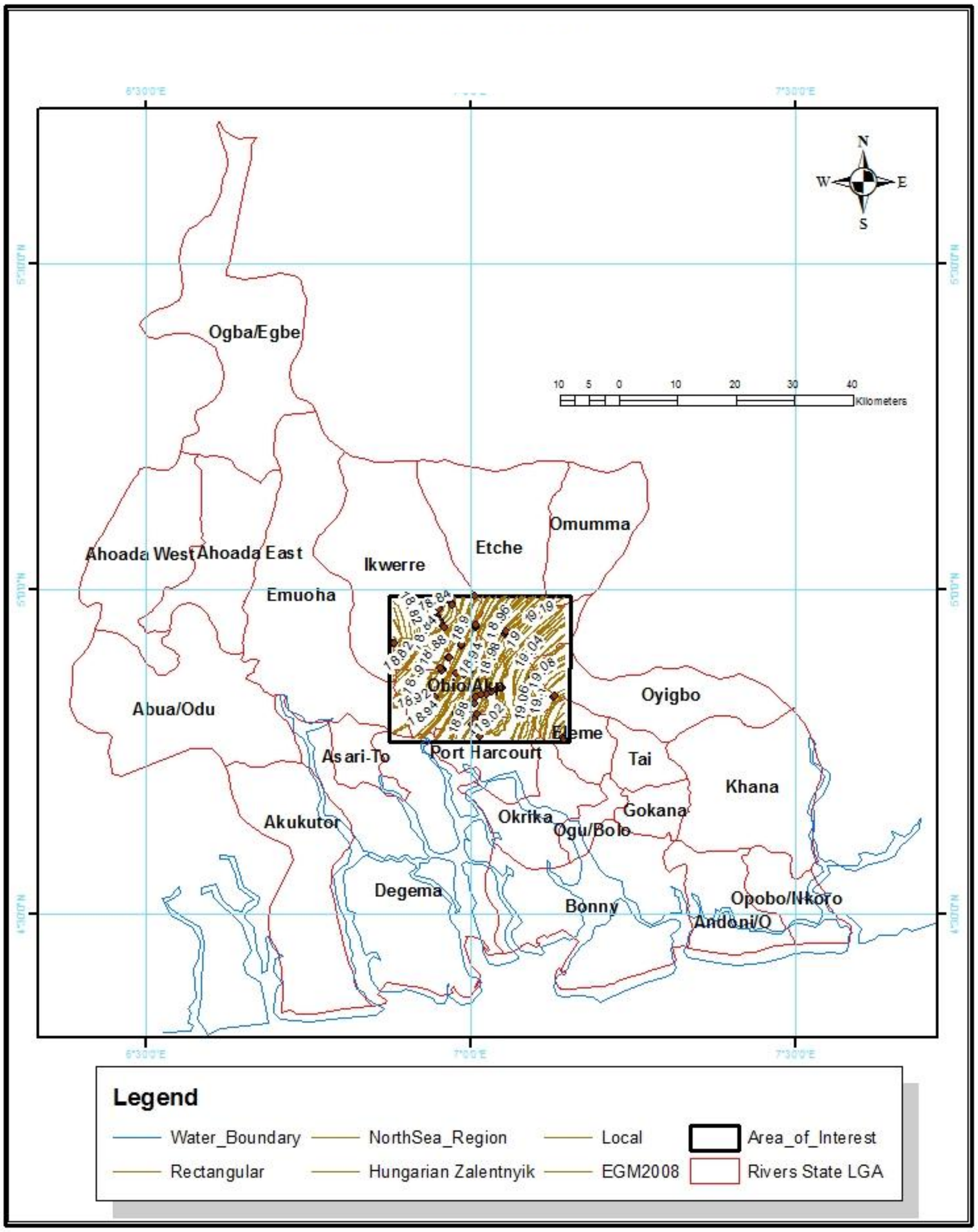


Figure 4.13: Local Government Geoidal Map Plotted From Some of the Existing and the New 'Satlevel' Collocation Geoid Models for Port Harcourt (Source: Author, October, 2009)

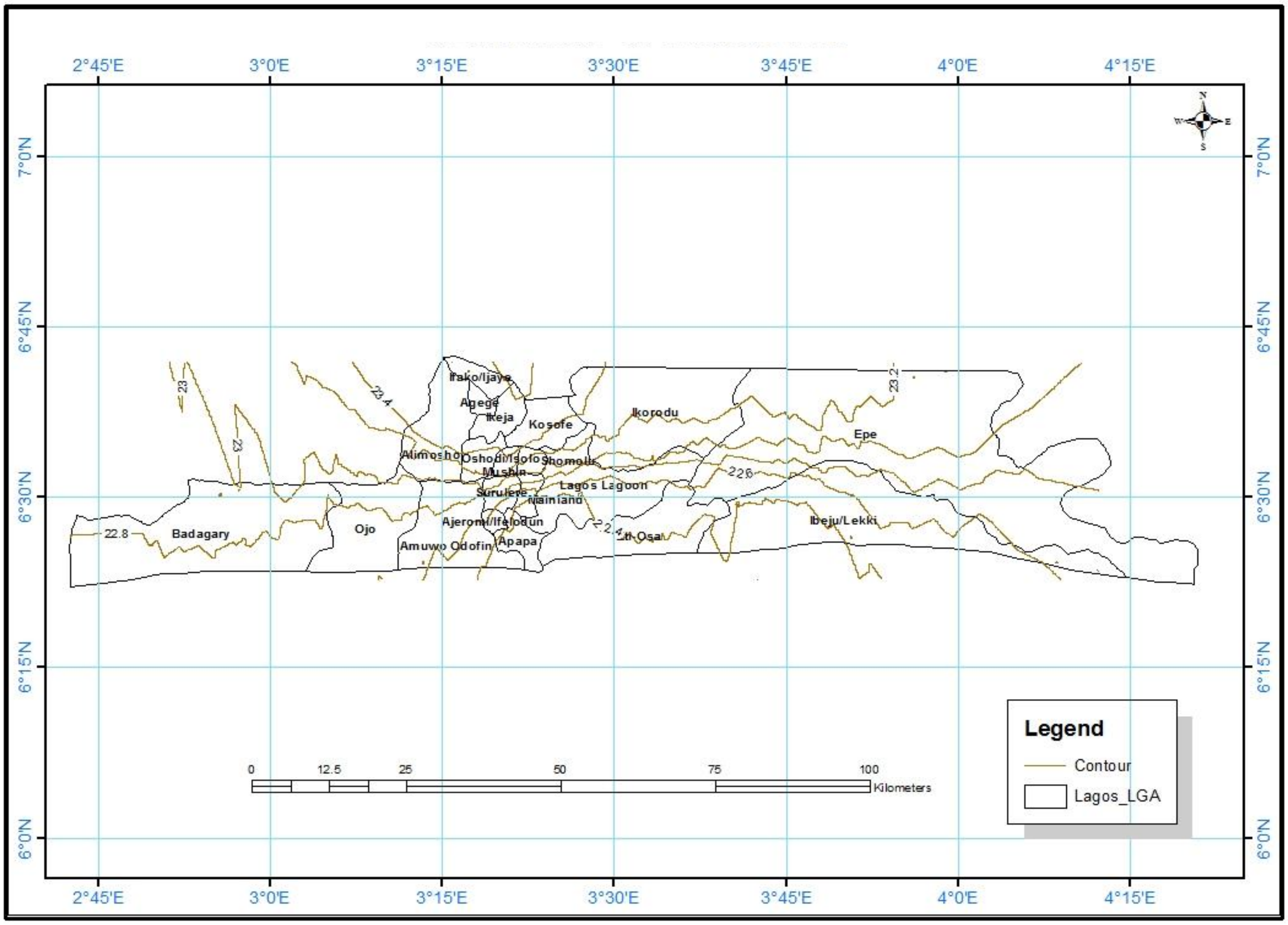


Figure 4.14a: Geoidal Map Plotted From Local Undulation for Lagos State (Source: Author, October, 2009)

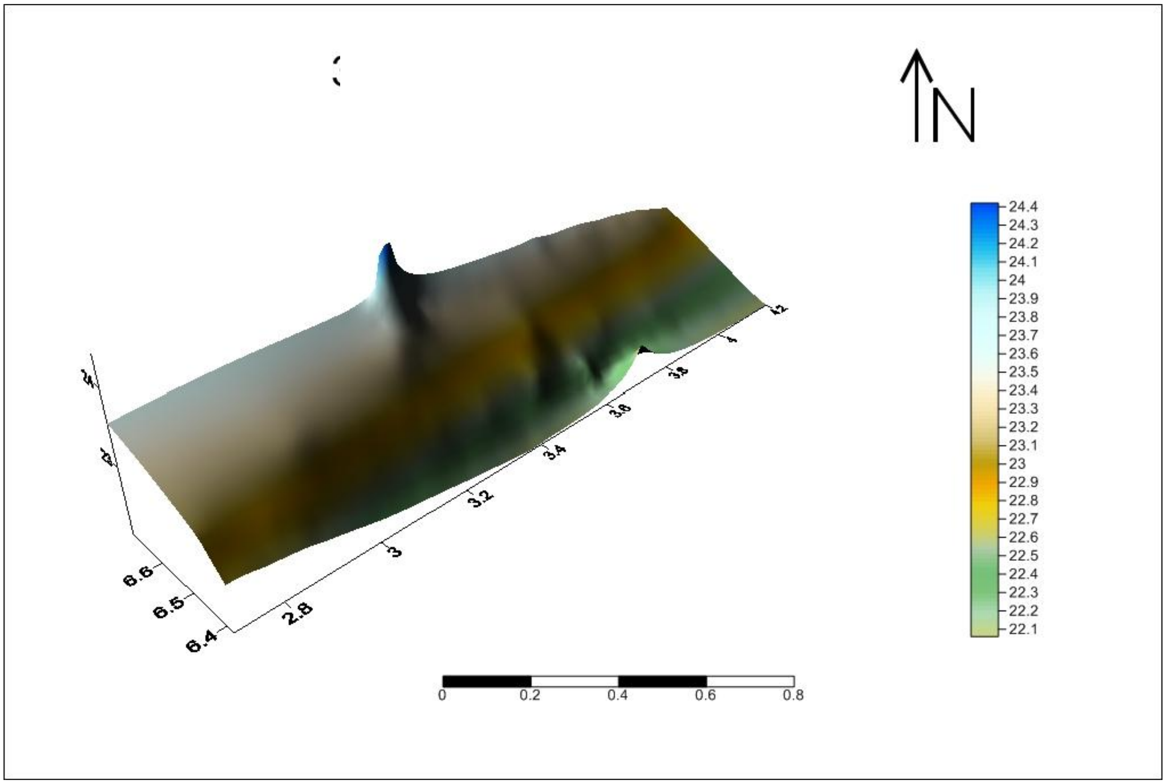


Figure 4.14b: Three Dimensional Surface Modelling Plotted From Local Undulation for Lagos State (Source: Author, October, 2009)

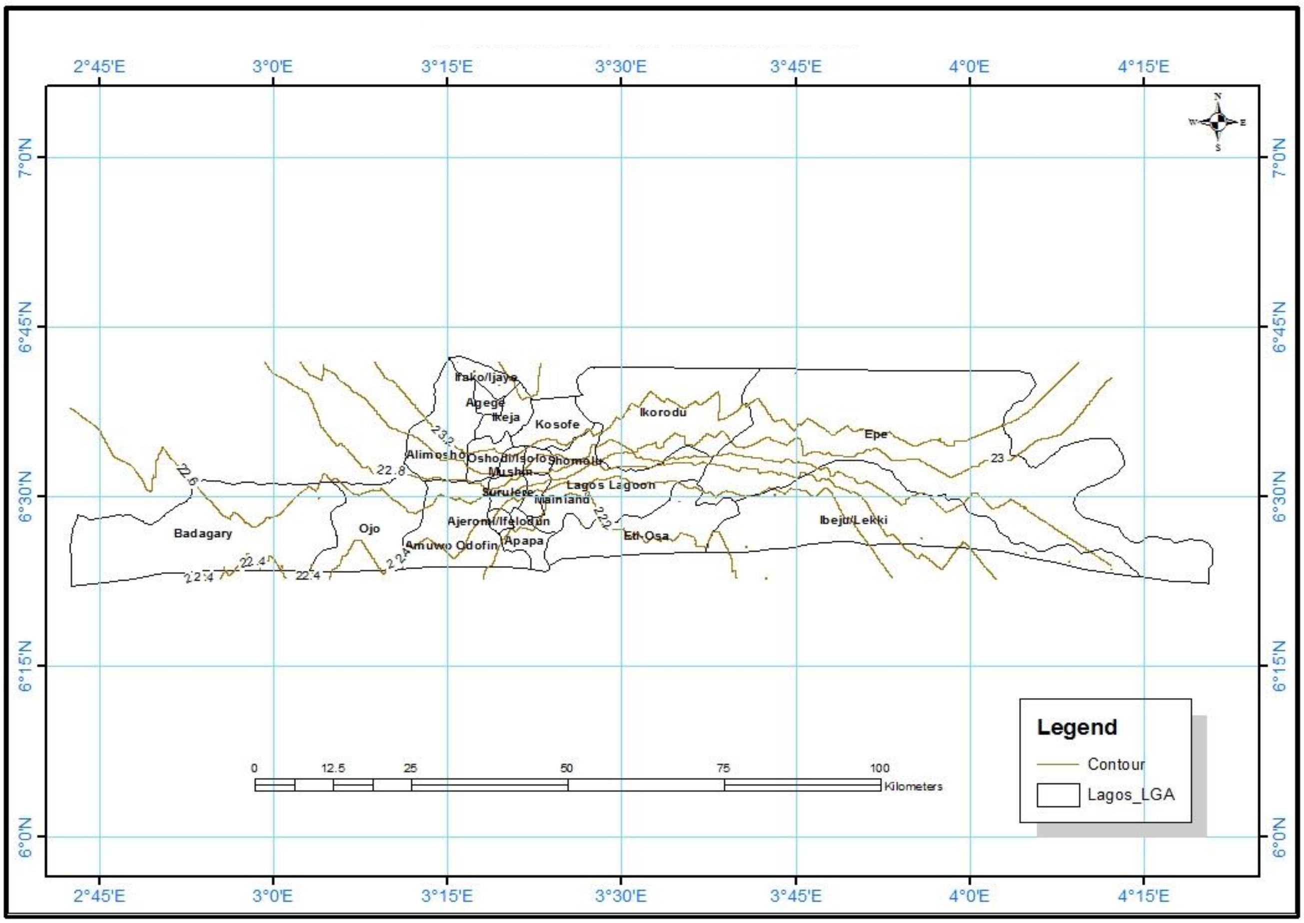


Figure 4.14c: Geoidal Map Plotted From GEM2008 Undulation for Lagos State (Source: Author, October, 2009)

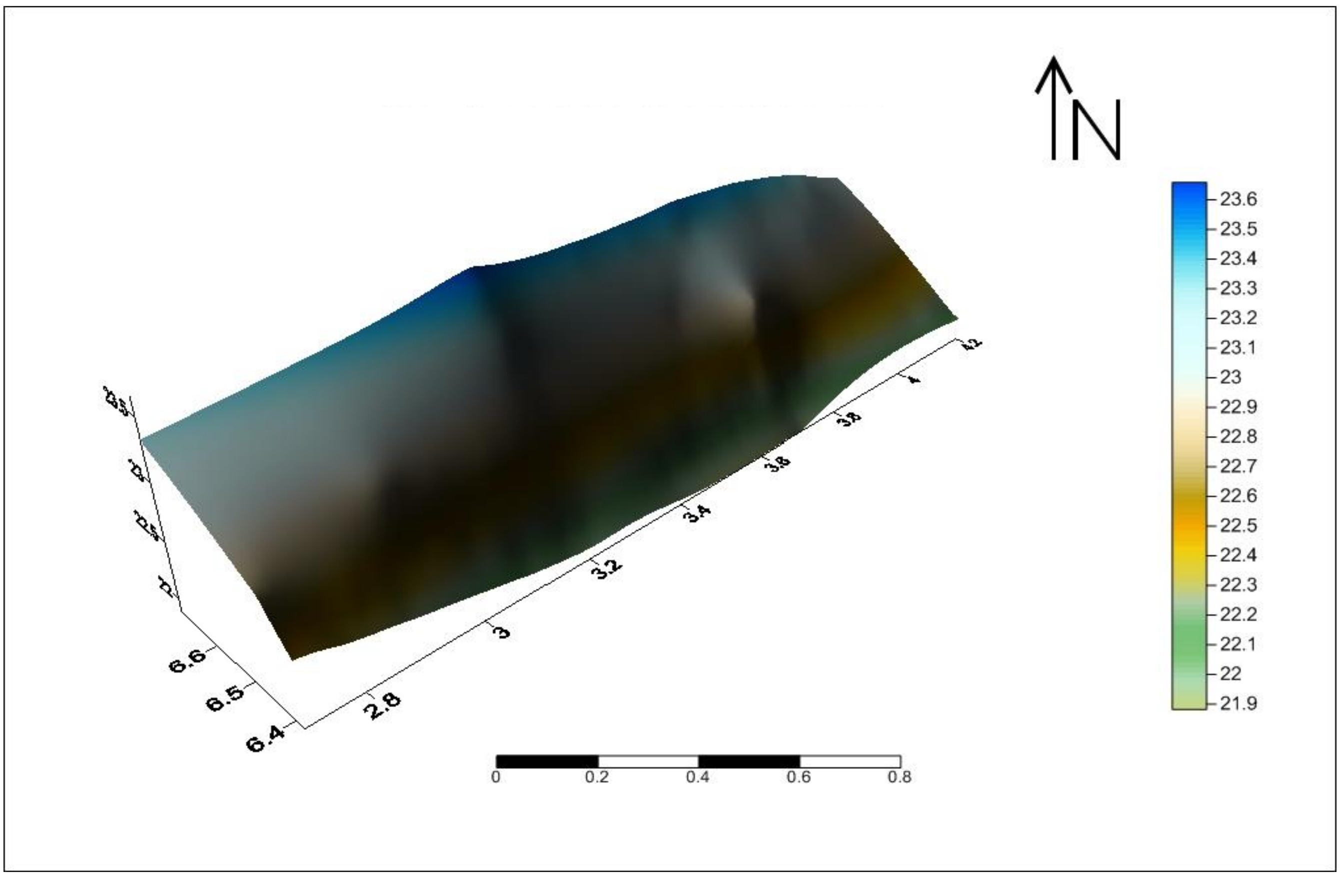


Figure 4.14d: Three Dimensional Surface Modelling Plotted From GEM2008 Undulation for Lagos State

(Source: Author, October, 2009)

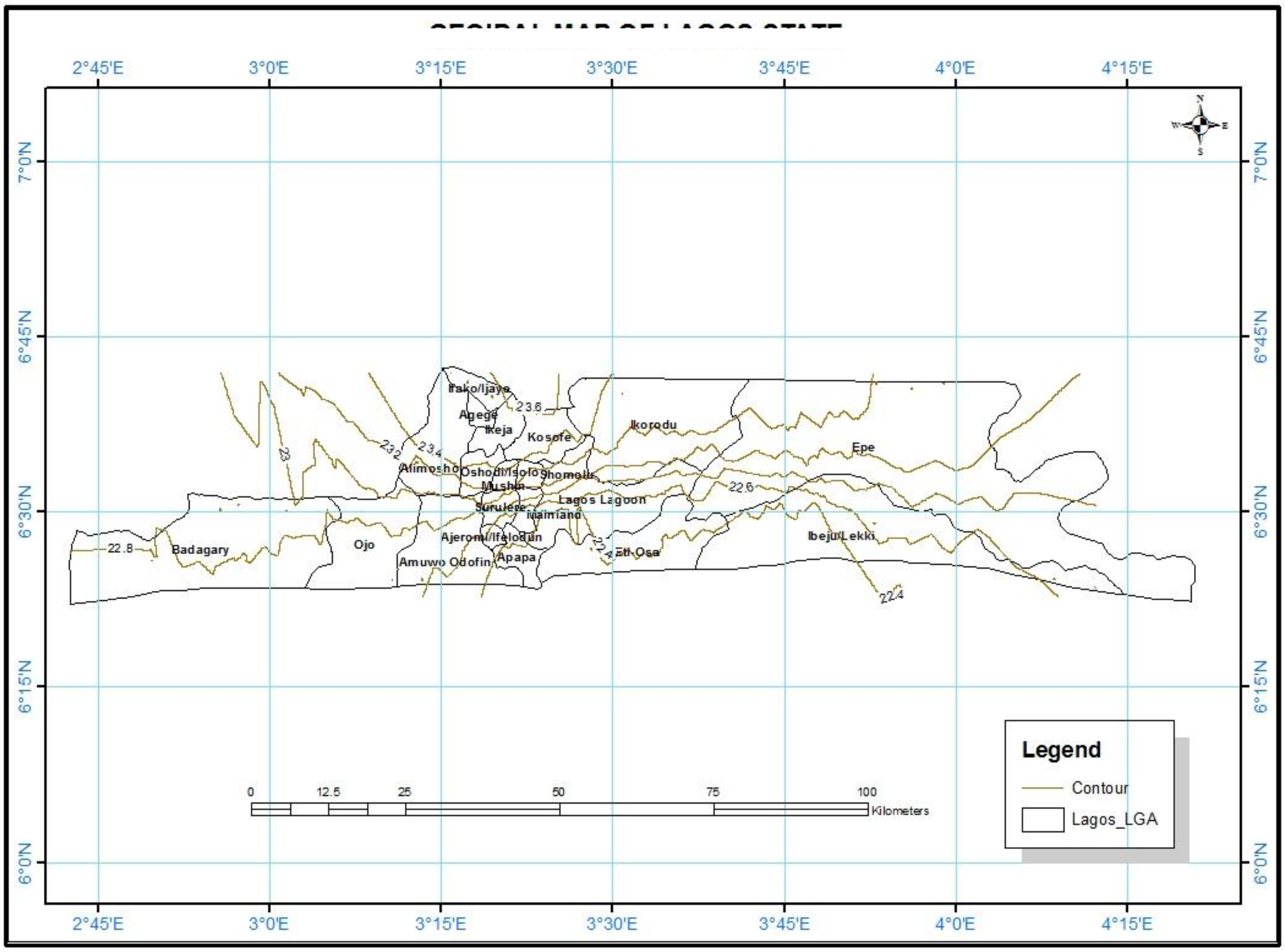


Figure 4.14e: Geoidal Map Plotted From Zanlentyik Hungarian Model for Lagos State

(Source: Author, October, 2009)

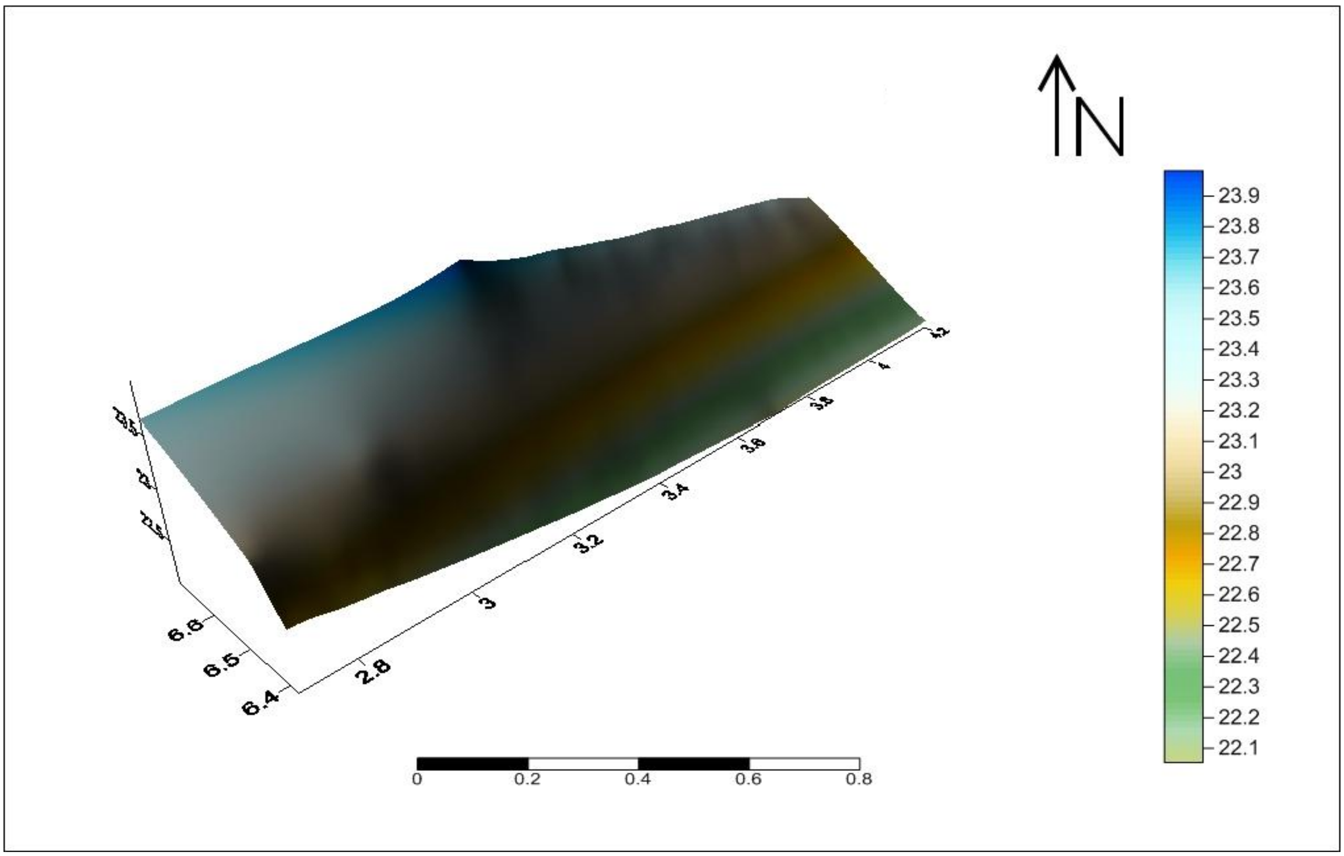


Figure 4.14f: 3D Surface Modelling Plotted From Zanlentyik Hungarian Model for Lagos State

(Source: Author, October, 2009)

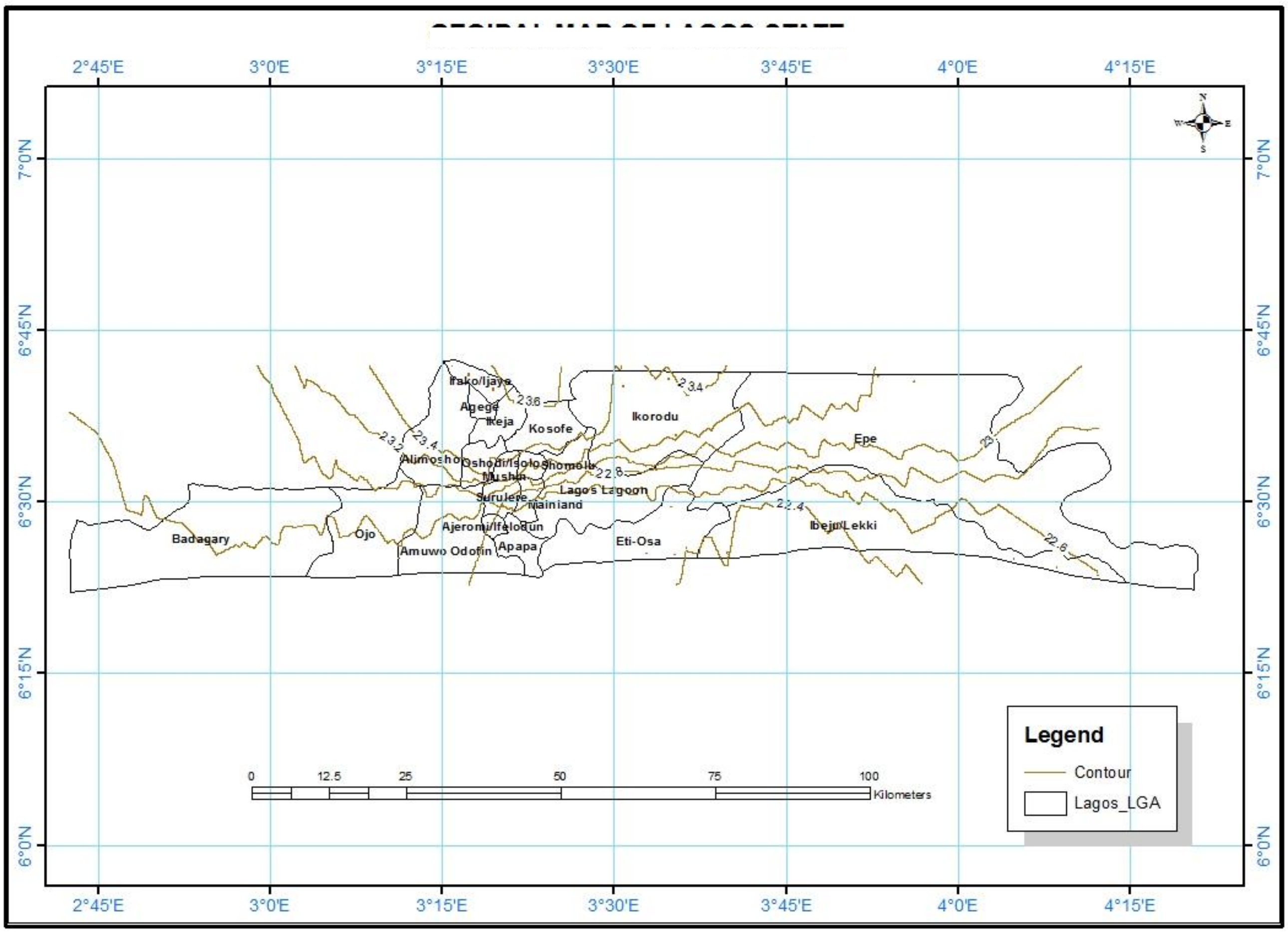


Figure 4.14g: Geoidal Map Plotted From North Sea Region Model for Lagos State
(Source: Author, October, 2009)

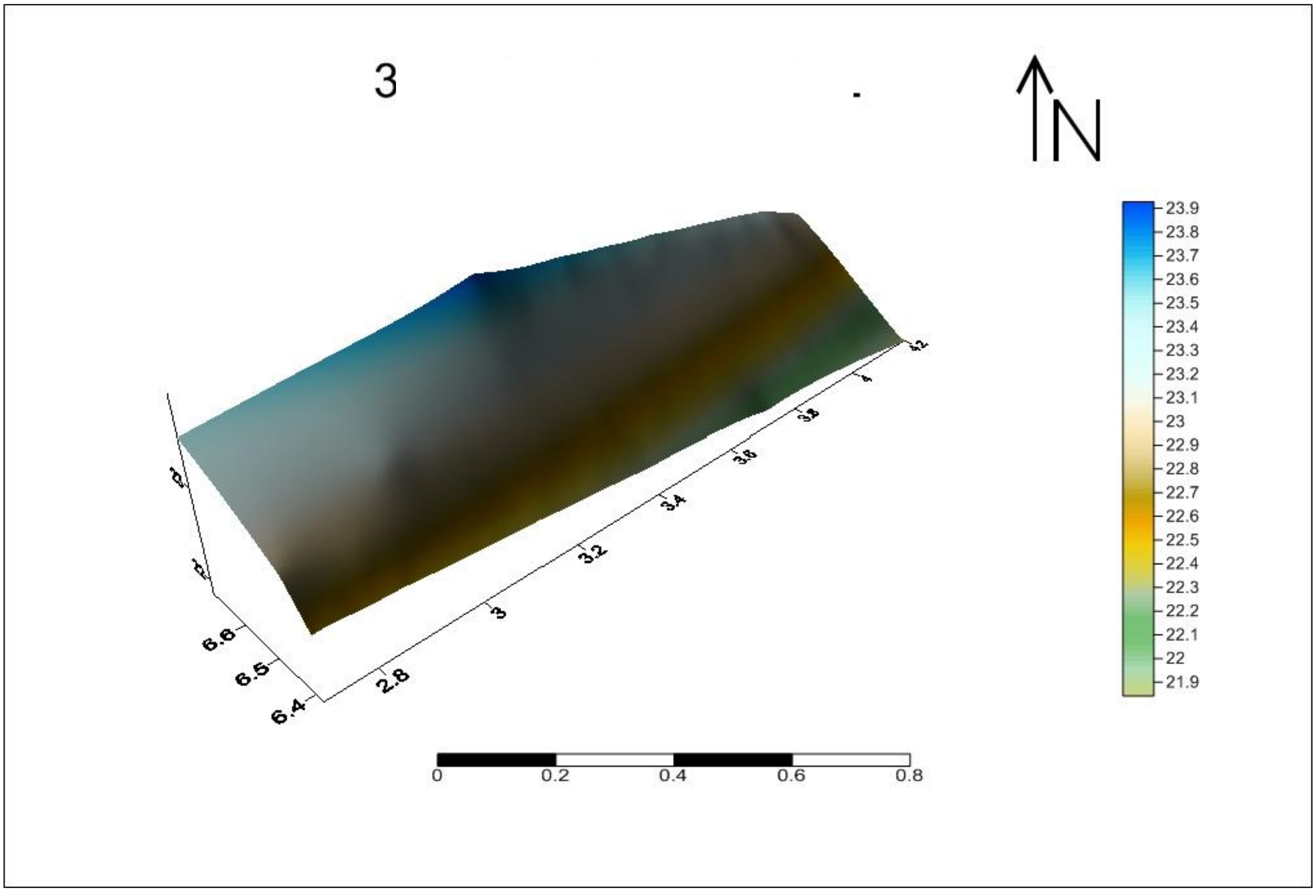


Figure 4.14h: 3D Surface Modelling Plotted From North Sea Region Model for Lagos State
(Source: Author, October, 2009)

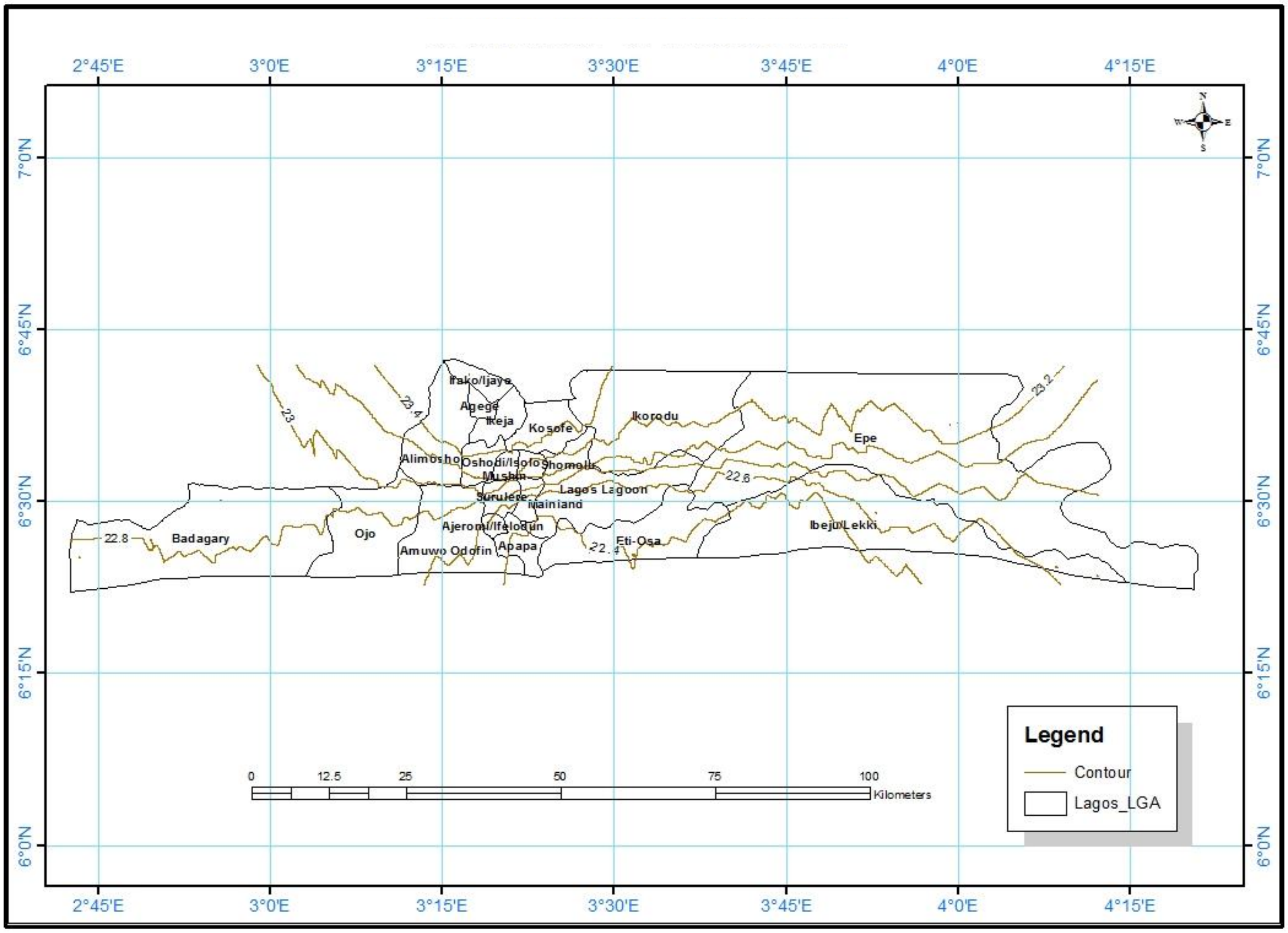


Figure 4.14i: : Geoidal Map Plotted From Spherical Satlevel Model for Lagos State
(Source: Author, October, 2009)

