THE RELATIONSHIP BETWEEN KNOWLEDGE OF MATHEMATICAL CONCEPTS AND PROBLEM-SOLVING ABILITY IN SCHOOL MATHEMATICS

BY

ADEAGBO, ADENIKE OLAJUMOKE [MRS]
B.Sc (MATHEMATICS) EDUCATION, LAGOS, 1981
M.Ed (CURRICULUM STUDIES), LAGOS, 1983

A THESIS SUBMITTED TO THE DEPARTMENT OF CURRICULUM STUDIES
SCHOOL OF POST GRADUATE STUDIES
UNIVERSITY OF LAGOS
AKOKA-YABA
LAGOS

IN PARTIAL FULFILMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

NOVEMBER 1989

DEDICATION

To my loving husband and children Remi, Seun, Deoye and Bayo

SCHOOL OF POSTGRADUATE STUDIES
UNIVERSITY OF LAGOS

CERTIFICATION

THIS IS TO CERTIFY THAT THE THESIS -

THE RELATIONSHIP BETWEEN KNOWLEDGE OF MATHEMATICAL CONCEPTS AND PROBLEM-SOLVING ABILITY IN SCHOOL MATHEMATICS.

SUBMITTED TO THE SCHOOL OF POSTGRADUATE STUDIES UNIVERSITY OF LAGOS FOR THE AWARD OF THE DEGREE OF

DOCTOR OF PHILOSOPHY (MATHEMATICS EDUCATION)

IS A RECORD OF ORIGINAL RESEARCH CARRIED OUT BY

ADEAGBO, ADENIKE OLAJUMOKE (MRS.)

IN THE DEPARTMENT OF

CURRICULUM STUDIES

MRS. A. O. ADEAGBO AUTHOR'S NAME	A.O Adeagbo SIGNATURE	10/11/89 DATE
PROF. A. O. KALEJAIYE SUPERVISOR'S NAME	SIGNATURE	10/11/89 DATE
PROF. A. O. OSIYALE INTERNAL EXAMINER'S NAME	SIGNATURE	10-11-89 DATE
PROF. A. O. KALEJAIYE INTERNAL EXAMINER'S NAME	SIGNATURE	10/11/89 DATE
DR. G. A. BADMUS EXTERNAL EXAMINER'S NAME	SIGNATURE	10/11/89 DATE

DECLARATION

THE RELATIONSHIP BETWEEN KNOWLEDGE OF MATHEMATICAL CONCEPTS AND PROBLEM-SOLVING ABILITY IN SCHOOL MATHEMATICS

I declare that the above named thesis has been composed by me, the work of which it is a record has been done by me, and it has not been accepted in any previous application for a higher degree.

A-O-Adeagbo

ADEAGBO, ADENIKE OLAJUMOKE (MRS)

(CANDIDATE)

ACKNOWLEDGEMENT

I am grateful to God whose Grace has been my sufficiency throughout the period of this study.

I would like to express my profound gratitude to my able supervisor, Prof. A. O. Kalejaiye who stimulated my interest in the topic. His subsequent supervision and diligence throughout the duration of the study have made possible any good quality with which this thesis may be accreditted.

I wish to thank Prof. Segun Adesina for granting me one year study leave to complete the programme.

I am also greatly indebted to the co-ordinator of Curriculum Development Centre of N.E.R.D.C. Dr. U. M. O. Iwovi, for his advice and constant encouragement which enabled me to complete this study in time.

I would like to express my profound gratitude particularly to Doctors Adegoke, Ajeyalemi, Odunusi, Soyibo and Baiyelo for their constant support, invaluable advice and benefit of their experience.

My thanks are due to those who have in one way or the other influenced my achievements so far. I am extremely grateful to Dr. Ola. Adeniyi, Dr. (Mrs) Bene Ikegulu, Mr. J. S. O. Oludotun, Mrs. Agnes Amodeni, Dr. (Mrs) Osanyin and Miss Lola Osunkiyesi.

I am also grateful to my parents, Prof. and Mrs. M. S. Olayinka for their concern and prayers over my success.

Special thanks go to my brothers, Bimbo and 'Segun and my in-laws, Sunday and 'Segun for their tremendious help in very many ways.

My thanks also go to my typists, Mr. J. O. Elufidipe, Mr. E. O. Okafor, Mr. Patrick Idiege and Mr. 'Dare Elufidipe for being so useful. The co-operation received from the staff and students of the secondary schools used for this study contributed immensely to its success. I appreciate it all with many thanks.

Finally, I would like to express my indebtedness to my loving husband and children for their love, trust and moral support.

God Bless you all.

ABSTRACT

The purpose of this study is twofold. Firstly, it seeks to investigate the relationship between students' knowledge of mathematical concepts and their problem-solving ability with due consideration for their computational ability and comprehension of mathematics language. Secondly, it intends to find out whether teachers' methods of teaching mathematical concepts would have any positive effect on the improvement of problem-solving ability of students.

The study was carried out in Lagos State, using form four students of selected schools as subjects.

For the first part of the study, five hundred and eighty-eight (588) Form Four students in selected schools' Management Committees were used as subjects while a new group of students numbering two hundred and forty (240) participated in the second part.

Four tests were constructed to méasure the performance of students in problem-solving in mathematics. The tests included knowledge of concepts, comprehension of mathematical language; computation and mathematics problem-solving. A matrix of bivariate correlations was obtained among the four variables.

Multiple regression analysis technique was applied in testing the significance of the F-ratio and the beta weights. Problem solving was the dependent variable and the remaining three variables formed the independent variables.

Significant positive correlations were found between problem-solving performance and each of the three variables. A significant positive bivariate correlation of 0.91 was obtained between knowledge of concepts and problem-solving ability. The F-ratio in the multiple regression analysis was significant at $P < .05$. Thus, knowledge of concepts contributed significantly to the prediction of students' problem-solving performance in the presence of computational ability and mathematical language. The three independent variables had significant correlation coefficients and accounted for about 83% of the variation in problem-solving scores.

For the second part of the study, that is experimenting with teaching methods, a concept-teaching approach combined with practice in problem-solving was used in teaching one experimental group of students while practice in problem-solving alone was used on students in the second experimental group for the five weeks. The students in the control group were not taught but were asked to practice solving problems on the topics covered with the two experimental groups. Both experimental groups were divided into three ability groups and administered the pre-test, treatment and post-test. The control group was also divided into three ability groups and were given the same pre-test and post-test. A concept-teaching approach combined with practice in prolem-solving was significantly more effective than just asking students to practice on their own at the three ability groups.

However, a concept teaching approach with practice in problem-solving was found to be more effective with lower achievers. There was no significant difference between the higher and average achievers of the two experimental groups.

The study highlighted some of the implications of these findings on mathematics instruction in Nigerian Secondary Schools. It also suggested various activities which the teacher might provide in order to improve the concept acquisition and problem-solving ability of secondary school students.

TABLE OF CONTENT

LIST OF TABLES

...

Tables

LIST OF FIGURES

CHAPTER I

STATEMENT OF PROBLEM

1.1 BACKGROUND TO THE STUDY

The objectives of mathematics teaching in schools have long been emphasized among Nigerian mathematics educators. In particular, the Mathematics Conference held in Benin in 1976 was of the view that Mathematics is taught in schools to:-

i. generate interest in mathematics and provide a solid foundation for everyday living;

ii. develop computational skills;

iii. foster the desire and ability to be accurate to a degree relevant to the problem at hand;

iv. develop precise, logical and abstract thinking;

v. develop ability to recognise problems and solve them with related mathematical knowledge;

vi. provide necessary mathematical background for further education;

vii. stimulate and encourage creativity.

Twelve years after the declaration of the Benin Conference, it would appear that these objectives are hardly being achieved in the majority of schools as students continue to perform poorly in mathematics. Table I shows the numbers and percentages of students passing West African School Certificate mathematics subjects in grades 1 to 6 (distinctions and credits) for four consecutive years. It is clear from this Table that mathematics which is made compulsory

TABLE I

NUMBER AND PERCENTAGE OF STUDENTS PASSING WASC EXAMINATIONS IN GRADES 1 - 6 IN MATHEMATICS SUBJECTS FOR 1980,81, 83 & 84

		1980		1981		1983		1984	
SUBJECT	SEX	TOTAL NO OF CANDIDATES	NO AND % IN 1 - 6	TOTAL NO OF CANDIDATES	NO AND % IN 1 - 6	TOTAL NO OF CANDIDATES	NO AND % IN 1 - 6	TOTAL NO OF CANDIDATES	NO AND % IN 1 - 6
Mathematics	Male	105,320	20,551 (19.58)	145,021	11,370 (7.84)	212,351	23,349 (11.08)	247,328	32,268 (13.04)
	Female	50,261	4,385 (8.72)	70,242	2,104 (2.99)	129,313	6,910 (5.34)	156,576	10,096 (6.44)
Statistics	Male	894	152	1,141	469 (41.10)	1,796	427 (23.77)	1,922	511 (28.66)
	Female	231	16 (6.92)	240	56 (23.33)	518	77 (14.86)	620	87 (14.04)
Additional Mathematics	Male	4,306	1,984 (46.07)	5,307	1,424 (26.83)	7,160	1,693 (23.64)	7,422	2,546 (34.30)
	Female	535	188 (35.14)	629	143 (22.73)	1,083	237 (21.88)	1,264	374 (29.58)

N. B. % are in Parenthesis

Source: Adeyegbe and Olamousi (1987)

for all students attracts the worst performance compared with statistics and additional mathematics. There is a general assumption that mathematics is difficult and uninteresting. Such negative assumptions contribute to students' inability to perform well in the subject. Factors attributed to students' poor performance in mathematics include among others, students' negative attitude towards the subject (Aiken and Dregger, 1961; Brown and Abel, 1965; McDermott, 1966; Denga, 1967, Neale, 1969); lack of teachers (N. E. R. C, 1977; Ale, 1980c); difficulty associated with the specialised language of the subject (Johnson, 1944; Aiken, 1972; Munro, 1979); and poor learning environment (Aiyedun, 1981; Chacko, 1976; Fakuade, 1983; Adeagbo, 1987; Koyejo, 1987).

Furthermore, the central activity in any mathematics classroom is problem solving. Hence, a major cause of students' poor performance in mathematics could be attributed to their inability to solve mathematical problems. It is therefore imperative to foster the acquisition of problem solving skills in order to improve students' performance in mathematics. Several researchers have attempted to investigate the nature of a problem and problem solving with a view to making recommendations on how to improve pupils' performance. For example, Bruecknel et al (1957) were of the opinion that a student encounters a mathematical problem when he confronts a situation which he cannot answer in a habitual manner.

This implies that what is a problem for one student may not be a problem for another student . Kantowski (1981) defined a problem as

> *"a situation that differs from an exercise in that the problem solver does not have a procedure or algorithm which will certainly lead to a solution"*

Polya (1981) classified mathematical problems as:

(a) One rule under your nose,

- when the problem can be solved by simply applying an algorithm just presented.

(b) Application with some choice,

- when the suitable algorithm must be selected among others previously studied.

(c) Choice of a combination,

- when in order to reach the solution some of the algorithms previously learnt must be suitably combined.

(d) Approaching research level,

- when the elaboration of a new algorithm is required.

An analysis of the two definitions above, indicates that Kantowski does not agree that an exercise is a problem while an exercise could be classified as a problem in Polya's classification (a) i.e. one rule under your nose, when the problem can be solved by simply applying an algorithm just presented.

This shows that there are different views on the notion of a problem. A look at mathematics textbooks also shows that there are various views held by authors on the notion of a problem. Borasi (1986) in his bid to define a problem identified four structural elements which problems have as follows:

(a) The formulation of the problem
- the explicit definition of the task to be performed.

(b) The context of the problem
- the situation in which the problem itself is embedded.

(c) The set of solution(s) that could be considered acceptable for the problem given.

(d) The methods of approach that could be used to reach the solution.

Based on the four structural elements, Borasi classified problems into seven categories which are; exercise, word-problem, puzzle problem, proof of a conjecture, real life problem, problematic situation and situation. Borasi's classification shows the complexity of the notion of problem.

To become a better problem solver and to be prepared to handle future problems in both real and academic life, students need to be exposed to as many different problems as possible and also to a variety of conceptions of a problem.

In the same vein, Koliagin (1981) stresses that

> *"The effective organisation and management of teaching pupils to solve different mathematical problems is a most important way of developing a high mathematical culture in pupils, and of making the teaching of mathematics more active. It is precisely in solving mathematical problems that pupils master consciously and firmly, the system of mathematical knowledge, ability and skill. More than this, pupils in the most natural way, can form qualities of personal creativity which are necessary for them to take an active part in the creation of material and spiritual values in the future, regardless of the profession they will choose". Pp 82.*

In connection with this, it is instructive to recall the saying of a well known mathematician, Polya (1970):

> "What does mastery of Mathematics means? It is the ability to solve problems, not just the standard ones, but also those requiring a known independence of thought common sense, originality, inventiveness"

Therefore, the first duty of a mathematics course for secondary schools is to lay emphasis on the process of solving problems.

1.11 Problem Solving In Mathematics:

Gagne (1965) defined problem solving as a process by which the learner discovers a combination of previously learned rules which can be applied to solve a problem in a new situation. Since one of the major reasons for learning rules is to use them in solving problems, one would no doubt expect teachers to lay much emphasis on the acquisition of problem solving skills. It has however been observed that teachers find it easier to help students acquire computational skills than to help them acquire skills in problem solving (Chapman, 1966).

This is evident in the results of National Assessment of Educational Progress (NAEP) in U.S.A which indicated that although students are learning many algorithms or computational skills, they have difficulty applying these skills to solve simple non routine problems. (Carpenter and Reys, 1980).

Leblanc (1977) gave a reason for the difficulty in teaching and learning problem solving which is the fact that there is no specific content as in computational skills or concept attainment.

This may be due to the fact that problem solving runs across all mathematical learning and therefore a basic skill in need of attention in the mathematics curriculum.

The importance of problem solving in mathematics cannot be over-emphasized. This researcher agrees with National Council of Supervisors of Mathematics (1977) which asserted that learning to solve problems is the principal reason for studying mathematics and with the National Council of Teachers of Mathematics (1980) which recommended that problem solving be the focus of school mathematics in the 1980's.

Generally, it is observed that pupils are not motivated to solve a variety of problems in mathematics because of the fear that they will not obtain the right answer. A good understanding of mathematical concepts may help in allaying the fears of students in problem solving.

1.12 Concept In Mathematics:

Various definitions have been ascribed to the word "concept". Holland (1983) described a concept as a word for an idea or mental impression of qualities. Klausmeier et al (1974) defined concept as ordered information about the properties of one or more things - objects, events, or processes that enables any particular thing or class of things to be differentiated from and related to other things or classes of things.

Lovell (1961) defined a concept as a generalisation about data which have a relation or pattern. Lovell stresses the importance of concepts as an aid to thinking when he opined that concepts enable words to stand for a whole class of objects qualities or events and areof enormous help to us in thinking.

Wilson (1966) noted that having a concept of something is tantamount to understanding the use of the correlated term. Bruner (1956) considered a concept as a category determined by a collection of defining attributes. Johnson and Rising (1967) defined a mathematical concept as a mental construct.

In general, it seems reasonable to suggest that a concept involves a label i.e. a name or an expression. It is also apparent that the label has a reference which is either a set of objects or properties associated with a set of objects.

Gagne (1977) defined a mathematical concept as an abstract idea that enables a student to classify objects or events with examples and non examples. Brown's (1978) definition of concepts requires that a student who attains a mathematical concept should not only exhibit examples and non-examples but also develop a wide network of relationship between the new concepts attained and those already in the repertoire.

Concept learning could therefore be defined as learning in which an individual responds to a group of objects and classifies them in terms of some abstract quality which they all share; or as that learning in which an individual is able to decide for a new item whether it is a member of a particular classification or not.