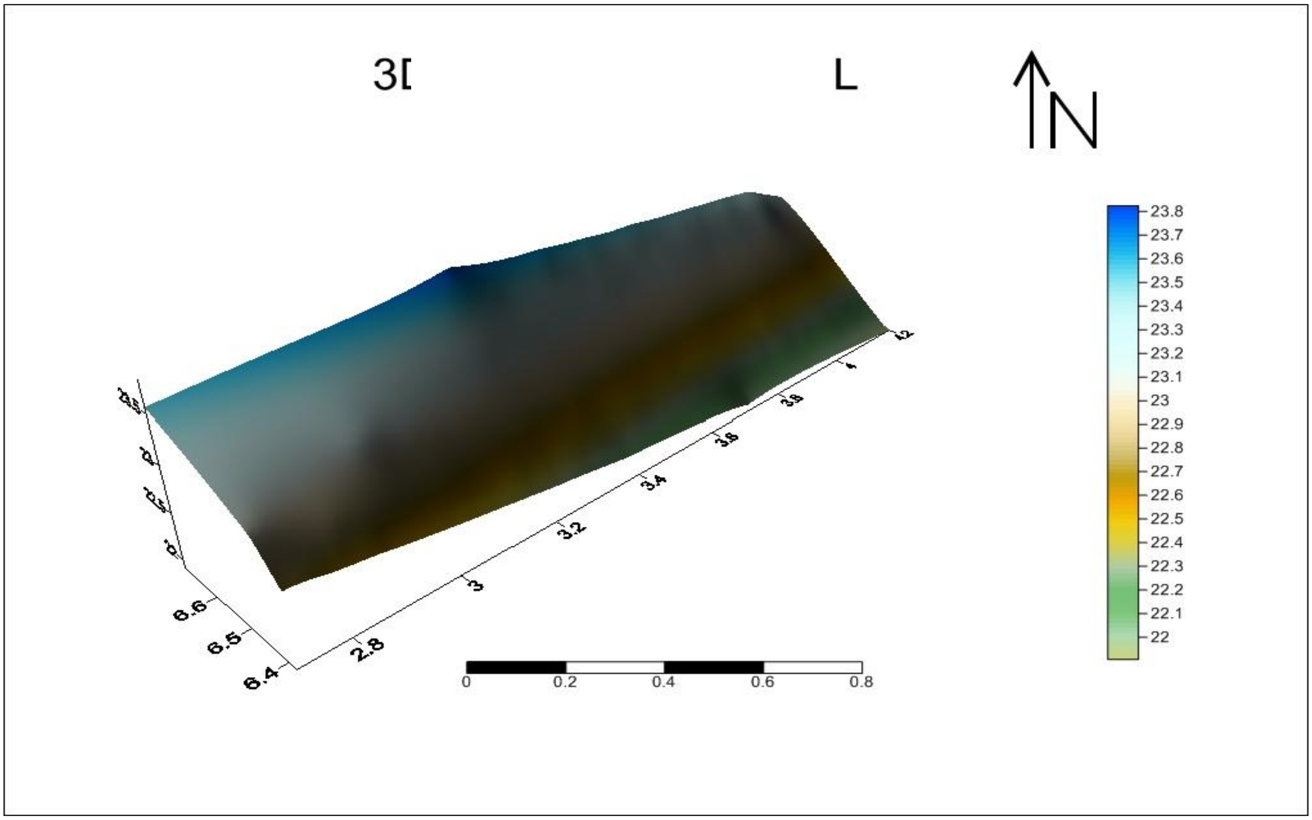


Figure 4.14j: 3D Surface Modelling Plotted From Spherical Satlevel Model for Lagos State
(Source: Author, October, 2009)

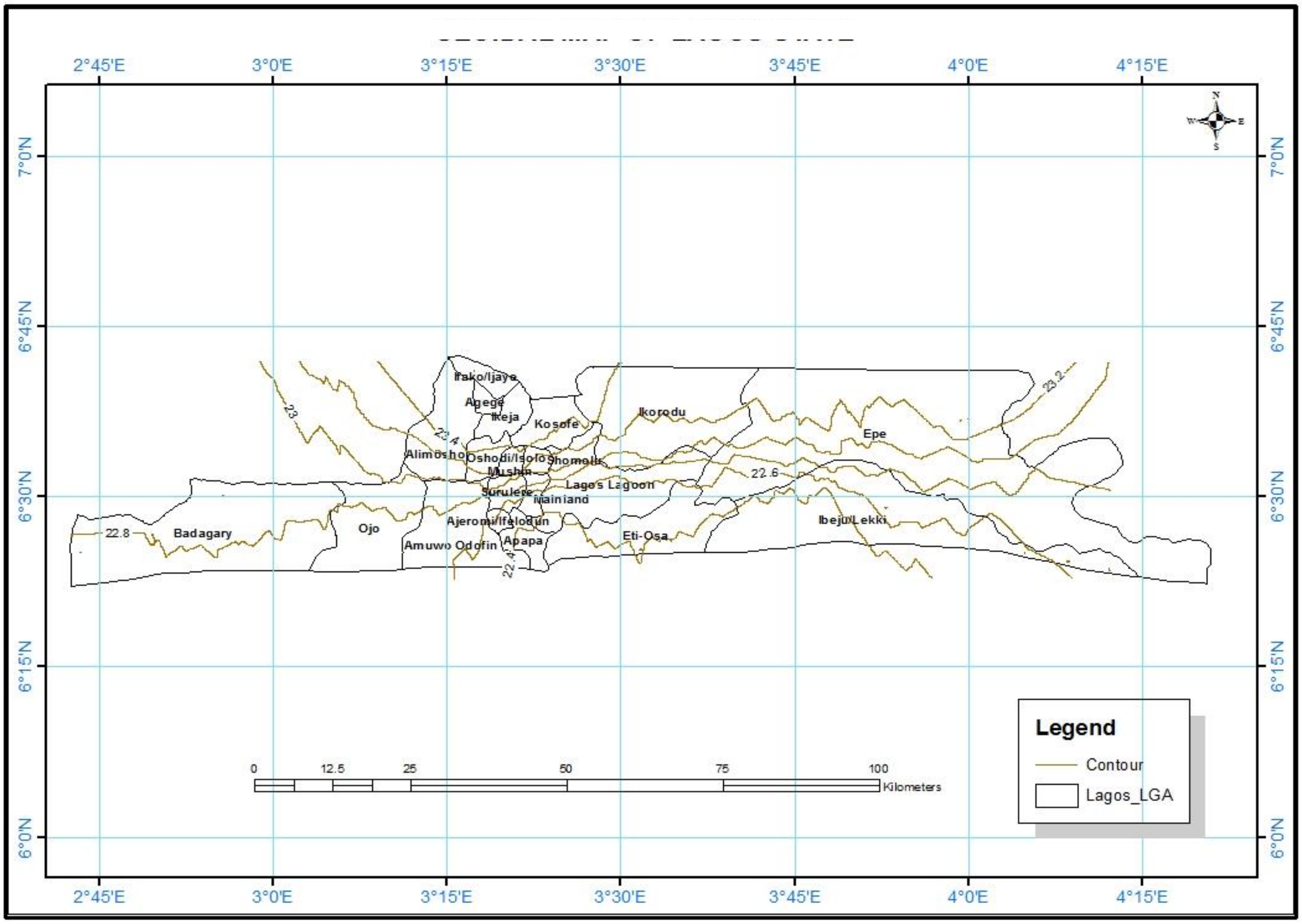


Figure 4.14k: 3D Surface Modelling Plotted From Rectagular Satlevel Model for Lagos State
(Source: Author, October, 2009)

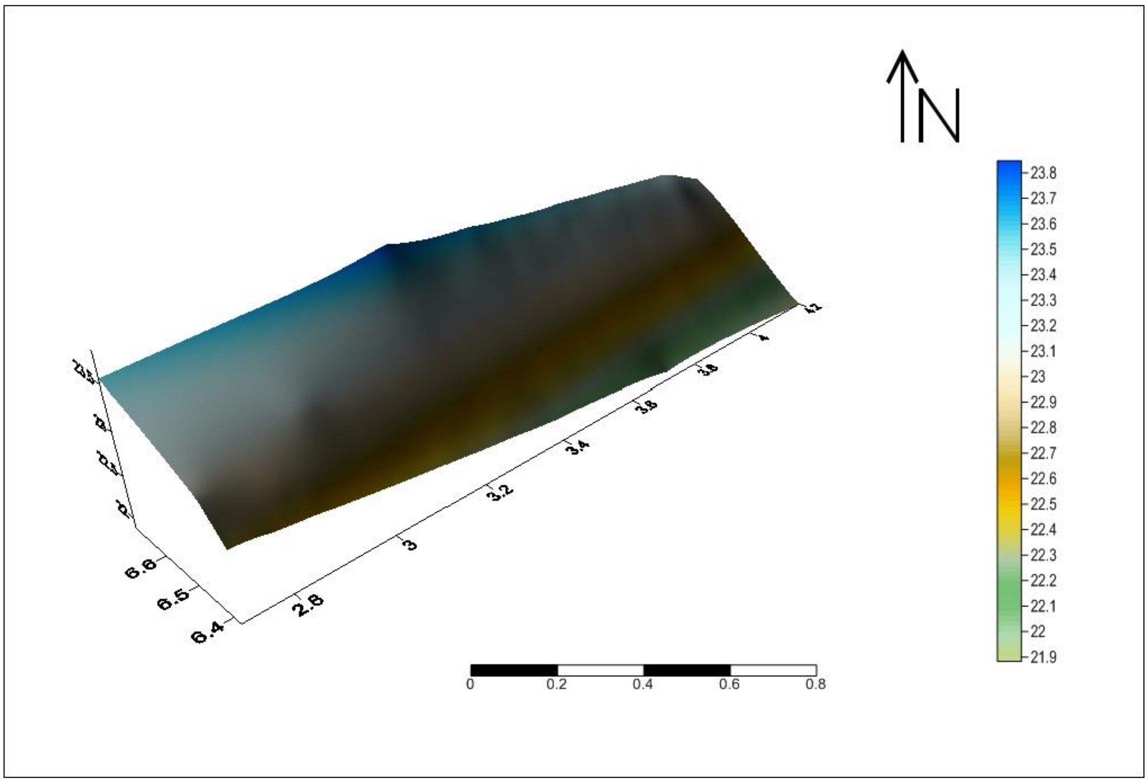


Figure 4.14l: 3D Surface Modelling Plotted From Rectagular Satlevel Model for Lagos State
(Source: Author, October, 2009)

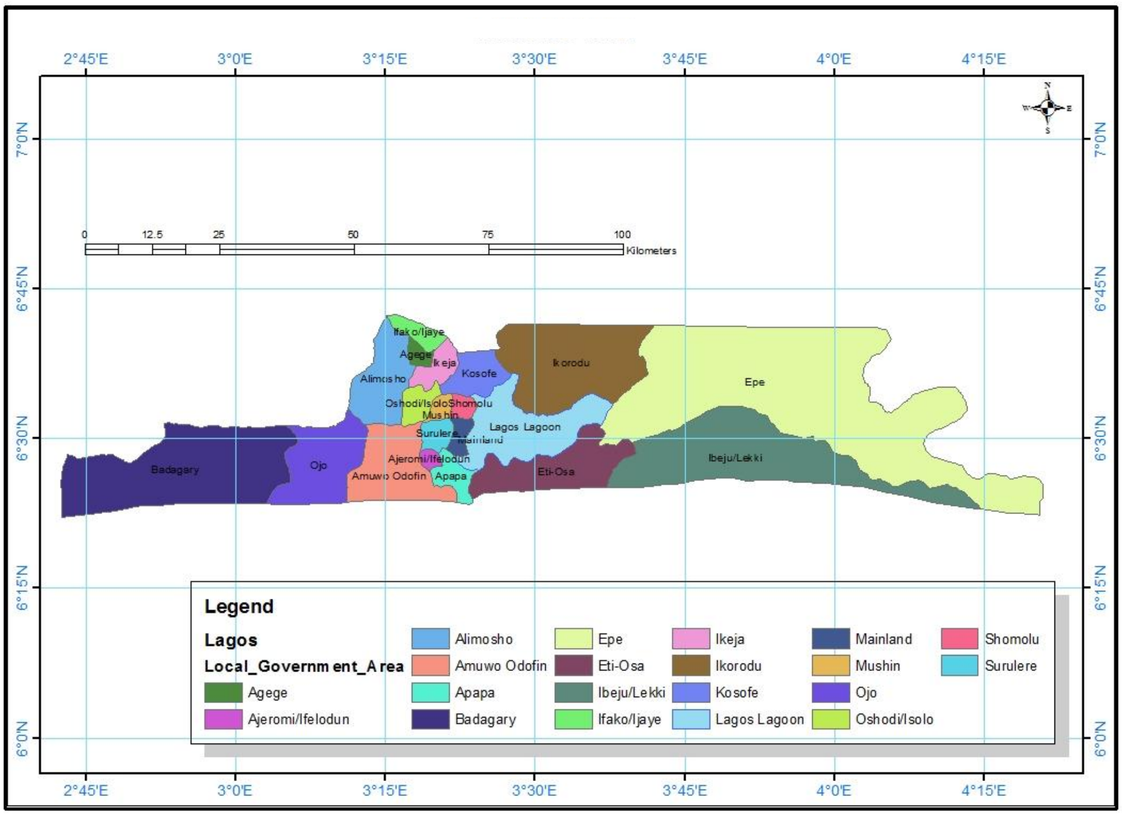


Figure 4.14m: Geoidal Map Plotted From Spherical Satlevel Model Overlaid on the Local Government Map of Lagos State (Source: Author, October, 2009)

4.20 DISCUSSIONS

4.21 Derivation of Optimal Empirical Geoidal Undulation models

Different geoid models have been developed by different authors in different locality using different approaches. Each of these approaches has its own advantages and disadvantages. For example, deterministic or classical approaches compute absolute geoid and require data all over the Earth to compute geoidal undulation. Some of these existing empirical models are given inconsistence results. Also, the fact that, there is no geoid model for Nigeria has made the Nigerian geodetic coordinates undefined uniquely. An attempt so solve the problem is derivation of new model. This research developed 'Satlevel' Collocation Models which has the advantage of using data format that is most common on maps and GNSS devices to produce Geoidal Undulation with accuracy comparable to most of the existing models (See Tables 4.10a and 4.10b). The models use four parameters to get precised results unlike the Zanletnyik Hungarian Polynomial Model which is over parameterised with 26 coefficients and still give inconsistency values as a result of deterioration of conditions of equations. Zanletnyik Hungarian Polynomial Model (Equation 2.27) in its original form did not fit Nigerian environment accurately, while 7-Parameter Similarity Datum Shift Model (Equation 2.26) also deviates for more than 2m from the observed Geoidal Undulation (See Tables 4.5a and 4.5b) in the study areas. These models fit where they were developed and tested. Therefore, model developed and fit in a particular locality may not necessarily fit in another place. 'Satlevel' Collocation Models developed in this research satisfied the necessary requirements in the study areas as shown from the statistics (Tables 4.15a and 4.15b). These Models fit Nigerian environment and was used in predicting the local Orthometric Heights for the study area.

4.22 **Adapting the Global geoid to Local Geoid**

The recent success in the determination of reliable geoid model for the entire Earth (global) resulted in Geopotential Earth Model (GEM). The latest version available to the public is GEM2008. It is easy to obtain GEM2008 since it is available on the INTERNET for all geographical location. GEM is global in nature; it generalizes the geoid in the locality. To use the Global geoid in local environment, it is always advisable to test the fit in the locality for accurate usage. With the Global geoid, opportunities now exist for a relative geoid determination from the global model. This enables the user to calculate the differences between the Global geoid and the local Geoidal Undulation. As earlier discussed, GEM96 a version of global geoid was used as long wavelength component in determination of Indonesian geoid (Heliani et al., 2004). GEM96 has a very low accuracy, but produced a good result, when combining with GTOPO. The similar approach is used here with 'Satlevel' collocation model. GEM2008 has sub-meter accuracy and was used as the long wavelength component in 'Satlevel' collocation model and therefore computes the differences between GEM2008 value and the values at the point of interest. The difference will then be applied to give the actual local value, thereby transforming the global undulation (GEM2008) to its local equivalents.

4.23 **Predicting the Local Orthometric Heights from GNSS Ellipsoidal Height**: The Geoidal Undulation computed from 'Satlevel' collocation model is on the same system with GNSS coordinates of which ellipsoidal height is a component. Ellipsoidal height observed with any GNSS receiver can be substituted in Equation (1.3) with Geoidal Undulation computed using 'Satlevel' collocation model to get the Orthometric Height. This becomes necessary because Orthometric Height is a natural height which the height users are always preferred.

4.24 The Use of GNSS Ellipsoidal Heights in place of Orthometric Heights for Engineering Applications and Other Purposes

Orthometric Height of two different benchmarks will serve the same purpose as the ellipsoidal height of the same points. When levelling operation is done between any two points, it is the difference in elevation that are observed and added to the reduced level of the benchmark to get that of the following point. If the value of the benchmark is Orthometric Height, the value of the other point automatically becomes the Orthometric Height too. Ellipsoidal height can equally be obtained by addition of height differences to the ellipsoidal height value of the benchmark. The value of the benchmark and difference in elevation determines the height system of the subsequent points. Though, Orthometric Height is governed by gravity, which has a negligible difference within a locality; it is almost constant in the coastal areas making it easy to substitute the ellipsoidal height differences for Orthometric Height differences. The average of differences between changes in elevation is computed to be 1.6mm with average mean square error of 2.38mm and average of differences between changes in elevation 3.1mm and average mean square error of 10.5mm over an average distance of 12.862km and 89.650km in Port Harcourt and Lagos State respectively. The work met the third order accuracy of $1.2mm\sqrt{K}$ where; K is kilometers is the requirement for engineering applications. These showed that ellipsoidal height differences can replace the Orthometric Height differences for engineering applications.

4.25 Easier Way for the Users to Get Orthometric Height from GNSS Ellipsoidal Height than the Manual Computation

Stoke's function and other existing models are readily available but the requirement of gravity data all the earth is one of the major problems for the implementation of the model. This is a major challenge to the surveyors and other height users who are in need of using the geoid information for their activities. A user interactive program will make necessary information available to the users, such that the user will be able to implement the program with little effort. Problems experienced with regard to the number of decimal places causing rounding errors in manual computation were solved in this software as data were stored in the computer memory location to eliminate the copying error. It is on the basis of accessibility and availability that this research developed a user-interactive program that computes the Geoidal Undulation and transforms the global Orthometric Height to its local equivalent.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 CONCLUSION

In this study, levelled heights were established along with GPS observation in some parts of Port Harcourt metropolis to model the geoid in the study area. Some of the benchmarks have been coordinated and collocated with both GPS and Geodetic levelling in Port Harcourt metropolis. This thesis also developed optimal predictive geoid models (Spherical 'Satlevel' and Rectangular 'Satlevel') for deriving Orthometric Height from ellipsoidal heights on WGS 84 reference ellipsoid. Analysis were done and the computed residuals as tabulated in Tables 4.11a and 4.11b show that there are no significant differences between the values obtained with the derived modelled, existing and the observed values. Some other existing methods such as: North Sea Region Model, Zanletnyik Hungarian Polynomial model fitting and GEM2008 were used to estimate the geoid of the study areas, from which the Orthometric Heights were computed. The results compared favourably with the new models. Some of the models fit perfectly with the existing models but 'Satlevel' has the following advantages:

- The computational process is less cumbersome when using the 'Satlevel' than in Zanletnyik Hungarian model.
- The mean corrected observation can be applied to Spherical 'Satlevel' to take care of inversion of large numbers for normal matrices when using Least Squares to determine the coefficients.
- 'Satlevel' Collocation takes data format of GNSS

Among the new models, Spherical 'Satlevel' is the best because it satisfies all the tests performed. The coefficient of determination, corresponding moment correlation coefficient, and F Distribution showed that the models satisfied 95% confidence in the study areas. Other results were shown in Tables 4.15a.and 4.15b. Therefore, the new 'Satlevel' geoid models developed in Nigeria could meet the requirements of potential users for converting GNSS heights into their corresponding Mean Sea

Level heights. The fact that geographic coordinates (global) are the input is ‘Satlevel’ collocation is another advantage.

Also, ellipsoidal height differences and Orthometric Height differences were compared. The average between the two height differences were 1.6mm and 3.1mm in Port Harcourt and Lagos State respectively. The average of the Mean Square Error of the two height differences was 2.382mm in Port Harcourt metropolis. Therefore, there is no significant difference between the elevation obtained from ellipsoidal height differences and Orthometric Height differences for any two points in Port Harcourt metropolis. This shows that GPS ellipsoidal height differences can be substituted for Orthometric Height differences in the Port Harcourt metropolis.

‘Satlevel’ collocation was developed to provide a cheap and convenient way to obtain Orthometric Height. It also adapts the Global (GEM2008) Orthometric Heights to their local equivalent. The shift in reference model GEM96 to GEM2008 as observed by Roman (2009) on the Geoid of United States of America has been corrected. This has corrected the geoid in the study area in Nigeria using ‘Satlevel’ collocation models.

From the analysis as observed in Figure 4.9a, it can be concluded that GEM2008 fits perfectly in the Port Harcourt Coastal area of Nigeria and therefore adapted for Orthometric Height with the use of ‘Satlevel’ Collocation Models developed in this research.

Evidently, there is a need for the incorporation of a corrector surface function to model the local discrepancies in the Nigerian height system. The present situation where; different height systems are scattered all over the country is unprofessional, unacceptable and therefore should be discouraged. This will involve the use of different models as correcting factor in different parts of the country. This task may require collocated GNSS and Spirit levelling observations. ‘Satlevel’ collocation has satisfied the necessary requirements in the study area and therefore suitable for geodetic applications.

5.2 **SUMMARY OF FINDINGS:** The findings are tabulated against each of the objectives as shown below:

Table 4.16: Summary of Findings

Objectives	**Findings**
1. To derive optimal empirical Geoidal Undulation-models for transforming Global undulation to local values.	i. Satlevel' Collocation model was found to be optimal models for transforming Global undulation to its local equivalent.
2. To compute the local Orthometric Heights from GNSS ellipsoidal height.	i. Satlevel' Collocation model compute the local Orthometric Heights from GNSS ellipsoidal heights. The results compared favourably with the observed values.
3. To compare ellipsoidal height differences with Orthometric Heights differences.	i. Ellipsoidal and Orthometric Heights differences have been compared. The differences between the two are neglible and therefore can be substituted for each other.
4. To validate the adequacy of the developed models on some data sets.	i. Explanatory variables made significant contributions to the variability of Geoidal Undulation ii. Statistical analysis of the model satisfied 95% significant level of the goodness of fit in the study areas.
5. To develop a user friendly software for computation of Geoidal Undulation.	i. A user-friendly interactive program called 'Orthometric Height on fly' was developed to compute the local Geoidal Undulation and Orthometric Height from "Satlevel' collocation model. ii. There is no significant difference between the

	results obtained from manual computation and the results obtained from the computer program

5.3 CONTRIBUTIONS TO KNOWLEDGE

The contributions to knowledge are as follows:

1. A predictive geoid model for transforming the global Orthometric heights to local Orthometric heights was developed.
2. The thesis established that ellipsoidal Height differences can substitute Orthometric heights differences over a non-rolling terrain.
3. A user-friendly interactive software for computing Orthometric heights from observed geodetic Coordinates was developed.

5.4 RECOMMENDATIONS

1. T The work is therefore recommended to be used for transformation of Global Orthometric to local height

2. It is recommended that the Office of the Surveyor General of the Federation should commence work on the re-observation of Nigeria Vertical control network to be integrated with GNSS observation so that the general geoid model for the entire country can be determined. This is to correct some of the inadequacies in Nigerian Geodetic Network which can be corrected with geoid model as ‘Satlevel’ provides a quick method for geoid determination. Availability of data throughout the country will enhance effective use of this method and other researches.

3. The Nigerian Vertical Control network needs to be properly integrated into continental based datum as planned by the African Geoid Project (AGP) which will be integrated to global vertical datum. Such unification is dependent on the proper determination of the geoid.

4. It is recommended that the area of coverage of the data be extended as funds become available so that improvement can be made on the models. It will be necessary to carry differential levelling to other parts of Rivers state and into other parts of Nigeria to serve the need of surveyors and engineers.

5. The office of Surveyor General of the Federation should set all machinery in motion to set up tidal observation gauge stations, so that changes in Mean Sea Level can be detected in view of global changing climatic and weather conditions.

6. The geoid can be improved in the study area close to the sea with determination of the geoid on the Atlantic Ocean using the satellite altimetric data tied to on-shore tide-gauge locations. This in turn will improve the overall precision of the calibration process for the tide gauge.

7. The gravimetric method requires the availability of gravity measurements from all over the Earth, with good spatial distribution; Gravity data is not available in Nigeria. Therefore there is a need to establish a good gravity network in Nigeria.

8. The current situation in Nigeria is the determination of geoid by individual state. There is a need to be properly integrated the geoid from various states in Nigeria to have a uniform geoid for the country. It is therefore recommended that each state should have the geoid extended to neighbouring state so as to make integration smooth.

9. Furthermore, the Office of the Surveyor General of the Federation (OSGoF) should as a matter of urgency embark on the determination of a National Geoid Model in order to stem the current trend in Nigeria whereby each state is determining their own geoid model.

5.5 Suggestion for Further Studies

The astrogeodetic method of geoid determination is NOT investigated. It is therefore suggested for further studies.

A combination of different models in a uniform manner across the country is suggested for further studies so as to aid effective use of Orthometric Height.

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LIST OF APPENDICES

APPENDIX A

Sample of Microsoft Excel Worksheet used for the Computation of Spherical 'Satlevel' Collocation Model in Port Harcourt

STATIONS	LATITUDE	LONGITUDES	ELLIPSOIDAL	ORTHOMETRI	GEOID(N)
AP4	4.868335803	6.989905397	35.849	16.92611	18.92289
AP1	4.869537347	6.977927531	33.72	14.80812	18.91188
PT.3 EMM/	4.790218708	7.00227435	25.195	6.22827	18.96673
PHCS 1s	4.772389314	7.013525022	30.796	11.798	18.998
PT.4 EMM/	4.798391819	7.005574083	30.693	11.69056	19.00244
PT.8 EMM/	4.833761764	7.007032608	26.789	7.8509	18.9381
PT.4 ABDL	4.837173481	7.022857481	32.842	13.8392	19.0028
PT.5 EMM/	4.806938314	7.009407025	29.374	10.3801	18.9939
PT.7 EMM/	4.823872525	7.006017658	33.379	14.37161	19.00739
PT.9 EMM/	4.836566356	7.015292797	29.141	10.16598	18.97502
PT.2 ABDL	4.844335522	7.039518178	32.64	13.65394	18.98606
PT.3 ABDL	4.840752114	7.031318094	26.75	7.76967	18.98033
GPS 02	4.988341858	7.005441514	42.542	23.638	18.904
GPS 03	4.981133603	6.949840522	40.065	21.24	18.825
GPS 04	4.972244803	6.951180808	38.771	19.938	18.833
GPS 05	4.988165797	6.959676808	41.357	22.523	18.834
GPS 06	4.976870211	6.950525386	39.485	20.657	18.828
GPS 07	4.968417417	6.950765697	38.351	19.516	18.835
GPS 08	4.956065461	6.949389547	36.427	17.585	18.842
GPS 09	4.95495015	6.947081147	34.627	15.787	18.84
GPS 10	4.953781161	6.944284003	36.819	17.983	18.836
GPS 11	4.978015694	6.968921853	38.155	19.301	18.854
GPS 12	4.976619567	6.970370336	39.661	20.804	18.857
GPS 13	4.975173192	6.971955836	40.589	21.728	18.861
GPS 14	4.953134586	6.950453306	35.359	16.514	18.845
GPS 15	4.949708683	6.952838769	34.766	15.915	18.851
GPS 16	4.946587319	6.955108775	34.756	15.9	18.856
GPS 17	4.943006336	6.957377311	34.79	15.929	18.861
GPS 18	4.939244417	6.957961819	34.784	15.919	18.865
GPS 19	4.893158592	6.964717458	29.266	10.362	18.904
GPS 20	4.89404995	6.964342617	29.87	10.967	18.903
GPS 21	4.893297169	6.966278353	30.338	11.432	18.906
GPS 22	4.875097889	6.955985178	32.335	13.428	18.907
GPS 23	4.875640256	6.954831264	33.256	14.351	18.905
GPS 24	4.873833222	6.955013361	33.065	14.158	18.907
GPS 25	4.876598708	6.952834056	33.532	14.63	18.902
GPS 26	4.832460906	6.945637275	20.18	1.25	18.93
GPS 27	4.832444461	6.9448869	19.557	0.627	18.93
GPS 28	4.832327742	6.944121753	20.699	1.77	18.929
GPS 29	4.836480189	6.928271461	20.239	1.326	18.913
GPS 30	4.837388344	6.928477733	20.984	2.072	18.912
GPS 31	4.838183467	6.929087211	23.319	4.407	18.912
GPS 32	4.940823194	7.007985167	37.527	18.592	18.935
GPS 33	4.942280164	7.008015719	38.369	19.435	18.934
GPS 34	4.943984306	7.007760989	39.567	20.634	18.933
GPS 35	4.930137067	7.052698958	40.67	21.666	19.004
GPS 36	4.931735783	7.052849775	40.87	21.867	19.003
GPS 37	4.935097586	7.053556919	38.757	19.753	19.004
GPS 38	4.890883953	7.076113975	34.478	15.431	19.047
GPS 39	4.892411842	7.076911742	36.043	16.995	19.048
GPS 40	4.8946095	7.07747475	37.128	18.08	19.048
GPS 41	4.862920831	7.093361511	37.962	18.886	19.076

GPS 42	4.863447247	7.095125922	38.177	19.099	19.078
GPS 43	4.863901311	7.09699115	36.294	17.214	19.08
GPS 45	4.833776561	7.127300578	33.432	14.311	19.121
GPS 46	4.835730717	7.127621192	31.881	12.759	19.122
GPS 47	4.769962542	7.140300147	32.793	13.653	19.14
GPS 48	4.769413628	7.141166558	33.017	13.877	19.14
GPS 49	4.7683703	7.14278445	33.822	14.68	19.142
GPS 50	4.912119492	6.985296881	35.117	16.199	18.918
GPS 51	4.913761719	6.984875258	35.499	16.582	18.917
GPS 53	4.807930044	6.977191642	29.078	10.102	18.976
GPS 54	4.807218517	6.976286997	29.336	10.36	18.976
GPS 55	4.806990144	6.977222258	29.173	10.197	18.976
GPS 56	4.781655028	7.006075439	28.033	9.015	19.018
GPS 57	4.782321533	7.005458108	27.536	8.519	19.017
GPS 58	4.783296731	7.005240433	27.441	8.425	19.016
GPS 59	4.916896858	6.880102978	20.494	1.703	18.791
GPS 60	4.91610835	6.881154569	20.982	2.189	18.793
XSV 662	4.873506919	6.99841315	27.603	8.648	18.955
ZVS 3003	4.847971022	7.047811589	32.308	13.282	19.026
	346.5339571	496.851457	2318.499	973.29246	1345.20654
	4.880759959	6.997907846	32.65491549	13.70834451	18.94657099

DESIGNED MATRIX

1	2.010816977	0.243811871	0.17159744
1	2.01079428	0.243396451	0.171635729
1	2.010732223	0.244227238	0.168835701
1	2.010730369	0.244614464	0.168208379
1	2.010750394	0.244343165	0.169126244
1	2.010803045	0.244400018	0.170379181
1	2.010840236	0.244949697	0.170505611
1	2.0107702	0.24447766	0.169430207
1	2.010787047	0.244363051	0.170028639
1	2.010823892	0.24468712	0.170481425
1	2.010884506	0.245529026	0.170765175
1	2.010862624	0.245243883	0.17063535
1	2.011021027	0.244372633	0.175853658
1	2.010897458	0.242441765	0.17557808
1	2.010887281	0.242486673	0.17526371
1	2.01092762	0.242784413	0.175830746
1	2.010892659	0.242464763	0.175427311
1	2.010880895	0.242471575	0.17512799
1	2.01086024	0.242421587	0.174689988
1	2.010853951	0.242341273	0.174649653
1	2.010846597	0.242243987	0.174607241
1	2.010931653	0.243103417	0.175474554
1	2.010932572	0.243153432	0.175425624
1	2.010933697	0.243208193	0.175374965
1	2.010858164	0.242457977	0.174586561
1	2.010858055	0.242540147	0.174466078
1	2.010858157	0.242618365	0.174356341
1	2.010857598	0.242696449	0.174230323
1	2.010853366	0.242716057	0.174097293
1	2.010801032	0.242942248	0.172467532

1	2.010801543	0.242929399	0.172498965
1	2.010804402	0.24299644	0.172472996
1	2.010757579	0.242636004	0.171824847
1	2.010756008	0.242596057	0.171843644
1	2.010753809	0.242602056	0.171779719
1	2.010753319	0.242526918	0.171876875
1	2.01067623	0.242269379	0.170311462
1	2.010674685	0.242243337	0.170310616
1	2.01067297	0.242216764	0.170306215
1	2.010646735	0.241667439	0.170447675
1	2.010648434	0.241674756	0.170479901
1	2.010650789	0.241696047	0.170508267
1	2.010957495	0.244452255	0.174171265
1	2.010959655	0.244453579	0.174222885
1	2.010961589	0.244445048	0.174283157
1	2.011033758	0.246001801	0.173808956
1	2.011036364	0.246007324	0.173865644
1	2.011042648	0.246032472	0.173984992
1	2.011025822	0.246807056	0.172426985
1	2.011029644	0.24683501	0.172481395
1	2.011033938	0.246854941	0.172559445
1	2.011021713	0.247400357	0.171442731
1	2.011026106	0.247461661	0.171462014
1	2.011030607	0.24752645	0.17147877
1	2.011050885	0.248572427	0.170422671
1	2.011054303	0.248583901	0.170492002
1	2.010988659	0.249011893	0.168167212
1	2.0109897	0.249041847	0.168148081
1	2.010991618	0.249097777	0.168111708
1	2.010869999	0.243659782	0.173146407
1	2.010871492	0.243645446	0.17320442
1	2.01070585	0.24336003	0.169453992
1	2.01070301	0.243328515	0.169428482
1	2.010704594	0.243360927	0.169420724
1	2.010728039	0.24435762	0.168533828
1	2.010727707	0.244336317	0.168557209
1	2.010728622	0.244328936	0.168591661
1	2.01066407	0.240009839	0.173278091
1	2.010665051	0.2400462	0.173250542
1	2.010841677	0.244108007	0.171783601
1	2.010906675	0.245817414	0.170896887

INVERSE OF DESIGNED MATRIX

1	1	1	1	1	1	1	1	1	1
2.010817	2.01079428	2.010732223	2.010730369	2.010750394	2.010803045	2.010840236	2.0107702	2.010787047	2.010824
0.243812	0.243396451	0.244227238	0.244614464	0.244343165	0.244400018	0.244949697	0.24447766	0.244363051	0.244687
0.171597	0.171635729	0.168835701	0.168208379	0.169126244	0.170379181	0.170505611	0.169430207	0.170028639	0.170481

1	1	1	1	1	1	1	1	1	1	1
2.010884506	2.01086	2.011021	2.0108975	2.01088728	2.0109276	2.0108927	2.0108809	2.0108602	2.010854	2.0108466
0.245529026	0.24524	0.2443726	0.2424418	0.24248667	0.2427844	0.2424648	0.2424716	0.2424216	0.2423413	0.242244
0.170765175	0.17064	0.1758537	0.1755781	0.17526371	0.1758307	0.1754273	0.175128	0.17469	0.1746497	0.1746072

1	1	1	1	1	1	1	1	1	1	1
2.0109317	2.0109326	2.0109337	2.0108582	2.0108581	2.0108582	2.0108576	2.0108534	2.010801	2.0108015	2.0108044
0.2431034	0.2431534	0.2432082	0.242458	0.2425401	0.2426184	0.2426964	0.2427161	0.2429422	0.2429294	0.2429964
0.1754746	0.1754256	0.175375	0.1745866	0.1744661	0.1743563	0.1742303	0.1740973	0.1724675	0.172499	0.172473

1	1	1	1	1	1	1	1	1	1	1
2.0107576	2.010756	2.0107538	2.0107533	2.0106762	2.0106747	2.010673	2.0106467	2.0106484	2.0106508	2.0109575
0.242636	0.2425961	0.2426021	0.2425269	0.2422694	0.2422433	0.2422168	0.2416674	0.2416748	0.241696	0.2444523
0.1718248	0.1718436	0.1717797	0.1718769	0.1703115	0.1703106	0.1703062	0.1704477	0.1704799	0.1705083	0.1741713

1	1	1	1	1	1	1	1	1	1	1
2.0109597	2.0109616	2.0110338	2.0110364	2.0110426	2.0110258	2.0110296	2.0110339	2.0110217	2.0110261	2.0110306
0.2444536	0.244445	0.2460018	0.2460073	0.2460325	0.2468071	0.246835	0.2468549	0.2474004	0.2474617	0.2475265
0.1742229	0.1742832	0.173809	0.1738656	0.173985	0.172427	0.1724814	0.1725594	0.1714427	0.171462	0.1714788

1	1	1	1	1	1	1	1	1	1	1
2.0110509	2.0110543	2.0109887	2.0109897	2.0109916	2.01087	2.0108715	2.0107058	2.010703	2.0107046	2.010728
0.2485724	0.2485839	0.2490119	0.2490418	0.2490978	0.2436598	0.2436454	0.24336	0.2433285	0.2433609	0.2443576
0.1704227	0.170492	0.1681672	0.1681481	0.1681117	0.1731464	0.1732044	0.169454	0.1694285	0.1694207	0.1685338