According to Johnson and Rising (1967), a typical sequence for learning a mathematical concept progresses from perception to differentiation, abstraction, integration and deduction.

TABLE 2

CONCEPT FORMATION FLOWCHART

PERCEPTION	DIFFERENTIATION	ABSTRACTION	INTEGRATION	DEDUCTION
Involves sensory motor or additional experiences with objects, events or ideas.	Results from perception of the elements of the experience or structure.	Depends on identifications of common elements, relationships and structure	Results in a generalization which applies to the objects, events or ideas involved	The generalization can be established by a deductive proof.

(Source: Johnson & Rising, 1967) pp 51

After the first concepts are learnt, additional concepts are learnt by deriving meanings from the context in which words are used and by learning definitions of concepts in terms of other words (Clifford and Richard. 1971). Like other kinds of learning, concepts can be learnt well or poorly, accurately or inaccurately. Ehrenberg (1981) explained that understanding of concepts involves not only recall but also comprehension and application. Brown (1979) ranked conceptual learning as the third in the hierarchy of mathematical learning of which fact is the lowest followed by skill then concept and finally problem solving. Bidwell (1983) however revealed that students hardly encounter problems on exercises on facts and skills. Students' difficulty in Mathematics may therefore be intimately related to poor knowledge of concepts and non possession of problem-solving techniques.

This research is concerned with finding out whether there is any relationship between knowledge of concepts and problem solving ability in school mathematics.

1.2 THE PROBLEM:

Mathematics educators agree that the development of the ability to solve problems is an important goal of mathematics instruction since what features most in mathematics classroom is problem solving. It is a well recognised fact that tests and examinations contain problems to be solved, thus one can easily ascertain students' difficulty from the results obtained from tests and examinations. A look at students' written works show that most of them perform unsatisfactorily in problem solving. (Bidwell, 1983). Results of West African School Certificate Examinations are other sources of information which confirm that most students do indeed have difficulty in problem solving. (WAEC, Chief Examiners Report, 1983). Some students have even complained that they had never succeeded in solving a mathematics problem on their own except those already solved by the teacher which they then memorized. (Ale, 1981).

With the overwhelming evidence pointing to the fact that students perform poorly in problem solving, a situation which impairs their achievement, the need for a systematic investigation of the situation is imperative. This research is concerned with finding ways and means of improving the learning conditions of students and thereby improve achievement in mathematics.

1.3 PURPOSE OF THE STUDY:

Various factors had been identified which affect the teaching and learning of problem solving skills. Some of these factors are reading ability, mathematical language, computational ability, teachers' method of teaching, exposure to solving problems and intelligence.

Since the ability to solve problems is one important goal of mathematics instruction, it is highly desirable to continue seeking other factors that may be related to problem solving and hence help in the improvement of teaching and learning of problem solving. The purpose of this study is to investigate the relationship between mathematics concepts and problem solving ability. The specific aims are to:

1. find out the relationship between knowledge of mathematical concepts and students' performance in problem solving;
2. investigate the relationship between mathematical problem solving ability and knowledge of concepts in the presence of computational ability and comprehension of mathematical language;

3. formulate a concept teaching strategy in mathematics;

4. test the effectiveness of this strategy on an experimental group of form four students;

5. test the effectiveness of this strategy on an experimental group of students of three different ability groupings.

1.4 DEFINITIONS OF TERMS:

In this study, the following terms will be referred to with the understated definitions:

1. Knowledge of Mathematics Concepts:
   Is the extent to which students can define, give examples, non-examples and attributes of a concept.

2. Computational Ability:
   It refers to students' knowledge of specific facts and the ability to carry out algorithms.

3. Comprehension of Mathematical Language:
   Will refer to the score obtained on identification and interpretation of mathematical diagrams, knowledge of principles, rules and terms.

4. Ability Groups:
   The three ability groups in the study are higher achievers, average achievers and lower achievers. The ability groups were chosen on the basis of students' performances in the pretest.

5. Concept - Teaching Strategy:
   This is the approach used by the researcher in teaching the experimental group. It consists of giving examples, non-examples, identifying the attributes and defining concepts.

1.5 VARIABLES:

The two major variables in this study are: students' problem-solving ability, the dependent variable, and the independent variables of computational ability, knowledge of concepts, comprehension of mathematical language and teachers' method of teaching concepts.

1.6 RESEARCH QUESTIONS:

In this study, effort has been made to answer the following questions:

1. Is there any relationship between students' knowledge of mathematical concepts and problem-solving ability?
2. Is there any relationship between students' knowledge of mathematical concepts and problem-solving ability in the presence of computational ability and comprehension of mathematical language?
3. Is there any difference between the performances of the experimental and control groups in the post-test?
4. Is there any difference between the performances of higher achievers in the experimental and control groups in the post-test?
5. Is there any difference between the performances of average achievers in the experimental and control groups in the post-test?
6. Is there any difference between the performances of lower achievers in the experimental and control groups in the post-test?
7. Is there any interaction between students' ability levels and the instructional approach they are exposed to?

1.7 RESEARCH HYPOTHESES:

1. There will be no significant relationship between students' knowledge of mathematical concepts and their problem-solving ability.

2. There will be no significant relationship between students' knowledge of mathematical concepts and performance in solving mathematical problems in the presence of the following factors namely: computational ability and comprehension of mathematical language.

3. There will be no significant difference in the mathematical problem-solving performance of students in the experimental and control groups.

4. There will be no significant difference in the mathematical problem-solving performance of higher achievers in the experimental and control groups.

5. There will be no significant difference in the mathematical problem-solving performance of average achievers in the experimental and control groups.

6. There will be no significant difference in the mathematical problem-solving performance of lower achievers in the experimental and control groups.

7. There will be no significant interaction between students' ability levels and the instructional approach they are exposed to.

1.8 SIGNIFICANCE OF THE STUDY

Very little effort has been made in the area of concept formation in mathematics. In this research, attempt is made to find out the extent to which knowledge of concepts affects problem solving ability. Apart from this, a concept teaching approach in mathematics is proposed which could help in the improvement of mathematics problem-solving ability.

With the relationship between concepts and problem solving established, useful suggestions could be advanced which could enable students make maximum use of their academic potentialities.

It is anticipated that the results of the study will be beneficial to authors of mathematics textbooks in the provision of richer sources of problems of various kinds.

It is envisaged that the proposed concept teaching approach used in this study would be emulated by classroom teachers to help students in the learning of mathematics.

The Ministry of Education and Curriculum Development Centres whose responsibility it is to provide mathematics curricula could benefit from the knowledge of the relationship between concepts and problem solving and so be motivated to sponsor more research into the factors influencing concept formation and problem solving in mathematics.

Lecturers at teacher-training colleges and teachers will be adequately informed and hence emphasis will be laid on the appropriate areas during problem solving sessions.

Finally, members of the community (parents) would be able to know the importance of concepts in mathematics. They would be aware that little encouragements on their part, like responding to their children's questions, allowing them to compare objects, play with toys or objects at home and so on, in their formative years would go a long way in concept formation which could be a strong motivator to make their children perform well in mathematics.

CHAPTER TWO

REVIEW OF LITERATURE

The purpose of this study is to investigate the relationship between knowledge of concepts and mathematical problem-solving ability of students.

The literature review is presented in five parts; Problem Solving in Mathematics, Concept Learning in Mathematics, Factors related to Problem Solving, Methods of Teaching Concepts and Methods of Developing Mathematics Problem-Solving Ability. A summary of findings was also highlighted at the end of the review.

2.1 PROBLEM SOLVING IN MATHEMATICS

All learning situations in mathematics involve problems. Problem-solving therefore occurs when an individual seeks to answer a question for which that individual has no readily available strategy for determining the answer.

Devault (1981) described problem-solving in mathematics like writing in the language arts, which requires competence with certain basic skills or tools. Clearly, one cannot solve many mathematical problems without some understanding of the basic facts, competence in computation and understanding of operations.