1	1	1	1	1	1
2.0107277	2.0107286	2.0106641	2.0106651	2.0108417	2.0109067
0.2443363	0.2443289	0.2400098	0.2400462	0.244108	0.2458174
0.1685572	0.1685917	0.1732781	0.1732505	0.1717836	0.1708969

NORMAL MATRIX

71	142.7704956	17.33050803	12.21487367
142.7705	287.0903449	34.84910026	24.56231099
17.33051	34.84910026	4.230534555	2.981421987
12.21487	24.56231099	2.981421987	2.101820343

INVERSE OF NORMAL MATRIX

7.22E+10	-36265361976	2126034826	1445764196
-3.6E+10	18226696991	-1068529634	-726630603
2.13E+09	-1068529634	62645807.25	42599636.94
1.45E+09	-726630603	42599636.95	28971239.37

U-VECTOR

1345.20654
2705.011628
328.3654295
231.4202208

SOLUTION VECTOR

12559.38861
-6305.379486
402.0375862
236.0263758

DATUM	RESIDUES
18.9476	0.024661977
18.9327	0.020808935
18.9971	0.030376976
19.0164	0.01841168
18.9977	-0.004724941
18.9843	0.046214747
19.0006	-0.002156271
18.9986	0.004746717
18.9876	-0.019803866
18.9924	0.017402843
19.0157	0.02961873
19.0084	0.028044692
18.891	-0.013036808
18.8288	0.003792157
18.8368	0.003813236
18.836	0.002001366
18.8327	0.004710152
18.839	0.003976195
18.8457	0.003736596
18.8436	0.003581218
18.8408	0.004832009
18.8548	0.000751581
18.8575	0.000519757
18.8605	-0.00051671
18.849	0.004049744
18.8543	0.003330626
18.8592	0.003234534
18.8644	0.003409428
18.8676	0.002579897
18.9038	-0.000168518
18.9029	-0.00013229
18.9057	-0.000337211
18.903	-0.003990387
18.9013	-0.003710537
18.9025	-0.004519677
18.8983	-0.003705428
18.9114	-0.018646977
18.9104	-0.019579196
18.9095	-0.019484693
18.8875	-0.025524044
18.8873	-0.024686984
18.8877	-0.02428114
18.9265	-0.008517812
18.9256	-0.008422462
18.9242	-0.008820674
18.9831	-0.020925081
18.9822	-0.020756993
18.9809	-0.023101848
19.0307	-0.01632434
19.0307	-0.017344535
19.03	-0.017982707
19.0628	-0.013193657

19.0643	-0.013700266
19.0659	-0.014076533
19.1093	-0.011682332
19.1087	-0.013253968
19.146	0.006012431
19.147	0.006977366
19.1488	0.00678006
18.9177	-0.000323425
18.9162	-0.000807118
18.9607	-0.015316532
18.9599	-0.016102576
18.9611	-0.014890854
19.0047	-0.013344349
19.0037	-0.013293097
19.0031	-0.012900471
18.7798	-0.011193599
18.7817	-0.01126328
18.9548	-0.000191812
19.0229	-0.003075478
	-0.22143583

SUM OF SQUARES

REGRESSIOI	RESIDUALS	TOTAL
1.6808E-05	0.000608213	0.000422803
0.00011585	0.000433012	0.000996802
0.00287884	0.000922761	0.000541857
0.00532309	0.00033899	0.002975466
0.00294446	2.23251E-05	0.003479564
0.00166975	0.002135803	2.86457E-05
0.00327087	4.64951E-06	0.003522165
0.00304644	2.25313E-05	0.002544983
0.00194781	0.000392193	0.004088046
0.00239813	0.000302859	0.000996528
0.00521668	0.000877269	0.001815427
0.00421493	0.000786505	0.001359974
0.00275509	0.000169958	0.001556474
0.01314692	1.43805E-05	0.014030917
0.01137186	1.45408E-05	0.012199682
0.01154568	4.00546E-06	0.011979778
0.01226379	2.21855E-05	0.013329204
0.01091523	1.58101E-05	0.011761873
0.00954833	1.39621E-05	0.010292543
0.00997421	1.28251E-05	0.010702352
0.0105309	2.33483E-05	0.011545969
0.00786779	5.64874E-07	0.008001691
0.00738438	2.70147E-07	0.007473978
0.00688384	2.6699E-07	0.006798361
0.00891182	1.64004E-05	0.00969283
0.00794265	1.10931E-05	0.008547404
0.00709261	1.04622E-05	0.007647882
0.00624776	1.16242E-05	0.006798361
0.0057566	6.65587E-06	0.006154743
0.0015698	2.83984E-08	0.001556474
0.0016471	1.75005E-08	0.001636378
0.00142804	1.13711E-07	0.001402665
0.0016356	1.59232E-05	0.001328761
0.00177769	1.37681E-05	0.001478569
0.00167869	2.04275E-05	0.001328761
0.00203921	1.37302E-05	0.001718283
0.00103036	0.00034771	0.000180961
0.00109107	0.000383345	0.000180961
0.00115171	0.000379653	0.000208865
0.00313334	0.000651477	0.000927335
0.0031516	0.000609447	0.000989239
0.0031062	0.000589574	0.000989239
0.00028798	7.25531E-05	7.14392E-05
0.0003195	7.09379E-05	8.93435E-05
0.00037144	7.78043E-05	0.000109248
0.00156996	0.000437859	0.00366604
0.00150473	0.000430853	0.003545944
0.0014022	0.000533695	0.00366604
0.00760794	0.000266484	0.010722153

0.00760441	0.000300833	0.010930248
0.00749352	0.000323378	0.010930248
0.01424542	0.000174073	0.017568927
0.01460413	0.000187697	0.018103118
0.01499922	0.000198149	0.018645309
0.02751136	0.000136477	0.031523231
0.02732206	0.000175668	0.031879327
0.04103066	3.61493E-05	0.038631049
0.0414225	4.86836E-05	0.038631049
0.04215954	4.59692E-05	0.03942124
0.00066438	1.04604E-07	0.000647813
0.00074307	6.51439E-07	0.000699717
0.00029692	0.000234596	0.001059361
0.00027045	0.000259293	0.001059361
0.00031177	0.000221738	0.001059361
0.00374587	0.000178072	0.005557379
0.00363063	0.000176706	0.005409283
0.00355781	0.000166422	0.005263187
0.02677994	0.000125297	0.023241665
0.02615189	0.000126861	0.022635856
0.00012896	3.67919E-08	0.000133352
0.00631585	9.45857E-06	0.006814144
0.53167764	0.015237177	0.546927228
0.00748842	0.000214608	0.0077032

VARIANCE COVARIANCE MATRIX FOR PORT HARCOURT

NORMAL MATRIX								
71	142.7704956	17.33050803	12.21487367		0.000227421	0	0	0
142.7704956	287.0903449	34.84910026	24.56231099		0	0.000227421	0	0
17.33050803	34.84910026	4.230534555	2.981421987		0	0	0.000227421	0
12.21487367	24.56231099	2.981421987	2.101820343		0	0	0	0.000227421

VARIANCE COVARIANCE MATRIX FOR SPHERICAL 'SATLEVEL'			
0.01614686	0.03246895	0.0039413	0.0027779
0.032468946	0.06529025	0.0079254	0.005586
0.003941314	0.0079254	0.0009621	0.000678
0.002777913	0.00558597	0.000678	0.000478

NORMAL MATRIX								
71	70.21886841	8.619192991	6.000693056		0.000225	0	0	0
70.21887	69.44633182	8.524359373	5.934670834		0	0.0002248	0	0
8.619193	8.524359373	1.046420646	0.728433742		0	0	0.0002248	0
6.000693	5.934670834	0.728433742	0.507247532		0	0	0	0.00022481

VARIANCE COVARIANCE MATRIX FOR RECTANGULAR 'SATLEVEL'			
0.015961701	0.01578609	0.00194	0.00135
0.015786092	0.01561242	0.00192	0.00133
0.001937704	0.00191638	0.00024	0.00016
0.001349032	0.00133419	0.00016	0.00011

VARIANCE COVARIANCE MATRIX FOR LAGOS STATE

110	220.89383	13.1316	25.1582448		0.015225742	0	0	0
220.8938264	443.5826	26.3706	50.5210588		0	0.0152257	0	0
13.13159286	26.370565	1.58725	3.00549148		0	0	0.01523	0
25.15824482	50.521059	3.00549	5.75546752		0	0	0	0.01522574

VARIANCE COVARIANCE MATRIX FOR SPHERICAL 'SATLEVEL IN LAGOS STATE

1.67483164	3.3632725	0.1999382	0.38305295
3.363272451	6.7538742	0.4015114	0.76922062
0.199938247	0.4015114	0.0241671	0.04576084
0.383052949	0.7692206	0.0457608	0.08763126

NORMAL MATRIX										
	110	695849925.2	41476496.77	79047563		0.0106072	0	0	0
	695849925.2	4.40188E+15	2.62363E+14	5E+14		0	0.0106072	0	0
	41476496.77	2.62363E+14	1.58345E+13	2.983E+13		0	0	0.01061	0
	79047562.7	5.00045E+14	2.98252E+13	5.682E+13		0	0	0	0.0106072

VARIANCE COVARIANCE MATRIX FOR RECTANGULAR 'SATLEVEL' IN LAGOS STATE			
1.16678892	7381019.326	439949.4965	838473.3071
7380999.86	4.66917E+13	2.78293E+12	5.30408E+12
439948.336	2.78293E+12	1.6796E+11	3.16362E+11
838471.095	5.30408E+12	3.16362E+11	6.02688E+11

COVARIANCE MATRIX OF ADJUSTED PARAMETERS SPHERICAL SATLEVEL PORTHARCOURT			
0.425347843	0.425336203	0.425302746	0.425301326
0.425336203	0.425324564	0.425291107	0.425289687
0.425302746	0.425291107	0.425257653	0.425256234
0.425301326	0.425289687	0.425256234	0.425254814

COVARIANCE MATRIX OF ADJUSTED PARAMETERS RECTANGULAR SATLEVEL			
0.0638467	0.063846725	0.0638467	0.06384667
0.0638467	0.063846724	0.0638467	0.06384667
0.0638467	0.063846687	0.0638466	0.06384663
0.0638467	0.063846669	0.0638466	0.06384661

Appendix B1: Least Squares Adjustment Program Listing Using MATLAB

CODE

```
format long
load dmat
load Lmat
% A is the designed matrix
A = dmat(:,1:4);
% At is the transpose of designed matrix
At = A'
% Normal matrix
Normal_mat = At*A
% Inverse of Normal matrix
(AtA)inv = inv(At*A)
% Lb is the observation data
Lb = Lmat(:,1)
% U vector
U_vector = At*Lb
% Solution vector
Solution_vector = inv(At*A)*(At*Lb)
```

Appendix B2:
Program for Conversion of Geodetic Coordinates to Space Rectangular Coordinates used in 'Satlevel' Rectangular Model

```
C     COMPUTATION OF RECTANGULAR COORDINATES AND SHIFT OF THE ELLIPSOID
      IMPLICIT REAL (A-H,O-Z)
      OPEN(1,FILE='RES',STATUS='OLD',FORM='FORMATTED ')
      DATA PI/3.141592653589793/
      DATA F,A/298.257223563,6378137.0/
      WRITE(*,*)'       '
      WRITE(*,*)'       '
      WRITE(*,*)'THIS PROGRAM COMPUTES:'
      WRITE(*,*)'1. THE RECTANGULAR COORDINATES FROM GEOGRAPHICAL '
      WRITE(*,*)'   COORDINATES USING WGS84 ELLIPSOIDAL PARAMETERS'
      WRITE(*,*)'FOR FURTHER DETAILS CONTACT :'
      WRITE(*,*)'          '
      WRITE(*,*)'          '
      WRITE(*,*)'        K. F. A. ALEEM,'
      WRITE(*,*)'DEPARTMENT OF SURVEYING AND GEOINFORMATICS,'
      WRITE(*,*)'       UNIVERSITY OF LAGOS,'
      WRITE(*,*)'          AKOKA - LAGOS,'
      WRITE(*,*)'             NIGERIA. '
      WRITE(*,*)'   '
      WRITE(*,*)'   '
      WRITE(*,*)'   '
      WRITE(*,*)'          '
      WRITE(*,*)'          '
      WRITE(*,*)'DO YOU WANT TO CONTINUE ?   YES=1,  NO=2    '
      READ(*,*)CON
      IF (CON.EQ.1) GOTO 1
      IF (CON.EQ.2) GOTO 99
  1   CONTINUE
      WRITE(1,2)
  2   FORMAT(T14,'****************************************************')
      WRITE(1,3)
  3   FORMAT(T14,'*COMPUTATION OF RECTANGULAR COORDINATES FROM GIVEN *')
      WRITE (1,4)
  4   FORMAT(T14,'*          GPS GEODETIC COORDINATES              *')
      WRITE(1,5)
  5   FORMAT(T14,'*AUTHOR:-K.F.ALEEM MAIN SUPERVISOR:PROF.J.B.OLALEYE *')
      WRITE(1,6)
  6   FORMAT(T14,'*   DEPARTMENT OF SURVEYING AND GEOINFORMATICS     *')
      WRITE(1,7)
  7   FORMAT(T14,'*     UNIVERSITY OF LAGOS     AKOKA - LAGOS       *')
      WRITE(1,8)
  8   FORMAT(T14,'****************************************************')
      WRITE(1,9)
  9   FORMAT(T14,'*                Ph.D. RESEARCH                  *')
      WRITE(1,10)
 10   FORMAT(T14,'****************************************************')
      WRITE(1,11)
 11   FORMAT(//,T5,'RECTANGULAR COORDINATES FROM WGS 84 GEODETIC ')
      WRITE(*,*)'   '
      WRITE(1,12)F
 12   FORMAT (T23,'FLATTENING          =  1/'F11.7)
      WRITE(1,13) A
 13   FORMAT (T23,'SEMI-MAJOR AXIS       = 'F18.3)
      F=1.D0/F
      WRITE(*,*)'   '
```

```
 14 CONTINUE
    WRITE(*,16)
 16 FORMAT(T2,'ENTER THE STATION NUMBER')
    READ(*,17)SN
 17 FORMAT(2X,A8)
    WRITE(1,18)SN
 18 FORMAT(T2,' STATION NUMBER:-'A8)
    WRITE(1,21)
 21FORMAT (T3,'COMPUTATION OF RECTANGULAR COORDINATES')
C   CONVERSION OF SECONDS OF ARC TO RADIAN
    RAD=PI/180.D0
    CDEG=180.D0*PI
    WRITE(*,24)
 24 FORMAT(T2,'ENTER THE LATITUDE OF THE STATION DEG,MIN,SEC')
    READ(*,*)IDEG,MINP,SECP
    WRITE(1,25)IDEG,MINP,SECP
 25 FORMAT(T23,' LATITUDE   ='I8,I3,F7.3)
    SLAT=(IDEG+(MINP/60.D0)+(SECP/3600.D0))
    SLAT= SLAT*RAD
    WRITE(*,26)
 26 FORMAT(T2,'ENTER THE LONGITUDE OF THE STATION DEG,MIN,SEC')
    READ(*,*)MDEG,MINLG,SECLG
    WRITE(1,27)MDEG,MINLG,SECLG
 27 FORMAT(T23,' LONGITUDE ='I8,I3,F7.3)
    SLONG=(MDEG+(MINLG/60.D0)+(SECLG/3600.D0))
    SLONG= SLONG*RAD
    WRITE(*,28)
 28 FORMAT(T2,'ENTER THE HEIGHT OF THE STATION ')
    READ(*,*)HI
    WRITE(1,29)HI
 29 FORMAT(T23,' HEIGHT     ='F18.4)
    SMB=A*(1.D0-F)
    ECS=(2.D0*F)-(F**2)
    EN=A/(SQRT(1.D0-(ECS*SIN(SLAT)**2)))
    X=(EN+HI)*COS(SLONG)*COS(SLAT)
    Y=(EN+HI)*SIN(SLONG)*COS(SLAT)
    Z=(EN*(1.D0-ECS)+HI)*SIN(SLAT)
30 FORMAT(T12,3F18.4)
    WRITE(1,30)X,Y,Z
    WRITE(*,*)'DO YOU WANT TO COMPUTE FOR OTHER POINTS? YES=1, NO=2'
    READ(*,*)CON
    IF (CON.EQ.1) GOTO 14
    IF (CON.EQ.2) GOTO 99
 99 STOP
    END
```

Appendix B3:
‘Satlevel’ Collocation Program Listing in FORTRAN Programming Language

```
C       PROGRAM SATLEVEL
C       PROGRAM TO COMPUTE GEOIDAL UNDULATION FROM GEODETIC COORDINATES
        IMPLICIT REAL*8 (A-H,O-Z)
        OPEN(1,FILE='RESULTS',FORM='FORMATTED',STATUS='OLD')
        WRITE(*,*)'ORTHOMETRIC HEIGHT ON FLY'
        WRITE(*,*)' '
        WRITE(*,*)' '
        WRITE(*,*)'                   ORTHOMETRIC HEIGHT ON FLY'
        WRITE(*,*)' '
        WRITE(*,*)' '
        WRITE(*,*)'ORTHOMETRIC HEIGHT ON FLY'
        WRITE(*,*)' '
        WRITE(*,*)' '
        WRITE(*,*)'THIS PROGRAM COMPUTES:  '
        WRITE(*,*)' '
        WRITE(*,*)' 1. THE GEOIDAL UNDULATION FROM GEODETIC COORDINATES'
        WRITE(*,*)'     2. THE ORTHOMETRIC HEIGHT OF THE POINT'
        WRITE(*,*)'      '
        WRITE(*,*)' FOR FURTHER DETAILS'
        WRITE(*,*)' '
        WRITE(*,*)' '
        WRITE(*,*)'                 CONTACT:- '
        WRITE(*,*)' '
        WRITE(*,*)' '
        WRITE(*,*)' THE AUTHOR:-      K. F. ALEEM'
        WRITE(*,*)'                  MATRIC NO: 069045005'
        WRITE(*,*)' '
        WRITE(*,*)'      DEPARTMENT OF SURVEYING and GEOINFORMATICS,'
        WRITE(*,*)'             SCHOOL OF POSYGRADUATES STUDIES'
        WRITE(*,*)'              UNIVERSITY OF LAGOS'
        WRITE(*,*)'                 AKOKA- LAGOS'
        WRITE(*,*)'    E-MAIL: akfaleem@yahoo.com'
        WRITE(*,*)'      '
        WRITE(*,*)'        OR'
        WRITE(*,*)'      '
        WRITE(*,*)' THE SUPERVISORS:-'
        WRITE(*,*)'      '
        WRITE(*,*)'  '
        WRITE(*,*)'  '
        WRITE(*,*)'               PROF. J. B. OLALEYE'
        WRITE(*,*)'  '
        WRITE(*,*)'               DR. O. T. BADEJO'
        WRITE(*,*)'  '
        WRITE(*,*)'               DR. J.O. OLUSINA   '
        WRITE(*,*)' '
        WRITE(*,*)'      DEPARTMENT OF SURVEYING and GEOINFORMATICS,'
        WRITE(*,*)'             SCHOOL OF POSYGRADUATES STUDIES'
        WRITE(*,*)'              UNIVERSITY OF LAGOS'
        WRITE(*,*)'                 AKOKA- LAGOS'
        WRITE(*,*)'      '
        WRITE(*,*)'      '
        WRITE(*,*)'DO YOU WANT TO CONTINUE? YES=1,NO=2'
```

```
      READ (*,*)ICON
      IF (ICON.EQ.1) GOTO 1
      IF (ICON.EQ.2) GOTO 99
1     CONTINUE
      PI=3.141592665358979
      PHID=0
      WRITE(1,3)
3     FORMAT(T23,'********************************************')
      WRITE(1,4)
4     FORMAT(T23,'*                                          *')
      WRITE(1,5)
5     FORMAT(T23,'*      ORTHOMETRIC HEIGHT ON FLY          *')
      WRITE(1,6)
6     FORMAT(T23,'*    COMPUTATION OF SATLEVEL GEODETIC     *')
      WRITE(1,7)
7     FORMAT(T23,'* AUTHOR: K.F. ALEEM * MATRIC NO:069045005  *')
      WRITE(1,8)
8     FORMAT(T23,'* MAJOR SUPERVISOR :- PROF. J. B. OLALEYE   *')
      WRITE(1,9)
9     FORMAT(T23,'*          SUPERVISOR 1 :- DR. O. T. BADEJO    *')
      WRITE(1,10)
 10   FORMAT(T23,'*          SUPERVISOR 2:- DR. J. O. OLUSINA    *')
      WRITE(1,11)
11    FORMAT(T23,'* DEPARTMENT OF SURVEYING and GEOINFORMATICS  *')
      WRITE(1,12)
12    FORMAT(T23,'*    UNIVERSITY OF LAGOS, AKOKA-LAGOS      *')
      WRITE(1,4)
      WRITE(1,13)
13    FORMAT(T23,'********************************************')
      WRITE(1,14)
14    FORMAT(T23,'*   Ph.D FINAL THESIS PRESENTATION        *')
      WRITE(1,13)
      WRITE(1,*)'        '
      WRITE(1,*)'        '
      WRITE(1,*)'STATION NO. LATITUDE      LONGITUDE    UNDULATION   GP
     $S HEIGHT ORTHOMETRIC HEIGHT'
17    CONTINUE
      WRITE(*,*)'INPUT THE STATION NUMBER'
      READ(*,15)SN
15    FORMAT(A8)
      WRITE(*,*)'INPUT THE LATITUDE OF THE POINT IN DEG,MINS,SEC'
      READ(*,*)LDPHI,MPHI,SPHI
      PHI=LDPHI+(MPHI/60)+(SPHI/3600)
      WRITE(*,*)'INPUT THE LONGITUDE OF THE POINT IN DEG,MINS,SEC'
      READ(*,*)LONGD,MLONG,SLONG
      WRITE(*,*)'INPUT THE ELLIPSOIDAL HEIGHTS OF THE POINT'
      READ(*,*)ELHT
      STLONG=LONGD+(MLONG/60.D0)+(SLONG/3600.D0)
      RAD=PI/180.D0
      DEG=180.D0/PI
      PHI=PHI*RAD
      STLONG=STLONG*RAD
      AM=5.300744666
      BM=-0.025066704
      CM=-64.718629
      DM=30.3237149
```

```
      SATMEAN=AM+(BM*DCOS(PHI)*DCOS(STLONG))+(CM*DCOS(PHI)*DSIN(STLONG))
     $+(DM*DSIN(PHI))
      AC=13.64582533
      BC=0.025066704
      CC=64.7186291
      DC=-30.323715
      SATCUV=AC+(BC*DCOS(PHI)*DCOS(STLONG))+(CC*DCOS(PHI)*DSIN(STLONG))+
     $(DC*DSIN(PHI))
      ORTHO=ELHT-SATCUV
      PHI=PHI*DEG
      STLONG=STLONG*DEG
      NPHID=ABS(PHI)
      PHIS=(PHI-PHID)*60.D0
      MINPH=ABS(PHIS)
      SECPH=(PHIS-MINPH)*60.D0
      LONG=ABS(STLONG)
      SMINLG=(STLONG-LONG)*60.D0
      MINLG=ABS(SMINLG)
      SECLG=(SMINLG-MINLG)*60.D0
      WRITE(1,94)SN,LDPHI,MPHI,SPHI,LONGD,MLONG,SLONG,SATCUV,ELHT,ORTHO
   94 FORMAT(A8,1X,I5,I3,F7.3,I5,I3,F7.3,2F12.6,F15.6)
      WRITE(*,*)'DO YOU WANT TO END? YES=1,NO=2'
      READ(*,*)IEND
      IF (IEND.EQ.2) GOTO 17
      IF (IEND.EQ.1) GOTO 99
99    CONTINUE
      WRITE(1,13)
      STOP
      END
```

Appendix C1:
Full Data Set for Table 3.2a - Local Geoidal Undulations for Port Harcourt.

Table 3.2a: Local Geoidal Undulation for Port Harcourt.

Stations	Latitude [º]	Longitude [º]	Ellipsoidal Heights (h) [m]	Orthometric Heights (H) [m]	Geoid (N) [m]
AP4	4.868335803	6.989905397	35.849	16.926	18.923
AP1	4.869537347	6.977927531	33.720	14.808	18.912
PT.3 EMMA	4.790218708	7.00227435	25.195	6.228	18.967
PHCS 1s	4.772389314	7.013525022	30.796	11.798	18.998
PT.4 EMMA	4.798391819	7.005574083	30.693	11.691	19.002
PT.8 EMMA	4.833761764	7.007032608	26.789	07.851	18.938
PT.4 ABDUL	4.837173481	7.022857481	32.842	13.839	19.003
PT.5 EMMA	4.806938314	7.009407025	29.374	10.380	18.994
PT.7 EMMA	4.823872525	7.006017658	33.379	14.372	19.007
PT.9 EMMA	4.836566356	7.015292797	29.141	10.166	18.975
PT.2 ABDUL	4.844335522	7.039518178	32.640	13.654	18.986
PT.3 ABDUL	4.840752114	7.031318094	26.750	07.770	18.980
GPS 02	4.9883341858	7.005441514	42.542	23.638	18.904
GPS 03	4.981133603	6.949840522	40.065	21.24	18.825
GPS 04	4.972244803	6.951180808	38.771	19.938	18.833
GPS 05	4.988165797	6.959676808	41.357	22.523	18.834
GPS 06	4.976870211	6.950525386	39.485	20.657	18.828
GPS 07	4.968417417	6.950765697	38.351	19.516	18.835
GPS 08	4.956065461	6.949389547	36.427	17.585	18.842
GPS 09	4.95495015	6.947081147	34.627	15.787	18.840
GPS 10	4.953781161	6.944284003	36.819	17.983	18.836
GPS 11	4.978015694	6.968921853	38.155	19.301	18.854
GPS 12	4.976619567	6.970370336	39.661	20.804	18.857
GPS 13	4.975173192	6.971955836	40.589	21.728	18.861
GPS 14	4.953134586	6.950453306	35.359	16.514	18.845
GPS 15	4.949708683	6.952838769	34.766	15.915	18.851
GPS 16	4.946587319	6.955108775	34.756	15.900	18.856
GPS 17	4.943006336	6.957377311	34.790	15.929	18.861
GPS 18	4.939244417	6.957961819	34.784	15.919	18.865
GPS 19	4.893158592	6.964717458	29.266	10.362	18.904
GPS 20	4.89404995	6.964342617	29.870	10.967	18.903
GPS 21	4.893297169	6.966278353	30.338	11.432	18.906
GPS 22	4.875097889	6.955985178	32.335	13.428	18.907
GPS 23	4.875640256	6.954831264	33.256	14.351	18.905
GPS 24	4.873833222	6.955013361	33.065	14.158	18.907
GPS 25	4.876598708	6.952834056	33.532	14.630	18.902
GPS 26	4.832460906	6.945637275	20.180	01.250	18.930
GPS 27	4.832444461	6.9448869	19.557	00.627	18.930
GPS 28	4.832327742	6.944121753	20.699	01.770	18.929

Stations	Latitude [º]	Longitude [º]	Ellipsoidal Heights (h) [m]	Orthometric Heights (H) [m]	Geoid (N) [m]
GPS 29	4.836480189	6.928271461	20.239	01.326	18.913
GPS 30	4.837388344	6.928477733	20.984	02.072	18.912
GPS 31	4.838183467	6.929087211	23.319	04.407	18.912
GPS 32	4.940823194	7.007985167	37.527	18.592	18.935
GPS 33	4.942280164	7.008015719	38.369	19.435	18.934
GPS 34	4.943984306	7.007760989	39.567	20.634	18.933
GPS 35	4.930137067	7.052698958	40.670	21.666	19.004
GPS 36	4.931735783	7.052849775	40.870	21.867	19.003
GPS 37	4.935097586	7.053556919	38.757	19.753	19.004
GPS 38	4.890883953	7.076113975	34.478	15.431	19.047
GPS 39	4.892411842	7.076911742	36.043	16.995	19.048
GPS 40	4.8946095	7.07747475	37.128	18.080	19.048
GPS 41	4.862920831	7.093361511	37.962	18.886	19.076
GPS 42	4.863447247	7.095125922	38.177	19.099	19.078
GPS 43	4.863901311	7.09699115	36.294	17.214	19.080
GPS 45	4.833776561	7.127300578	33.432	14.311	19.121
GPS 46	4.835730717	7.127621192	31.881	12.759	19.122
GPS 47	4.769962542	7.140300147	32.793	13.653	19.140
GPS 48	4.769413628	7.141166558	33.017	13.877	19.140
GPS 49	4.7683703	7.14278445	33.822	14.680	19.142
GPS 50	4.912119492	6.985296881	35.117	16.199	18.918
GPS 51	4.913761719	6.984875258	35.499	16.582	18.917
GPS 53	4.807930044	6.977191642	29.078	10.102	18.976
GPS 54	4.807218517	6.976286997	29.336	10.360	18.976
GPS 55	4.806990144	6.977222258	29.173	10.197	18.976
GPS 56	4.781655028	7.006075439	28.033	09.015	19.018
GPS 57	4.782321533	7.005458108	27.536	08.519	19.017
GPS 58	4.783296731	7.005240433	27.441	08.425	19.016
GPS 59	4.916896858	6.880102978	20.494	01.703	18.791
GPS 60	4.91610835	6.881154569	20.982	02.189	18.793
XSV 662	4.873506919	6.99841315	27.603	08.648	18.955
ZVS 3003	4.847971022	7.047811589	32.308	13.282	19.026

Appendix C2:

Full Data Set for Table 3.2b - Local Geoidal Undulation for Lagos State.

Table 3.2b: Local Geoidal Undulation for Lagos State

Stations	Latitude [^{o}]	Longitude [^{o}]	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Geoidal Undulations (N) [m]
XST 237	6.454802139	3.470396222	25.8360	3.2720	22.5640
XST44	6.422368909	3.473378551	26.4830	4.2290	22.2540
YTT78A	6.470008869	3.646457902	27.3350	4.8610	22.4740
XST245	6.433911612	3.603378587	29.0220	6.5310	22.4910
XST244	6.426005944	3.631051025	27.4720	5.2480	22.2240
FGPLA-Y-003	6.427041234	2.890722633	27.0450	4.2620	22.7830
CFPA21	6.440896094	2.919119213	30.9400	8.1120	22.8280
XST 55	6.37965975	2.706952389	30.0470	7.3470	22.7000
YTT1703A	6.419998574	2.712921902	25.0470	2.1350	22.9120
XST46	6.443881271	2.709402845	25.6840	2.6400	23.0440
XST50	6.430888353	2.826984239	29.1860	6.3060	22.8800
LWBC5-61P	6.504592611	2.926533297	26.0300	2.8440	23.1860
YTT19-54	6.510901227	2.954208526	37.7640	14.5740	23.1900
XST75	6.498898805	3.063821936	36.4430	13.4200	23.0230
CFPA40	6.385017233	2.78113861	28.3150	5.6600	22.6550
CFPB36	6.39047864	2.824224997	27.5300	4.8810	22.6490
XST60	6.395764278	2.928216261	27.3760	4.8370	22.5390
XST72	6.399500358	3.053622142	27.1670	4.7710	22.3960
XST76	6.400752663	3.095451055	27.1060	4.7410	22.3650
XST44	6.422368909	3.490045221	26.4830	4.2290	22.2540
YTT2-18A	6.425548341	3.546123013	24.5220	2.2640	22.2580
XST156	6.426882584	3.678521952	27.6630	5.4460	22.2170
ZTT2-57A	6.438082356	3.77811817	26.8840	4.6100	22.2740
YTT2-66A	6.441722983	3.84345449	26.8840	4.6140	22.2700
YTT2-80	6.439486058	3.930290799	26.1150	3.8740	22.2410
XST224	6.418510123	4.080058618	27.1950	5.0350	22.1600
ZTT35-14	6.405233422	4.142532315	27.1800	5.0610	22.1190
XST149	6.383583508	4.255296272	26.1530	0000	26.1530
MCS1188T-A	6.378977398	44.60666667	25.7940	0000	25.7940
XST42	6.665776816	4.088917185	29.2460	6.0780	23.1680
YTT13-1A	6.679592549	4.062929161	33.7240	10.4780	23.2460
ZTT34-10A	6.665085385	4.002523018	43.6820	20.4450	23.2370
XST135	6.684094578	3.981722921	79.5500	56.2210	23.3290
XST218	6.676580001	3.935228307	42.6040	19.2830	23.3210
XST209	6.684599219	3.882579673	34.1190	10.7090	23.4100
XST201	6.683016458	3.838593558	44.7510	21.3140	23.4370
XST203	6.682724793	3.749736646	25.2830	1.8230	23.4600

Stations	Latitude [º]	Longitude [º]	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Geoidal Undulations (N) [m]
XST177	6.690429036	3.712051242	70.0000	46.665	23.3350
YTT22-1	6.670187596	3.67055825	53.7920	30.3450	23.4470
XST159	6.680407761	3.577680739	71.5940	48.0650	23.5290
ZTT31-70	6.669253338	3.512980517	69.5080	46.0020	23.5060
XST131	6.683364347	3.461519509	35.0790	11.4890	23.5900
XST127	6.643481664	3.466548326	24.5030	1.0980	23.4050
XST133	6.639145304	3.41173667	25.7390	2.3270	23.4120
XST128	6.640816598	3.372557102	63.8060	40.3870	23.4190
YTT28-117	6.643671475	3.3393115	41.4419	17.9704	23.4716
MCS1174S-A	6.665027289	3.323236155	73.1510	49.5700	23.5810
YTT28-96	6.685802442	3.288081883	82.3486	57.7276	24.6210
XST41	6.699541552	3.264344748	74.3330	50.5559	23.7780
YTT28-89	6.654158664	3.242408083	43.9890	20.38926	23.5997
YTT28-87	6.621962881	3.247943681	49.2344	25.73163	23.5028
YTT28-67	6.599941767	3.238823975	58.3284	34.90225	23.4260
YTT28-65	6.571270847	3.214430689	45.8025	22.49444	23.3087
YTT28-47	6.5237964	3.209969817	30.3179	7.31250	23.0054
XST87	6.510439635	3.173555949	25.6370	2.6570	22.9800
YTT28-30	6.502141856	3.169837828	29.1474	6.1958	22.9516
YTT28-1	6.497681789	3.115275631	28.3025	5.3304	22.9720
XST71	6.501847826	3.024295615	42.2200	19.1490	23.0710
YTT19-7	6.497953867	3.008970191	40.2970	17.2350	23.0620
YTT19-54	6.510901225	2.954208526	37.7640	14.5740	23.1900
XST59	6.502318363	2.926581412	28.0940	4.9210	23.1730
XST120	6.4233642	3.457259185	26.5160	4.2470	22.2690
CFPA31	6.394388887	2.890307941	27.1840	4.6040	22.5800
XST64	6.396820427	2.96973699	26.7090	4.2250	22.4840
XST68	6.397824759	3.011246563	27.3560	4.9270	22.4290
XST76	6.400752663	3.095451055	27.1060	4.7410	22.3650
XST83	6.403513554	3.177978425	27.1370	4.8160	22.3210
XST84	6.404556548	3.220993917	27.0360	4.7540	22.2820
XST99A	6.40434195	3.302776744	25.7630	3.5480	22.2150
XST241	6.401641891	3.343845061	26.0650	3.8900	22.1750
XST107	6.397472254	3.380957804	25.4700	3.3400	22.1300
XST114	6.422654072	3.420188491	26.2180	3.9330	22.2850
XST44	6.422368909	3.490045221	26.4830	4.2290	22.2540
YTT2-14A	6.422859232	3.527906685	25.0230	2.7750	22.2480
YTT2-25A	6.424395468	3.586657712	25.4180	3.1750	22.2430
YTT2-37A	6.426411593	3.664612708	27.3180	5.1000	22.2180

Stations	Latitude [^{o}]	Longitude [^{o}]	Ellipsoidal Height (h) [m)	Orthometric Height (H) [m]	Geoidal Undulations (N) [m]
YTT2-48A	6.429279172	3.718083886	26.6820	4.4510	22.2310
XST55	6.37965975	3.706952389	30.0470	7.3470	22.7000
YTT17-08A	6.419892501	2.722609701	28.4460	5.5410	22.9050
XST53	6.431164111	2.868855608	28.5270	5.6880	22.8390
FGPLA-Y-008	6.441898015	2.948674497	30.5720	7.7810	22.7910
XST59	6.502318362	2.926581412	28.0940	4.9210	23.1730
CFPA18	6.457021906	2.95957542	27.4715	4.6070	22.8645
XST69	6.436063964	3.031327624	27.0360	4.3790	22.6570
YTT28-1	6.497681789	3.115275631	28.3029	5.3304	22.9726
ZTT45-200	6.484483843	3.143460993	28.5930	5.7190	22.8740
MCS1144S-A	6.460816877	3.204114125	29.6720	6.9990	22.6730
YTT28-151	6.455414556	3.330872553	25.7547	3.2189	22.5358
YTT28-134	6.529735401	3.529742897	27.1228	4.1703	22.9525
ZTT6-53	6.569916795	3.269374699	55.1210	31.9280	23.1930
YTT27-33	6.635802535	3.337821171	73.0000	49.0840	23.9160
YTT27-41	6.634425861	3.353201574	59.7990	36.3770	23.4220
YTT16-76A	6.551977817	3.388735983	28.8272	000000	28.8272
XST121	6.460263853	3.440859348	24.4600	1.9710	22.4890
YTT28-200	6.447630558	3.467725678	25.3818	2.9555	22.4263
XT101	6.628991651	3.510495332	62.3990	39.0800	23.3190
ZTT30-5	6.5986892	3.588452971	50.4540	27.3150	23.1390
MCS1178T-A	6.474988831	3.56779892	25.5580	3.0310	22.5270
YTT9-73A	6.464696263	3.670838479	27.7320	5.2940	22.4380
XST165	6.614877133	3.645515546	47.3230	24.1220	23.2010
XST126	6.65058152	3.708116618	59.3190	35.9600	23.3590
YTT9-29A	6.484681321	3.880715476	26.0500	3.57000	22.4800
XST215	6.605924865	3.925565174	25.6900	2.6600	23.0300
ZTT35-26	6.394128552	4.202842114	26.9560	4.8720	22.0840
ZTT34-34	6.644054924	4.036229785	30.9890	7.8590	23.1300
YTT13-27	6.61492692	3.999839731	53.4290	30.3870	23.0420
XT161	6.585103161	3.955504287	48.0910	25.1640	22.9270
XST202	6.622775589	3.875495858	26.0590	2.9390	23.1200
YTT13-30	6.612424233	3.98731709	56.5500	33.5130	23.0370
XST204	6.433572854	3.988969653	27.1270	4.9060	22.2210
ZTT35-2A	6.416279493	4.089807093	26.9580	4.8070	22.1510
YTT16-76A	6.503491991	3.719303861	29.3680	6.7340	22.6340
XST149	6.565506766	3.588484489	37.3090	14.3260	22.9830
MCS1188T-A	6.493459685	3.582388693	25.3970	2.7750	22.6220

Appendix C3:
Full Data Set for Table 4.1a - Comparison between the Differences in Elevation of Ellipsoidal and Orthometric Heights for Port Harcourt:

Table 4.1a: Differences in Elevation between Successive Points Computed from Ellipsoidal and Orthometric Heights for Port Harcourt

Stations	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Ellipsoidal Changes in Elevation (Dh) [m]	Orthometric Changes in Elevation (DH) [m]	Difference Between (Diff) [m]	Mean Square Error (MSE) [m]
AP4	35.8490	16.9260				
AP1	33.7200	14.8080	02.1290	02.1180	0.0110	9.27369E-05
P10 BALOGUN	36.0840	17.1810	-02.3640	-02.3730	0.0092	6.17796E-05
PT.3 EMMA	25.1950	06.2283	10.8890	10.9500	-0.0640	0.004286321
PHCS 1s	30.7960	11.7980	-05.6010	-05.5700	-0.0310	0.001066022
PT.4 EMMA	30.6930	11.6910	00.1030	00.1070	-0.0040	3.38724E-05
PT.8 EMMA	26.7890	07.8509	03.9040	03.8400	0.0643	0.003963962
PT.4 ABDUL	32.8420	13.8390	-06.0530	-05.9880	-0.0650	0.004366566
PT.5 EMMA	29.3740	10.3800	03.4680	03.4590	0.0089	5.65504E-05
PT.7 EMMA	33.3790	14.3720	-04.0050	-03.9920	-0.0130	0.000221117
PT.9 EMMA	29.1410	10.1660	04.2380	04.2060	0.0324	0.00096038
PT.2 ABDUL	32.6400	13.6540	-03.4990	-03.4880	-0.0110	0.000154256
PT.3 ABDUL	26.7500	07.7697	5.89000	05.8840	0.0057	1.89225E-05
GPS 02	42.5420	23.6380	-15.7900	-15.8700	0.0763	0.005617503
GPS 03	40.0650	21.2400	02.4770	02.3980	0.0790	0.006024864
GPS 04	38.7710	19.9380	01.2940	01.3020	-0.0080	8.79844E-05
GPS 05	41.3570	22.5230	-02.5860	-02.5850	-01E-03	5.6644E-06
GPS 06	39.4850	20.6570	01.8720	01.8660	0.0060	2.13444E-05
GPS 07	38.3510	19.5160	01.1340	01.1410	-0.0070	7.02244E-05
GPS 08	36.4270	17.5850	01.9240	01.9310	-0.0070	7.02244E-05
GPS 09	34.6270	15.7870	01.8000	01.7980	0.0020	003.844E-07
GPS 10	36.8190	17.9830	-02.1920	-02.1960	0.0040	6.8644E-06
GPS 11	38.1550	19.3010	-01.3360	-01.3180	-0.0180	0.000375584
GPS 12	39.6610	20.8040	-01.5060	-01.5030	-0.0030	1.91844E-05
GPS 13	40.5890	21.7280	-00.9280	-00.9240	-0.0040	2.89444E-05
GPS 14	35.3590	16.5140	05.2300	05.2140	0.0160	0.000213744
GPS 15	34.7660	15.9150	00.5930	00.5990	-0.0060	5.44644E-05
GPS 16	34.7560	15.9000	00.0100	00.0150	-0.0050	4.07044E-05
GPS 17	34.7900	15.9290	-00.0340	-00.0290	-0.0050	4.07044E-05
GPS 18	34.7840	15.9190	00.0060	00.0100	-0.0040	2.89444E-05
GPS 19	29.2660	10.3620	05.5180	05.5570	-0.0390	0.001630544
GPS 20	29.8700	10.9670	-00.6040	-00.6050	0.0010	1.444E-07
GPS 21	30.3380	11.4320	-00.4680	-00.4650	-0.0030	1.91844E-05
GPS 22	32.3350	13.4280	-01.9970	-01.9960	-01E-03	5.6644E-06
GPS 23	33.2560	14.3510	-00.9210	-00.9230	0.0020	3.844E-07

Stations	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Ellipsoidal Changes in Elevation (Dh) [m]	Orthometric Changes in Elevation (DH) [m]	Difference Between (Diff) [m]	Mean Square Error (MSE) [m]
GPS 24	33.0650	14.1580	00.1910	00.1930	-0.0020	1.14244E-05
GPS 25	33.5320	14.6300	-00.4670	-00.4720	0.0050	1.31044E-05
GPS 26	20.1800	01.2500	13.3520	13.3800	-0.0280	0.000863184
GPS 27	19.5570	00.6270	00.6230	00.6230	01E-15	1.9044E-06
GPS 28	20.6990	01.7700	-01.1420	-01.1430	0.0010	1.444E-07
GPS 29	20.2390	01.3260	00.4600	00.4440	0.0160	0.000213744
GPS 30	20.9840	02.0720	-00.7450	-00.7460	0.0010	1.444E-07
GPS 31	23.3190	04.4070	-02.3350	-02.3350	0.0000	1.9044E-06
GPS 32	37.5270	18.5920	-14.2100	-14.1900	-0.0230	0.000594384
GPS 33	38.3690	19.4350	-00.8420	-00.8430	0.0010	1.444E-07
GPS 34	39.5670	20.6340	-01.1980	-01.1990	0.0010	1.444E-07
GPS 35	40.6700	21.6660	-01.1030	-01.0320	-0.0710	0.005238864
GPS 36	40.8700	21.8670	-00.2000	-00.2010	0.0010	1.444E-07
GPS 37	38.7570	19.7530	02.1130	02.1140	-0.0010	5.6644E-06
GPS 38	34.4780	15.4310	04.2790	04.3220	-0.0430	0.001969584
GPS 39	36.0430	16.9950	-01.5650	-01.5640	-01E-03	5.6644E-06
GPS 40	37.1280	18.0800	-01.0850	-01.0850	-04E-15	1.9044E-06
GPS 41	37.9620	18.8860	-00.8340	-00.8060	-0.0280	0.000863184
GPS 42	38.1770	19.0990	-00.2150	-00.2130	-0.0020	1.14244E-05
GPS 43	36.2940	17.2140	01.8830	01.8850	-0.0020	1.14244E-05
GPS 44	34.4110	15.2900	01.8830	01.9240	-0.0410	0.001796064
GPS 45	33.4320	14.3110	00.9790	00.9790	0.0000	1.9044E-06
GPS 46	31.8810	12.7590	01.5510	01.5520	-01E-03	5.6644E-06
GPS 47	32.7930	13.6530	-00.9120	-00.8940	-0.0180	0.000375584
GPS 48	33.0170	13.8770	-00.2240	-00.2240	-04E-15	1.9044E-06
GPS 49	33.8220	14.6800	-00.8050	-00.8030	-0.0020	1.14244E-05
GPS 50	35.1170	16.1990	-01.2950	-01.5190	0.2240	0.049559664
GPS 51	35.4990	16.5820	-00.3820	-00.3830	0.0010	1.444E-07
GPS 52	35.2540	16.3390	00.2450	00.2430	0.0020	3.844E-07
GPS 53	29.0780	10.1020	06.1760	06.2370	-0.0610	0.003891264
GPS 54	29.3360	10.3600	-00.2580	-00.2580	0.0000	1.9044E-06
GPS 55	29.1730	10.1970	00.1630	00.1630	0.0000	1.9044E-06
GPS 56	28.0330	09.0150	01.1400	01.1820	-0.0420	0.001881824
GPS 57	27.5360	08.5190	00.4970	00.4960	0.0010	1.444E-07
GPS 58	27.4410	08.4250	00.0950	00.0940	0.0010	1.444E-07
GPS 59	20.4940	01.7030	06.9470	06.7220	0.2250	0.050005904

Stations	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Ellipsoidal Changes in Elevation (Dh) [m]	Orthometric Changes in Elevation (DH) [m]	Difference Between (Diff) [m]	Mean Square Error (MSE) [m]
GPS 60	20.9820	02.1890	-00.4880	-00.4860	-0.0020	1.14244E-05
GPS 61	20.6720	01.8770	00.3100	00.3120	-0.0020	1.14244E-05
XSV 662	27.6030	08.6480	-06.9310	-06.7710	-0.160	0.026043504
ZVS 3003	32.3080	13.2820	-04.7050	-04.6340	-0.0710	0.005238864
RHS 8A	23.5290	04.4860	08.7790	08.7960	-0.0170	0.000337824

Appendix C4:
Full Data Set for Table 4.1b: Comparison between the Differences in Elevation of Ellipsoidal and Orthometric Heights for Lagos State

Table 4.1b: Comparison between the Differences in Elevation of Successive Ellipsoidal and Orthometric Heights for Lagos State

Stations	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Ellipsoidal Changes in Elevation (Dh) [m]	Orthometric Changes in Elevation (DH) [m]	Difference Between Changes in Elevations (Diff) [m]	Mean Square Error (MSE) [m]
XST 237	25.8360	3.2720				
XST44	26.4830	4.2290	-0.6470	-0.9570	0.3100	0.094158893
YTT78A	27.3350	4.8610	-0.8520	-0.6320	-0.2200	0.049794489
XST245	29.0220	6.5310	-1.6870	-1.6700	-0.0170	0.000405893
XST244	27.4720	5.2480	1.5500	1.2830	0.2670	0.069618517
FGPLA-Y-003	27.0450	4.2620	0.4270	0.9860	-0.5590	0.316009012
CFPA21	30.9400	8.1120	-3.8950	-3.8500	-0.0450	0.002318113
XST 55	30.0470	7.3470	0.8930	0.7650	0.1280	0.015588324
YTT1703A	25.0470	2.1350	50.0000	5.2120	-0.2120	0.046288141
XST46	25.6840	2.6400	-0.6370	-0.5050	-0.1320	0.018264655
XST50	29.1860	6.3060	-3.5020	-3.6660	0.1640	0.025873755
LWBC5-61P	26.0300	2.8440	3.1560	3.4620	-0.3060	0.095571737
YTT19-54	37.7640	14.5740	-11.730	-11.7300	-0.0040	5.10766E-05
XST75	36.4430	13.4200	1.3210	1.1540	0.1670	0.026847875
CFPA40	28.3150	5.6600	8.1280	7.7600	0.3680	0.133117866
CFPB36	27.5300	4.8810	0.7850	0.7790	0.0060	8.14081E-06
XST60	27.3760	4.8370	0.1540	0.0440	0.1100	0.011417609
XST72	27.1670	4.7710	0.2090	0.0660	0.1430	0.019558921
XST76	27.1060	4.7410	0.0610	0.0300	0.0310	0.000775801
XST44	26.4830	4.2290	0.6230	0.5120	0.1110	0.011632315
YTT2-18A	24.5220	2.2640	1.9610	1.9650	-0.0040	5.10766E-05
XST156	27.6630	5.4460	-3.1410	-3.1820	0.0410	0.001432866
ZTT2-57A	26.8840	4.6100	0.7790	0.8360	-0.0570	0.003617636
YTT2-66A	26.8840	4.6140	00000	-0.0040	0.0040	7.27969E-07
YTT2-80	26.1150	3.8740	0.7690	0.7400	0.0290	0.000668389
XST224	27.1950	5.0350	-1.0800	-1.1610	0.0810	0.006061122
ZTT35-14	27.1800	5.0610	0.0150	-0.0260	0.0410	0.001432866
XST149	37.3090	14.3260	-10.1300	-9.2650	-0.8640	0.751943554
MCS1188T-A	25.3970	2.7750	11.9120	11.5500	0.3610	0.128058921
XST42	29.2460	6.0780	-3.8490	-3.3030	-0.5460	0.301562196
YTT13-1A	33.7240	10.4780	-4.4780	-4.4000	-0.0780	0.006584801
ZTT34-10A	43.6820	20.4450	-9.9580	-9.9670	0.0090	3.42601E-05
XST135	79.5500	56.2210	-35.8700	-35.7800	-0.0920	0.009052911
XST218	42.6040	19.2830	36.9460	36.9400	0.0080	2.35537E-05

Stations	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Ellipsoidal Changes in Elevation (Dh) [m]	Orthometric Changes in Elevation (DH) [m]	Difference Between Changes in Elevations (Diff) [m]	Mean Square Error (MSE) [m]
XST209	34.1190	10.7090	8.4850	8.5740	-0.0890	0.008491031
XST201	44.7510	21.3140	-10.6300	-10.6100	-0.0270	0.000908829
XST203	25.2830	1.8230	19.4680	19.4900	-0.0230	0.000683655
XST177	70.0000	46.6650	-44.9300	-44.8400	-0.0890	0.008491031
YTT22-1	53.7920	30.3450	16.4220	16.3200	0.1020	0.009771957
XST159	71.5940	48.0650	-17.8000	-17.7200	-0.0820	0.007249976
ZTT31-70	69.5080	46.0020	2.0860	2.06300	0.0230	0.00039415
XST131	35.0790	11.4890	34.4290	34.5100	-0.0840	0.007594563
XST127	24.5030	1.0980	10.5760	10.3900	0.1850	0.03307059
XS22T133	25.7390	2.3270	-1.2360	-1.2290	-0.0070	0.000102957
XST128	63.8060	40.3870	-38.0700	-38.0600	-0.0070	0.000102957
YTT28-117	41.4420	17.97040	22.3640	22.4200	-0.0530	0.003102132
MCS1174S-A	73.1510	49.570	-31.7100	-31.6000	-0.1090	0.012678037
YTT28-96	82.3490	57.7276	-9.1980	-8.1580	-1.0400	1.088217813
XST41	74.3330	50.5550	8.0156	7.1730	0.8430	0.705403808
YTT28-89	43.9890	20.3893	30.3440	30.1700	0.1783	0.030664637
YTT28-87	49.2340	25.7316	-5.2450	-5.3420	0.0970	0.008802795
YTT28-67	58.3280	34.9023	-9.0940	-9.1710	0.0766	0.005398313
YTT28-65	45.8030	22.4944	12.5260	12.4100	0.1181	0.013211942
YTT28-47	30.3180	7.3125	15.4850	15.1800	0.3027	0.089708164
XST87	25.6370	2.6570	4.6809	4.6560	0.0254	0.000495205
YTT28-30	29.1470	6.1958	-3.5100	-3.5390	0.0284	0.000638735
YTT28-1	28.3030	5.3304	0.8449	0.8650	-0.0200	0.000559171
XST71	42.2200	19.1490	-13.9200	-13.8200	-0.0990	0.010417629
YTT19-7	40.2970	17.2350	1.9230	1.9140	0.0090	3.42601E-05
YTT19-54	37.7640	14.5740	2.5330	2.6610	-0.1280	0.01719948
XST59	28.0940	4.9210	9.6700	9.6530	0.0170	0.000191911
XST120	26.5160	4.2470	1.5780	0.6740	0.9040	0.811536508
CFPA31	27.1840	4.6040	-0.6680	-0.3570	-0.3110	0.098688205
XST64	26.7090	4.2250	0.4750	0.3790	0.0960	0.008621719
XST68	27.3560	4.9270	-0.6470	-0.7020	0.0550	0.002688755
XST76	27.1060	4.7410	0.2500	0.1860	0.0640	0.003703113
XST83	27.1370	4.8160	-0.0310	-0.0750	0.0440	0.001668985
XST84	27.0360	4.7540	0.1010	0.0620	0.0390	0.001285453
XST99A	25.7630	3.5480	1.2730	1.2060	0.0670	0.004077233
XST241	26.0650	3.8900	-0.3020	-0.3420	0.0400	0.001358159

Stations	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Ellipsoidal Changes in Elevation (Dh) [m]	Orthometric Changes in Elevation (DH) [m]	Difference Between Changes in Elevations (Diff) [m]	Mean Square Error (MSE) [m]
XST107	25.4700	3.3400	0.5950	0.5500	0.0450	0.001751691
XST114	26.2180	3.9330	-0.7480	-0.5930	-0.1550	0.025010407
XST44	26.4830	4.2290	-0.2650	-0.2960	0.0310	0.000775801
YTT2-14A	25.0230	2.7750	1.4600	1.4540	0.0060	8.14081E-06
YTT2-25A	25.4180	3.1750	-0.3950	-0.4000	0.0050	3.43439E-06
YTT2-37A	27.3180	5.1000	-1.9000	-1.9250	0.0250	0.000477563
YTT2-48A	26.6820	4.4510	0.6360	0.6490	-0.0130	0.000260719
XST55	30.0470	7.3470	-3.3650	-2.8960	-0.4690	0.22292259
YTT17-08A	28.4460	5.5410	1.6010	1.8060	-0.2050	0.043325086
XST53	28.5270	5.6880	-0.0810	-0.1470	0.0660	0.003950526
FGPLA-Y-008	30.5720	7.7810	-2.0450	-2.0930	0.0480	0.002011811
XST59	28.0940	4.9210	2.4780	2.8600	-0.3820	0.148338049
CFPA18	27.4720	4.6070	0.6225	0.3140	0.3085	0.093232644
XST69	27.0360	4.3790	0.4355	0.2280	0.2075	0.041765548
YTT28-1	28.3030	5.3304	-1.2670	-0.9510	-0.3150	0.101523031
ZTT45-200	28.5930	5.7190	-0.2900	-0.3890	0.0985	0.009088421
MCS1144S-A	29.6720	6.9990	-1.0790	-1.2800	0.2010	0.039145893
YTT28-151	25.7550	3.2189	3.9173	3.7800	0.1372	0.017972945
YTT28-134	27.1230	4.1703	-1.3680	-0.9510	-0.4170	0.176279723
ZTT6-53	55.1210	31.9280	-28.0000	-27.7600	-0.2410	0.059363758
YTT27-33	73.0000	49.0840	-17.4000	-17.1600	-0.2460	0.062074122
YTT27-41	59.7990	36.3770	12.7240	12.7100	0.0170	0.000191911
YTT16-76A	29.3680	6.7340	30.4310	29.6400	0.7880	0.615994563
XST121	24.4600	1.9710	4.9080	4.7630	0.1450	0.020122333
YTT28-200	25.3820	2.9555	-0.9220	-0.9850	0.0627	0.003548967
XT101	62.3990	39.0800	-37.0200	-36.1200	-0.8930	0.802577304
ZTT30-5	50.4540	27.3150	11.9450	11.7700	0.1800	0.031277058
MCS1178T-A	25.5580	3.0310	24.8960	24.2800	0.6120	0.370702233
YTT9-73A	27.7320	5.2940	-2.1740	-2.2630	0.0890	0.007370774
XST165	47.3230	24.1220	-19.590	-18.8300	-0.7630	0.586980902
XST126	59.3190	35.9600	-12.0000	-11.8400	-0.1580	0.025968288
YTT9-29A	26.0500	3.5700	33.2690	32.3900	0.8790	0.767118847
XST215	25.6900	2.6600	0.3600	0.9100	-0.5500	0.30597137
ZTT35-26	26.9560	4.8720	-1.2660	-2.2120	0.9460	0.888972178
ZTT34-34	30.9890	7.8590	-4.0330	-2.9870	-1.0460	1.100708985
YTT13-27	53.4290	30.3870	-22.4400	-22.5300	0.0880	0.007200067

Stations	Ellipsoidal Height (h) [m]	Orthometric Height (H) [m]	Ellipsoidal Changes in Elevation (Dh) [m]	Orthometric Changes in Elevation (DH) [m]	Difference Between Changes in Elevations (Diff) [m]	Mean Square Error (MSE) [m]
XT161	48.0910	25.1640	5.3380	5.2230	0.1150	0.012511141
XST202	26.0590	2.9390	22.0320	22.2300	-0.1930	0.038473563
YTT13-30	56.5500	33.5130	-30.4900	-30.5700	0.0830	0.006376535
XST204	27.1270	4.9060	29.4230	28.6100	0.8160	0.660730343

Appendix C5:
Full Data Set for Table 4.3 – Geoidal Undulations Computed Using each of the Degree of the Zanletnyik Hungarian Polynomial Model

Table 4.3: Undulation Computed Using Each Degree of the Zanletnyik Hungarian Polynomial Model

Stations	1st Degree [m]	2^{nd} Degree [m]	3rd Degree [m]	4th Degree [m]	5th Degree [m]	6th Degree [m]	7th Degree [m]	8th Degree [m]
AP4	18.9443	18.9408	21.1712	11.2761	14.2913	18.5150	166.8048	402.7213
AP1	18.9303	18.9278	21.1583	11.2900	13.8830	18.4890	167.1157	403.5774
PT.3 EMMA	19.0001	18.9980	21.1594	10.7564	13.8509	18.2854	168.6081	405.1638
PHCS 1s	19.0223	19.0170	21.1653	10.6370	14.0435	18.2426	168.7446	405.1322
PT.4 EMMA	18.9994	18.9960	21.1624	10.8043	14.0536	18.3238	168.3134	404.5598
PT.8 EMMA	18.9820	18.9770	21.1741	11.0329	14.5001	18.4521	167.322	402.8974
PT.4 ABDUL	18.9979	18.9910	21.1922	11.0465	15.0957	18.5051	166.7754	401.5306
PT.5 EMMA	18.9991	18.9940	21.1670	10.8555	14.2807	18.3645	167.9875	403.8965
PT.7 EMMA	18.9862	18.9810	21.1693	10.9680	14.3527	18.4161	167.6256	403.4062
PT.9 EMMA	18.9898	18.9800	21.1838	11.0467	14.8221	18.4828	167.0085	402.1426
PT.2 ABDUL	19.0127	19.0060	21.2160	11.0873	15.7708	18.5757	166.084	399.8983
PT.3 ABDUL	19.0054	18.9990	21.2038	11.0665	15.4369	18.5404	166.4281	400.7091
GPS 02	18.8972	18.9120	21.2469	12.1190	16.2482	18.7604	162.1648	396.4643
GPS 03	18.8388	18.8260	21.1540	12.0725	14.1345	18.6206	164.2545	401.2933
GPS 04	18.8451	18.8330	21.1551	12.0101	14.0835	18.6171	164.5372	401.5328
GPS 05	18.8461	18.8370	21.1707	12.1212	14.5705	18.6470	163.6755	400.2405
GPS 06	18.8419	18.8290	21.1546	12.0426	14.1116	18.6192	164.3894	401.4048
GPS 07	18.8467	18.8350	21.1538	11.9833	14.0256	18.6128	164.6887	401.7133
GPS 08	18.8518	18.8400	21.1494	11.8970	13.8381	18.5964	165.1703	402.2971
GPS 09	18.8498	18.8380	21.1457	11.8895	13.7427	18.5898	165.2810	402.5172
GPS 10	18.8473	18.8350	21.1413	11.8816	13.6294	18.5820	165.4084	402.7757
GPS 11	18.8619	18.8560	21.1845	12.0492	14.7917	18.6631	163.7580	399.9024
GPS 12	18.8642	18.8590	21.1866	12.0392	14.8285	18.6657	163.7634	399.8418
GPS 13	18.8668	18.8620	21.1888	12.0289	14.8696	18.6684	163.7662	399.7721
GPS 14	18.8546	18.8440	21.1504	11.8764	13.8436	18.5953	165.2397	402.3287
GPS 15	18.8591	18.8490	21.1532	11.8521	13.8909	18.5963	165.2848	402.2782
GPS 16	18.8633	18.8540	21.1558	11.8299	13.9374	18.5973	165.3225	402.2250
GPS 17	18.8678	18.8590	21.1582	11.8045	13.9787	18.5975	165.3757	402.1903
GPS 18	18.8705	18.8620	21.1580	11.7781	13.9574	18.5932	165.4863	402.2926
GPS 19	18.9028	18.8990	21.1517	11.4567	13.6816	18.5182	166.7827	403.6064
GPS 20	18.9019	18.8980	21.1515	11.4630	13.6783	18.5194	166.7660	403.5987
GPS 21	18.9045	18.9010	21.1536	11.4571	13.7383	18.5222	166.7325	403.4836
GPS 22	18.9027	18.9030	21.1366	11.3379	13.1751	18.4530	167.5826	404.9779

Stations	1st Degree [m]	2^{nd} Degree [m]	3rd Degree [m]	4th Degree [m]	5th Degree [m]	6th Degree [m]	7th Degree [m]	8th Degree [m]
GPS 23	18.9012	18.9010	21.1355	11.3421	13.1407	18.4518	167.5997	405.0406
GPS 24	18.9023	18.9030	21.1353	11.3299	13.1273	18.4475	167.6477	405.0996
GPS 25	18.8984	18.8990	21.1336	11.3494	13.0813	18.4499	167.6288	405.1482
GPS 26	18.9141	18.9280	21.1239	11.0674	12.3607	18.3086	169.0578	407.4344
GPS 27	18.9133	18.9280	21.1234	11.0678	12.3349	18.3071	169.0787	407.4875
GPS 28	18.9125	18.9270	21.1229	11.0677	12.3075	18.3052	169.1025	407.5456
GPS 29	18.8925	18.9130	21.1121	11.1053	11.809	18.2870	169.4245	408.4742
GPS 30	18.8922	18.9130	21.1121	11.1108	11.8254	18.2901	169.3954	408.4242
GPS 31	18.8925	18.9120	21.1123	11.1152	11.8543	18.2936	169.3582	408.3512
GPS 32	18.9255	18.9310	21.2337	11.7807	15.7823	18.7224	163.8678	398.2526
GPS 33	18.9248	18.9300	21.2344	11.7911	15.8006	18.7247	163.8145	398.1888
GPS 34	18.9236	18.9290	21.2349	11.8033	15.8114	18.7265	163.7613	398.138
GPS 35	18.9813	19.0040	21.2993	11.6977	17.2919	18.8355	162.8236	394.9586
GPS 36	18.9806	19.0040	21.3006	11.7093	17.3172	18.8387	162.7601	394.8739
GPS 37	18.9796	19.0050	21.3041	11.7336	17.3846	18.8464	162.6133	394.6621
GPS 38	19.0286	19.0460	21.3072	11.4092	17.6662	18.8277	163.4755	394.7368
GPS 39	19.0287	19.0470	21.3098	11.4209	17.7147	18.8343	163.3975	394.5971
GPS 40	19.0281	19.0480	21.3127	11.4363	17.7631	18.8416	163.3033	394.4462
GPS 41	19.0629	19.0740	21.3097	11.2032	17.9467	18.8094	163.8877	394.5712
GPS 42	19.0646	19.0760	21.3131	11.2068	18.0182	18.8174	163.8160	394.3938
GPS 43	19.0664	19.0790	21.3167	11.2099	18.0925	18.8255	163.7435	394.2107
GPS 45	19.1165	19.1230	21.3379	10.9887	18.8220	18.8425	163.8057	393.0411
GPS 46	19.1158	19.1240	21.3406	11.0028	18.8592	18.8507	163.7327	392.9149
GPS 47	19.1654	19.1360	21.2950	10.5401	18.4831	18.6408	165.3515	395.1417
GPS 48	19.1666	19.1370	21.2959	10.5363	18.5078	18.6421	165.3423	395.0949
GPS 49	19.1690	19.1390	21.2976	10.5290	18.5536	18.6444	165.3256	395.0083
GPS 50	18.9156	18.9120	21.1860	11.5819	14.6284	18.6109	165.5605	401.2612
GPS 51	18.9143	18.9100	21.1862	11.5936	14.6322	18.6132	165.5191	401.2269
GPS 53	18.9626	18.9690	21.1448	10.8887	13.1818	18.2935	168.8356	406.2161
GPS 54	18.9619	18.9690	21.1443	10.8851	13.1433	18.2891	168.8780	406.3113
GPS 55	18.9631	18.9700	21.1448	10.8829	13.1728	18.2903	168.8586	406.2539
GPS 56	19.0090	19.0060	21.1613	10.7004	13.8885	18.2615	168.7177	405.2646
GPS 57	19.0079	19.0060	21.1609	10.7050	13.8746	18.2627	168.7178	405.2802
GPS 58	19.0072	19.0050	21.1608	10.7112	13.8776	18.2659	168.6998	405.2529
GPS 59	18.7954	18.7990	21.0533	11.6406	10.9692	18.3920	168.5115	408.7272
GPS 60	18.7970	18.8000	21.0548	11.6352	10.9978	18.3925	168.5052	408.6847

Stations	1st Degree [m]	2nd Degree [m]	3rd Degree [m]	4th Degree [m]	5th Degree [m]	6th Degree [m]	7th Degree [m]	8th Degree [m]
XSV 662	18.9510	18.9470	21.1836	11.3079	14.6514	18.5509	166.3992	401.8501
ZVS 3003	19.0200	19.0150	21.2291	11.1090	16.1105	18.6122	165.7315	399.0682

Appendix C6:
Full Data Set for Table 4.4a: Summary of the Results from the Local and Existing Geoid Models for Port Harcourt

Table 4.4a: Summary of the Results from the Local and Existing Models for Port Harcourt Area

STATIONS	Observed Undulation [m]	North Sea Region Model [m]	4-parameters Similarity Datum Shift [m]	5-parameters Similarity Datum Shift [m]	7-parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM 2008 [m]
AP4	18.9229	18.9482	18.9542	18.9463	19.5078	18.9408	18.9229	18.947
AP1	18.9119	18.9336	18.9393	18.9319	19.5232	18.9278	18.9444	18.934
PT.3 EMMA	18.9667	19.0024	19.0038	18.9894	19.5801	18.9984	18.9309	19.011
PHCS 1s	18.9980	19.0214	19.0232	19.0060	19.5965	19.0171	18.9936	19.033
PT.4 EMMA	19.0024	19.0014	19.0044	18.9910	19.5819	18.9958	19.0139	19.008
PT.8 EMMA	18.9381	18.9852	18.9910	18.9811	19.5723	18.9770	18.9934	18.986
PT.4 ABDUL	19.0028	19.0004	19.0073	18.9976	19.5889	18.9910	18.9788	19.000
PT.5 EMMA	18.9939	19.0009	19.0054	18.9929	19.5839	18.9938	18.9944	19.006
PT.7 EMMA	19.0074	18.9891	18.9943	18.9836	19.5747	18.9814	18.9937	18.992
PT.9 EMMA	18.9750	18.9926	18.9991	18.9893	19.5806	18.9836	18.9822	18.993
PT.2 ABDUL	18.9861	19.0154	19.0223	19.0134	19.6046	19.0064	18.9865	19.013
PT.3 ABDUL	18.9803	19.0080	19.0151	19.0057	19.5969	18.9986	19.0093	19.006
GPS 02	18.9040	18.9060	18.8974	18.8812	19.4711	18.9117	18.9040	18.900
GPS 03	18.8250	18.8270	18.8361	18.8234	19.4134	18.8257	18.8010	18.825
GPS 04	18.8330	18.8349	18.8440	18.8325	19.4227	18.8329	18.8795	18.832
GPS 05	18.8340	18.8372	18.8432	18.8288	19.4187	18.8366	18.8826	18.835
GPS 06	18.8280	18.8308	18.8400	18.8279	19.4179	18.8292	18.8658	18.829
GPS 07	18.8350	18.8368	18.8462	18.8352	19.4254	18.8346	18.8779	18.834
GPS 08	18.8420	18.8430	18.8529	18.8434	19.4339	18.8403	18.8767	18.840
GPS 09	18.8400	18.8405	18.8508	18.8416	19.4320	18.8379	18.8724	18.838
GPS 10	18.8360	18.8373	18.8480	18.8392	19.4297	18.8350	18.8717	18.835
GPS 11	18.8540	18.8573	18.8617	18.8483	19.4384	18.8559	18.9025	18.853
GPS 12	18.8570	18.8602	18.8645	18.8511	19.4413	18.8588	18.8776	18.856
GPS 13	18.8610	18.8634	18.8674	18.8542	19.4444	18.8619	18.8778	18.859
GPS 14	18.8450	18.8464	18.8562	18.8469	19.4374	18.8435	18.8517	18.843
GPS 15	18.8510	18.8520	18.8614	18.8523	19.4429	18.8488	18.8795	18.848
GPS 16	18.8560	18.8572	18.8663	18.8573	19.4480	18.8537	18.8792	18.853
GPS 17	18.8610	18.8626	18.8714	18.8627	19.4533	18.8588	18.8793	18.858
GPS 18	18.8650	18.8658	18.8746	18.8661	19.4568	18.8619	18.8771	18.861
GPS 19	18.9040	18.9036	18.9106	18.9039	19.4950	18.8990	18.9040	18.904
GPS 20	18.9030	18.9026	18.9096	18.9030	19.4941	18.8980	18.9094	18.903
GPS 21	18.9060	18.9054	18.9124	18.9056	19.4968	18.9005	18.9084	18.906
GPS 22	18.9070	18.9045	18.9097	18.9036	19.4947	18.9027	18.9112	18.908
GPS 23	18.9050	18.9027	18.9080	18.9020	19.4931	18.9012	18.9112	18.907
GPS 24	18.9070	18.9041	18.9092	18.9032	19.4942	18.9027	18.9095	18.908
GPS 25	18.9020	18.8998	18.9051	18.8992	19.4903	18.8986	18.9109	18.904
GPS 26	18.9300	18.9210	18.9179	18.9107	19.5015	18.9281	18.9065	18.932

STATIONS	Observed Undula tion [m]	North Sea Region Model [m]	4-parameters Similarity Datum Shift [m]	5-parameters Similarity Datum Shift [m]	7-parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM 2008 [m]
GPS 27	18.9300	18.9202	18.9170	18.9098	19.5006	18.9276	18.9276	18.932
GPS 28	18.9290	18.9195	18.9161	18.9090	19.4998	18.9272	18.9267	18.931
GPS 29	18.9130	18.8997	18.8941	18.8887	19.4793	18.9133	18.9259	18.915
GPS 30	18.9120	18.8993	18.8939	18.8886	19.4792	18.9126	18.9044	18.914
GPS 31	18.9120	18.8994	18.8943	18.8890	19.4796	18.9122	18.904	18.914
GPS 32	18.9350	18.9333	18.9330	18.9225	19.5133	18.9309	18.9434	18.924
GPS 33	18.9340	18.9326	18.9321	18.9214	19.5122	18.9304	18.9286	18.924
GPS 34	18.9330	18.9314	18.9307	18.9199	19.5106	18.9295	18.9277	18.923
GPS 35	19.0040	18.9984	18.9892	18.9803	19.5709	19.0042	18.9406	18.989
GPS 36	19.0030	18.9981	18.9884	18.9793	19.5700	19.0042	18.875	18.988
GPS 37	19.0040	18.9978	18.9870	18.9777	19.5683	19.0050	18.8753	18.988
GPS 38	19.0470	19.0414	19.0370	19.0309	19.6217	19.0460	18.9262	19.035
GPS 39	19.0480	19.0420	19.0370	19.0309	19.6217	19.0471	19.0427	19.036
GPS 40	19.0480	19.0420	19.0363	19.0302	19.6210	19.0478	19.0426	19.036
GPS 41	19.0760	19.0695	19.0694	19.0644	19.6551	19.0738	19.0418	19.075
GPS 42	19.0780	19.0714	19.0709	19.0660	19.6568	19.0764	19.0592	19.077
GPS 43	19.0800	19.0734	19.0725	19.0678	19.6585	19.0792	19.0609	19.079
GPS 45	19.1210	19.1127	19.1163	19.1135	19.7037	19.1234	19.0627	19.121
GPS 46	19.1220	19.1128	19.1157	19.1131	19.7033	19.1241	19.1094	19.121
GPS 47	19.1400	19.1336	19.1541	19.1453	19.7348	19.1364	19.1089	19.146
GPS 48	19.1400	19.1345	19.1551	19.1463	19.7358	19.1374	19.1527	19.147
GPS 49	19.1420	19.1360	19.1570	19.1482	19.7376	19.1393	19.1539	19.149
GPS 50	18.9180	18.9183	18.9244	18.9162	19.5073	18.9119	19.0819	18.915
GPS 51	18.9170	18.9168	18.9229	18.9146	19.5057	18.9105	18.9211	18.913
GPS 53	18.9760	18.9686	18.9673	18.9558	19.5466	18.9689	19.1561	18.978
GPS 54	18.9760	18.9681	18.9665	18.9549	19.5458	18.9688	18.9582	18.978
GPS 55	18.9760	18.9692	18.9677	18.9561	19.5469	18.9696	18.9576	18.979
GPS 56	19.0180	19.0102	19.0114	18.9957	19.5863	19.0064	18.9587	19.020
GPS 57	19.0170	19.0093	19.0105	18.9948	19.5854	19.0056	19.0016	19.019
GPS 58	19.0160	19.0086	19.0098	18.9943	19.5850	19.0048	19.0006	19.018
GPS 59	18.7910	18.7789	18.7874	18.7890	19.3790	18.7988	18.9194	18.786
GPS 60	18.7930	18.7809	18.7893	18.7908	19.3808	18.8005	18.7934	18.795
XSV 662	18.9550	18.9553	18.9614	18.9534	19.5447	18.9473	18.7952	18.953
ZVS 3003	19.0260	19.0230	19.0296	19.0211	19.6123	19.0150	18.9625	19.020