The nature of problem solving can be understood better by examining the relationships between an individual's success in problem solving and other characteristics of his thinking and personality. Tate et al (1964) analysed the performance of good and poor problem solvers on tests of critical thinking and practical judgement. They found that the poor problem solvers tended to avoid the judgement 'not enough facts' and to make unqualified 'true' or 'false' judgements. This finding is not surprising since poor problem solvers cannot think critically and make correct judgement using the available evidence.

The tactics of problem solving according to Polya (1967) is that mathematical problems should be used to implant in the minds of students whatever attitudes and procedures that may be generally useful for solving any kind of problem. It is generally believed that those who are mathematically inclined can apply problem solving techniques to any field of their endeavour.

Gagne (1965) was of the opinion that four sequences were involved in problem solving. These are:

(a) presentation of the problem;

(b) learner's definition of the problem;

(c) learner's formulation of hypotheses

(d) learner's verification of the hypotheses until learner finds one that achieves the solution.

Burton (1980) however was of the opinion that problem solving procedures could be seen as being dynamic and that it could be divided into three categories namely: entry procedure, attack procedure and extension procedure. Entry procedure enables the pupils to come to grips with the problem. It includes such techniques as trial and error, defining terms and relationship and information ordering. Attack procedure includes techniques such as working backwards, trying related problem and systematic control of variables. Extension procedure increases the solvers understanding of the problems and helps him to place problems in a known context or to develop understanding of a new context.

Problem solving ability is best nurtured in a situation which motivates pupils to develop an enquiring mind and a desire to find solutions to problems. Therefore, for a satisfactory problem solving lesson, classroom climate and teaching methodology are of prime importance.

2.2 CONCEPT LEARNING IN MATHEMATICS

Piaget (1969) theorized that there exists four stages through which the learner proceeds in the development of knowledge. These stages are based upon the development in the learner, of a well-defined set of operations:

(i) Sensori-motor stage (from bith to 2 years);

(ii) Pre-operational stage (2 to 7 years);

(iii) Concrete operation stage (7 to 11 years);

(iv) Formal operational stage (11 to 16 years).

The first stage is the sensori-motor which usually lasts from birth to the age of two years. In this stage the child lays emphasis on immediate perceptional and physical response to aspects of the environment. This is observed by the fact that very young children do not hunt for object once it is out of sight. To them "out of sight" means "out of existence". However, as they grow older, they learn to look for an object which has been hidden that is, an object does not cease to exist just because it is no longer visibly present.

The second stage is the pre-operational stage. This stage usually lasts from two to seven year. This period is sub-divided into two - the symbolic phase (two to four) and intuitive phase (four to seven). At the symbolic stage, the child can use symbols and the child is very ego-centric, that is, he cannot understand the viewpoints of others, other than his own. The child uses himself as the standard of judgement.

At the intuitive stage, the child thinks in terms of classes and sees relationship. The child can classify material on the basis of objective similarity. For example, when presented with a group of squares and triangles, he can classify the objects on the basis of shapes or colours. Children at this stage are unable to understand transformations. They can focus on an object as it goes through a succession of individual states but cannot understand the process by which one stage is changed into another. If such children are asked to draw the changing position of an object, falling from a high shelf onto the ground, they will draw the beginning and end states but be unable to draw the object in a state of transition.

The concrete operation stage usually occurs from age seven to age eleven. At this stage the child can conserve and his reasoning begins to appear logical. The child can also classify objects according to the hierarchies of classes or arrange items in their increasing values. The child can perform such operations as substitution but all this thought is bound to the concrete items, that is, to actual, visible materials and objects. The child cannot manipulate possible relations among absent objects.

The final stage or formal operational usually occurs from age eleven to age sixteen. At this stage the child is truly logical and he is able to take the final steps to abstract thinking and to conceptualize. The child is able to combine mentally several rules or operations in solving problem To be precise, the child has reached adult thinking.

Piaget's theory shows that learning occurs before school age, during school and later in life. Similarly, mathematical concepts are not only formed in the classrooms but also in the shop, at home or during playtime. (Zammareli and Bolton , 1974).

Gagne (1977) showed that discriminations are pre-requisite to the learning of concepts. He used the concepts of a straight line to illustrate.

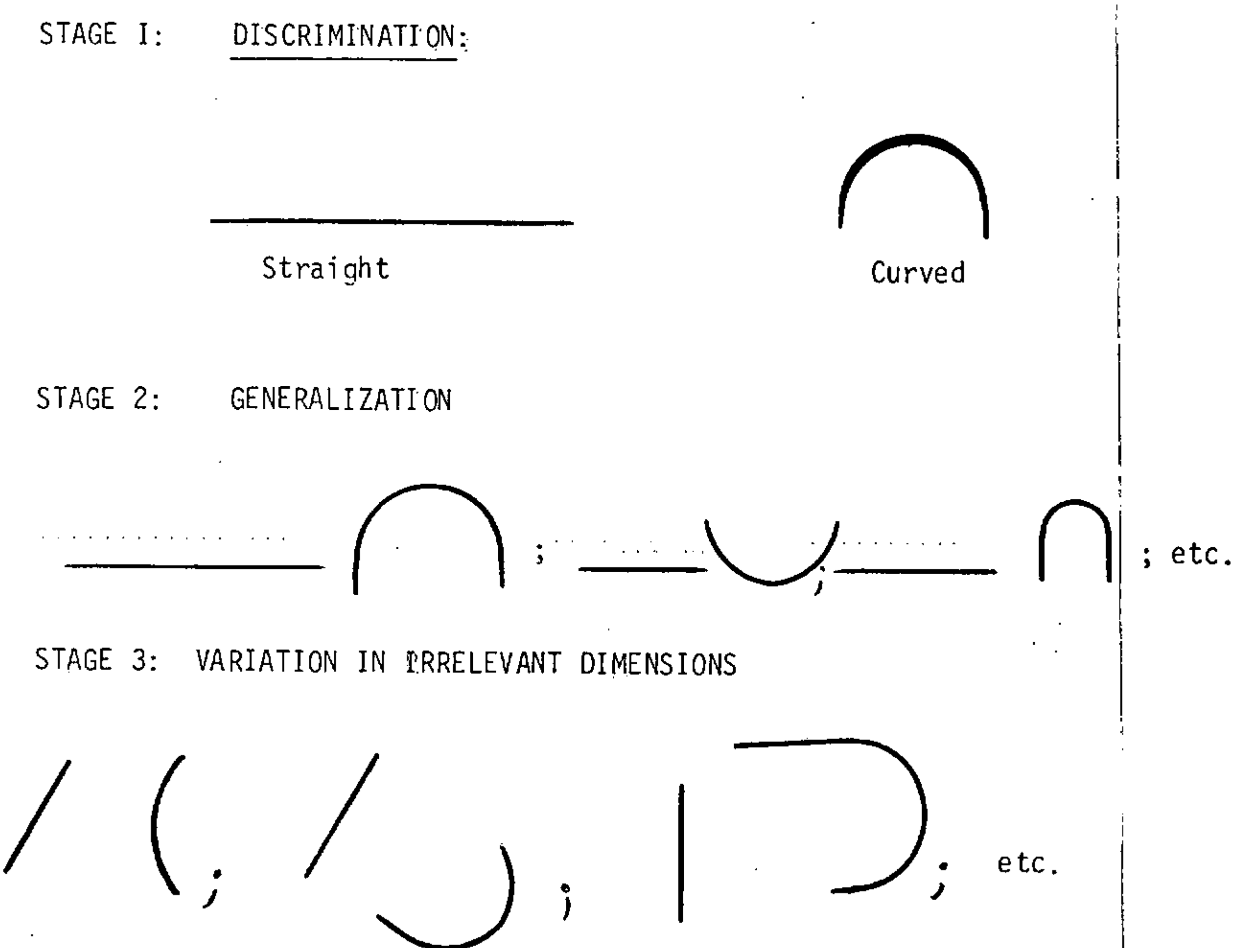


Figure 1:- Stage of learning beginning with a discrimination and leading to the learning of a concept.