Appendix C7:
Full Data Set for Table 4.4b: Summary of The Results from the Local and Existing Geoid Models for Lagos State

Table 4.4b: Summary of the Results from the Local and Existing Geoid Models for Lagos State

Stations	Model 1 [m]	Model 2 [m]	Model 3 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 8 [m]
XST 237	22.5640	22.5444	22.5389	22.4898	20.3034	22.46340	22.5640	22.2640
XST44	22.2540	22.3805	22.3754	22.3222	20.1455	22.30554	22.6056	22.0660
YTT78A	22.4740	22.5187	22.5086	22.4782	20.3016	22.4617	22.8674	22.3580
XST245	22.4910	22.3672	22.3564	22.3141	20.1505	22.3105	22.6140	22.1350
XST244	22.2240	22.3137	22.3015	22.2606	20.1065	22.2665	22.7004	22.0970
FGPLA-Y-003	22.7830	22.7186	22.7376	22.7760	20.5955	22.7555	23.1494	22.4460
CFPA21	22.8280	22.7793	22.7891	22.8211	20.6477	22.8077	22.7958	22.4870
XST 55	22.7000	22.5487	22.6120	22.7104	20.5108	22.6709	22.5935	22.4540
YTT1703A	22.9120	22.7746	22.8066	22.9127	20.7507	22.9108	22.9243	22.6340
XST46	23.0440	22.9121	22.9259	23.0387	20.9039	23.0639	22.8570	22.7670
XST50	22.8800	22.7744	22.7936	22.8554	20.6862	22.8462	22.6300	22.54700
LWBC5-61P	23.1860	23.1247	23.0975	23.1376	21.0207	23.1807	23.0213	22.8570
YTT19-54	23.1900	23.1423	23.1123	23.1448	21.0278	23.1878	22.7630	22.8690
XST75	23.0230	23.0105	22.9896	22.9918	20.8408	23.0008	22.6386	22.7120
CFPA40	22.6550	22.5418	22.5951	22.6634	20.4619	22.6219	22.3994	22.3700
CFPB36	22.6490	22.5506	22.5968	22.6498	20.4491	22.6092	22.7510	22.3450
XST60	22.5390	22.5268	22.5622	22.5809	20.3778	22.5379	22.7178	22.2660
XST72	22.3960	22.4826	22.5075	22.4929	20.2893	22.4493	22.6995	22.1630
XST76	22.3650	22.4677	22.4893	22.4657	20.2630	22.4230	22.7332	22.1160
XST44	22.2540	22.3716	22.3657	22.3130	20.1385	22.2985	22.6346	22.0630
YTT2-18A	22.2580	22.3572	22.3487	22.2996	20.1316	22.2916	22.7340	22.0800
XST156	22.2170	22.2923	22.2782	22.2439	20.0983	22.2583	22.6852	22.1230
ZTT2-57A	22.2740	22.2915	22.2752	22.2615	20.1276	22.2876	22.7461	22.2800
YTT2-66A	22.2700	22.2726	22.2551	22.2572	20.1349	22.2949	22.7311	22.3420
YTT2-80	22.2410	22.2143	22.1936	22.2195	20.1218	22.2818	22.6938	22.3730
XST224	22.1600	22.0381	22.0034	22.0775	20.0567	22.2167	22.5777	22.2670
ZTT35-14	22.1190	21.9463	21.9019	21.9989	20.0245	22.1846	22.6582	22.2090
XST149	22.9830	23.0202	23.0112	22.9844	20.7905	22.9506	23.7510	22.8870
MCS1188T-A	22.6220	22.6708	22.6610	22.6271	20.4377	22.5978	22.4360	22.4840
XST42	23.1680	23.1355	23.2123	23.3084	21.0572	23.2171	23.2384	23.2230
YTT13-1A	23.2460	23.217	23.2953	23.3798	21.1275	23.2874	22.8246	23.3090
ZTT34-10A	23.2370	23.1973	23.2592	23.3227	21.0807	23.2406	22.7183	23.3230
XST135	23.3290	23.2995	23.3646	23.4192	21.1780	23.3379	22.8447	23.4040
XST218	23.3210	23.3006	23.3547	23.3951	21.1617	23.3217	22.7417	23.4150
XST209	23.4100	23.3779	23.4247	23.4488	21.2259	23.3859	22.8133	23.4770
XST201	23.4370	23.4043	23.4425	23.4552	21.2413	23.4013	22.7664	23.4930
XST203	23.4600	23.471	23.4928	23.4858	21.2928	23.4528	22.7961	23.5090
XST177	23.5490	23.5368	23.5525	23.5378	21.3583	23.5183	22.8040	23.5460

Stations	Model 1 [m]	Model 2 [m]	Model 3 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 8 [m]
YTT22-1	23.4470	23.4712	23.4773	23.4580	21.2801	23.4401	24.2842	23.4560
XST159	23.5290	23.5915	23.5815	23.5504	21.4052	23.5653	23.0241	23.4780
ZTT31-70	23.5060	23.5855	23.5644	23.5299	21.3973	23.5573	22.9107	23.4240
XST131	23.5900	23.6951	23.6636	23.6260	21.5231	23.6832	23.0447	23.4920
XST127	23.4050	23.4915	23.4649	23.4306	21.2931	23.4531	22.7584	23.3100
XST133	23.4120	23.5097	23.4755	23.4413	21.3168	23.4768	22.9469	23.3010
XST128	23.4190	23.5467	23.5065	23.4732	21.3625	23.5225	22.9788	23.3260
YTT28-117	23.4720	23.5855	23.5399	23.5080	21.4110	23.5710	22.9850	23.3580
MCS1174S-A	23.5810	23.707	23.6541	23.6218	21.5526	23.7127	23.0832	23.4640
YTT28-96	24.6210	23.8407	23.7765	23.7450	21.7151	23.8751	23.0801	23.5810
XST41	23.7780	23.9302	23.8577	23.8269	21.8261	23.9861	23.0428	23.6610
YTT28-89	23.6000	23.7111	23.6477	23.6238	21.5741	23.7341	22.7292	23.5020
YTT28-87	23.5030	23.5395	23.4865	23.4635	21.3758	23.5358	22.7991	23.3380
YTT28-67	23.4260	23.4313	23.3837	23.3621	21.2556	23.4156	22.8531	23.2300
YTT28-65	23.3080	23.2982	23.2572	23.2381	21.1129	23.2729	22.8178	23.0680
YTT28-47	23.0050	23.0527	23.0267	23.0051	20.8460	23.0060	22.7181	22.7920
XST87	22.9800	23.0051	22.9824	22.9647	20.8033	22.9633	22.8991	22.7320
YTT28-30	22.9520	22.9636	22.9438	22.9257	20.7598	22.9199	22.9259	22.6830
YTT28-1	22.9720	22.973	22.9537	22.9448	20.7843	22.9443	22.9462	22.6850
XST71	23.0710	23.0503	23.0271	23.0392	20.898	23.058	22.9920	22.7560
YTT19-7	23.0620	23.0385	23.0169	23.0325	20.8910	23.0510	22.9492	22.7440
YTT19-54	23.1900	23.1423	23.1123	23.1448	21.0278	23.1878	23.0386	22.8690
XST59	23.1730	23.1121	23.0863	23.1261	21.0068	23.1668	22.9244	22.8440
XST120	22.2690	22.3941	22.3897	22.3364	20.1574	22.3174	22.5512	22.0760
CFPA31	22.5800	22.5386	22.5775	22.6081	20.4059	22.5660	22.8162	22.2900
XST64	22.4840	22.5113	22.5432	22.5497	20.3459	22.5060	22.9828	22.2300
XST68	22.4290	22.4954	22.5240	22.5194	20.3153	22.4754	22.9752	22.1940
XST76	22.3650	22.4677	22.4893	22.4657	20.2630	22.4230	22.9854	22.1160
XST83	22.3210	22.4396	22.4548	22.4168	20.2172	22.3772	22.9845	22.0790
XST84	22.2820	22.4227	22.4349	22.3909	20.1937	22.3537	22.9790	22.0710
XST99A	22.2150	22.379	22.3862	22.3335	20.1426	22.3026	22.9652	22.0840
XST241	22.1750	22.3439	22.3491	22.2928	20.1068	22.2668	22.9556	22.0530
XST107	22.1300	22.3035	22.3070	22.2480	20.0679	22.2279	22.9478	22.0100
XST114	22.2850	22.4104	22.4078	22.3542	20.1712	22.3313	23.1035	22.0830
XST44	22.2540	22.3716	22.3657	22.3130	20.1385	22.2985	22.9684	22.0630
YTT2-14A	22.2480	22.3537	22.3461	22.2952	20.1259	22.2859	22.9725	22.0630
YTT2-25A	22.2430	22.3297	22.3194	22.2734	20.1125	22.2725	22.8353	22.0750
YTT2-37A	22.2180	22.2975	22.2840	22.2476	20.0995	22.2596	22.8259	22.1130
YTT2-48A	22.2310	22.2824	22.2669	22.2396	20.1003	22.2603	22.8197	22.1620
XST55	22.7000	22.0505	22.0297	21.9881	19.8913	22.0513	22.7899	21.8810

Stations	Model 1 [m]	Model 2 [m]	Model 3 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 8 [m]
YTT17-08A	22.9050	22.7689	22.8004	22.9022	20.7385	22.8986	22.5532	22.6190
XST53	22.8390	22.7532	22.7706	22.8173	20.6427	22.8027	22.8463	22.4970
FGPLA-Y-008	22.7910	22.7685	22.7768	22.7999	20.6237	22.7837	22.8285	22.4610
XST59	23.1730	23.1121	23.0863	23.1261	21.0068	23.1668	22.8134	22.8440
CFPA18	22.8650	22.845	22.8447	22.8675	20.701	22.861	22.8027	22.5400
XST69	22.6570	22.6913	22.7000	22.6993	20.5116	22.6716	22.8188	22.3480
YTT28-1	22.9720	22.973	22.9537	22.9448	20.7843	22.9443	22.8418	22.6850
ZTT45-200	22.8740	22.88615	22.8725	22.85648	20.6840	22.844	22.8091	22.5870
MCS1144S-A	22.6730	22.72629	22.7210	22.69136	20.50339	22.6634	22.8152	22.4060
YTT28-151	22.5360	22.6263	22.6206	22.5770	20.3854	22.5454	22.8371	22.3250
YTT28-134	22.9530	22.882	22.8698	22.8360	20.6437	22.8038	22.8754	22.6900
ZTT6-53	23.1930	23.2545	23.2186	23.1929	21.0532	23.2133	22.7453	23.0060
YTT27-33	23.4390	23.5463	23.5021	23.4708	21.3668	23.5268	22.8387	23.3220
YTT27-41	23.4220	23.5282	23.4864	23.4543	21.3440	23.5041	22.8085	23.3060
YTT16-76A	22.6340	22.6368	22.6306	22.6163	20.4310	22.5911	22.8706	22.8450
XST121	22.4890	22.5886	22.5804	22.5344	20.3454	22.5055	22.7205	22.3040
YTT28-200	22.4260	22.51	22.5028	22.4547	20.2697	22.4297	22.8089	22.2210
XT101	23.3190	23.3873	23.3682	23.3361	21.1773	23.3374	22.8596	23.2350
ZTT30-5	23.1390	23.1828	23.1741	23.1481	20.9606	23.1207	22.8182	23.0920
MCS1178T-A	22.5270	22.5887	22.5788	22.5409	20.3556	22.5157	22.7697	22.3760
YTT9-73A	22.4380	22.479	22.4683	22.4404	20.2693	22.4294	22.8297	22.3370
XST165	23.2010	23.2218	23.2204	23.2006	21.0081	23.1682	22.8340	23.1840
XST126	23.3590	23.3488	23.3592	23.3474	21.1529	23.3130	22.8299	23.3660
YTT9-29A	22.4800	22.4514	22.4443	22.4639	20.3101	22.4702	22.8108	22.5860
XST215	23.0300	22.9828	23.0135	23.0546	20.8335	22.9935	22.8457	23.1540
ZTT35-26	22.0840	21.8665	21.8122	21.9339	20.0065	22.1665	22.8276	22.1250
ZTT34-34	23.1300	23.0773	23.1363	23.2135	20.9738	23.1338	22.8204	23.2220
YTT13-27	23.0420	22.9718	23.0145	23.0797	20.8527	23.0128	22.7881	23.1490
XT161	22.9270	22.8667	22.8939	22.9442	20.7310	22.8911	22.7859	23.0590
XST202	23.1200	23.0961	23.1254	23.1517	20.9299	23.0900	22.7926	23.2360
YTT13-30	23.0370	22.9693	23.0095	23.0704	20.8448	23.0049	22.8320	23.1490
XST204	22.2210	22.1551	22.1304	22.1741	20.1008	22.2608	22.7624	22.3520
SUM	2514	2514.3	2514.04	2514.04	2276.43	2514.04	2514.04	2497.10
MEAN	22.8550	22.8573	22.8549	22.8549	20.6949	22.8549		22.7010

Appendix C8:
Full Data Set for Table 4.5a - Residuals for the Existing Geoid Models for Port Harcourt

Table 4.5a: Residuals for the Existing Geoid Models for Port Harcourt

Stations	Model 2 [m]	Model 3 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model7 [m]	Model 8 [m]
AP4	-0.0253	-0.0313	-0.0234	-0.5850	-0.0179	-0.0241	0
AP1	-0.0217	-0.0275	-0.0200	-0.6113	-0.0159	-0.0221	-0.0325
PT.3 EMMA	-0.0357	-0.0371	-0.0226	-0.6134	-0.0317	-0.0443	0.03587
PHCS 1s	-0.0234	-0.0252	-0.0080	-0.5985	-0.0191	-0.0350	0.00444
PT.4 EMMA	0.0011	-0.0020	0.01144	-0.5794	0.00664	-0.0056	-0.0115
PT.8 EMMA	-0.0471	-0.0529	-0.0430	-0.6342	-0.0389	-0.0479	-0.0553
PT.4 ABDUL	0.0024	-0.0045	0.00517	-0.5861	0.0118	0.0028	0.02398
PT.5 EMMA	-0.0070	-0.0115	0.00098	-0.5900	1E-04	-0.0121	-0.0005
PT.7 EMMA	0.0183	0.01314	0.02384	-0.5673	0.02599	0.01539	0.01373
PT.9 EMMA	-0.0176	-0.0241	-0.0143	-0.6056	-0.0086	-0.018	-0.0072
PT.2 ABDUL	-0.0293	-0.0363	-0.0273	-0.6186	-0.0203	-0.0269	-0.0004
PT.3 ABDUL	-0.0276	-0.0347	-0.0254	-0.6166	-0.0183	-0.0257	-0.0289
GPS 02	-0.0020	0.00656	0.02276	-0.5671	-0.0077	0.0040	0.0000
GPS 03	-0.0020	-0.0111	0.00158	-0.5884	-0.0007	o.0000	0.0240
GPS 04	-0.0019	-0.0110	0.0005	-0.5897	1E-04	0.0010	-0.0465
GPS 05	-0.0032	-0.0092	0.00523	-0.5847	-0.0026	-0.0010	-0.0486
GPS 06	-0.0028	-0.012	0.00014	-0.5899	-0.0012	-0.0010	-0.0378
GPS 07	-0.0018	-0.0112	-0.0002	-0.5904	0.0004	0.0010	-0.0429
GPS 08	-0.0010	-0.0109	-0.0014	-0.5919	0.0017	0.0020	-0.0347
GPS 09	-0.0005	-0.0108	-0.0016	-0.5920	0.0021	0.0020	-0.0324
GPS 10	-0.0013	-0.0120	-0.0032	-0.5937	0.001	0.0010	-0.0357
GPS 11	-0.0033	-0.0077	0.00571	-0.5844	-0.0019	0.0010	-0.0485
GPS 12	-0.0032	-0.0075	0.00585	-0.5843	-0.0018	0.0010	-0.0206
GPS 13	-0.0024	-0.0064	0.0068	-0.5834	-0.0009	0.0020	-0.0168
GPS 14	-0.0014	-0.0112	-0.0019	-0.5924	0.0015	0.0020	-0.0067
GPS 15	-0.0010	-0.0104	-0.0013	-0.5919	0.0022	0.0030	-0.0285
GPS 16	-0.0012	-0.0103	-0.0013	-0.5920	0.0023	0.0030	-0.0232
GPS 17	-0.0016	-0.0104	-0.0017	-0.5923	0.0022	0.0030	-0.0183
GPS 18	-0.0008	-0.0096	-0.0011	-0.5918	0.0031	0.0040	-0.0121
GPS 19	0.0004	-0.0066	8.6E-05	-0.5910	0.0050	0.0000	0.0000
GPS 20	0.0004	-0.0066	3.6E-05	-0.5911	0.0050	0.0000	-0.0064
GPS 21	0.0006	-0.0064	0.0004	-0.5908	0.0055	0.0000	-0.0024
GPS 22	0.0025	-0.0027	0.0034	-0.5877	0.0043	-0.0010	-0.0042
GPS 23	0.0023	-0.0030	0.0030	-0.5881	0.0038	-0.0020	-0.0062
GPS 24	0.0029	-0.0022	0.0038	-0.5872	0.0043	-0.0010	-0.0025
GPS 25	0.0023	-0.0031	0.0028	-0.5883	0.0034	-0.0020	-0.0089
GPS 26	0.0090	0.0121	0.0193	-0.5715	0.0019	-0.0020	0.0235
GPS 27	0.0098	0.0130	0.0202	-0.5706	0.0024	-0.0020	0.0024
GPS 28	0.0095	0.0129	0.0200	-0.5708	0.0018	-0.0020	0.0023

Stations	Model 1 [m]	Model 2 [m]	Model 3 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 8 [m]
GPS 29	0.0133	0.0189	0.0243	-0.5663	-0.0003	-0.0020	-0.0129
GPS 30	0.0127	0.0181	0.0234	-0.5672	-0.0006	-0.0020	0.0077
GPS 31	0.0126	0.0177	0.0230	-0.5676	-0.0002	-0.0020	0.0080
GPS 32	0.0017	0.0020	0.0126	-0.5783	0.0041	0.0110	-0.0084
GPS 33	0.0014	0.0019	0.0126	-0.5782	0.0036	0.0100	0.0054
GPS 34	0.0016	0.0023	0.0131	-0.5776	0.0035	0.0100	0.0053
GPS 35	0.0056	0.0148	0.0238	-0.5669	-0.0002	0.0150	0.0634
GPS 36	0.0050	0.0146	0.0237	-0.5670	-0.0012	0.0150	0.1200
GPS 37	0.0062	0.0170	0.0263	-0.5643	-0.0010	0.0160	0.1288
GPS 38	0.0056	0.0100	0.0161	-0.5747	0.0010	0.0120	0.1208
GPS 39	0.0060	0.0111	0.0171	-0.5737	0.0009	0.0120	0.0053
GPS 40	0.0060	0.0117	0.0178	-0.5730	0.0002	0.0120	0.0054
GPS 41	0.0065	0.0066	0.0116	-0.5791	0.0022	0.0010	0.0342
GPS 42	0.0066	0.0071	0.0120	-0.5788	0.0016	0.0010	0.0188
GPS 43	0.0066	0.0075	0.0122	-0.5785	0.0008	0.0010	0.0191
GPS 45	0.0083	0.0047	0.0075	-0.5827	-0.0024	0.0000	0.0583
GPS 46	0.0093	0.0063	0.0090	-0.5813	-0.0021	0.0010	0.0126
GPS 47	0.0064	-0.0141	-0.0053	-0.5948	0.0036	-0.0060	0.0311
GPS 48	0.0055	-0.0151	-0.0063	-0.5958	0.0026	-0.0070	-0.0127
GPS 49	0.0060	-0.015	-0.0062	-0.5956	0.0027	-0.0070	-0.0119
GPS 50	-0.0003	-0.0064	0.0018	-0.5893	0.0061	0.0030	-0.1639
GPS 51	0.0002	-0.0059	0.0024	-0.5887	0.0065	0.0040	-0.0040
GPS 53	0.0074	0.0088	0.0202	-0.5706	0.0071	-0.0020	-0.1801
GPS 54	0.0079	0.0095	0.0211	-0.5698	0.0072	-0.0020	0.0178
GPS 55	0.0068	0.0083	0.0199	-0.5709	0.0064	-0.0030	0.0184
GPS 56	0.0078	0.0066	0.0223	-0.5683	0.0116	-0.0020	0.0593
GPS 57	0.0077	0.0066	0.0222	-0.5684	0.0114	-0.0020	0.0154
GPS 58	0.0074	0.0062	0.0217	-0.5690	0.0112	-0.0020	0.0154
GPS 59	0.0121	0.0036	0.0020	-0.5880	-0.0078	0.0050	-0.1284
GPS 60	0.0121	0.0037	0.0022	-0.5878	-0.0075	-0.0020	-0.0004
XSV 662	-0.0003	-0.0064	0.0016	-0.5897	0.0077	0.0020	0.1598
ZVS 3003	0.0030	-0.0036	0.0049	-0.5863	0.0110	0.0060	0.0635
sum	1E-05	-0.2625	0.37551	-41.53	0.00073	-0.1315	-0.0114
Mean	1E-07	-0.0037	0.00529	-0.585	1E-05	-0.0019	-0.0002

Appendix C9:
Full Data Set for Table 4.5b: Residuals for the Existing Geoid Models for Lagos State

Table 4.5b: Result of the Differences Between Observed Undulation and the Existing Models (Residuals) for Lagos State

Stations	North Sea Region Model [m]	4-parameters Similarity Datum Shift [m]	5-parameters Similarity Datum Shift [m]	7-parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM2008 [m]
XST 237	0.0196	0.0251	0.0742	2.2606	0.1006	-0.0770	0.3000
XST44	-0.1265	-0.1210	-0.0680	2.1085	-0.0520	-0.3980	0.1880
YTT78A	-0.0447	-0.0350	-0.0040	2.1724	0.0123	-0.1970	0.1160
XST245	0.1238	0.1346	0.1769	2.3405	0.1805	-0.1870	0.3560
XST244	-0.0897	-0.0780	-0.0370	2.1175	-0.0430	-0.4820	0.1270
FGPLA-Y-003	0.0644	0.0454	0.0070	2.1875	0.0275	-0.6180	0.3370
CFPA21	0.0487	0.0389	0.0069	2.1803	0.0203	-0.0680	0.3410
XST 55	0.1513	0.088	-0.0100	2.1892	0.0291	-0.1550	0.2460
YTT1703A	0.1374	0.1054	-7E-04	2.1613	0.0012	0.1128	0.2780
XST46	0.1319	0.1181	0.0053	2.1401	-0.0200	0.1570	0.2770
XST50	0.1056	0.0864	0.0246	2.1938	0.0338	0.0722	0.3330
LWBC5-61P	0.0613	0.0885	0.0484	2.1653	0.0053	0.4497	0.3290
YTT19-54	0.0477	0.0777	0.0452	2.1622	0.0022	0.3818	0.3210
XST75	0.0125	0.0334	0.0312	2.1822	0.0222	0.2366	0.3110
CFPA40	0.1132	0.0599	-0.0080	2.1931	0.0331	-0.0730	0.2850
CFPB36	0.0984	0.0522	-8E-04	2.1999	0.0398	-0.0340	0.3040
XST60	0.0122	-0.0230	-0.0420	2.1612	0.0011	-0.3190	0.2730
XST72	-0.0866	-0.1120	-0.0970	2.1067	-0.0530	-0.3770	0.2330
XST76	-0.1027	-0.1240	-0.1010	2.102	-0.0580	-0.8590	0.2490
XST44	-0.1176	-0.1120	-0.0590	2.1155	-0.0450	-0.4910	0.1910
YTT2-18A	-0.0992	-0.0910	-0.0420	2.1264	-0.0340	-0.5050	0.1780
XST156	-0.0753	-0.0610	-0.0270	2.1187	-0.0410	-0.4550	0.0940
ZTT2-57A	-0.0175	-0.0010	0.0125	2.1464	-0.0140	-0.4090	-0.0060
YTT2-66A	-0.0026	0.0150	0.0128	2.1351	-0.0250	-0.4800	-0.0720
YTT2-80	0.0267	0.0474	0.0215	2.1192	-0.0410	-0.5120	-0.1320
XST224	0.1219	0.1566	0.0825	2.1033	-0.0570	-0.5490	-0.1070
ZTT35-14	0.1727	0.2171	0.1201	2.0945	-0.0660	-0.6030	-0.0900
XST149	-0.0372	-0.0280	-0.0010	2.1925	0.0324	0.2131	0.0960
MCS1188T-A	-0.0488	-0.0390	-0.0050	2.1843	0.0242	-0.1720	0.1380
XST42	0.0325	-0.0440	-0.1400	2.1108	-0.0490	0.3857	-0.0550
YTT13-1A	0.029	-0.0490	-0.1340	2.1185	-0.0410	0.4567	-0.0630
ZTT34-10A	0.0397	-0.0220	-0.0860	2.1563	-0.0040	0.2565	-0.0860
XST135	0.0295	-0.0360	-0.0900	2.151	-0.0090	0.5593	-0.0750
XST218	0.0204	-0.0340	-0.0740	2.1593	-7E-04	0.5225	-0.0940

Stations	North Sea Region Model [m]	4-parameters Similarity Datum Shift [m]	5-parameters Similarity Datum Shift [m]	7-parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM2008 [m]
XST209	0.0321	-0.0150	-0.0390	2.1841	0.0241	0.6105	-0.0670
XST201	0.0327	-0.0060	-0.0180	2.1957	0.0357	0.6631	-0.0560
XST203	-0.011	-0.0330	-0.0260	2.1672	0.0072	0.6614	-0.0490
XST177	0.0122	-0.0040	0.0112	2.1907	0.0307	0.7293	-0.2110
YTT22-1	-0.0242	-0.0300	-0.0110	2.1669	0.0069	-0.1770	-0.0090
XST159	-0.0625	-0.0520	-0.0210	2.1238	-0.0360	-0.3410	0.0510
ZTT31-70	-0.0795	-0.0580	-0.0240	2.1087	-0.0510	-0.1380	0.0820
XST131	-0.1051	-0.0740	-0.0360	2.0669	-0.0930	-0.2830	0.0980
XST127	-0.0865	-0.0600	-0.0260	2.1119	-0.0480	-0.4990	0.0950
XST133	-0.0977	-0.0640	-0.0290	2.0952	-0.0650	-0.1270	0.1110
XST128	-0.1277	-0.0880	-0.0540	2.0565	-0.1030	0.1140	0.0930
YTT28-117	-0.114	-0.0680	-0.0360	2.0606	-0.0990	-0.0060	0.1136
MCS1174S-A	-0.126	-0.0730	-0.0410	2.0284	-0.1320	0.2858	0.1170
YTT28-96	0.7803	0.8446	0.8761	2.906	0.7460	1.3895	1.0400
XST41	-0.1522	-0.0800	-0.0490	1.9519	-0.2080	0.3045	0.1170
YTT28-89	-0.1113	-0.0480	-0.0240	2.0256	-0.1340	-0.6210	0.0977
YTT28-87	-0.0367	0.0163	0.0393	2.127	-0.0330	0.1383	0.1648
YTT28-67	-0.0051	0.0424	0.0641	2.1705	0.0105	0.1168	0.1962
YTT28-65	0.0099	0.0509	0.0700	2.1952	0.0352	-0.2790	0.2401
YTT28-47	-0.0473	-0.0210	0.0003	2.1594	-6E-04	-0.5050	0.2134
XST87	-0.0251	-0.0020	0.0153	2.1767	0.0167	-0.4490	0.2480
YTT28-30	-0.0121	0.0078	0.0259	2.1917	0.0317	-0.2320	0.2686
YTT28-1	-0.0009	0.0184	0.0273	2.1878	0.0278	-0.2050	0.2871
XST71	0.0207	0.0439	0.0318	2.173	0.0130	-0.2120	0.3150
YTT19-7	0.0235	0.0451	0.0295	2.171	0.0110	-0.2590	0.3180
YTT19-54	0.0477	0.0777	0.0452	2.1622	0.0022	-0.3030	0.3210
XST59	0.0609	0.0867	0.0469	2.1662	0.0062	-0.3180	0.3290
XST120	-0.1251	-0.1210	-0.0670	2.1116	-0.0480	-1.3710	0.1930
CFPA31	0.0414	0.0025	-0.0280	2.1741	0.0140	-1.1060	0.2900
XST64	-0.0273	-0.0590	-0.0660	2.1381	-0.0220	-0.4880	0.2540
XST68	-0.0664	-0.0950	-0.0900	2.1137	-0.0460	-0.9550	0.2350
XST76	-0.1027	-0.1240	-0.1010	2.102	-0.0580	0.1750	0.2490
XST83	-0.1186	-0.1340	-0.0960	2.1038	-0.0560	-0.1550	0.2420
XST84	-0.1407	-0.1530	-0.1090	2.0883	-0.0720	0.3900	0.2110
XST99A	-0.164	-0.1710	-0.1180	2.0724	-0.0880	0.4486	0.1310
XST241	-0.1689	-0.1740	-0.1180	2.0682	-0.0920	0.4145	0.1220
XST107	-0.1735	-0.1770	-0.1180	2.0621	-0.0980	0.5398	0.1200

Stations	North Sea Region Model [m]	4-parameters Similarity Datum Shift [m]	5-parameters Similarity Datum Shift [m]	7-parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM2008 [m]
XST114	-0.1254	-0.1230	-0.0690	2.1138	-0.0460	0.7268	0.2020
XST44	-0.1176	-0.1120	-0.0590	2.1155	-0.0450	0.8107	0.1910
YTT2-14A	-0.1057	-0.0980	-0.0470	2.1221	-0.0380	0.6685	0.1850
YTT2-25A	-0.0867	-0.0760	-0.0300	2.1305	-0.0300	-0.4850	0.1680
YTT2-37A	-0.0795	-0.0660	-0.0300	2.1185	-0.0420	-0.3570	0.1050
YTT2-48A	-0.0514	-0.0360	-0.0090	2.1307	-0.0290	-0.4400	0.0690
XST55	0.6495	0.6703	0.7119	2.8087	0.6487	-0.1350	0.8190
YTT17-08A	0.1361	0.1046	0.0028	2.1665	0.0064	0.2829	0.2860
XST53	0.0858	0.0684	0.0217	2.1963	0.0363	0.1069	0.3420
FGPLA-Y-008	0.0225	0.0142	-0.0090	2.1673	0.0073	0.0630	0.3300
XST59	0.0609	0.0867	0.0469	2.1662	0.0062	0.4077	0.3290
CFPA18	0.0195	0.0198	-0.0030	2.1635	0.0035	0.0763	0.3245
XST69	-0.0343	-0.0430	-0.0420	2.1454	-0.0150	-0.1470	0.3090
YTT28-1	-0.0005	0.0188	0.0277	2.1882	0.0282	0.1999	0.2875
ZTT45-200	-0.0122	0.0015	0.0175	2.1900	0.0300	0.0879	0.2870
MCS1144S-A	-0.0533	-0.0480	-0.0180	2.1696	0.0096	-0.1040	0.2670
YTT28-151	-0.0905	-0.0850	-0.0410	2.1504	-0.0100	-0.1970	0.2108
YTT28-134	0.0705	0.0827	0.1165	2.3088	0.1487	0.2133	0.2625
ZTT6-53	-0.0615	-0.0260	8E-05	2.1398	-0.0200	0.4600	0.1870
YTT27-33	-0.1073	-0.0630	-0.0320	2.0722	-0.0880	0.7789	0.5940
YTT27-41	-0.1062	-0.0640	-0.0320	2.0780	-0.0820	0.8476	0.1160
YTT16-76A	-0.0028	0.0034	0.0177	2.2030	0.0429	0.0715	-0.2110
XST121	-0.0996	-0.0910	-0.0450	2.1436	-0.0160	-0.0960	0.1850
YTT28-200	-0.0837	-0.0760	-0.0280	2.1566	-0.0030	-0.1810	0.2053
XT101	-0.0683	-0.0490	-0.0170	2.1417	-0.0180	0.6796	0.0840
ZTT30-5	-0.0438	-0.0350	-0.0090	2.1784	0.0183	0.4364	0.0470
MCS1178T-A	-0.0617	-0.0520	-0.0140	2.1714	0.0113	-0.1930	0.1510
YTT9-73A	-0.0410	-0.0300	-0.0020	2.1687	0.0086	-0.3160	0.1010
XST165	-0.0208	-0.0190	0.0004	2.1929	0.0328	0.3378	0.0170
XST126	0.0102	-02E-04	0.0116	2.2061	0.0460	0.4359	-0.0070
YTT9-29A	0.0286	0.0357	0.0161	2.1699	0.0098	-0.2090	-0.1060
XST215	0.0472	0.0165	-0.0250	2.1965	0.0365	0.3614	-0.1240
ZTT35-26	0.2175	0.2718	0.1501	2.0775	-0.0830	-0.6110	-0.0410
ZTT34-34	0.0527	-0.0060	-0.0840	2.1562	-0.0040	0.4386	-0.0920
YTT13-27	0.0702	0.0275	-0.0380	2.1893	0.0292	0.3265	-0.1070
XT161	0.0603	0.0331	-0.0170	2.1960	0.0359	0.2228	-0.1320
XST202	0.0239	-0.0050	-0.0320	2.1901	0.0300	0.4158	-0.1160

Stations	North Sea Region Model [m]	4-parameters Similarity Datum Shift [m]	5-parameters Similarity Datum Shift [m]	7-parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM2008 [m]
YTT13-30	0.0677	0.0275	-0.0330	2.1922	0.0321	0.3137	-0.1120
XST204	0.0659	0.0906	0.0469	2.1202	-0.0400	-0.5100	-0.1310

Appendix C10:
Full Data Set for Table 4.6a - Curvilinear and Space Rectangular Coordinates of the Points used for Port Harcourt.