The learner first masters stage 1 so that he responds positively to the straight line and negatively to the curved line. Stage 2 introduces the same straight line, paired with various curved lines. Practice is continued until the learner shows that he discriminates straight from not straight. Varied contrast practice is again continued in stage 3. Once the learning of this phase has been mastered, there is convincing proof that the learner has acquired the concept of line straightness.

In order to check on the learning of the concept, it is appropriate to choose a new example which has not been used during learning. A new straight line may be drawn, with a different non straight line. The concept has been acquired when the learner can identify the new example. Hence, concept learning can be referred to as that learning in which an individual is able to decide for a new item whether it is a member of a particular classification or not.

For example, if a child can say rectangle for each of a group of large or small or brown or black or blue plane shapes made of four sides joined together and four right angles, then he is considered to have learned the concept "rectangle". However, if all that he can do is to define rectangle as a shape with four sides without being able to apply the definition to real objects that are rectangular in shape, then he is not considered to have learnt the concept rectangle.

The instructional sequence for concept learning according to Gagne (1965) and Eggen, Kauchak and Harder (1979) is summarized below:-

(i) the teacher should ensure that the prerequisite learning required for learning the new concept has occurred.

(ii) the teacher should present an example of the concept at the same time as he presents an item which is not an example of the concept.

(iii) the teacher should present a new example and ask the learner to identify it.

2.3 FACTORS RELATED TO PROBLEM SOLVING

Many factors may be responsible for students' poor performance in problem-solving. In this review however, the extent to which reading ability, mathematical language, computational ability, teachers' method of teaching, exposure to solving-problems, lengths of problem and pupils' giftedness affect problem solving were looked into.

2.3.1 Reading Ability and Problem Solving

Brenman and Dunlop (1978) defined reading as the meaningful interpretation of printed or written verbal symbols. Reading of mathematical materials is a meaningful interpretation of printed symbols/pictures and lof the arrangement of symbols in expressive charts, graphs and tables.

Before a student can attempt to solve a problem, he must be able to read and understand the problem. Balow (1964) conducted a study in which he aimed at determining whether the level of general reading ability is significantly associated with problem solving ability. One thousand and four hundred sixth grade students (primary six pupils) were involved in the study. The results showed that general reading ability does have a positive effect on problem-solving ability.

Kilpatrick (1969) also confirmed that success in solving word problems in mathematics depends upon skills in reading. Reading skills are important to the child if he is to succeed in verbal problem solving in school.

According to Newman (1977), associated with any given word problem are a number of hurdles which have to be overcome if a correct solution is to be obtained. In his method of analyzing errors, a person confronted with a one-step written problem has to read the problem, comprehend what he has read, carry out the transformation from words to the selection of an appropriate mathematical model, apply the necessary process skills and then encode the answer. Failure at any level of the hierarchy prevents the person from obtaining the correct answer.

Casey (1978) however modified Newman's method. Casey produced a more general 'hierarchy' which can be applied to the analysis of errors made on many-step verbal problems in mathematics. Casey's approach emphasized that anyone who attempts to solve a many-step problem has to identify, sequence, and solve an appropriate set of sub-problems. In moving towards the overall solution the person often returns to lower stages of the hierarchy, not only after each sub-problem has been solved but also while attempting to solve any particular sub-problem. For example, in the middle of a complicated calculation someone might decide to re-read the question in order to check that all relevant information has been taken into account.

Clement (1980) administered a test to 542 pupils comprising 55 pupils in grade 5, 207 in grade 6 and 280 pupils in grade 7. After the test, the pupils were interviewed by teachers who had earlier been trained to use Newman's error analysis guideline. The results suggested that reading and comprehension difficulties caused fewer errors in higher grades but errors occurring at the transformation, process skills or errors due to carelessness were common in grade 7.

In order to find out whether grade 2 (third year of primary school) children encountered similar difficulties as older children, Watson (1980) administered a test of sixteen arithmetic problems on thirty students and interviewed fifteen of them which included 10 less able and 5 more able students. The results indicated that majority of initial errors were made at the stages of reading and comprehension. As expected less able students made more mistakes than the able group. It was also found that children performed better at interview situation than solving the problems on their own. This study shows that Newmån's mode of sequence of steps in solving mathematical problems provides a useful framework for the diagnosis of students' strengths and weaknesses. It helps the teacher assess the child's skills at all the stages of solving a problem. The interview allows the teacher to hear and know how the child goes about solving a problem. Due to time factor, it would be suggested that few children having persistent difficulties in mathematics be helped in this way. Teachers are therefore encouraged to use diagnostic interview with an error analysis such as Newman's.

The studies reviewed above show that students in higher grades have less problems with reading of mathematics materials. However, pupils in lower grades (one and two) have problems with reading and comprehension of mathematics materials.

An approach that could be used to help solve students' reading difficulties is Taschows' directed reading ability.

Taschow (1969) described a procedure to help students in mathematical reading. This includes administration of a Group Informal Reading Inventory designed to identify students who are able to read problems and think through them and those who cannot. This is followed by the Directed Reading Activity (DRA) which consists of five phases namely:

(i) Readiness
(ii) Guided Silent Reading
(iii) Questions
(iv) Oral Re-reading (if necessary)
(v) Application.

Taschow pointed out that the five phases are not separate entities but that they are inter-related. A careful study of the DRA indicates that students' active participation is involved, which will be beneficial in solving verbal problems.

2.3.2 Mathematical Language and Problem Solving:

In a research conducted by Otternburn and Nicholson (1976), thirty-six words in common usage in Certificate of Secondary Education (CSE) mathematics course were listed. Three hundred students who had attended the course were invited to explain what each of the 36 terms meant, and to illustrate each explanation by a diagram or example if they wished. The results showed that students were having serious difficulty with many of the terms as revealed by students' *"blank"* or confused responses. The study shows that students do not understand the meaning of some words which mathematics teachers and examiners assume were comprehensible. It should be observed that the 36 words are part of the language used in setting examination questions. Lack of understanding of these words causes difficulty in solving verbal problems.

Clement (1982) described test data showing that a large proportion of science-oriented college students were unable to solve a very simple kind of algebra word problem. His analysis of students' work shows that most errors were due to a difficulty in translating word to algebraic equations. Fajemidagba (1986) conducted a similar study in Nigeria. Fajemidagba investigated the types of errors committed by pre-service mathematics teachers when solving mathematical word problem. The same problem which Clement administered in Massachuchetts was administered to thirty three undergraduate students at University of Ilorin. The results revealed that the students committed two types of errors in the transformation of the mathematical statement to algebraic equations. The two types of errors syntactic and semantic were caused through word order matching and misunderstanding of the problem. He concluded that teaching students standard methods for solving problems may not help much but that students have to learn how to formulate equations in a meaningful way. This study shows that undergraduates were unsuccessful in solving the problem due to their inability to comprehend the mathematical terms or phrases involved in the problem.

Kalejaiye (1979) identified basic skills which might foster verbal problem solving among primary six pupils. These are comprehension of English Language and Mathematics Language. Kalejaiye obtained a positive correlation co-efficient of 0.64 between verbal problem-solving and comprehension of mathematical language. He recommended that comprehension of mathematical language and English language be taught intensively at the primary school level to improve pupils' performance in solving word problem. In a similar study, Tade (1982) using form four students found a significant positive correlation co-efficient of 0.74 between comprehension of mathematical language and mathematics problem solving ability.

Linville (1976) carried out a study to find out whether the degree of syntax used in the sentences which state verbal arithmetic problems and the level of vocabulary used in the statement of the problems are factors which contribute significantly to the degree of difficulty of the problems when computational operations are held constant. In his study four hundred and twenty-two (422) fourth grade pupils were involved. Four tests comprising of ten verbal arithmetic problems each were developed as follows.

Test 1 "Easy Syntax; Easy Vocabulary"

Test 2 "Easy Syntax; Difficult Vocabulary"

Test 3 "Difficult Syntax; Easy Vocabulary"

Test 4 "Difficult Syntax; Difficult Vocabulary".