Table 4.6a: Curvilinear and Space Rectangular Coordinates of the Point used for Port Harcourt

Station Name	Latitude [°]	Longitude [°]	X [m]	Y [m]	Z [m]	Distance [m]
AP4	4°52'06.00889"	6°59'23.6594"	6308080.8210	773406.8196	537681.2220	
AP1	4°52'10.33445"	6°58'40.5391"	6308229.0670	772086.4504	537813.4320	1335.2270
P10 BALOGUN	4°51'59.85353"	6°59'58.6000"	6307965.8740	774477.3606	537492.8470	2426.6230
PW401 JB	4°51'22.06657"	7°03'59.8975"	6307157.4270	781868.7906	536336.6680	7524.8640
RPCS 209p	4°46'17.86345"	7°00'47.8189"	6308650.6760	776089.5453	527024.4120	11061.080
HS 8	4°45'18.49512"	7°00'59.6229"	6308752.7370	776468.6023	525206.7530	1859.5660
RPCS 146p	4°52'21.66037"	7°01'42.1522"	6307519.3940	777637.0714	538160.2430	13064.430
ZVS 3003	4°50'52.69616"	7°02'52.12122"	6307484.3480	779804.9848	535437.2560	3480.7660
PT.1 EMMA	4°45'53.09752"	7°00'59.92051"	6308668.1830	776467.4348	526266.3250	9830.9040
PT.2 EMMA	4°46'46.65319"	7°00'25.11678"	6308663.7820	775386.3089	527905.7540	1963.8180
PT.3 EMMA	4°47'24.78735"	7°00'08.18766"	6308624.980	774855.9449	529072.5940	1282.3050
PHCS 1s	4°46'20.60153"	7°00'48.69008"	6308641.3540	776115.4473	527108.3030	2333.4640
PT.4 EMMA	4°47'54.21055"	7°00'20.06670"	6308510.8090	775210.7202	529973.6940	3007.6640
PT.8 EMMA	4°50'01.54235"	7°00'25.31739"	6308161.2870	775330.776	533870.8580	3914.6470
PT.4 ABDUL	4°50'13.82453"	7°01'22.28693"	6307921.3260	777069.8853	534247.3020	1795.4920
PT.5 EMMA	4°48'24.97793"	7°00'33.86529"	6308379.0960	775622.9254	530915.3590	3661.2970
PT.7 EMMA	4°49'25.94109"	7°00'21.66357"	6308272.9060	775231.0610	532781.7160	1910.0060
PT.9 EMMA	4°50'11.63888"	7°00'55.05407"	6308025.8250	776237.2933	534180.0920	1740.4050
PT.6 EMMA	4°48'55.94619"	7°00'35.10111"	6308300.3450	775651.6099	531863.7030	2405.0040
PT.2 ABDUL	4°50'39.60788"	7°02'22.26544"	6307628.5710	778895.8798	535036.4590	4587.2590
PT.3 ABDUL	4°50'26.70761"	7°01'52.74514"	6307767.3550	777996.5102	534641.1130	992.1820
UNIPORT GATE	4°53'37.49584"	6°54'52.00249"	6308850.5060	765068.6973	540480.7560	14226.840
PP 9	4°53'17.70060"	7°08'40.10360"	6305783.3680	790397.8310	539875.240	25521.340
PP 5	4°52'12.92745"	7°06'31.90002"	6306446.5230	786499.9167	537893.2260	4422.8820
GPS 01	5°02'18.51328"	7°00'09.83198"	6306306.7920	774622.2454	556426.6210	22013.300
GPS 02	4°59'18.03069"	7°00'19.58945"	6306745.9460	774979.0352	550903.3820	5552.1460
GPS 03	4°58'52.08097"	6°56'59.42588"	6307561.3340	768866.5585	550109.0690	6217.5690
GPS 04	4°58'20.08129"	6°57'04.25091"	6307626.7100	769024.2710	549129.7090	994.1292
GPS 05	4°59'17.39687"	6°57'34.83651"	6307363.4530	769941.3701	550883.8840	1996.8740
GPS 06	4°58'36.73276"	6°57'01.89139"	6307592.1860	768946.8351	549639.3370	1609.4460
GPS 07	4°58'06.30270"	6°57'02.75651"	6307668.2660	768982.9588	548708.0190	935.1185
GPS 08	4°57'21.83566"	6°56'57.80237"	6307802.1120	768845.522	547347.0530	1374.4210
GPS 09	4°57'17.82054"	6°56'49.49213"	6307841.8780	768592.4569	547224.0230	284.1821
GPS 10	4°57'13.61218"	6°56'39.42241"	6307892.6430	768286.1249	547095.4250	336.0861
GPS 11	4°58'40.85650"	6°58'08.11867"	6307332.7440	770970.5361	549765.4150	3827.3220
GPS 12	4°58'35.83044"	6°58'13.33321"	6307328.0360	771131.7979	549611.740	222.8087
GPS 13	4°58'30.62349"	6°58'19.04101"	6307321.3830	771308.1313	549452.4780	237.7016
GPS 14	4°57'11.28451"	6°57'01.63190"	6307814.5660	768965.8916	547024.06600	3409.7660

Station Name	Latitude [º]	Longitude [º]	X [m]	Y [m]	Z [m]	Distance [m]
GPS 15	4°56'58.95126"	6°57'10.21957"	6307814.4160	769232.3984	546646.5810	462.0828
GPS 16	4°56'47.71435"	6°57'18.39159"	6307813.4780	769485.9115	546302.6970	427.2312
GPS 17	4°56'34.82281"	6°57'26.55832"	6307816.9220	769739.7974	545908.1770	469.1649
GPS 18	4°56'21.27990"	6°57'28.66255"	6307844.6300	769808.4870	545493.7170	421.0256
GPS 19	4°53'35.37093"	6°57'52.98285"	6308181.8790	770604.5195	540415.6840	5151.0990
GPS 20	4°53'38.57982"	6°57'51.63342"	6308179.1710	770562.3034	540513.9460	106.9803
GPS 21	4°53'35.86981"	6°57'58.60207"	6308160.6460	770776.3424	540431.0440	230.2792
GPS 22	4°52'30.35240"	6°57'21.54664"	6308471.0760	769664.0627	538425.9930	2313.8200
GPS 23	4°52'32.30492"	6°57'17.39255"	6308482.4270	769536.5063	538485.8310	141.3506
GPS 24	4°52'25.79960"	6°57'18.04810"	6308496.6480	769558.5891	538286.7110	200.8452
GPS 25	4°52'35.75535"	6°57'10.20260"	6308500.5770	769315.5484	538591.4590	389.8154
GPS 26	4°49'56.85926"	6°56'44.29419"	6308993.9650	768571.4867	533726.960	4945.7470
GPS 27	4°49'56.80006"	6°56'41.59284"	6309003.5660	768488.8043	533725.0960	83.2589
GPS 28	4°49'56.37987"	6°56'38.83831"	6309016.0370	768404.8207	533712.3310	85.8587
GPS 29	4°50'11.32868"	6°55'41.77726"	6309189.4970	766654.7432	534169.8470	1817.1900
GPS 30	4°50'14.59804"	6°55'42.51984"	6309179.0680	766676.5252	534269.9780	103.0024
GPS 31	4°50'17.46048"	6°55'44.71396"	6309165.860	766743.0245	534357.7880	110.9380
GPS 32	4°56'26.96350"	7°00'28.74660"	6307157.9180	775313.8953	545667.8910	14332.130
GPS 33	4°56'32.20859"	7°00'28.85659"	6307144.5630	775315.6677	545828.4810	161.1541
GPS 34	4°56'38.34350"	7°00'27.93956"	6307133.0780	775285.7914	546016.3330	190.5592
GPS 35	4°55'48.49344"	7°03'09.71625"	6306654.9440	780248.6501	544490.8370	5213.9920
GPS 36	4°55'54.24882"	7°03'10.25919"	6306638.0070	780263.4094	544666.9910	177.5809
GPS 37	4°56'06.35131"	7°03'12.80491"	6306594.5590	780337.0616	545037.1910	379.9478
GPS 38	4°53'27.18223"	7°04'34.01031"	6306698.1600	782870.9317	540165.5090	5492.2230
GPS 39	4°53'32.68263"	7°04'36.88227"	6306674.5090	782957.1610	540333.9860	190.7334
GPS 40	4°53'40.59420"	7°04'38.90910"	6306647.3160	783016.7115	540576.2160	250.9210
GPS 41	4°51'46.51499"	7°05'36.10144"	6306726.5270	784802.2694	537084.7560	3922.3440
GPS 42	4°51'48.41009"	7°05'42.45332"	6306697.6710	784995.9002	537142.7770	204.1862
GPS 43	4°51'50.04472"	7°05'49.16814"	6306666.0260	785200.4526	537192.6480	212.9089
GPS 44	4°49'55.37439"	7°07'36.24289"	6306551.0870	788510.9178	533682.7100	4826.1850
GPS 45	4°50'01.59562"	7°07'38.28208"	6306526.3470	788571.1475	533873.0480	201.1674
GPS 46	4°50'08.63058"	7°07'39.43629"	6306502.3290	788603.9857	534088.2450	219.0084
GPS 47	4°46'11.86515"	7°08'25.08053"	6306932.7680	790075.3312	526841.0400	7407.5710
GPS 48	4°46'09.88906"	7°08'28.19961"	6306926.0500	790171.358	526780.5690	113.6797
GPS 49	4°46'06.13308"	7°08'34.02402"	6306914.0480	790350.7421	526665.6610	213.3693
GPS 50	4°54'43.63017"	6°59'07.06877"	6307732.6120	772849.1508	542505.26600	23619.250
GPS 51	4°54'49.54219"	6°59'05.55093"	6307723.2400	772800.8891	542686.2340	187.5273
GPS 52	4°54'55.12338"	6°59'01.63988"	6307723.0720	772679.4708	542857.02300	209.5503
GPS 53	4°48'28.54816"	6°58'37.88991"	6308804.7890	772074.6731	531024.6170	11897.130
GPS 54	4°48'25.98666"	6°58'34.63319"	6308823.7790	771975.8952	530946.2320	127.5216

Station Name	Latitude [º]	Longitude [º]	X [m]	Y [m]	Z [m]	Distance [m]
GPS 55	4°48'25.16452"	6°58'38.00013"	6308813.1160	772079.1139	530921.0540	106.7790
GPS 56	4°46'53.95810"	7°00'21.87158"	6308654.7880	775284.4491	528129.1390	4253.7080
GPS 57	4°46'56.35752"	7°00'19.64919"	6308656.5510	775215.6669	528202.5450	100.6107
GPS 58	4°46'59.86823"	7°00'18.86556"	6308650.4780	775190.5913	528310.0010	110.5103
GPS 59	4°55'00.82869"	6°52'48.37072"	6309081.5200	761259.8173	543030.3660	20271.690
GPS 60	4°54'57.99006"	6°52'52.15645"	6309075.4480	761376.5660	542943.5330	145.6264
GPS 61	4°54'50.33509"	6°52'51.17259"	6309098.7740	761348.8490	542709.2300	237.0870
XSV 662	4°52'24.62491"	6°59'54.28734"	6307909.5620	774336.5702	538250.2940	13783.220
ZVS 3003	4°50'52.69568"	7°02'52.12172"	6307481.7270	779804.6764	535437.0170	6164.2320
RHS 8A	4°45'18.49317"	7°00'59.62433"	6308750.2650	776468.3413	525206.4860	10835.320
						12862.350

Appendix C11:
Full Data Set for Table 4.6b - Curvilinear and Space Rectangular Coordinates of the Points used for Lagos State

Table 4.6b: Curvilinear and Space Rectangular Coordinates of the Point used for Lagos State

Station Name	Latitude [^{o}]	Longitude [^{o}]	X [m]	Y [m]	Z [m]	Distance [m]
XST 237	6.45480214	3.470396222	6326376.79	383656.9222	712259.3733	
XST44	6.42236891	3.473378551	6326758.915	384010.625	708695.3187	3601.88979
YTT78A	6.47000887	3.646457902	6324980.599	403083.1823	713930.5512	19857.8072
XST245	6.43391161	3.603378587	6325731.23	398355.7656	709964.0705	6216.50092
XST244	6.42600594	3.631051025	6325634.269	401416.9931	709095.1187	3183.64447
FGPLA-Y-003	6.42704123	2.890722633	6330279.633	319650.5076	709208.8429	81898.4157
CFPA21	6.44089609	2.919119213	6329952.831	322779.2959	710731.8211	3495.07918
XST 55	6.37965975	2.706952389	6331859.013	299372.9251	704002.0136	24429.1227
YTT1703A	6.41999857	2.712921902	6331326.144	300008.8548	708434.6703	4509.63434
XST46	6.44388127	2.709402845	6331049.66	299606.034	711059.2738	2669.69124
XST50	6.43088835	2.826984239	6330585.856	312605.9799	709631.8551	13086.2994
LWBC5-61P	6.50459261	2.926533297	6329113.107	323557.5998	717730.4949	13700.18
YTT19-54	6.51090123	2.954208526	6328888.762	326611.2003	718424.9939	3139.60765
XST75	6.49889881	3.063821936	6328401.177	338726.4535	717106.051	12196.586
CFPA40	6.38501723	2.78113861	6331398.578	307567.8447	704590.6254	33711.7071
CFPB36	6.39047864	2.824224997	6331097.606	312325.6154	705190.7575	4804.90637
XST60	6.39576428	2.928216261	6330455.157	323812.6405	705771.6364	11519.6313
XST72	6.39950036	3.053622142	6329685.072	337665.1458	706182.208	13879.9677
XST76	6.40075266	3.095451055	6329421.397	342285.2194	706319.8288	4629.63755
XST44	6.42236891	3.490045221	6326646.943	385850.9886	708695.3187	43718.6093
YTT2-18A	6.42554834	3.546123013	6326225.056	392040.4049	709044.5007	6213.59728
XST156	6.42688258	3.678521952	6325288.875	406657.1104	709191.4774	14647.3928
ZTT2-57A	6.43808236	3.77811817	6324433.208	417642.4318	710422.1614	11087.1112
YTT2-66A	6.44172298	3.84345449	6323907.79	424851.0975	710822.2334	7238.85231
YTT2-80	6.43948606	3.930290799	6323283.552	434436.8405	710576.3297	9609.19401
XST224	6.41851012	4.080058618	6322386.505	450982.5877	708271.3368	16729.5976
ZTT35-14	6.40523342	4.142532315	6322054.527	457887.9128	706812.2669	7065.59334
XST229A	6.38358351	4.255296272	6321406.003	470349.1775	704432.8146	12702.9719
XST230	6.3789774	44.60666667	4512969.945	4451428.725	703926.548	4372577.68
XST42	6.66577682	4.088917185	6319211.228	451738.0943	735438.522	4388738.51
YTT13-1A	6.67959255	4.062929161	6319242.828	448859.5385	736956.5729	3254.46781
ZTT34-10A	6.66508539	4.002523018	6319908.36	442210.702	735364.2491	6869.16697
XST135	6.68409458	3.981722921	6319860.272	439901.863	737456.4041	3116.11338
XST218	6.67658000	3.935228307	6320275.000	434777.3687	736626.7082	5207.76691
XST209	6.68459922	3.882579673	6320560.506	428961.9795	737506.5451	5888.49543
XST201	6.68301646	3.838593558	6320918.821	424111.614	737333.9346	4866.64459
XST203	6.68272479	3.749736646	6321553.402	414307.3192	737299.6329	9824.8698
XST177	6.69042904	3.712051242	6321770.075	410145.7933	738151.083	4253.25899
YTT22-1	6.6701876	3.67055825	6322309.026	405583.154	735925.8609	5104.876

Station Name	Latitude [º]	Longitude [º]	X [m]	Y [m]	Z [m]	Distance [m]
XST159	6.68040776	3.577680739	6322844.7	395326.9516	737050.5205	10331.5775
ZTT31-70	6.66925334	3.512980517	6323428.127	388195.4017	735825.0656	7259.55437
XST131	6.68336435	3.461519509	6323559.014	382502.7516	737371.0206	5900.28592
XST127	6.64348166	3.466548326	6324025.774	383088.0751	732988.9694	4445.54163
XST133	6.6391453	3.41173667	6324445.947	377041.4338	732512.774	6079.89945
XST128	6.6408166	3.372557102	6324718.706	372717.5783	732700.7642	4336.52682
YTT28-117	6.64367148	3.3393115	6324875.277	369044.2068	733011.7793	3689.83771
MCS1174S-A	6.66502729	3.323236155	6324736.803	367255.5981	735361.288	2956.09324
YTT28-96	6.68580244	3.288081883	6324703.436	363360.1436	737644.3198	4515.29773
XST41	6.69954155	3.264344748	6324668.696	360729.3073	739152.4506	3032.64981
YTT28-89	6.65415866	3.242408083	6325358.849	358339.0739	734164.0517	5574.3743
YTT28-87	6.62196288	3.247943681	6325740.314	358973.8067	730627.9961	3612.76773
YTT28-67	6.59994177	3.238823975	6326086.304	357983.2914	728209.9214	2635.88995
YTT28-65	6.57127085	3.214430689	6326588.775	355309.6632	725058.6939	4163.05161
YTT28-47	6.5237964	3.209969817	6327198.767	354849.7535	719841.0001	5273.32308
XST87	6.51043964	3.173555949	6327585.758	350837.5059	718372.901	4289.89595
YTT28-30	6.50214186	3.169837828	6327715.812	350432.8286	717461.5653	1005.58972
YTT28-1	6.49768179	3.115275631	6328101.57	344409.8388	716971.4056	6055.20196
XST71	6.50184783	3.024295615	6328602.21	334358.9892	717430.7383	10073.788
YTT19-7	6.49795387	3.008970191	6328738.189	332668.667	717002.6585	1748.98024
YTT19-54	6.51090123	2.954208526	6328888.762	326611.2003	718424.9937	6224.035
XST59	6.50231836	2.926581412	6329143.332	323564.4738	717480.8402	3199.80848
XST120	6.4233642	3.457259185	6326854.443	382229.9241	708804.6994	59347.7
CFPA31	6.39438889	2.890307941	6330684.743	319625.0272	705620.459	62802.7364
XST64	6.39682043	2.96973699	6330205.181	328399.3593	705887.6331	8791.48826
XST68	6.39782476	3.011246563	6329953.885	332984.7554	705998.0813	4593.60485
XST76	6.40075266	3.095451055	6329421.397	342285.2194	706319.8288	9321.24965
XST83	6.40351355	3.177978425	6328887.851	351399.7116	706623.2512	9135.13575
XST84	6.40455655	3.220993917	6328609.305	356150.3719	706737.8634	4760.19925
XST99A	6.40434195	3.302776744	6328095.876	365183.4135	706714.1375	9047.65233
XST241	6.40164189	3.343845061	6327866.037	369721.1148	706417.438	4553.19559
XST107	6.39747225	3.380957804	6327675.942	373822.8422	705959.1308	4131.62794
XST114	6.42265407	3.420188491	6327108.898	378136.8402	708726.6269	5156.66098
XST44	6.42236891	3.490045221	6326646.943	385850.9886	708695.3187	7728.03129
YTT2-14A	6.42285923	3.527906685	6326383.087	390031.1355	708749.0392	4188.81061
YTT2-25A	6.42439547	3.586657712	6325961.246	396516.8308	708917.9071	6501.59285
YTT2-37A	6.42641159	3.664612708	6325392.886	405121.9087	709139.6797	8626.67849
YTT2-48A	6.42927917	3.718083886	6324975.995	411022.5492	709454.737	5923.73345
XST55	6.37965975	3.706952389	6325669.862	409833.5061	704002.0136	5623.8304
YTT17-08A	6.4198925	2.722609701	6331280.011	301079.5991	708423.3936	108988.232

Station Name	Latitude [º]	Longitude [º]	X [m]	Y [m]	Z [m]	Distance [m]
XST53	6.43116411	2.868855608	6330351.65	317232.0417	709662.0852	16226.448
FGPLA-Y-008	6.44189802	2.948674497	6329772.709	326043.8186	710841.8816	8909.23705
XST59	6.50231836	2.926581412	6329143.332	323564.4738	717480.8402	7114.70563
CFPA18	6.45702191	2.95957542	6329519.871	327238.2375	712503.482	6197.775
XST69	6.43606396	3.031327624	6329364.524	335178.2516	710200.3742	8268.75217
YTT28-1	6.49768179	3.115275631	6328101.57	344409.8388	716971.4056	11517.9912
ZTT45-200	6.48448384	3.143460993	6328096.414	347531.827	715521.2471	3442.35329
MCS1144S-A	6.46081688	3.204114125	6328020.619	354247.0845	712920.7472	7201.599
YTT28-151	6.45541456	3.330872553	6327284.611	368249.6927	712326.6612	14034.5176
YTT28-134	6.5297354	3.529742897	6325042.106	390151.9375	720493.1694	23482.5256
ZTT6-53	6.5699168	3.269374699	6326271.489	361377.9109	724910.9995	29137.1449
YTT27-33	6.63580254	3.337821171	6325016.141	368887.3453	732150.9825	10506.4198
YTT27-41	6.63442586	3.353201574	6324921.833	370585.5006	731998.2852	1707.6129
YTT28-188	6.55197782	3.388735983	6325705.12	374568.1603	722937.1241	9928.73396
XST121	6.46026385	3.440859348	6326504.56	380391.3658	712859.3909	11666.599
YTT28-200	6.44763056	3.467725678	6326483.159	383367.4132	711471.2478	3283.93929
XT101	6.62899165	3.510495332	6323952.484	387952.2597	731401.6362	20606.9288
ZTT30-5	6.5986892	3.588452971	6323792.141	396579.7929	728071.4129	9249.34729
MCS1178T-A	6.47498883	3.56779892	6325464.293	394395.5304	714477.5692	13869.3793
YTT9-73A	6.46469626	3.670838479	6324874.938	405778.8219	713346.8196	11454.4864
XST165	6.61487713	3.645515546	6323185.385	402864.3544	729849.3857	16842.9036
XST126	6.65058152	3.708116618	6322298.226	409744.0675	733772.8942	7969.40538
YTT9-29A	6.48468132	3.880715476	6323095.81	428927.3669	715542.659	26475.9625
XST215	6.60592487	3.925565174	6321233.005	433772.1402	728863.4487	14296.339
ZTT35-26	6.39412855	4.202842114	6321705.341	464552.2967	705591.8227	38590.2801
ZTT34-34	6.64405492	4.036229785	6319903.38	445946.6724	733052.6908	33219.2053
YTT13-27	6.61492692	3.999839731	6320578.546	441960.1409	729855.5584	5154.60364
XT161	6.58510316	3.955504287	6321291.525	437094.9322	726578.5969	5909.06692
XST202	6.62277559	3.875495858	6321395.866	428233.5287	730714.6009	9779.66711
YTT13-30	6.61242423	3.98731709	6320709.882	440581.1284	729580.9874	12418.4892
XST204	6.43357285	3.988969653	6322909.443	440917.7035	709926.6313	19779.9157
ZTT35-2A	6.41627949	4.089807093	6322336.948	452060.2404	708026.1735	11317.9336
YTT16-76A	6.50349199	3.719303861	6324047.59	411097.4419	717609.9399	42103.7497
XST149	6.56550677	3.588484489	6324198.692	396608.7811	724424.4592	16011.9267
MCS1188T-A	6.49345969	3.582388693	6325133.279	395991.808	716507.156	7996.11148
YTT2-11A	6.42250489	3.513237463	6326488.494	388411.7561	708710.2553	10958.351
XST126	6.62386157	3.528768937	6323856.395	389970.8915	730833.7411	22333.9981
XST136	6.46823267	3.56529207	6325570.244	394124.3001	713735.7025	17678.5464
XST137	6.42635852	3.580480429	6325981.371	395833.3884	709133.8213	4926.18698
XST225	6.42348209	3.531541184	6326352.054	390432.0548	708817.6473	5423.2625

Station Name	Latitude [º]	Longitude [º]	X [m]	Y [m]	Z [m]	Distance [m]
XST83	6.40351355	3.177978425	6328887.851	351399.7116	706623.2512	39176.1338

Appendix C12:
Full Data Set for Table 4.8a: Summary of the Results obtained from the Local Geoidal Undulation and each of the New ‘Satlevel’ Collocation Geoid Models for Port Harcourt.

Table 4.8a: Summary of the Results Obtained from Local Geoid and New 'Satlevel' Collocation Geoid Models for Port Harcourt

STATIONS	Local [m]	Spherical 'Satlevel' [m]	Rectangular 'Satlevel' [m]
Model Number	Model 1	SATLEVEL 1	SATLEVEL 2
Equation Number	Equation 1.2	Equation 3.77	3.79
AP4	18.9229	18.9476	18.9497
AP1	18.9119	18.9327	18.9345
PT.3 EMMA	18.9667	18.9971	18.9994
PHCS 1s	18.9980	19.0164	19.0232
PT.4 EMMA	19.0024	18.9977	19.0018
PT.8 EMMA	18.9381	18.9843	18.9831
PT.4 ABDUL	19.0028	19.0006	19.0017
PT.5 EMMA	18.9939	18.9986	19.0010
PT.7 EMMA	19.0074	18.9876	18.9906
PT.9 EMMA	18.9750	18.9924	18.9919
PT.2 ABDUL	18.9861	19.0157	19.0157
PT.3 ABDUL	18.9803	19.0084	19.0058
GPS 02	18.9040	18.8910	18.8953
GPS 03	18.8250	18.8288	18.8361
GPS 04	18.8330	18.8368	18.8429
GPS 05	18.8340	18.8360	18.8433
GPS 06	18.8280	18.8327	18.8394
GPS 07	18.8350	18.8390	18.8448
GPS 08	18.8420	18.8457	18.8504
GPS 09	18.8400	18.8436	18.8475
GPS 10	18.8360	18.8408	18.8461
GPS 11	18.8540	18.8548	18.8591
GPS 12	18.8570	18.8575	18.8625
GPS 13	18.8610	18.8605	18.8657
GPS 14	18.8450	18.8490	18.8530
GPS 15	18.8510	18.8543	18.8576
GPS 16	18.8560	18.8592	18.8623
GPS 17	18.8610	18.8644	18.8672
GPS 18	18.8650	18.8676	18.8703
GPS 19	18.9040	18.9038	18.9035
GPS 20	18.9030	18.9029	18.9029
GPS 21	18.9060	18.9057	18.9057
GPS 22	18.9070	18.9030	18.9056
GPS 23	18.9050	18.9013	18.9044

STATIONS	Local [m]	Spherical 'Satlevel' [m]	Rectangular 'Satlevel' [m]
Model Number	Model 1	SATLEVEL 1	SATLEVEL 2
Equation Number	Equation 1.2	Equation 3.77	3.79
GPS 24	18.9070	18.9025	18.9055
GPS 25	18.9020	18.8983	18.9017
GPS 26	18.9300	18.9114	18.9110
GPS 27	18.9300	18.9104	18.9098
GPS 28	18.9290	18.9095	18.9096
GPS 29	18.9130	18.8875	18.8885
GPS 30	18.9120	18.8873	18.8887
GPS 31	18.9120	18.8877	18.8902
GPS 32	18.9350	18.9265	18.9269
GPS 33	18.9340	18.9256	18.9265
GPS 34	18.9330	18.9242	18.9257
GPS 35	19.0040	18.9831	18.9835
GPS 36	19.0030	18.9822	18.9828
GPS 37	19.0040	18.9809	18.9803
GPS 38	19.0470	19.0307	19.0288
GPS 39	19.0480	19.0307	19.0295
GPS 40	19.0480	19.0300	19.0293
GPS 41	19.0760	19.0628	19.0643
GPS 42	19.0780	19.0643	19.0659
GPS 43	19.0800	19.0659	19.0666
GPS 45	19.1210	19.1093	19.1119
GPS 46	19.1220	19.1087	19.1104
GPS 47	19.1400	19.1460	19.1564
GPS 48	19.1400	19.1470	19.1576
GPS 49	19.1420	19.1488	19.1601
GPS 50	18.9180	18.9177	18.9184
GPS 51	18.9170	18.9162	18.9171
GPS 53	18.9760	18.9607	18.9644
GPS 54	18.9760	18.9599	18.9639
GPS 55	18.9760	18.9611	18.9650
GPS 56	19.0180	19.0047	19.0092
GPS 57	19.0170	19.0037	19.0079
GPS 58	19.0160	19.0031	19.0072
GPS 59	18.7910	18.7798	18.7845
GPS 60	18.7930	18.7817	18.7865
XSV 662	18.9550	18.9548	18.9521
ZVS 3003	19.0260	19.0229	19.0224

Appendix C13:
Full Data Set for Table 4.8b: Summary of the Results obtained from the Local Geoidal Undulations and each of the New ‘Satlevel’ Collocation Geoid Models for Lagos State

Table 4.8b: Summary of the Results Obtained from Local Geoidal Undulation and each of the New 'Satlevel' Collocation Geoid Models for Lagos State

STATIONS	Local [m]	Spherical 'Satlevel' [m]	Rectangular 'Satlevel' [m]
Model Number	Model 1	SATLEVEL 1	SATLEVEL 2
Equation Number	Equation 1.2	Equation 3.22	3.24
XST 237	22.5640	22.4859	22.4944
XST44	22.2540	22.3242	22.3308
YTT78A	22.4740	22.4719	22.4768
XST245	22.4910	22.3137	22.3173
XST244	22.2240	22.2617	22.2645
FGPLA-Y-003	22.7830	22.7769	22.7766
CFPA21	22.8280	22.8196	22.8199
XST 55	22.7000	22.7223	22.7102
YTT1703A	22.9120	22.9149	22.9061
XST46	23.0440	23.0365	23.0284
XST50	22.8800	22.8556	22.8521
LWBC5-61P	23.1860	23.1291	23.1336
YTT19-54	23.1900	23.1360	23.1386
XST75	23.0230	22.9836	22.9893
CFPA40	22.6550	22.6740	22.6666
CFPB36	22.6490	22.6590	22.6541
XST60	22.5390	22.5888	22.5884
XST72	22.3960	22.5000	22.5036
XST76	22.3650	22.4726	22.4771
XST44	22.2540	22.3150	22.3213
YTT2-18A	22.2580	22.3009	22.3067
XST156	22.2170	22.2448	22.2461
ZTT2-57A	22.2740	22.2599	22.2581
YTT2-66A	22.2700	22.2549	22.2502
YTT2-80	22.2410	22.2173	22.2081
XST224	22.1600	22.0792	22.0587
ZTT35-14	22.1190	22.0036	21.9775
XST149	22.9830	22.9747	22.9833
MCS1188T-A	22.6220	22.6185	22.6268
XST42	23.1680	23.3097	23.3024
YTT13-1A	23.2460	23.3847	23.3786
ZTT34-10A	23.2370	23.3247	23.319
XST135	23.3290	23.4261	23.4129
XST218	23.3210	23.4004	23.3993

STATIONS	Local [m]	Spherical 'Satlevel' [m]	Rectangular 'Satlevel' [m]
Model Number	Model 1	SATLEVEL 1	SATLEVEL 2
Equation Number	Equation 1.2	Equation 3.22	3.24
XST209	23.4100	23.4567	23.4608
XST201	23.4370	23.4630	23.4663
XST203	23.4600	23.4942	23.5062
XST177	23.5490	23.5486	23.5503
YTT22-1	23.4470	23.4637	23.4702
XST159	23.5290	23.5593	23.5637
ZTT31-70	23.5060	23.5363	23.542
XST131	23.5900	23.6365	23.6524
XST127	23.4050	23.4315	23.4488
XST133	23.4120	23.4416	23.4588
XST128	23.4190	23.4740	23.4813
YTT28-117	23.4720	23.5095	23.5229
MCS1174S-A	23.5810	23.6281	23.6338
YTT28-96	24.6210	23.7570	23.7607
XST41	23.7780	23.8432	23.8492
YTT28-89	23.6000	23.6280	23.6405
YTT28-87	23.5030	23.4613	23.4714
YTT28-67	23.4260	23.3567	23.3635
YTT28-65	23.3080	23.2300	23.2388
YTT28-47	23.0050	22.9956	23.0066
XST87	22.9800	22.9557	22.967
YTT28-30	22.9520	22.9172	22.9271
YTT28-1	22.9720	22.9366	22.9456
XST71	23.0710	23.0308	23.0341
YTT19-7	23.0620	23.0243	23.0275
YTT19-54	23.1900	23.1360	23.1386
XST59	23.1730	23.1178	23.1216
XST120	22.2690	22.3382	22.345
CFPA31	22.5800	22.6163	22.6145
XST64	22.4840	22.5574	22.5587
XST68	22.4290	22.5269	22.5293
XST76	22.3650	22.4726	22.4771
XST83	22.3210	22.4231	22.429
XST84	22.2820	22.3970	22.4034
XST99A	22.2150	22.3396	22.3468
XST241	22.1750	22.2997	22.3065
XST107	22.1300	22.2560	22.2625

STATIONS	Local [m]	Spherical ‘Satlevel’ [m]	Rectangular ‘Satlevel’ [m]
Model Number	Model 1	SATLEVEL 1	SATLEVEL 2
Equation Number	Equation 1.2	Equation 3.22	3.24
XST114	22.2850	22.3562	22.3635
XST44	22.2540	22.3150	22.3213
YTT2-14A	22.2480	22.2971	22.3031
YTT2-25A	22.2430	22.2749	22.2794
YTT2-37A	22.2180	22.2486	22.2504
YTT2-48A	22.2310	22.2400	22.2402
XST55	22.7000	22.0009	21.9977
YTT17-08A	22.9050	22.9044	22.8953
XST53	22.8390	22.8174	22.816
FGPLA-Y-008	22.7910	22.7982	22.7997
XST59	23.1730	23.1178	23.1216
CFPA18	22.8650	22.8635	22.8669
XST69	22.6570	22.6986	22.7034
YTT28-1	22.9720	22.9366	22.9456
ZTT45-200	22.8740	22.8493	22.8581
MCS1144S-A	22.6730	22.6868	22.6951
YTT28-151	22.5360	22.5732	22.5829
YTT28-134	22.9530	22.8258	22.8366
ZTT6-53	23.1930	23.1846	23.1913
YTT27-33	23.4390	23.4707	23.4754
YTT27-41	23.4220	23.4539	23.4621
YTT16-76A	22.6340	22.6066	22.6102
XST121	22.4890	22.5298	22.5393
YTT28-200	22.4260	22.4520	22.4603
XT101	23.3190	23.3340	23.3401
ZTT30-5	23.1390	23.1412	23.1477
MCS1178T-A	22.5270	22.5341	22.5418
YTT9-73A	22.4380	22.4348	22.4384
XST165	23.2010	23.1956	23.2022
XST126	23.3590	23.3483	23.3514
YTT9-29A	22.4800	22.4552	22.4514
XST215	23.0300	23.0467	23.0472
ZTT35-26	22.0840	21.9413	21.9097
ZTT34-34	23.1300	23.2107	23.2053
YTT13-27	23.0420	23.0724	23.0618
XT161	22.9270	22.9340	22.9258
XST202	23.1200	23.1465	23.1503

STATIONS	Local [m]	Spherical ‘Satlevel’ [m]	Rectangular ‘Satlevel’ [m]
Model Number	Model 1	SATLEVEL 1	SATLEVEL 2
Equation Number	Equation 1.2	Equation 3.22	3.24
YTT13-30	23.0370	23.0629	23.0521
XST204	22.2210	22.1729	22.1595
SUM	2514.00	2514.03	2514.3
MEAN	22.8550	22.8548	22.8573

Appendix C14:
Full Data Set for Table 4.9a: Computed Residuals from the New Geoid Model for Port Harcourt

Table 4.9a: Computed Residuals from the New Satlevel Collocation Geoid Model for Port Harcourt

STATIONS	Spherical 'Satlevel' [m]	Rectangular 'Satlevel' [m]
AP4	-0.0247	-0.0268
AP1	-0.0208	-0.0226
PT.3 EMMA	-0.0304	-0.0326
PHCS 1s	-0.0184	-0.0252
PT.4 EMMA	0.0047	0.0007
PT.8 EMMA	-0.0462	-0.0450
PT.4 ABDUL	0.0022	0.0011
PT.5 EMMA	-0.0048	-0.0071
PT.7 EMMA	0.0198	0.0168
PT.9 EMMA	-0.0174	-0.0169
PT.2 ABDUL	-0.0296	-0.0296
PT.3 ABDUL	-0.0280	-0.0255
GPS 02	0.0130	0.0087
GPS 03	-0.0038	-0.0111
GPS 04	-0.0038	-0.0099
GPS 05	-0.0020	-0.0093
GPS 06	-0.0047	-0.0114
GPS 07	-0.0040	-0.0100
GPS 08	-0.0037	-0.0084
GPS 09	-0.0036	-0.0075
GPS 10	-0.0048	-0.0101
GPS 11	-0.0008	-0.0051
GPS 12	-0.0005	-0.0055
GPS 13	0.0005	-0.0047
GPS 14	-0.0041	-0.0080
GPS 15	-0.0033	-0.0066
GPS 16	-0.0032	-0.0063
GPS 17	-0.0034	-0.0062
GPS 18	-0.0026	-0.0053
GPS 19	0.0002	0.0005
GPS 20	0.0001	0.0002
GPS 21	0.0003	0.0003
GPS 22	0.0040	0.0015
GPS 23	0.0037	0.0006
GPS 24	0.0045	0.0015
GPS 25	0.0037	0.0003
GPS 26	0.0187	0.0190
GPS 27	0.0196	0.0202
GPS 28	0.0195	0.0194

STATIONS	Spherical 'Satlevel' [m]	Rectangular 'Satlevel' [m]
GPS 29	0.0255	0.0245
GPS 30	0.0247	0.0233
GPS 31	0.0243	0.0218
GPS 32	0.0085	0.0081
GPS 33	0.0084	0.0075
GPS 34	0.0088	0.0073
GPS 35	0.0209	0.0205
GPS 36	0.0208	0.0202
GPS 37	0.0231	0.0237
GPS 38	0.0163	0.0182
GPS 39	0.0173	0.0185
GPS 40	0.0180	0.0187
GPS 41	0.0132	0.0117
GPS 42	0.0137	0.0121
GPS 43	0.0141	0.0134
GPS 45	0.0117	0.0091
GPS 46	0.0133	0.0116
GPS 47	-0.0060	-0.0164
GPS 48	-0.0070	-0.0176
GPS 49	-0.0068	-0.0181
GPS 50	0.0003	-0.0004
GPS 51	0.0008	-0.0001
GPS 53	0.0153	0.0116
GPS 54	0.0161	0.0121
GPS 55	0.0149	0.0110
GPS 56	0.0133	0.0088
GPS 57	0.0133	0.0091
GPS 58	0.0129	0.0088
GPS 59	0.0112	0.0065
GPS 60	0.0113	0.0065
XSV 662	0.0002	0.0030
ZVS 3003	0.0031	0.0036

Appendix C15:
Full Data Set for Table 4.9b - Computed Residuals from the New ‘Satlevel’ Collocation Geoid Models for Lagos State

Table 4.9b: Computed Residuals from the New 'Satlevel' Collocation Geoid Models for Lagos State

STATIONS	Spherical 'Satlevel' [m]	Rectangular 'Satlevel' [m]
XST 237	0.0781	0.0696
XST44	-0.0700	-0.0768
YTT78A	0.0021	-0.0028
XST245	0.1773	0.1737
XST244	-0.0380	-0.0405
FGPLA-Y-003	0.0061	0.0065
CFPA21	0.0084	0.0081
XST 55	-0.0220	-0.0102
YTT1703A	-0.0030	0.0059
XST46	0.0075	0.0156
XST50	0.0244	0.0279
LWBC5-61P	0.0569	0.0524
YTT19-54	0.0540	0.0514
XST75	0.0394	0.0337
CFPA40	-0.0190	-0.0116
CFPB36	-0.0100	-0.0051
XST60	-0.0500	-0.0494
XST72	-0.1040	-0.1076
XST76	-0.1080	-0.1121
XST44	-0.0610	-0.0673
YTT2-18A	-0.0430	-0.0487
XST156	-0.0280	-0.0291
ZTT2-57A	0.0141	0.0159
YTT2-66A	0.0151	0.0198
YTT2-80	0.0237	0.0329
XST224	0.0808	0.1013
ZTT35-14	0.1154	0.1415
XST149	0.0083	-0.0003
MCS1188T-A	0.0035	-0.0048
XST42	-0.1420	-0.1344
YTT13-1A	-0.1390	-0.1326
ZTT34-10A	-0.0880	-0.0820
XST135	-0.0970	-0.0839
XST218	-0.0790	-0.0783
XST209	-0.0470	-0.0508
XST201	-0.0260	-0.0293
XST203	-0.0340	-0.0462
XST177	0.0004	-0.0013
STATIONS	Spherical	Rectangular

	'Satlevel' [m]	'Satlevel' [m]
YTT22-1	-0.0170	-0.0232
XST159	-0.0300	-0.0347
ZTT31-70	-0.0300	-0.0360
XST131	-0.0460	-0.0624
XST127	-0.0270	-0.0438
XST133	-0.0300	-0.0468
XST128	-0.0550	-0.0623
YTT28-117	-0.0380	-0.0514
MCS1174S-A	-0.0470	-0.0528
YTT28-96	0.8640	0.8603
XST41	-0.0650	-0.0712
YTT28-89	-0.0280	-0.0407
YTT28-87	0.0415	0.0314
YTT28-67	0.0695	0.0627
YTT28-65	0.0781	0.0693
YTT28-47	0.0098	-0.0012
XST87	0.0243	0.0130
YTT28-30	0.0344	0.0245
YTT28-1	0.0355	0.0265
XST71	0.0402	0.0369
YTT19-7	0.0377	0.0345
YTT19-54	0.0540	0.0514
XST59	0.0552	0.0514
XST120	-0.0690	-0.0760
CFPA31	-0.0360	-0.0345
XST64	-0.0730	-0.0747
XST68	-0.0980	-0.1003
XST76	-0.1080	-0.1121
XST83	-0.1020	-0.1080
XST84	-0.1150	-0.1214
XST99A	-0.1250	-0.1318
XST241	-0.1250	-0.1315
XST107	-0.1260	-0.1325
XST114	-0.0710	-0.0785
XST44	-0.0610	-0.0673
YTT2-14A	-0.0490	-0.0551
YTT2-25A	-0.0320	-0.0364
YTT2-37A	-0.0310	-0.0324
YTT2-48A	-0.0090	-0.0092
XST55	0.6991	0.7023
STATIONS	Spherical 'Satlevel'	Rectangular 'Satlevel'