The results revealed that:

(i) easy vocabulary scores were significantly higher than difficult vocabulary scores across difficulty levels of syntax;

(ii) easy syntax scores were significantly higher than difficult syntax scores across difficulty levels of vocabulary.

He concluded that syntax and vocabulary levels are both determinants of difficulty in verbal arithmetic problems. Linville also expressed the opinion that vocabulary level could be more crucial in determinig success in problem solving than syntax.

Although Linvillee stressed that computational operations were held constant in all the four tests, this researcher feels that the structure of the problems in each of the test would have been altered in the process of changing the syntax and vocabulary levels.

Jackson and Phillips (1983) conducted a study to find out whether the mathematics achievement of seventh graders (Form 1 Students) could be improved through the use of vocabulary-oriented instruction in a typical classroom setting. In the study, III students were in the three experimental classes and 102 students in the three control classes. They received similar instructions for four weeks except that the control classes did not have any vocabulary-oriented activity. The vocabulary-oriented activities were employed for 5 - 10 minutes daily with the experimental groups, whereas the control classes spent the time working computational problems. The remainder of the 50 minutes period was devoted to mathematics learning activities in both experimental and control classes. With the vocabulary-oriented activities, emphasis was placed on:

(a) recognising and identifying of terms and symbols;

(b) attaching literal meaning to terms and symbols;

(c) identifying examples and non-examples of concepts represented by terms and symbols.

The mean score for students in the experimental classes was found to be significantly higher than that for students in the control classes when set of computational items and verbal items were considered. Jackson and Phillips concluded that concentration on the meaning of relatively few essential terms and symbols for only a few minutes daily can result in increased achievement.

The studies reviewed on mathematical language and problem solving showed that comprehension of mathematical language is crucial to the mathematics problem-solving ability of students.

2.3.3 Computational Ability and Problem Solving:

Results of the mathematical assessment of the National Assessment of Educational Progress (NAEP) indicated that although students are learning many basic algorithms or computational skills they have difficulty applying these skills to solve simple non-routine problems (Carpenter and Reys, 1980). In their study, representative samples of 13 and 17 year olds were given a multiple choice estimation exercise. The students were asked to estimate the sum of two fractions, each fraction less than one. The result showed that rather than estimating the sum of two fractions less than one, many students attempted to find some calculation involving the numbers given in the exercise. Most of the students at each age group simply added the numbers in the numerator or denominator with no concern for the reasonableness of the result. The assessment result showed that many students merely memorize computational skills taught in school and this hinders them from being able to apply the mathematics skills to new or unfamiliar situations. From the assessment results, Capenter et al concluded that problem solving is the basic skill in need of attention in the mathematics curriculum. The assessment result indicates that mere memorization of computational skills without being able to apply them hinders problem solving ability.

Knifong and Holtan (1976) administered the word problem section of the Metropolitan Achievement Test to thirty five sixth grade children. Their written work were analysed to determine typical errors so that teachers would have a factual basis for planning instruction.

The errors were classified into

(a) Clerical and Computational errors, and

(b) other errors.

In their study inaccurate computation played the major role in children's failure on these problems. Computational errors were positively identified as the single deterrent to the children's success on 49% of the incorrectly solved problems.

In order to find out whether the use of calculator can be of help to students in solving problems, Wheatley (1980) compared the problem solving performances of elementary school pupils whose mathematics achievement was above average using calculators with those of pupils not using calculators. He observed that the calculator group used significantly more processes than the non-calculator group and that the non-calculator group committed more errors than the calculator group. This study shows that calculators can be used to improve students' problem-solving ability. However, students should also be encouraged to practise solving problems mechanically on their own without using calculators.

In a similar study conducted by Johnson and Harding (1979) to find out whether students problem-solving ability could be improved by participating in computing courses, it was discovered that the computing group performed significantly better than the control group.

This result shows that certain areas in the mathematics could be learned better when students write and run programme which implies that computer facilities aid problem solving ability.

2.3.4 Teachers' Method of Teaching Problem Solving:

It is generally believed that for a satisfactory problem solving lesson, classroom climate and teaching methodology are of prime importance. This may be due to the fact that students look up to the teacher to explain meanings of difficult words, to correct them when they are making computational mistakes and to guide them until they succeed in solving a problem.

Schoefeld (1979) studied the impact of instruction in heuristics on some students' performances in problem solving. He found that differences in performances of students who received the instruction and those who did not, was statistically significant in favour of students who received heuristic instruction. This study shows that under appropriate circumstances, explicit instructions in general problem-solving strategies (heuristics) does have an impact on students' problem-solving performances.

Schoefeld (1982) further studied the effect of a one-month long intensive problem solving course on students' performance in solving non-routine college mathematics problems. There were two groups, experimental and control. The experimental and control groups' performances on the pretest were quite similar while their performances on the post test differ significantly favouring the experimental group. He concluded that much of the experimental groups' improvement was due to their having learned to use certain problem solving heuristics during the course.

An exploratory study conducted by Lucas (1974) also supports Schoefeld's findings. Lucas investigated heuristic usage and problem solving performance and analyzed the influence of heuristic-oriented teaching on first year University calculus students. He discovered that there were significant differences in favour of the heuristic group over the control group on the problem solving performances, although there was no significant difference between the groups in the total time spent in solving the problems.

Kantowski (1977) conducted a similar study to uncover information about processes involved in solving complex, non-routine problems. Eight students of above average ability were used. A pretest containing eight problems was first administered followed by a readiness instruction to acquaint the students with the heuristic method of instruction and to introduce the students to the use of heuristic in problem solving. Later, the students were taught geometry using heuristic instructional techniques. Finally a post-test was administered. During each of the stages, the subjects were asked to think aloud and protocols were recorded on cassette tapes which werelater analysed.

The result indicated that students' tendency to use goal-oriented heuristics increased as problem solving ability develops.

Lovelace and McKnight (1980) examined the effects of two teaching techniques on art students' ability to solve calculus problems. The techniques were:

(1) a non-traditional method which focusses on;

(a) peer-tutoring through heterogeneous grouping combined with

(b) instruction in specific reading skills through instructor's development of study guides

(c) students' reading of mathematics text for initial exposure to problem solving techniques;

(2) a traditional lecture method which focusses on lectures by instructors with little or no instructor interaction.

The result showed that little differences were observed on short term basis but suggested that long term problem solving ability is influenced by the experimental approach. This study indicated that the peer tutoring mathematics reading instrustion approach is a useful alternative to the traditional lecture method usually emplored by teachers or instructors.

Kalejaiye (1981) compared the effectiveness of a new method of teaching verbal problems with a conventional method. The new method, called Verbal Problem Solving with Pictures (VPSP) encouraged the pupils to formulate word problems from pictures and solve other word problems similar to those illustrated in the picture. The conventional method was characterised mainly by pupils' practice in solving the same word problems as in the VPSP. The researcher concluded that the VPSP method of teaching verbal problems was more effective than the conventional methods of giving practice only.

Adeagbo (1985) formulated a problem-solving approach which was used on an experimental group of students in solving problems on simultaneous equations. The problem-solving approach encouraged students to read the problem in order to gain an understanding of the problem before translating the two sentences into mathematical equations before eventually solving the problems. The results showed that the experimental group performed better than those in the control group who were given only practice exercises.

In order to find out whether a method of teaching could improve primary three pupils performance in number sentence problem, Obioma and Ohuche, (1985) used a mathematical game - the mathematical scrabble (MASC) to teach number sentences to a random sample of 60 primary three pupils. Their post test scores were compared with those of an equated group who were exposed to the same mathematical unit but without the use of MASC. Results showed that the sample taught by the aid of MASC performed significantly better than their other counterparts in number sentence problems.

Obioma and Adibe (1987) compared the effect of small group modelling instructional technique in presenting some junior secondary mathematics tasks to slow learners. Sixty four slow learners were identified and randomly assigned to the two groups. The small group instructional technique subjects were subdivided in small groups of four members each and taught some mathematics concepts for four weeks. Their counterparts in the control group were presented with the same tasks without small grouping.