	[m]	[m]
YTT17-08A	0.0006	0.0097
XST53	0.0216	0.0230
FGPLA-Y-008	-0.0070	-0.0087
XST59	0.0552	0.0514
CFPA18	0.0010	-0.0024
XST69	-0.0420	-0.0464
YTT28-1	0.0359	0.0269
ZTT45-200	0.0247	0.0159
MCS1144S-A	-0.0140	-0.0221
YTT28-151	-0.0370	-0.0471
YTT28-134	0.1267	0.1159
ZTT6-53	0.0084	0.0017
YTT27-33	-0.0320	-0.0364
YTT27-41	-0.0320	-0.0401
YTT16-76A	0.0274	0.0238
XST121	-0.0410	-0.0503
YTT28-200	-0.0260	-0.0340
XT101	-0.0150	-0.0211
ZTT30-5	-0.0020	-0.0087
MCS1178T-A	-0.0070	-0.0148
YTT9-73A	0.0032	-0.0004
XST165	0.0054	-0.0012
XST126	0.0107	0.0076
YTT9-29A	0.0248	0.0286
XST215	-0.0170	-0.0172
ZTT35-26	0.1427	0.1743
ZTT34-34	-0.0810	-0.0753
YTT13-27	-0.0300	-0.0198
XT161	-0.0070	0.0012
XST202	-0.0270	-0.0303
YTT13-30	-0.0260	-0.0151
XST204	0.0481	0.0615
SUM	0.0036	-0.2634
MEAN	3E-05	-0.0024

Appendix C16:
Full Data Set for Table 4.10a: Summary of the Results Obtained from the Local, Existing and New 'Satlevel' Collocation Geoid Models for Port Harcourt

Table 4.10a: Summary of the Results from the Local, Existing Geoid and New 'Satlevel' Collocation Models for Port Harcourt

Stations	Model 1 **[m]**	Model 2 **[m]**	Model 4 **[m]**	Model 4 **[m]**	Model 5 **[m]**	Model 6 **[m]**	Model 7 **[m]**	Model 8 **[m]**	SATLEVEL 1 [m]	SATLEVEL 2 [m]
AP4	18.9229	18.9482	18.9542	18.9463	19.5078	18.94083	18.9229	18.9470	18.9476	18.9522
AP1	18.9119	18.9336	18.9393	18.9319	19.5232	18.9278	18.94441	18.9340	18.9327	18.9373
PT.3 EMMA	18.9667	19.0024	19.0038	18.9894	19.5801	18.9984	18.9309	19.0110	18.9971	19.0019
PHCS 1s	18.9980	19.0214	19.0232	19.0060	19.5965	19.0171	18.9935	19.0330	19.0164	19.0214
PT.4 EMMA	19.0024	19.0014	19.0044	18.9910	19.5819	18.9958	19.0139	19.0080	18.9977	19.0025
PT.8 EMMA	18.9381	18.9852	18.9910	18.9811	19.5723	18.9770	18.9934	18.9860	18.9843	18.9890
PT.4 ABDUL	19.0028	19.0004	19.0073	18.9976	19.5889	18.9910	18.9788	19.0000	19.0006	19.0053
PT.5 EMMA	18.9939	19.0009	19.0054	18.9929	19.5839	18.9938	18.9944	19.0060	18.9987	19.0034
PT.7 EMMA	19.0074	18.9891	18.9943	18.9836	19.5747	18.9814	18.9937	18.9920	18.9876	18.9923
PT.9 EMMA	18.9750	18.9926	18.9991	18.9893	19.5806	18.9836	18.9822	18.9930	18.9924	18.9971
PT.2 ABDUL	18.9861	19.0154	19.0223	19.0134	19.6046	19.0064	18.9865	19.0130	19.0157	19.0204
PT.3 ABDUL	18.9803	19.0080	19.0151	19.0057	19.5969	18.9986	19.0093	19.0060	19.0084	19.0131
GPS 02	18.9040	18.9060	18.8974	18.8812	19.4711	18.9117	18.9040	18.9000	18.8910	18.8955
GPS 03	18.8250	18.8270	18.8361	18.8234	19.4134	18.8257	18.8010	18.8250	18.8288	18.8342
GPS 04	18.8330	18.8349	18.8440	18.8325	19.4227	18.8329	18.8794	18.8320	18.8368	18.8421
GPS 05	18.8340	18.8372	18.8432	18.8288	19.4187	18.8366	18.8826	18.8350	18.8360	18.8413
GPS 06	18.8280	18.8308	18.8400	18.8279	19.4179	18.8292	18.8658	18.8290	18.8327	18.8381
GPS 07	18.8350	18.8368	18.8462	18.8352	19.4254	18.8346	18.8779	18.8340	18.8390	18.8443
GPS 08	18.8420	18.8430	18.8529	18.8434	19.4339	18.8403	18.8767	18.8400	18.8457	18.8509
GPS 09	18.8400	18.8405	18.8508	18.8416	19.432	18.8379	18.8724	18.8380	18.8436	18.8488
GPS 10	18.8360	18.8373	18.8480	18.8392	19.4297	18.8350	18.8717	18.8350	18.8408	18.8461
GPS 11	18.8540	18.8573	18.8617	18.8483	19.4384	18.8559	18.9025	18.8530	18.8548	18.8598
GPS 12	18.8570	18.8602	18.8645	18.8512	19.4413	18.8588	18.8776	18.8560	18.8575	18.8626
GPS 13	18.8610	18.8634	18.8674	18.8542	19.4444	18.8619	18.8778	18.8590	18.8605	18.8655
GPS 14	18.8450	18.8464	18.8562	18.8469	19.4374	18.8435	18.8517	18.8430	18.8491	18.8542
GPS 15	18.8510	18.8520	18.8614	18.8524	19.4429	18.8488	18.8795	18.8480	18.8543	18.8595
GPS 16	18.8560	18.8572	18.8663	18.8574	19.448	18.8537	18.8792	18.8530	18.8592	18.8643
GPS 17	18.8610	18.8626	18.8714	18.8627	19.4533	18.8588	18.8793	18.8580	18.8644	18.8694
GPS 18	18.8650	18.8658	18.8746	18.8661	19.4568	18.8619	18.8771	18.8610	18.8676	18.8726
GPS 19	18.9040	18.9036	18.9106	18.9039	19.495	18.8990	18.9040	18.9040	18.9038	18.9086
GPS 20	18.9030	18.9026	18.9096	18.9030	19.4941	18.8980	18.9094	18.9030	18.9029	18.9076
GPS 21	18.9060	18.9054	18.9124	18.9057	19.4968	18.9005	18.9084	18.9060	18.9057	18.9104
GPS 22	18.9070	18.9045	18.9097	18.9036	19.4947	18.9027	18.9112	18.9080	18.9030	18.9077
GPS 23	18.9050	18.9027	18.9080	18.9020	19.4931	18.9012	18.9112	18.9070	18.9013	18.9060
GPS 24	18.9070	18.9041	18.9092	18.9032	19.4942	18.9027	18.9095	18.9080	18.9025	18.9072
GPS 25	18.9020	18.8998	18.9051	18.8992	19.4903	18.8986	18.9109	18.9040	18.8983	18.9030
GPS 26	18.9300	18.9210	18.9179	18.9107	19.5015	18.9281	18.9064	18.9320	18.9114	18.9159

Stations	Model 1 **[m]**	Model 2 **[m]**	Model 4 **[m]**	Model 4 **[m]**	Model 5 **[m]**	Model 6 **[m]**	Model 7 **[m]**	Model 8 **[m]**	SATLEVEL 1 [m]	SATLEVEL 2 [m]
GPS 27	18.9300	18.9202	18.9170	18.9098	19.5006	18.9276	18.9276	18.9320	18.9104	18.9150
GPS 28	18.9290	18.9195	18.9161	18.9089	19.4998	18.9272	18.9267	18.9310	18.9095	18.9141
GPS 29	18.9130	18.8997	18.8941	18.8887	19.4793	18.9133	18.9259	18.9150	18.8875	18.8921
GPS 30	18.9120	18.8993	18.8939	18.8887	19.4792	18.9126	18.9044	18.9140	18.8873	18.8919
GPS 31	18.9120	18.8994	18.8943	18.8890	19.4796	18.9122	18.9040	18.9140	18.8877	18.8923
GPS 32	18.9350	18.9333	18.9330	18.9225	19.5133	18.9309	18.9434	18.9240	18.9265	18.9310
GPS 33	18.9340	18.9326	18.9321	18.9214	19.5122	18.9304	18.9286	18.9240	18.9256	18.9301
GPS 34	18.9330	18.9314	18.9307	18.9199	19.5106	18.9295	18.9277	18.9230	18.9242	18.9287
GPS 35	19.0040	18.9984	18.9892	18.9803	19.5709	19.0042	18.9406	18.9890	18.9831	18.9872
GPS 36	19.0030	18.9981	18.9884	18.9793	19.57	19.0042	18.8750	18.9880	18.9822	18.9864
GPS 37	19.0040	18.9978	18.9870	18.9777	19.5683	19.0050	18.8753	18.9880	18.9809	18.9850
GPS 38	19.0470	19.0414	19.0370	19.0309	19.6217	19.0460	18.9262	19.0350	19.0307	19.0350
GPS 39	19.0480	19.0420	19.0370	19.0309	19.6217	19.0471	19.0427	19.0360	19.0307	19.0349
GPS 40	19.0480	19.0420	19.0363	19.0302	19.621	19.0478	19.0426	19.0360	19.0300	19.0343
GPS 41	19.0760	19.0695	19.0694	19.0644	19.6551	19.0738	19.0418	19.0750	19.0628	19.0674
GPS 42	19.0780	19.0714	19.0709	19.0660	19.6568	19.0764	19.0592	19.0770	19.0643	19.0688
GPS 43	19.0800	19.0734	19.0725	19.0678	19.6585	19.0792	19.0609	19.0790	19.0659	19.0705
GPS 45	19.1210	19.1127	19.1163	19.1135	19.7037	19.1234	19.0627	19.1210	19.1093	19.1143
GPS 46	19.1220	19.1128	19.1157	19.1131	19.7033	19.1241	19.1094	19.1210	19.1088	19.1137
GPS 47	19.1400	19.1336	19.1541	19.1453	19.7348	19.1364	19.1089	19.1460	19.1460	19.1522
GPS 48	19.1400	19.1345	19.1551	19.1463	19.7358	19.1374	19.1527	19.1470	19.1470	19.1532
GPS 49	19.1420	19.1360	19.1570	19.1482	19.7376	19.1393	19.1539	19.1490	19.1488	19.1550
GPS 50	18.9180	18.9183	18.9244	18.9162	19.5073	18.9119	19.0819	18.9150	18.9177	18.9224
GPS 51	18.9170	18.9168	18.9229	18.9146	19.5057	18.9105	18.9211	18.9130	18.9162	18.9209
GPS 53	18.9760	18.9686	18.9673	18.9558	19.5466	18.9689	19.1561	18.9780	18.9607	18.9653
GPS 54	18.9760	18.9681	18.9665	18.9549	19.5458	18.9688	18.9582	18.9780	18.9599	18.9645
GPS 55	18.9760	18.9692	18.9677	18.9561	19.5469	18.9696	18.9576	18.9790	18.9611	18.9657
GPS 56	19.0180	19.0102	19.0114	18.9957	19.5863	19.0064	18.9587	19.0200	19.0047	19.0095
GPS 57	19.0170	19.0093	19.0105	18.9948	19.5854	19.0056	19.0016	19.0190	19.0037	19.0085
GPS 58	19.0160	19.0086	19.0098	18.9944	19.585	19.0048	19.0006	19.0180	19.0031	19.0079
GPS 59	18.7910	18.7789	18.7874	18.7890	19.379	18.7988	18.9195	18.7860	18.7798	18.7854
GPS 60	18.7930	18.7809	18.7893	18.7908	19.3808	18.8005	18.7934	18.7950	18.7817	18.7873
XSV 662	18.9550	18.9553	18.9614	18.9534	19.5447	18.9473	18.7952	18.9530	18.9548	18.9594
ZVS 3003	19.0260	19.0230	19.0296	19.0211	19.6123	19.0150	18.9625	19.020	19.0229	19.0276
Sum	1345.207	1343.8	1345.47	1344.83	1386.738	1345.21	1345.22	1345.338	1345.207	1345.33
Mean	18.94657	18.9267	18.9503	18.9413	19.53152	18.9466	18.9467	18.94842	18.94657	18.9483

Appendix C17:
Full Data Set for Table 4.10b: Summary of the Results Obtained from Local, Existing and New 'Satlevel' Collocation Geoid Models for Lagos State

Table 4.10b: Summary of the Results from the Local, Existing Geoid and New 'Satlevel' Collocation Models for Lagos State

Stations	Model 1 [m]	Model 2 [m]	Model 3 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 8 [m]	SATLEVEL 1 [m]	SAT
XST 237	22.5640	22.5444	22.5389	22.4898	20.3034	22.4634	2.564	22.2640	22.4859	22
XST44	22.2540	22.3805	22.3754	22.3222	20.1455	22.3055	22.6056	22.0660	22.3242	22
YTT78A	22.4740	22.5187	22.5086	22.4782	20.3016	22.4617	22.8674	22.3580	22.4719	22
XST245	22.4910	22.3672	22.3564	22.3141	20.1505	22.3105	22.614	22.1350	22.3137	22
XST244	22.2240	22.3137	22.3015	22.2606	20.1065	22.2665	22.7004	22.0970	22.2617	22
FGPLA-Y-003	22.78300	22.7186	22.7376	22.776	20.5955	22.7555	23.1494	22.4460	22.7769	22
CFPA21	22.8280	22.7793	22.7891	22.8211	20.6477	22.8077	22.7958	22.4870	22.8196	22
XST 55	22.7000	22.5487	22.612	22.7104	20.5108	22.6709	22.5935	22.4540	22.7223	22
YTT1703A	22.9120	22.7746	22.8066	22.9127	20.7507	22.9108	22.9243	22.6340	22.9149	22
XST46	23.0440	22.9121	22.9259	23.0387	20.9039	23.0639	22.857	22.7670	23.0365	23
XST50	22.8800	22.7744	22.7936	22.8554	20.6862	22.8462	22.63	22.5470	22.8556	22
LWBC5-61P	23.1860	23.1247	23.0975	23.1376	21.0207	23.1807	23.0213	22.8570	23.1291	23
YTT19-54	23.1900	23.1423	23.1123	23.1448	21.0278	23.1878	22.763	22.8690	23.1360	23
XST75	23.0230	23.0105	22.9896	22.9918	20.8408	23.0008	22.6386	22.7120	22.9836	22
CFPA40	22.6550	22.5418	22.5951	22.6634	20.4619	22.6219	22.3994	22.3700	22.6740	22
CFPB36	22.6490	22.5506	22.5968	22.6498	20.4491	22.6092	22.751	22.3450	22.6590	22
XST60	22.5390	22.5268	22.5622	22.5809	20.3778	22.5379	22.7178	22.2660	22.5888	22
XST72	22.3960	22.4826	22.5075	22.4929	20.2893	22.4493	22.6995	22.1630	22.5000	22
XST76	22.3650	22.4677	22.4893	22.4657	20.263	22.423	22.7332	22.1160	22.4726	22
XST44	22.2540	22.3716	22.3657	22.313	20.1385	22.2985	22.6346	22.0630	22.3150	22
YTT2-18A	22.2580	22.3572	22.3487	22.2996	20.1316	22.2916	22.734	22.0800	22.3009	22
XST156	22.2170	22.2923	22.2782	22.2439	20.0983	22.2583	22.6852	22.1230	22.2448	22
ZTT2-57A	22.2740	22.2915	22.2752	22.2615	20.1276	22.2876	22.7461	22.2800	22.2599	22
YTT2-66A	22.2700	22.2726	22.2551	22.2572	20.1349	22.2949	22.7311	22.3420	22.2549	22
YTT2-80	22.2410	22.2143	22.1936	22.2195	20.1218	22.2818	22.6938	22.3730	22.2173	22
XST224	22.1600	22.0381	22.0034	22.0775	20.0567	22.2167	22.5777	22.2670	22.0792	22
ZTT35-14	22.1190	21.9463	21.9019	21.9989	20.0245	22.1846	22.6582	22.2090	22.0036	21
XST149	22.9830	23.0202	23.0112	22.9844	20.7905	22.9506	23.751	22.8870	22.9747	22
MCS1188T-A	22.6220	22.6708	22.661	22.6271	20.4377	22.5978	22.436	22.4840	22.6185	22
XST42	23.1680	23.1355	23.2123	23.3084	21.0572	23.2171	23.2384	23.2230	23.3097	23
YTT13-1A	23.2460	23.2170	23.2953	23.3798	21.1275	23.2874	22.8246	23.3090	23.3847	23
ZTT34-10A	23.2370	23.1973	23.2592	23.3227	21.0807	23.2406	22.7183	23.3230	23.3247	2
XST135	23.3290	23.2995	23.3646	23.4192	21.178	23.3379	22.8447	23.4040	23.4261	23
XST218	23.3210	23.3006	23.3547	23.3951	21.1617	23.3217	22.7417	23.4150	23.4004	23
XST209	23.4100	23.3779	23.4247	23.4488	21.2259	23.3859	22.8133	23.4770	23.4567	23
XST201	23.4370	23.4043	23.4425	23.4552	21.2413	23.4013	22.7664	23.4930	23.4630	23
XST203	23.4600	23.4710	23.4928	23.4858	21.2928	23.4528	22.7961	23.5090	23.4942	23

Stations	Model 1 [m]	Model 2 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 7 [m]	Model 8 [m]	SATLEVEL 1[m]	SAT
XST177	23.5490	23.5368	23.5525	23.5378	21.3583	23.5183	]22.804	23.5460	23.5486	23
YTT22-1	23.4470	23.4712	23.4773	23.458	21.2801	23.4401	24.2842	23.4560	23.4637	23
XST159	23.5290	23.5915	23.5815	23.5504	21.4052	23.5653	23.0241	23.4780	23.5593	23
ZTT31-70	23.5060	23.5855	23.5644	23.5299	21.3973	23.5573	22.9107	23.4240	23.5363	2
XST131	23.5900	23.6951	23.6636	23.626	21.5231	23.6832	23.0447	23.4920	23.6365	23
XST127	23.4050	23.4915	23.4649	23.4306	21.2931	23.4531	22.7584	23.3100	23.4315	23
XST133	23.4120	23.5097	23.4755	23.4413	21.3168	23.4768	22.9469	23.3010	23.4416	23
XST128	23.4190	23.5467	23.5065	23.4732	21.3625	23.5225	22.9788	23.3260	23.4740	23
YTT28-117	23.4720	23.5855	23.5399	23.508	21.411	23.571	22.985	23.3580	23.5095	23
MCS1174S-A	23.5810	23.7070	23.6541	23.6218	21.5526	23.7127	23.0832	23.4640	23.6281	23
YTT28-96	24.6210	23.8407	23.7765	23.745	21.7151	23.8751	23.0801	23.5810	23.7570	23
XST41	23.7780	23.9302	23.8577	23.8269	21.8261	23.9861	23.0428	23.6610	23.8432	23
YTT28-89	23.6000	23.7111	23.6477	23.6238	21.5741	23.7341	22.7292	23.5020	23.6280	23
YTT28-87	23.5030	23.5395	23.4865	23.4635	21.3758	23.5358	22.7991	23.3380	23.4613	23
YTT28-67	23.4260	23.4313	23.3837	23.3621	21.2556	23.4156	22.8531	23.2300	23.3567	23
YTT28-65	23.3080	23.2982	23.2572	23.2381	21.1129	23.2729	22.8178	23.0680	23.2300	23
YTT28-47	23.0050	23.0527	23.0267	23.0051	20.846	23.006	22.7181	22.7920	22.9956	23
XST87	22.9800	23.0051	22.9824	22.9647	20.8033	22.9633	22.8991	22.7320	22.9557	2
YTT28-30	22.9520	22.9636	22.9438	22.9257	20.7598	22.9199	22.9259	22.6830	22.9172	22
YTT28-1	22.9720	22.9730	22.9537	22.9448	20.7843	22.9443	22.9462	22.6850	22.9366	22
XST71	23.0710	23.0503	23.0271	23.0392	20.898	23.058	22.992	22.7560	23.0308	23
YTT19-7	23.0620	23.0385	23.0169	23.0325	20.891	23.051	22.9492	22.7440	23.0243	23
YTT19-54	23.1900	23.1423	23.1123	23.1448	21.0278	23.1878	23.0386	22.8690	23.1360	23
XST59	23.1730	23.1121	23.0863	23.1261	21.0068	23.1668	22.9244	22.8440	23.1178	23
XST120	22.2690	22.3941	22.3897	22.3364	20.1574	22.3174	22.5512	22.0760	22.3382	2
CFPA31	22.5800	22.5386	22.5775	22.6081	20.4059	22.566	22.8162	22.2900	22.6163	22
XST64	22.4840	22.5113	22.5432	22.5497	20.3459	22.506	22.9828	22.230	22.5574	22
XST68	22.4290	22.4954	22.524	22.5194	20.3153	22.4754	22.9752	22.1940	22.5269	22
XST76	22.3650	22.4677	22.4893	22.4657	20.263	22.423	22.9854	22.1160	22.4726	22
XST83	22.3210	22.4396	22.4548	22.4168	20.2172	22.3772	22.9845	22.0790	22.4231	2
XST84	22.2820	22.4227	22.4349	22.3909	20.1937	22.3537	22.979	22.0710	22.3970	22
XST99A	22.2150	22.3790	22.3862	22.3335	20.1426	22.3026	22.9652	22.0840	22.3396	22
XST241	22.1750	22.3439	22.3491	22.2928	20.1068	22.2668	22.9556	22.0530	22.2997	22
XST107	22.1300	22.3035	22.307	22.248	20.0679	22.2279	22.9478	22.0100	22.2560	22
XST114	22.2850	22.4104	22.4078	22.3542	20.1712	22.3313	23.1035	22.0830	22.3562	22
XST44	22.2540	22.3716	22.3657	22.313	20.1385	22.2985	22.9684	22.0630	22.3150	22
YTT2-14A	22.2480	22.3537	22.3461	22.2952	20.1259	22.2859	22.9725	22.0630	22.2971	22
YTT2-25A	22.2430	22.3297	22.3194	22.2734	20.1125	22.2725	22.8353	22.0750	22.2749	22
YTT2-37A	22.2180	22.2975	22.284	22.2476	20.0995	22.2596	22.8259	22.1130	22.2486	22
YTT2-48A	22.2310	22.2824	22.2669	22.2396	20.1003	22.2603	22.8197	22.1620	22.2400	22

Stations	Model 1 [m]	Model 2 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 7 [m]	Model 8 [m]	SATLEVEL 1[m]	SAT
XST55	22.7000	22.0505	22.0297	21.9881	19.8913	22.0513	22.7899	21.8810	22.0009	21
YTT17-08A	22.9050	22.7689	22.8004	22.9022	20.7385	22.8986	22.5532	22.6190	22.9044	22
XST53	22.8390	22.7532	22.7706	22.8173	20.6427	22.8027	22.8463	22.4970	22.8174	2
FGPLA-Y-008	22.7910	22.7685	22.7768	22.7999	20.6237	22.7837	22.8285	22.4610	22.7982	22
XST59	23.1730	23.1121	23.0863	23.1261	21.0068	23.1668	22.8134	22.8440	23.1178	23
CFPA18	22.8650	22.8450	22.8447	22.8675	20.701	22.861	22.8027	22.5400	22.8635	22
XST69	22.6570	22.6913	22.7	22.6993	20.5116	22.6716	22.8188	22.3480	22.6986	22
YTT28-1	22.9720	**22.9730**	**22.9537**	**22.9448**	**20.7843**	**22.9443**	22.8418	22.6850	22.9366	22
ZTT45-200	22.8740	22.88615	22.8725	22.85648	20.684	22.844	22.8091	22.5870	22.8493	22
MCS1144S-A	22.6730	22.72629	22.721	22.69136	20.50339	22.6634	22.8152	22.4060	22.6868	22
YTT28-151	22.5360	22.6263	22.6206	22.577	20.3854	22.5454	22.8371	22.3250	22.5732	22
YTT28-134	22.9530	22.8820	22.8698	22.836	20.6437	22.8038	22.8754	22.6900	22.8258	22
ZTT6-53	23.1930	23.2545	23.2186	23.1929	21.0532	23.2133	22.7453	23.0060	23.1846	23
YTT27-33	23.4390	23.5463	23.5021	23.4708	21.3668	23.5268	22.8387	23.3220	23.4707	23
YTT27-41	23.4220	23.5282	23.4864	23.4543	21.344	23.5041	22.8085	23.3060	23.4539	23
YTT16-76A	22.6340	22.6368	22.6306	22.6163	20.431	22.5911	22.8706	22.8450	22.6066	22
XST121	22.4890	22.5886	22.5804	22.5344	20.3454	22.5055	22.7205	22.3040	22.5298	22
YTT28-200	22.4260	22.5100	22.5028	22.4547	20.2697	22.4297	22.8089	22.2210	22.4520	22
XT101	23.3190	23.3873	23.3682	23.3361	21.1773	23.3374	22.8596	23.2350	23.3340	23
ZTT30-5	23.1390	23.1828	23.1741	23.1481	20.9606	23.1207	22.8182	23.0920	23.1412	23
MCS1178T-A	22.5270	22.5887	22.5788	22.5409	20.3556	22.5157	22.7697	22.3760	22.5341	22
YTT9-73A	22.4380	22.4790	22.4683	22.4404	20.2693	22.4294	22.8297	22.3370	22.4348	22
XST165	23.2010	23.2218	23.2204	23.2006	21.0081	23.1682	22.834	23.1840	23.1956	23
XST126	23.3590	23.3488	23.3592	23.3474	21.1529	23.313	22.8299	23.3660	23.3483	23
YTT9-29A	22.4800	22.4514	22.4443	22.4639	20.3101	22.4702	22.8108	22.5860	22.4552	22
XST215	23.0300	22.9828	23.0135	23.0546	20.8335	22.9935	22.8457	23.1540	23.0467	23
ZTT35-26	22.0840	21.8665	21.8122	21.9339	20.0065	22.1665	22.8276	22.1250	21.9413	21
ZTT34-34	23.1300	23.0773	23.1363	23.2135	20.9738	23.1338	22.8204	23.2220	23.2107	23
YTT13-27	23.0420	22.9718	23.0145	23.0797	20.8527	23.0128	22.7881	23.1490	23.0724	23
XT161	22.9270	22.8667	22.8939	22.9442	20.731	22.8911	22.7859	23.0590	22.9340	22
XST202	23.1200	23.0961	23.1254	23.1517	20.9299	23.09	22.7926	23.2360	23.1465	23
YTT13-30	23.0370	22.9693	23.0095	23.0704	20.8448	23.0049	22.832	23.1490	23.0629	23
XST204	22.2210	22.1551	22.1304	22.1741	20.1008	22.2608	22.7624	22.3520	22.1729	22
SUM	2514	2514.3	2514.04	2514.04	2276.43	2514.04		2497.10	2514.03	2
MEAN	22.855	22.8573	22.8549	22.8549	20.6949	22.8549		22.701	22.8548	22

Appendix C18:
Full Data Set for Table 4.11a: Residuals obtained from the Existing and New 'Satlevel' Collocation Models for Port Harcourt.

Table 4.11a: Residuals obtained of the Existing and New 'Satlevel' Collocation Models for Port Harcourt

Stations	Model 2 [m]	Model 3 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 8 [m]	SATLEVEL 1[m]	SATLEVEL 2[m]
AP4	-0.0253	-0.0313	-0.0234	-0.5850	-0.0179	0.0000	-0.0241	-0.0300	-0.0293
AP1	-0.0217	-0.0275	-0.0200	-0.6113	-0.0159	-0.0325	-0.0221	-0.0262	-0.0254
PT.3 EMMA	-0.0357	-0.0371	-0.0226	-0.6134	-0.0317	0.0359	-0.0443	-0.0363	-0.0352
PHCS 1s	-0.0234	-0.0252	-0.0080	-0.5985	-0.0191	0.0044	-0.0350	-0.0247	-0.0234
PT.4 EMMA	0.0011	-0.0020	0.0114	-0.5794	0.0066	-0.0115	-0.0056	-0.0011	-6E-05
PT.8 EMMA	-0.0471	-0.0529	-0.0430	-0.6342	-0.0389	-0.0553	-0.0479	-0.0517	-0.0509
PT.4 ABDUL	0.0024	-0.0045	0.0052	-0.5861	0.0118	0.0240	0.0028	-0.0033	-0.0025
PT.5 EMMA	-0.0070	-0.0115	0.0010	-0.5900	1E-04	-0.0005	-0.0121	-0.0105	-0.0095
PT.7 EMMA	0.0183	0.0131	0.0238	-0.5673	0.0260	0.0137	0.0154	0.0142	0.0151
PT.9 EMMA	-0.0176	-0.0241	-0.0143	-0.6056	-0.0086	-0.0072	-0.0180	-0.0229	-0.0221
PT.2 ABDUL	-0.0293	-0.0363	-0.0273	-0.6186	-0.0203	-0.0004	-0.0269	-0.0351	-0.0343
PT.3 ABDUL	-0.0276	-0.0347	-0.0254	-0.6166	-0.0183	-0.0289	-0.0257	-0.0335	-0.0328
GPS 02	-0.0020	0.0066	0.0228	-0.5671	-0.0077	0.0000	0.0040	0.0072	0.0085
GPS 03	-0.0020	-0.0111	0.0016	-0.5884	-0.0007	0.0240	0000	-0.0104	-0.0092
GPS 04	-0.0019	-0.0110	0.0005	-0.5897	1E-04	-0.0465	0.0010	-0.0102	-0.0091
GPS 05	-0.0032	-0.0092	0.0052	-0.5847	-0.0026	-0.0486	-0.0010	-0.0086	-0.0073
GPS 06	-0.0028	-0.0120	0.0001	-0.5899	-0.0012	-0.0378	-0.0010	-0.0112	-0.0101
GPS 07	-0.0018	-0.0112	-0.0002	-0.5904	0.0004	-0.0429	0.0010	-0.0103	-0.0093
GPS 08	-0.0010	-0.0109	-0.0014	-0.5919	0.0017	-0.0347	0.0020	-0.0099	-0.0089
GPS 09	-0.0005	-0.0108	-0.0016	-0.5920	0.0021	-0.0324	0.0020	-0.0098	-0.0088
GPS 10	-0.0013	-0.0120	-0.0032	-0.5937	0.0010	-0.0357	0.0010	-0.0110	-0.0101
GPS 11	-0.0033	-0.0077	0.0057	-0.5844	-0.0019	-0.0485	0.0010	-0.0070	-0.0058
GPS 12	-0.0032	-0.0075	0.0059	-0.5843	-0.0018	-0.0206	0.0010	-0.0067	-0.0056
GPS 13	-0.0024	-0.0064	0.0068	-0.5834	-0.0009	-0.0168	0.0020	-0.0056	-0.0045
GPS 14	-0.0014	-0.0112	-0.0019	-0.5924	0.0015	-0.0067	0.0020	-0.0102	-0.0092
GPS 15	-0.0010	-0.0104	-0.0013	-0.5919	0.0022	-0.0285	0.0030	-0.0094	-0.0085
GPS 16	-0.0012	-0.0103	-0.0013	-0.5920	0.0023	-0.0232	0.0030	-0.0092	-0.0083
GPS 17	-0.0016	-0.0104	-0.0017	-0.5923	0.0022	-0.0183	0.0030	-0.0093	-0.0084
GPS 18	-0.0008	-0.0096	-0.0011	-0.5918	0.0031	-0.0121	0.0040	-0.0084	-0.0076
GPS 19	0.0004	-0.0066	8.6E-05	-0.5910	0.0050	0.0000	0000	-0.0053	-0.0046
GPS 20	0.0004	-0.0066	3.6E-05	-0.5911	0.0050	-0.0064	00000	-0.0053	-0.0046
GPS 21	0.0006	-0.0064	0.0004	-0.5908	0.0055	-0.0024	00000	-0.0051	-0.0044
GPS 22	0.0025	-0.0027	0.0034	-0.5877	0.0043	-0.0042	-0.0010	-0.0014	-0.0007
GPS 23	0.0023	-0.0030	0.0030	-0.5881	0.0038	-0.0062	-0.0020	-0.0017	-0.0010
GPS 24	0.0029	-0.0022	0.0038	-0.5872	0.0043	-0.0025	-0.0010	-0.0009	-0.0002
GPS 25	0.0023	-0.0031	0.0028	-0.5883	0.0034	-0.0089	-0.0020	-0.0017	-0.0010
GPS 26	0.0090	0.0121	0.0193	-0.5715	0.0019	0.0235	-0.0020	0.0133	0.0141

Stations	Model 2 [m]	Model 3 [m]	Model 4 [m]	Model 5 [m]	Model 6 [m]	Model 7 [m]	Model 8 [m]	SATLEVEL 1[m]	SATLEVEL 2[m]
GPS 27	0.0098	0.0130	0.0202	-0.5706	0.0024	0.0024	-0.0020	0.0142	0.0150
GPS 28	0.0095	0.0129	0.0200	-0.5708	0.0018	0.0023	-0.0020	0.0141	0.0149
GPS 29	0.0133	0.0189	0.0243	-0.5663	-0.0003	-0.0129	-0.0020	0.0202	0.0209
GPS 30	0.0127	0.0181	0.0234	-0.5672	-0.0006	0.0077	-0.0020	0.0193	0.0201
GPS 31	0.0126	0.0177	0.0230	-0.5676	-0.0002	0.0080	-0.0020	0.0189	0.0197
GPS 32	0.0017	0.0020	0.0126	-0.5783	0.0041	-0.0084	0.0110	0.0031	0.0040
GPS 33	0.0014	0.0019	0.0126	-0.5782	0.0036	0.0054	0.0100	0.0030	0.0039
GPS 34	0.0016	0.0023	0.0131	-0.5776	0.0035	0.0053	0.0100	0.0034	0.0043
GPS 35	0.0056	0.0148	0.0238	-0.5669	-0.0002	0.0634	0.0150	0.0160	0.0168
GPS 36	0.0050	0.0146	0.0237	-0.5670	-0.0012	0.1277	0.0150	0.0158	0.0166
GPS 37	0.0062	0.0170	0.0263	-0.5643	-0.0010	0.1287	0.0160	0.0182	0.0190
GPS 38	0.0056	0.0100	0.0161	-0.5747	0.0010	0.1208	0.0120	0.0114	0.0120
GPS 39	0.0060	0.0111	0.0171	-0.5737	0.0009	0.0053	0.0120	0.0124	0.0131
GPS 40	0.0060	0.0117	0.0178	-0.5730	0.0002	0.0054	0.0120	0.0131	0.0137
GPS 41	0.0065	0.0066	0.0116	-0.5791	0.0022	0.0342	0.0010	0.0080	0.0086
GPS 42	0.0066	0.0071	0.0120	-0.5788	0.0016	0.0188	0.0010	0.0085	0.0092
GPS 43	0.0066	0.0075	0.0122	-0.5785	0.0008	0.0191	0.0010	0.0089	0.0095
GPS 45	0.0083	0.0047	0.0075	-0.5827	-0.0024	0.0583	00	0.0060	0.0067
GPS 46	0.0093	0.0063	0.0090	-0.5813	-0.0021	0.0126	0.0010	0.0076	0.0083
GPS 47	0.0064	-0.0141	-0.0053	-0.5948	0.0036	0.0311	-0.0060	-0.0134	-0.0122
GPS 48	0.0055	-0.0151	-0.0063	-0.5958	0.0026	-0.0127	-0.0070	-0.0144	-0.0132
GPS 49	0.0060	-0.0150	-0.0062	-0.5956	0.0027	-0.0119	-0.0070	-0.0143	-0.0130
GPS 50	-0.0003	-0.0064	0.0018	-0.5893	0.0061	-0.1639	0.0030	-0.0051	-0.0044
GPS 51	0.0002	-0.0059	0.0024	-0.5887	0.0065	-0.0040	0.0040	-0.0046	-0.0039
GPS 53	0.0074	0.0088	0.0202	-0.5706	0.0071	-0.1801	-0.0020	0.0097	0.0107
GPS 54	0.0079	0.0095	0.0211	-0.5698	0.0072	0.0178	-0.0020	0.0105	0.0115
GPS 55	0.0068	0.0083	0.0199	-0.5709	0.0064	0.0184	-0.0030	0.0093	0.0103
GPS 56	0.0078	0.0066	0.0223	-0.5683	0.0116	0.0593	-0.0020	0.0073	0.0085
GPS 57	0.0077	0.0066	0.0223	-0.5684	0.0114	0.0154	-0.0020	0.0072	0.0085
GPS 58	0.0074	0.0062	0.0217	-0.5690	0.0112	0.0154	-0.0020	0.0069	0.0081
GPS 59	0.0121	0.0036	0.0030	-0.5880	-0.0078	-0.1284	0.0050	0.0049	0.0056
GPS 60	0.0121	0.0037	0.0022	-0.5878	-0.0075	-0.0004	-0.0020	0.0050	0.0057
XSV 662	-0.0003	-0.0064	0.0016	-0.5897	0.0077	0.1598	0.0020	-0.0051	-0.0044
ZVS 3003	0.0030	-0.0036	0.0049	-0.5863	0.0110	0.0635	0.0060	-0.0023	-0.0016
Sum	1E-05	-0.2625	0.37551	-41.532	0.00073	-0.0114	-0.13150	-0.1857	-0.1225
Mean	1E-07	-0.0037	0.00529	-0.585	1E-05	-0.0002	-0.0019	-0.0026	-0.0017

Appendix C19:
Full Data Set for Table 4.11b: Summary of the Results from the Local, Existing Geoid and New ‘Satlevel’ Collocation Models for Lagos State.

Table 4.11b: Result of the Differences Between Observed Undulation and the Existing Models (Residuals) for Lagos State

Stations	North Sea Region Model [m]	4-parameters Similarity Datum Shift [m]	5-parameters Similarity Datum Shift [m]	7-parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM2008 [m]
XST 237	0.0196	0.0251	0.0742	2.2606	0.1006	-0.0770	0.3000
XST44	-0.1265	-0.1210	-0.0680	2.1085	-0.0520	-0.3980	0.1880
YTT78A	-0.0447	-0.0350	-0.0040	2.1724	0.0123	-0.1970	0.1160
XST245	0.1238	0.1346	0.1769	2.3405	0.1805	-0.1870	0.3560
XST244	-0.0897	-0.0780	-0.0370	2.1175	-0.0430	-0.4820	0.1270
FGPLA-Y-003	0.0644	0.0454	0.0070	2.1875	0.0275	-0.6180	0.3370
CFPA21	0.0487	0.0389	0.0069	2.1803	0.0203	-0.0680	0.3410
XST 55	0.1513	0.0880	-0.0100	2.1892	0.0291	-0.1550	0.2460
YTT1703A	0.1374	0.1054	-7E-04	2.1613	0.0012	0.1128	0.2780
XST46	0.1319	0.1181	0.0053	2.1401	-0.0200	0.1570	0.2770
XST50	0.1056	0.0864	0.0246	2.1938	0.0338	0.0722	0.3330
LWBC5-61P	0.0613	0.0885	0.0484	2.1653	0.0053	0.4497	0.3290
YTT19-54	0.0477	0.0777	0.0452	2.1622	0.0022	0.3818	0.3210
XST75	0.0125	0.0334	0.0312	2.1822	0.0222	0.2366	0.3110
CFPA40	0.1132	0.0599	-0.0080	2.1931	0.0331	-0.0730	0.2850
CFPB36	0.0984	0.0522	-8E-04	2.1999	0.0398	-0.0340	0.3040
XST60	0.0122	-0.0230	-0.0420	2.1612	0.0011	-0.3190	0.2730
XST72	-0.0866	-0.1120	-0.0970	2.1067	-0.0530	-0.3770	0.2330
XST76	-0.1027	-0.1240	-0.1010	2.102	-0.0580	-0.8590	0.2490
XST44	-0.1176	-0.1120	-0.0590	2.1155	-0.0450	-0.4910	0.1910
YTT2-18A	-0.0992	-0.0910	-0.0420	2.1264	-0.0340	-0.5050	0.1780
XST156	-0.0753	-0.0610	-0.0270	2.1187	-0.0410	-0.4550	0.0940
ZTT2-57A	-0.0175	-0.0010	0.0125	2.1464	-0.0140	-0.4090	-0.0060
YTT2-66A	-0.0026	0.0149	0.0128	2.1351	-0.0250	-0.4800	-0.0720
YTT2-80	0.0267	0.0474	0.0215	2.1192	-0.0410	-0.5120	-0.1320
XST224	0.1219	0.1566	0.0825	2.1033	-0.0570	-0.5490	-0.1070
ZTT35-14	0.1727	0.2171	0.1201	2.0945	-0.0660	-0.6030	-0.0900
XST149	-0.0372	-0.0280	-0.0010	2.1925	0.0324	0.2131	0.0960
MCS1188T-A	-0.0488	-0.0390	-0.0050	2.1843	0.0242	-0.1720	0.1380
XST42	0.0325	-0.0440	-0.1400	2.1108	-0.0490	0.3857	-0.0550
YTT13-1A	0.029	-0.0490	-0.1340	2.1185	-0.0410	0.4567	-0.0630
ZTT34-10A	0.0397	-0.0220	-0.0860	2.1563	-0.0040	0.2565	-0.0860
XST135	0.0295	-0.0360	-0.0900	2.151	-0.0090	0.5593	-0.0750

Stations	North Sea Region Model [m]	4-parameters Similarity Datum Shift [m]	5-parameters Similarity Datum Shift [m]	7-parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM2008 [m]
XST218	0.0204	-0.0340	-0.0740	2.1593	-7E-04	0.5225	-0.0940
XST209	0.0321	-0.0150	-0.0390	2.1841	0.0241	0.6105	-0.0670
XST201	0.0327	-0.0060	-0.0180	2.1957	0.0357	0.6631	-0.0560
XST203	-0.011	-0.0330	-0.0260	2.1672	0.0072	0.6614	-0.0490
XST177	0.0122	-0.0040	0.0112	2.1907	0.0307	0.7293	-0.2110
YTT22-1	-0.0242	-0.0300	-0.0110	2.1669	0.0069	-0.1770	-0.0090
XST159	-0.0625	-0.0520	-0.0210	2.1238	-0.0360	-0.3410	0.0510
ZTT31-70	-0.0795	-0.0580	-0.0240	2.1087	-0.0510	-0.1380	0.0820
XST131	-0.1051	-0.0740	-0.0360	2.0669	-0.0930	-0.2830	0.0980
XST127	-0.0865	-0.0600	-0.0260	2.1119	-0.0480	-0.4990	0.0950
XST133	-0.0977	-0.0640	-0.0290	2.0952	-0.0650	-0.1270	0.1110
XST128	-0.1277	-0.0880	-0.0540	2.0565	-0.1030	0.1140	0.0930
YTT28-117	-0.114	-0.0680	-0.0360	2.0606	-0.0990	-0.0060	0.1135
MCS1174S-A	-0.126	-0.0730	-0.0410	2.0284	-0.1320	0.2858	0.1170
YTT28-96	0.7803	0.8446	0.8761	2.906	0.7460	1.3895	1.0400
XST41	-0.1522	-0.0800	-0.0490	1.9519	-0.2080	0.3045	0.1170
YTT28-89	-0.1113	-0.0480	-0.0240	2.0256	-0.1340	-0.6210	0.0977
YTT28-87	-0.0367	0.0163	0.0393	2.127	-0.0330	0.1383	0.1647
YTT28-67	-0.0051	0.0424	0.0641	2.1705	0.0105	0.1168	0.1962
YTT28-65	0.0099	0.0509	0.0700	2.1952	0.0352	-0.2790	0.2400
YTT28-47	-0.0473	-0.0210	0.0003	2.1594	-6E-04	-0.5050	0.2130
XST87	-0.0251	-0.0020	0.0153	2.1767	0.0167	-0.4490	0.2480
YTT28-30	-0.0121	0.0078	0.0259	2.1917	0.0317	-0.2320	0.2686
YTT28-1	-0.0009	0.0184	0.0273	2.1878	0.0278	-0.2050	0.2870
XST71	0.0207	0.0439	0.0318	2.173	0.0130	-0.2120	0.3150
YTT19-7	0.0235	0.0451	0.0295	2.171	0.0110	-0.2590	0.3180
YTT19-54	0.0477	0.0777	0.0452	2.1622	0.0022	-0.3030	0.3210
XST59	0.0609	0.0867	0.0469	2.1662	0.0062	-0.3180	0.3290
XST120	-0.1251	-0.1210	-0.0670	2.1116	-0.0480	-1.3710	0.1930
CFPA31	0.0414	0.0025	-0.0280	2.1741	0.0140	-1.1060	0.2900
XST64	-0.0273	-0.0590	-0.0660	2.1381	-0.0220	-0.4880	0.2540
XST68	-0.0664	-0.0950	-0.0900	2.1137	-0.0460	-0.9550	0.2350
XST76	-0.1027	-0.1240	-0.1010	2.102	-0.0580	0.1750	0.2490
XST83	-0.1186	-0.1340	-0.0960	2.1038	-0.0560	-0.1550	0.2420
XST84	-0.1407	-0.1530	-0.1090	2.0883	-0.0720	0.3900	0.2110
XST99A	-0.164	-0.1710	-0.1180	2.0724	-0.0880	0.4486	0.1310
XST241	-0.1689	-0.1740	-0.1180	2.0682	-0.0920	0.4145	0.1220

Stations	North Sea Region Model [m]	4-parameters Similarity Datum Shift [m]	5-parameters Similarity Datum Shift [m]	7-parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM2008 [m]
XST107	-0.1735	-0.1770	-0.1180	2.0621	-0.0980	0.5398	0.1200
XST114	-0.1254	-0.1230	-0.0690	2.1138	-0.0460	0.7268	0.2020
XST44	-0.1176	-0.1120	-0.0590	2.1155	-0.0450	0.8107	0.1910
YTT2-14A	-0.1057	-0.0980	-0.0470	2.1221	-0.0380	0.6685	0.1850
YTT2-25A	-0.0867	-0.0760	-0.0300	2.1305	-0.0300	-0.4850	0.1680
YTT2-37A	-0.0795	-0.0660	-0.0300	2.1185	-0.0420	-0.3570	0.1050
YTT2-48A	-0.0514	-0.0360	-0.0090	2.1307	-0.0290	-0.4400	0.0690
XST55	0.6495	0.6703	0.7119	2.8087	0.6487	-0.1350	0.8190
YTT17-08A	0.1361	0.1046	0.0028	2.1665	0.0064	0.2829	0.2860
XST53	0.0858	0.0684	0.0217	2.1963	0.0363	0.1069	0.3420
FGPLA-Y-008	0.0225	0.0142	-0.0090	2.1673	0.0073	0.0630	0.3300
XST59	0.0609	0.0867	0.0469	2.1662	0.0062	0.4077	0.3290
CFPA18	0.0195	0.0198	-0.0030	2.1635	0.0035	0.0763	0.3245
XST69	-0.0343	-0.0430	-0.0420	2.1454	-0.0150	-0.1470	0.3090
YTT28-1	-0.0005	0.0188	0.0277	2.1882	0.0282	0.1999	0.2875
ZTT45-200	-0.0122	0.0015	0.0175	2.19	0.0300	0.0879	0.2870
MCS1144S-A	-0.0533	-0.0480	-0.0180	2.1696	0.0096	-0.1040	0.2670
YTT28-151	-0.0905	-0.0850	-0.0410	2.1504	-0.0100	-0.1970	0.2107
YTT28-134	0.0705	0.0827	0.1165	2.3088	0.1487	0.2133	0.2625
ZTT6-53	-0.0615	-0.0260	8E-05	2.1398	-0.0200	0.4600	0.1870
YTT27-33	-0.1073	-0.0630	-0.0320	2.0722	-0.0880	0.7789	0.5940
YTT27-41	-0.1062	-0.0640	-0.0320	2.078	-0.0820	0.8476	0.1160
YTT16-76A	-0.0028	0.0034	0.0177	2.203	0.0429	0.0715	-0.2110
XST121	-0.0996	-0.0910	-0.0450	2.1436	-0.0160	-0.0960	0.1850
YTT28-200	-0.0837	-0.0760	-0.0280	2.1566	-0.0030	-0.1810	0.2052
XT101	-0.0683	-0.0490	-0.0170	2.1417	-0.0180	0.6796	0.0840
ZTT30-5	-0.0438	-0.0350	-0.0090	2.1784	0.0183	0.4364	0.0470
MCS1178T-A	-0.0617	-0.0520	-0.0140	2.1714	0.0113	-0.1930	0.1510
YTT9-73A	-0.041	-0.0300	-0.0020	2.1687	0.0086	-0.3160	0.1010
XST165	-0.0208	-0.0190	0.0004	2.1929	0.0328	0.3378	0.0170
XST126	0.0102	-2E-04	0.0116	2.2061	0.0460	0.4359	-0.0070
YTT9-29A	0.0286	0.0357	0.0161	2.1699	0.0098	-0.2090	-0.1060
XST215	0.0472	0.0165	-0.0250	2.1965	0.0365	0.3614	-0.1240
ZTT35-26	0.2175	0.2718	0.1501	2.0775	-0.0830	-0.6110	-0.0410
ZTT34-34	0.0527	-0.0060	-0.0840	2.1562	-0.0040	0.4386	-0.0920
YTT13-27	0.0702	0.0275	-0.0380	2.1893	0.0292	0.3265	-0.1070
XT161	0.0603	0.0331	-0.0170	2.196	0.0359	0.2228	-0.1320

Stations	North Sea Region Model [m]	4-parameters Similarity Datum Shift [m]	5-parameters Similarity Datum Shift [m]	7-parameters Similarity Datum Shift [m]	Zanletnyik Hungarian Polynomial [m]	Mosaic of Parametric Model [m]	GEM2008 [m]
XST202	0.0239	-0.0050	-0.0320	2.1901	0.0300	0.4158	-0.1160
YTT13-30	0.0677	0.0275	-0.0330	2.1922	0.0321	0.3137	-0.1120
XST204	0.0659	0.0906	0.0469	2.1202	-0.0400	-0.5100	-0.1310

Appendix C20:
Full Data Set for Table 4.13a: Results of Fitting the Local Geoid to GEM2008 Model for Port Harcourt.

Table 4.13a: Results of Fitting the Local Geoid to GEM2008 Model for Port Harcourt

Stations	Local Geoidal Undulation	GEM2008 Geoidal Undulations	Differences
Models	Equation (1.2)	Equation (3.86)	
AP4	18.9552	18.9470	0.0082
AP1	18.9408	18.9340	0.0068
PT.3 EMMA	19.0120	19.0110	0.0010
PHCS 1s	19.0327	19.0330	-0.0003
PT.4 EMMA	19.0117	19.0080	0.0037
PT.8 EMMA	18.9947	18.9860	0.0087
PT.4 ABDUL	19.0100	19.0000	0.0100
PT.5 EMMA	19.0116	19.0060	0.0056
PT.7 EMMA	18.9990	18.9920	0.0070
PT.9 EMMA	19.0022	18.9930	0.0092
PT.2 ABDUL	19.0236	19.0130	0.0106
PT.3 ABDUL	19.0170	19.0060	0.0110
GPS 02	18.8858	18.9000	-0.0142
GPS 03	18.8276	18.8250	0.0026
GPS 04	18.8364	18.8320	0.0044
GPS 05	18.8335	18.8350	-0.0015
GPS 06	18.8319	18.8290	0.0029
GPS 07	18.8389	18.8340	0.0049
GPS 08	18.8470	18.8400	0.0070
GPS 09	18.8450	18.8380	0.0070
GPS 10	18.8425	18.8350	0.0075
GPS 11	18.8528	18.8530	-0.0002
GPS 12	18.8556	18.8560	-0.0004
GPS 13	18.8586	18.8590	-0.0004
GPS 14	18.8505	18.8430	0.0075
GPS 15	18.8560	18.8480	0.0080
GPS 16	18.8610	18.8530	0.0080
GPS 17	18.8664	18.8580	0.0084
GPS 18	18.8699	18.8610	0.0090
GPS 19	18.9103	18.9040	0.0063
GPS 20	18.9093	18.9030	0.0063
GPS 21	18.9120	18.9060	0.0060
GPS 22	18.9116	18.9080	0.0036
GPS 23	18.9099	18.9070	0.0029
GPS 24	18.9112	18.9080	0.0032
GPS 25	18.9069	18.9040	0.0029
GPS 26	18.9244	18.9320	-0.0076

Stations	Local Geoidal Undulation	GEM2008 Geoidal Undulations	Differences
Models	Equation (1.2)	Equation (3.86)	
GPS 27	18.9235	18.9320	-0.0085
GPS 28	18.9226	18.9310	-0.0084
GPS 29	18.9009	18.9150	-0.0141
GPS 30	18.9006	18.9140	-0.0134
GPS 31	18.9009	18.9140	-0.0131
GPS 32	18.9260	18.9240	0.0021
GPS 33	18.9250	18.9240	0.0010
GPS 34	18.9234	18.9230	0.0004
GPS 35	18.9814	18.9890	-0.0076
GPS 36	18.9804	18.9880	-0.0076
GPS 37	18.9787	18.9880	-0.0093
GPS 38	19.0320	19.0350	-0.0030
GPS 39	19.0318	19.0360	-0.0042
GPS 40	19.0309	19.0360	-0.0051
GPS 41	19.0664	19.0750	-0.0086
GPS 42	19.0678	19.0770	-0.0092
GPS 43	19.0693	19.0790	-0.0097
GPS 45	19.1147	19.1210	-0.0063
GPS 46	19.1139	19.1210	-0.0071
GPS 47	19.1581	19.1460	0.0121
GPS 48	19.1591	19.1470	0.0121
GPS 49	19.1610	19.1490	0.0120
GPS 50	18.9213	18.9150	0.0063
GPS 51	18.9197	18.9130	0.0067
GPS 53	18.9748	18.9780	-0.0032
GPS 54	18.9741	18.9780	-0.0039
GPS 55	18.9753	18.9790	-0.0037
GPS 56	19.0203	19.0200	0.0003
GPS 57	19.0193	19.0190	0.0003
GPS 58	19.0186	19.0180	0.0006
GPS 59	18.7882	18.7860	0.0022
GPS 60	18.7902	18.7950	-0.0048
XSV 662	18.9616	18.9530	0.0086
ZVS 3003	19.0301	19.0200	0.0101

Appendix C21:
Full Data Set for Table 4.13b: Results of Fitting the Local Geoid to GEM2008 Model for Lagos State

Table 4.13b: Results of Fitting the Local Geoid to GEM2008 Model for Lagos State

Stations	Local Geoidal Undulation	GEM2008 Geoidal Undulations	Differences
Models	Equation (1.2)	Equation (3.86)	
XST 237	22.3910	22.2640	0.1270
XST44	22.2250	22.0660	0.1590
YTT78A	22.4218	22.3580	0.0638
XST245	22.2493	22.1350	0.1143
XST244	22.2017	22.0970	0.1047
FGPLA-Y-003	22.4057	22.4460	-0.0403
CFPA21	22.4686	22.4870	-0.0184
XST 55	22.2147	22.4540	-0.2393
YTT1703A	22.4185	22.6340	-0.2155
XST46	22.5412	22.7670	-0.2258
XST50	22.4427	22.5470	-0.1043
LWBC5-61P	22.7911	22.8570	-0.0659
YTT19-54	22.8157	22.8690	-0.0533
XST75	22.7248	22.7120	0.0128
CFPA40	22.2216	22.3700	-0.1484
CFPB36	22.2376	22.3450	-0.1074
XST60	22.2362	22.2660	-0.0298
XST72	22.2211	22.1630	0.0581
XST76	22.2162	22.1160	0.1002
XST44	22.2205	22.0630	0.1575
YTT2-18A	22.2219	22.0800	0.1419
XST156	22.1937	22.1230	0.0707
ZTT2-57A	22.2245	22.2800	-0.0555
YTT2-66A	22.2260	22.3420	-0.1160
YTT2-80	22.1919	22.3730	-0.1811
XST224	22.0461	22.2670	-0.2209
ZTT35-14	21.9623	22.2090	-0.2467
XST149	22.9237	22.8870	0.0367
MCS1188T-A	22.5583	22.4840	0.0742
XST42	23.3037	23.2230	0.0807
YTT13-1A	23.3809	23.3090	0.0719
ZTT34-10A	23.3226	23.3230	-0.0004
XST135	23.4249	23.4040	0.0209
XST218	23.3987	23.4150	-0.0163
XST209	23.4533	23.4770	-0.0237
XST201	23.4567	23.4930	-0.0363

Stations	Local Geoidal Undulation	GEM2008 Geoidal Undulations	Differences
Models	Equation (1.2)	Equation (3.86)	
XST203	23.4785	23.5090	-0.0305
XST177	23.5276	23.5460	-0.0184
YTT22-1	23.4354	23.4560	-0.0206
XST159	23.5120	23.4780	0.0340
ZTT31-70	23.4723	23.4240	0.0483
XST131	23.5579	23.4920	0.0659
XST127	23.3534	23.3100	0.0434
XST133	23.3459	23.3010	0.0449
XST128	23.3648	23.3260	0.0388
YTT28-117	23.3883	23.3580	0.0303
MCS1174S-A	23.5014	23.4640	0.0374
YTT28-96	23.6166	23.5810	0.0356
XST41	23.6930	23.6610	0.0320
YTT28-89	23.4677	23.5020	-0.0343
YTT28-87	23.3021	23.3380	-0.0359
YTT28-67	23.1924	23.2300	-0.0376
YTT28-65	23.0529	23.0680	-0.0151
YTT28-47	22.8122	22.7920	0.0202
XST87	22.7539	22.7320	0.0219
YTT28-30	22.7127	22.6830	0.0297
YTT28-1	22.7047	22.6850	0.0197
XST71	22.7505	22.7560	-0.0055
YTT19-7	22.7348	22.7440	-0.0092
YTT19-54	22.8157	22.8690	-0.0533
XST59	22.7795	22.8440	-0.0645
XST120	22.2343	22.0760	0.1583
CFPA31	22.2395	22.2900	-0.0505
XST64	22.2302	22.2300	0.0003
XST68	22.2241	22.1940	0.0301
XST76	22.2162	22.1160	0.1002
XST83	22.2080	22.0790	0.1290
XST84	22.0717	22.0710	0.0007
XST99A	22.1787	22.0840	0.0947
XST241	22.1539	22.0530	0.1009
XST107	22.1228	22.0100	0.1128
XST114	22.2406	22.0830	0.1576
XST44	22.2205	22.0630	0.1575
YTT2-14A	22.2130	22.0630	0.1500
YTT2-25A	22.2052	22.0750	0.1303

Stations	Local Geoidal Undulation	GEM2008 Geoidal Undulations	Differences
Models	Equation (1.2)	Equation (3.86)	
YTT2-37A	22.1949	22.1130	0.0819
YTT2-48A	22.1955	22.1620	0.0335
XST55	21.9456	21.8810	0.0646
YTT17-08A	22.4153	22.6190	-0.2037
XST53	22.4327	22.4970	-0.0643
FGPLA-Y-008	22.4656	22.4610	0.0046
XST59	22.7795	22.8440	-0.0645
CFPA18	22.5397	22.5400	-0.0003
XST69	22.4135	22.3480	0.0655
YTT28-1	22.7047	22.6850	0.0197
ZTT45-200	22.6298	22.5870	0.0428
MCS1144S-A	22.4929	22.4060	0.0869
YTT28-151	22.4314	22.3250	0.1064
YTT28-134	22.7570	22.6900	0.0670
ZTT6-53	23.0312	23.0060	0.0252
YTT27-33	23.3486	23.3220	0.0266
YTT27-41	23.3374	23.3060	0.0314
YTT16-76A	22.5732	22.8450	-0.2718
XST121	22.4267	22.3040	0.1227
YTT28-200	22.3552	22.2210	0.1342
XT101	23.2679	23.2350	0.0329
ZTT30-5	23.0928	23.0920	0.0008
MCS1178T-A	22.4680	22.3760	0.0920
YTT9-73A	22.3883	22.3370	0.0513
XST165	23.1602	23.1840	-0.0238
XST126	23.3256	23.3660	-0.0404
YTT9-29A	22.4351	22.5860	-0.1509
XST215	23.0412	23.1540	-0.1128
ZTT35-26	21.8902	22.1250	-0.2348
ZTT34-34	23.2067	23.2220	-0.0153
YTT13-27	23.0677	23.1490	-0.0813
XT161	22.9273	23.0590	-0.1317
XST202	23.1401	23.2360	-0.0959
YTT13-30	23.0582	23.1490	-0.0908
XST204	22.1465	22.3520	-0.2055

Appendix C22:
Full Data Set for Table 4.14a: Summary of the Result of GEM2008 Orthometric Height Computed from New 'Satlevel' Collacation for Port Harcourt.

Table 4.14a: Summary of the Result of GEM2008 Orthometric Height Computed from New 'Satlevel' Collacation for Port Harcourt

Stations	Local Geoidal Undulation	GEM2008 Geoidal Undulations	Differences
Models	Equation (1.2)	Equation (3.86)	
AP4	16.9020	16.8938	0.0083
AP1	14.7860	14.7792	0.0068
PT.3 EMMA	6.1840	6.1830	0.0010
PHCS 1s	11.7630	11.7633	-0.0003
PT.4 EMMA	11.6850	11.6813	0.0037
PT.8 EMMA	7.8030	7.7943	0.0087
PT.4 ABDUL	13.8420	13.8320	0.0100
PT.5 EMMA	10.3680	10.3624	0.0056
PT.7 EMMA	14.3870	14.3800	0.0070
PT.9 EMMA	10.1480	10.1388	0.0092
PT.2 ABDUL	13.6270	13.6164	0.0106
PT.3 ABDUL	7.7440	7.7330	0.0110
GPS 02	23.6420	23.6562	-0.0142
GPS 03	21.2400	21.2374	0.0026
GPS 04	19.9390	19.9346	0.0044
GPS 05	22.5220	22.5235	-0.0015
GPS 06	20.6560	20.6531	0.0029
GPS 07	19.5170	19.5121	0.0049
GPS 08	17.5870	17.5800	0.0070
GPS 09	15.7890	15.7820	0.0070
GPS 10	17.9840	17.9765	0.0075
GPS 11	19.3020	19.3022	-0.0002
GPS 12	20.8050	20.8054	-0.0004
GPS 13	21.7300	21.7304	-0.0004
GPS 14	16.5160	16.5085	0.0075
GPS 15	15.9180	15.9100	0.0080
GPS 16	15.9030	15.8950	0.0080
GPS 17	15.9320	15.9236	0.0084
GPS 18	15.9230	15.9141	0.0090
GPS 19	10.3620	10.3557	0.0063
GPS 20	10.9670	10.9607	0.0063
GPS 21	11.4320	11.4260	0.0060
GPS 22	13.4270	13.4234	0.0036
GPS 23	14.3490	14.3461	0.0029
GPS 24	14.1570	14.1538	0.0032
GPS 25	14.6280	14.6251	0.0030
GPS 26	1.2480	1.2556	-0.0076
GPS 27	0.6250	0.6335	-0.0085

Stations	Local Geoidal Undulation	GEM2008 Geoidal Undulations	Differences
Models	Equation (1.2)	Equation (3.86)	
GPS 29	1.3240	1.3381	-0.0141
GPS 28	1.7680	1.7764	-0.0084
GPS 30	2.0700	2.0834	-0.0134
GPS 31	4.4050	4.4181	-0.0131
GPS 32	18.6030	18.6010	0.0021
GPS 33	19.4450	19.4440	0.0010
GPS 34	20.6440	20.6436	0.0004
GPS 35	21.6810	21.6886	-0.0076
GPS 36	21.8820	21.8896	-0.0076
GPS 37	19.7690	19.7783	-0.0093
GPS 38	15.4430	15.4460	-0.0030
GPS 39	17.0070	17.0112	-0.0042
GPS 40	18.0920	18.0971	-0.0051
GPS 41	18.8870	18.8956	-0.0086
GPS 42	19.1000	19.1092	-0.0092
GPS 43	17.2150	17.2247	-0.0097
GPS 45	14.3110	14.3173	-0.0063
GPS 46	12.7600	12.7671	-0.0071
GPS 47	13.6470	13.6349	0.0121
GPS 48	13.8700	13.8579	0.0121
GPS 49	14.6730	14.6610	0.0120
GPS 50	16.2020	16.1957	0.0063
GPS 51	16.5860	16.5793	0.0067
GPS 53	10.1000	10.1032	-0.0032
GPS 54	10.3580	10.3619	-0.0039
GPS 55	10.1940	10.1977	-0.0037
GPS 56	9.0130	9.01274	0.0003
GPS 57	8.5170	8.51673	0.0003
GPS 58	8.4230	8.42243	0.0006
GPS 59	1.7080	1.70577	0.0022
GPS 60	2.1870	2.19182	-0.0048
XSV 662	8.6500	8.64139	0.0086
ZVS 3003	13.2880	13.2779	0.0101

Appendix C23:
Full Data Set for Table 4.14b: Summary of the Result of GEM2008 Orthometric Height Computed from New 'Satlevel' Collocation for Lagos State.

Table 4.14b: Summary of the Result of GEM2008 Orthometric Height Computed from New 'Satlevel' Collacation for Lagos State

Stations	GEM2008	Local	Differences
Models	Equation (1.2)	Equation (3.86)	
XST 237	3.4450	3.5720	-0.1270
XST44	4.2580	4.4170	-0.1590
YTT78A	4.9132	4.9770	-0.0638
XST245	6.7727	6.8870	-0.1143
XST244	5.2703	5.3750	-0.1047
FGPLA-Y-003	4.6393	4.5990	0.0403
CFPA21	8.4714	8.4530	0.0184
XST 55	7.8323	7.5930	0.2393
YTT1703A	2.6285	2.4130	0.2155
XST46	3.1428	2.9170	0.2258
XST50	6.7433	6.6390	0.1043
LWBC5-61P	3.2389	3.1730	0.0659
YTT19-54	14.948	14.8950	0.0533
XST75	13.718	13.7310	-0.0128
CFPA40	6.0934	5.9450	0.1484
CFPB36	5.2924	5.1850	0.1074
XST60	5.1398	5.1100	0.0298
XST72	4.9459	5.0040	-0.0581
XST76	4.8898	4.9900	-0.1002
XST44	4.2625	4.4200	-0.1575
YTT2-18A	2.3001	2.4420	-0.1419
XST156	5.4693	5.5400	-0.0707
ZTT2-57A	4.6595	4.6040	0.05547
YTT2-66A	4.6580	4.5420	0.11604
YTT2-80	3.9231	3.7420	0.18111
XST224	5.1489	4.9280	0.22088
ZTT35-14	5.2177	4.9710	0.24668
XST149	14.385	14.4220	-0.0367
MCS1188T-A	2.8387	2.9130	-0.0743
XST42	5.9423	6.0230	-0.0807
YTT13-1A	10.3430	10.4150	-0.0719
ZTT34-10A	20.3590	20.3590	0.0004
XST135	56.1250	56.1460	-0.0209
XST218	19.2050	19.1890	0.0163
XST209	10.6660	10.6420	0.0237
XST201	21.2940	21.2580	0.0363
XST203	1.8045	1.7740	0.0305
XST177	46.4720	46.4540	0.0184

Stations	GEM2008	Local	Differences
Models	Equation (1.2)	Equation (3.86)	
YTT22-1	30.3570	30.3360	0.0206
XST159	48.0820	48.1160	-0.0340
ZTT31-70	46.0360	46.0840	-0.0483
XST131	11.5210	11.5870	-0.0659
XST127	1.1496	1.1930	-0.0434
XST133	2.3931	2.4380	-0.0449
XST128	40.4410	40.4800	-0.0388
YTT28-117	18.0540	18.08390	-0.0303
MCS1174S-A	49.6500	49.6870	-0.0374
YTT28-96	58.7320	58.7676	-0.0356
XST41	50.6400	50.6720	-0.0320
YTT28-89	20.5210	20.4870	0.0343
YTT28-87	25.9320	25.8964	0.0359
YTT28-67	35.1360	35.0984	0.0376
YTT28-65	22.7500	22.7345	0.0151
YTT28-47	7.5057	7.5259	-0.0202
XST87	2.8831	2.9050	-0.0219
YTT28-30	6.4347	6.4644	-0.0297
YTT28-1	5.5978	5.6175	-0.0197
XST71	19.4690	19.4640	0.0055
YTT19-7	17.5620	17.5530	0.0092
YTT19-54	14.9480	14.8950	0.0533
XST59	5.3145	5.2500	0.0645
XST120	4.2817	4.4400	-0.1583
CFPA31	4.9445	4.8940	0.0505
XST64	4.4788	4.4790	-0.0002
XST68	5.1319	5.1620	-0.0301
XST76	4.8898	4.9900	-0.1002
XST83	4.9290	5.0580	-0.1290
XST84	4.9643	4.9650	-0.0007
XST99A	3.5843	3.6790	-0.0947
XST241	3.9111	4.0120	-0.1009
XST107	3.3472	3.4600	-0.1128
XST114	3.9774	4.1350	-0.1576
XST44	4.2625	4.4200	-0.1575
YTT2-14A	2.8100	2.9600	-0.1500
YTT2-25A	3.2128	3.3430	-0.1302
YTT2-37A	5.1231	5.2050	-0.0819
YTT2-48A	4.4865	4.5200	-0.0335
XST55	8.1014	8.1660	-0.0646
YTT17-08A	6.0307	5.8270	0.2037

Stations	GEM2008	Local	Differences
Models	Equation (1.2)	Equation (3.86)	
XST53	6.0943	6.0300	0.0643
FGPLA-Y-008	8.1064	8.1110	-0.0046
XST59	5.3145	5.2500	0.0645
CFPA18	4.9318	4.9315	0.0003
XST69	4.6225	4.6880	-0.0655
YTT28-1	5.5982	5.6179	-0.0197
ZTT45-200	5.9632	6.0060	-0.0428
MCS1144S-A	7.1791	7.2660	-0.0869
YTT28-151	3.3233	3.4297	-0.1064
YTT28-134	4.3658	4.4328	-0.0670
ZTT6-53	32.0900	32.1150	-0.0252
YTT27-33	49.6510	49.6780	-0.0266
YTT27-41	36.4620	36.4930	-0.0314
YTT16-76A	6.7948	6.5230	0.2718
XST121	2.0333	2.1560	-0.1227
YTT28-200	3.0266	3.1608	-0.1342
XT101	39.1310	39.1640	-0.0329
ZTT30-5	27.3610	27.3620	-0.0008
MCS1178T-A	3.0900	3.1820	-0.0920
YTT9-73A	5.3437	5.3950	-0.0513
XST165	24.1630	24.1390	0.0238
XST126	35.9930	35.9530	0.0404
YTT9-29A	3.6149	3.4640	0.1509
XST215	2.6488	2.5360	0.1128
ZTT35-26	5.0658	4.8310	0.2348
ZTT34-34	7.7823	7.7670	0.0153
YTT13-27	30.3610	30.2800	0.0813
XT161	25.1640	25.0320	0.1317
XST202	2.9189	2.8230	0.0959
YTT13-30	33.4920	33.4010	0.0908
XST204	4.9805	4.7750	0.2055

Appendix D:
Result from the Program “Orthometric Height on Fly” Predicted for a Selected Point for Port Harcourt.

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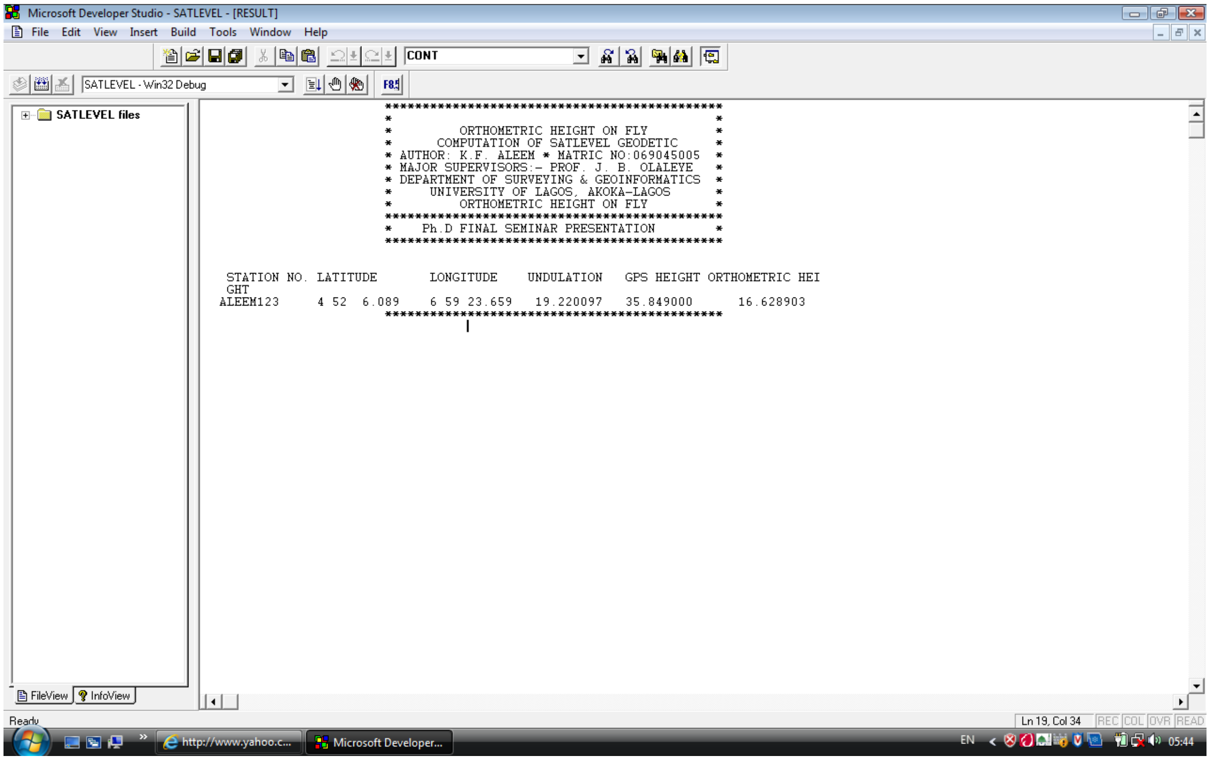
Microsoft Developer Studio - SATLEVEL - [RESULT]
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SATLEVEL - Win32 Debug
SATLEVEL files
ORTHOMETRIC HEIGHT ON FLY
COMPUTATION OF SATLEVEL GEODETIC
AUTHOR: K.F. ALEEM * MATRIC NO:069045005
MAJOR SUPERVISORS:- PROF. J. B. OLALEYE
DEPARTMENT OF SURVEYING & GEOINFORMATICS
UNIVERSITY OF LAGOS, AKOKA-LAGOS
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