Analysis of the post-test scores indicated that the small group instructional technique performed significantly better than the control. This result is not surprising, since the experimental group received more individual attention than the control group.

Bell and Bell (1985) investigated the effect of expository writing on students' problem-solving performance using two ninth-grade (Form 3) general mathematics classes. The experimental group was taught problem solving skills by using a method which combines traditional mathematics techniques with a structured expository writing component. The expository writing component involves discovering a topic, deciding what one needs to say about it, organizing and editing the finished product. The second class, the control group, was taught by using only the traditional mathematics method. Both classes were given the same assignments, examples, quizzes and tests during the four week period of study. The results of the post-test showed a significant difference in the performances of the two groups favouring the experimental group. The study showed that the writing component the experimental group underwent positively affected its progress in mathematics problem solving.

Beh-haim, Lappan and Houang (1985) examined the effect of instruction in spatial visualization activities on students' performance in geometrical concepts. A three phase model was used for the experiment. The first phase consisted of viewing flat projection from the front, right, left and back sides of the building. In order to acquire a formal understanding of this representational scheme, the students were required to match buildings with their flat view, draw the base and flat views of existing buildings, build a building from a given set of views, construct a building and evaluate another reconstruction. The second phase comprised the second representational scheme in which the buildings are turned and viewed from the corners so that the three faces of the buildings are seen. The third phase consisted of the summary of the unit wherein the teacher helps the students to deepen their understanding of both the mathematical ideas involved and the strategies used to solve it.

The results of the study showed that school students, grades five through eight, significantly improved their performance after three weeks of instruction in spatial visualization. This study shows that instruction in spatial visualization would go a long way in the improvement of students' performance in geometrical concepts.

Bassler, Bears and Richardson (1975) assessed the relative effects of two strategies of instructing students to solve verbal problems. The first strategy was derived from the work of Polya and is basically heuristic in nature; that is, the student is expected to read and understand the problem; plan for a solution of the problem, which includes identifying the unknown, identifying relations and operations to be performed and drawing analogies to similar problems that may have been previously solved; carry out the plan and finally examine the solution. This strategy is referred to as Polya's Method (PM). The second strategy is based on the work of Dahmus which is the DPPC method which stands for direct, pure, piecemeal and complete translation of the verbal statement into mathematical symbols. This second strategy is referred to as Dahmus Method (DM). Forty-eight ninth grade (Form 3)algebra students took part in the study. They were divided into three ability groups based on their performance in a test. Within each ability level, equal number of learners were assigned at random to one of the two treatment groups.

The results showed that the PM groups scored higher than the DM groups but there was no significant difference between the means of the two groups. Although the PM seems preferable, since there was no significant difference in the results, other studies are needed before one can conclude on which of the method PM or DM is better.

Moore and Schaut (1977) examined the effect of training teachers to use a problem-solving approach in modifying the behaviour of students in a group learning situation. In the control group, teachers were watched while training and the number of students not paying attention noted. For the experimental group, the same teachers used for the control group were trained and tested to be sure that they are acquainted with what they were expected to do. The teachers were then watched. The result showed that the mean number of students exhibiting inattention was less for the experimental group following training. Their study shows that the training on a problem-solving approach received by the teachers helped in reducing the number of students not paying attention in a mathematics class. With more students paying attention in class, their performance would no doubt improve in mathematics.

In order to find out the extent of influence of teachers' assistance on the problem solving performance of high school students, Flener (1979) chose a group of ninety-three students who were randomly divided into three groups. They were given specific sets of coloured rods and were then asked to try to form rectangles. During the study, each subject was given ten similar exercises. In the first group, the teacher gave information at the start of the experiment. (Information-Experiment I-E) while the same information was given after the students had worked on the problems for 20 minutes without any assistance (E-I-E) with the second group. However, with the third group, no information was given throughout the experiment. The results of the experiment indicated a significant difference in the three groups. However, there was no difference in the performance between the first and second groups but each performed significantly better than the control group. Flener's work shows that it is highly necessary for teachers to give students assistance during problem solving sessions.

With the studies reviewed above on method of teaching and problem solving performance, it becomes abundantly clear that teachers' methods of teaching affect students' problem solving ability. The role of the teacher is an important factor in problem solving. The teacher can help by presenting the problem situation so that the student can discover a pattern in solving the problem.

2.3.5 Exposure to Solving Problems and Problem Solving:

Mathematics educators believe that pupils can become better problem solvers by solving problems. This implies that practice would help in improving student's problem solving ability. Friendsen (1980) discussed a technique that could help gifted students become better problem solvers. The technique is one which challenges students to solve problems published in mathematics periodicals. He strongly believes that, in this way, problem solving skills of students will be further developed and previously learned mathematical concept and skills will be applied. Frienser's work also supports the view that students' problem-solving ability could be further enhanced by encouraging them to solve problems.

Afemikhe (1988) explored the effect of formative tests on problem solving habits of form two students in mathematics. There were three treatment groups which comprises formative test with remediation, formative test without remediation and no formative test. The three treatment groups shared a common curriculum. The results revealed that formative tests with remediation improved problem solving habits of students while the formative test without remediation had the least effect on problem solving. He concluded that teachers should endeavour to use formative tests for diagnostic purposes instead of using them as a series of summative test. The finding that exposure to problem with remediation aids problem solving habits implies that with improved interest in problem solving, performance in mathematics will improve.

Lemonye and Tremblay (1986) studied the effect of extra learning exercises on students problem solving ability. Three hypotheses were investigated as follows. Subjects who receive the information processing exercises referring to mathematical situations will improve their performance in solving:

(i) mathematical problems;

(ii) addition problems;

(iii) multiplication problems;

more that subjects who do not receive the exercises. The subjects were divided into experimental groups (29 subjects) and control group (19). Subgroups of 5 subjects from the experimental group were made to perform various learning exercises, the effect of which was evaluated, by a problem-solving pre-test and post-test. The results of the investigation showed that only the subjects who received the learning exercise obtained significantly better scores in the post-test than those obtained in the pretest. The three hypotheses were therefore accepted.

The researchers concluded that the information - processing exercises referring to various mathematical situations seemed to have helped to develop more effective problem solving strategies in more than half of the subjects of the experimental group.

Lemoyne and Tremblay's work confirms the general belief that additional exercises or homework given to students does help in the improvement of students' problem solving ability. Not only should students have as much experience as possible in solving problems, but they should be allowed, and even encouraged to engage in the process of formulating problems since the way in which the problem is stated often determines the form of mathematics to be applied in the solution of the problem.

2.3.6 Length of a Problem and Problem Solving:

Jerman (1974) investigated the effect the number of words may have on the difficulty of verbal arithmetic problems. Students in grades 4 - 8 from 3 schools were used for the study. A set of word problems were modified by adding articles or adjectives so that the number of words in each problem statement was a multiple of three. Three forms of each problem were prepared. Form 1 was the original set with the one-third fewer words in each problem. Form 2 was the original problem set. Form 3 contained one-third more words in each problem than the original problem set. The digits in each problem, the order of operations, the cues, and all the other aspects of the structure of each problem were held constant, except that extra clauses and modifiers were added to Form 2 problems to qualify them as Form 3 problems. Similarly, some rewriting was done to reduce the number of words in each problem statement to qualify it as a Form 1 problem. Apart from these modifications, all possible variables were preserved except for the actual number of words in the problem statements. Three tests sets were prepared. Each test was made up of 10 problems. Thus, a total of thirty problems of forms 1, 2 and 3 were set. He found that the variable, length entered in the first six steps for only the low group in Grade 7 and one of the test. It was significant at 0.05 level. On Form 2 of the test, the variable length entered among the first six variables entered for every group and was significant in every case. For Form 3 of the test, the variable length entered among the first six variables in only one case, the fourth-grade group and was significant at 0.05 level.

He concluded that the failure of the length variable to enter the regression consistently over all the forms of the test sets showed that it was not simply the number of words in the problem statement that influenced its difficulty but the number of words in relation to other factors.

For a study of the nature one would have thought that the three forms of the test sets would be similar. However, there was nothing mentioned about the similarity of the tests. The author just mentioned that the problems were arranged from simple to complex. Also in the process of adding extra clauses and modifiers, the structure of the problem may have changed. All these could have affected the results of the study.

From the review of Chief Examiner report in West African School Certificate in Mathematics (WAEC 1983), it was stated that students are often impatient to read through long worded questions. In most cases, students do not attempt such questions and the few who attempt them find it difficult putting the words into symbols in order to solve the problem. Other studies are therefore needed before a conclusive statement can be made on the effect of length of a problem on problem solving performance.

2.3.7 The Relationship Between Giftedness and Problem-Solving:

Span and Overtoom-Corsmit (1986) examined differences in the way gifted children process information when solving mathematical problem as compared to averagely gifted children. Fourteen relatively highly gifted pupils (7 boys and 7 girls) and fourteen averagely gifted pupils (7 boys and 7 girls) from second form of lower secondary participated in the study. Seven mathematical problems were presented individually by the experimenter. After each pupil had finished the problem, the experimenter interviewed them individually on the method of approach used in solving each problem. The results of the study revealed that the highly gifted pupils solved five out of seven problems more successfully than the averagely gifted pupils. The gifted pupils were faster in reaching a correct solution. The results also showed that gifted pupils take more time to orient themselves to the task and do it more thoroughly. They reflect beforehand what to do and plan their approach. The average pupils however immediately start to try before they understand what they are required to do. They concluded that gifted pupils solved the problem better, faster and needed less assistance than the average pupils. They urged teachers to try and teach average pupils to process information like the gifted pupils.

The number of students used in this study (twenty eight) seems too small. It would therefore not be appropriate to generalize these findings to all second form of lower secondary education. A similar study with a much larger sample is needed.

2.4 METHODS OF TEACHING CONCEPTS:

Tennyson and Cocchiarela (1986) opined that the purpose of concept teaching is to improve the learner's acquisition of concepts through instructional strategies that effectively aid in the formation of conceptual knowledge and the development of the corresponding procedural skills. Conceptual Knowledge was defined as the storage and integration of information while procedural knowledge refers to the retrieval of knowledge in the service of solving problems.

Tennyson et al (1983) tested third grade students to determine whether Conceptual Knowledge is established by means of clear cases (termed best examples) or a list of defining attributes. The learning task presented subjects with a definition followed by either a best example or a list of attributes. On an immediate posttest, subjects in the best-example conditions significantly outperformed subjects in the attribute-list conditions.

Corroborative evidence for the notion of best example is found in a study by Park (1984) using college students learning psychological concepts. He tested the hypothesis that presentation of information that elaborates the attributes within a concept would improve concept classification relative to a condition that instead focussed the instruction on use of a best example without the elaboration information. To test his hypotheses, Park presented two groups of 12th grade students with four psychological concepts.

One group (the example comparison strategy group - ECS) received best examples and the other (the attribute isolation strategy group - AIS) received an explanatory table showing the relationships of the critical attributes. Students were directed to study either the best examples or explanatory table and use them in the classification of instances presented during instruction. He found that students in the AIS group correctly classified more instances during the instructional programme than the ECS group. However, their scores deteriorated significantly on a retention test, while the scores of the ECS group did not.

Tennyson et al (1981) contrasted three presentation forms of examples: an expository-only form, an interrogatory - only form, and a combined expository - interrogatory form. In the experiment all three treatment conditions received the same training programme and definition of the concept of polygons. Subjects in the expository-only condition received an expository presentation of examples and non-examples. The interrogatory - only condition received an interrogatory presentation of examples and non-examples expected to develop procedural knowledge but minimal conceptual knowledge. Subjects in the third condition received a combined expository - interrogatory presentation of examples and non-examples expected to initially establish conceptual skill in memory followed by development of procedural knowledge.

They found that all the three groups attained the concept at the concrete and identity levels. However, the group receiving the combined expository - interrogatory instruction had significantly higher scores than the other two groups at the classificatory and formal levels. Furthermore, subjects' performance in the expository - only and interrogatory - only conditions showed deterioration one week later on a retention test, whereas subjects' scores in the combined treatments condition actually increased.

Charles (1980) examined the extent to which the number of exemplification moves, characterization moves used for instruction, and the clarity of the teacher's presentation are related to student achievement in rotational and bilateral symmetry. Exemplification move (E) was defined as the presentation of an example or a non example of a concept while charaterization move (C) referred to a statement about a relevant or irrelevant attribute of a concept.

Eighteen preservice teachers were randomly divided into two groups: trained and control. The results of the study showed that training preservice elementary teachers on the use of E and C moves related to bilateral and rotational symmetry facilitates the acquisition of these concepts by elementary pupils.

Johnson and Stratton (1966) evaluated five methods of teaching English concepts. The methods were by definition, classification, using it in a sentence, giving synonyms and a mixed method which is a combination of the first four. There was also a control group that received nothing. He found that the students in the mixed method performed significantly better than other groups that received a single kind of training. The single groups also performed significantly better than the control group. Within the groups that received a single kind of training there were no significant differences.

The review on methods of teaching concepts shows that concepts can be taught by defining, giving examples, nonexamples and by identifying attributes of the concept. The review also shows that concepts vary in nature. Hence different approaches could be used when teaching various concepts. Some concepts are learnt by giving examples, non-examples or listing attributes while other concepts are learnt by definition. A combination of two or more methods are however preferred when teaching concepts.

2.5 METHODS OF DEVELOPING MATHEMATICS PROBLEM SOLVING ABILITY:

Charles (1985) defined problem solving as the process of understanding a problem, selecting relevant data, choosing and implementating one or more solution strategies, answering the problem and evaluating the reasonabkness of the chosen answer.

In same vein, Ekenstam and Greger (1983) expressed the opinion that problem solving ability leans on the ability to:

(i) choose the correct arithmetic operations in problems involving one operation;

(ii) choose the correct operations in problems involving two operations;

(iii) judge whether an obtained answer was reasonable or not and to give reasonable answer;

(iv) choose information relevant to the solution of the problem; and

(v) make use of information in problems lacking one single answer.

From the views of various authors expressed above, one would no doubt agree that problem solving is a complex process. Due to the complex nature of problem solving researchers have identified various strategies that could help teachers during problem solving sessions which will also help in the improvement of students' problem solving ability.

Krulik (1977) gave seven suggestions the teacher can try to help students develop the attitudes and skill needed to become successful problem solvers. These include reading the problem carefully and analyzing what is really being asked for; encouraging students to make many suggestions towards solution of problem; helping the students examine data in a meaningful way; organizing the data carefully; allowing time for the problem solver to think; encouraging alternate solution if there is more than one possible answer to a question or more than one route to the answer; and looking for patterns within the data of the problem.

In the same manner Pottenger and Leth (1969) listed some suggestions that could help the teacher attain problem solving success. These are:

(i) Developing an enthusiastic attitude towards problem solving

(ii) Making sure the statement of the problem is clear to the students

(iii) Approaching a topic from as many different points of view as possible.

(iv) Asking many interesting and exciting questions.

(v) Giving the students time to think and time to answer questions.

(vi) Starting again, if the students become confused.

(vii) Capitalizing on errors so that students see a need for checking.

According to Riedesel (1969) problem solving performance can be improved by;

Providing problems of appropriate difficulty level;

Guiding pupils in the analysis of information;

Guiding pupils to use a method of getting started;

Encouraging pupils to proceed by praising them when they perform some processes correctly;

Encouraging pupils to verify their final solution;

Starting pupils with easy problems which they most certainly can get correct with a reasonable amount of effort;

These views were also shared by Gibney and Meiring, (1983).

It is however interesting to note that the various strategies, techniques or methods suggested by researchers in improving the problem solving ability of students can be grouped under Polya's four stages of problem solving. Polya (1967) identified four stages that are of utmost importance in problem solving. These were:

(i) understanding the problem;

(ii) planning its solution;

(iii) executing the plan, and

(iv) looking back to check the answer.

In order to ensure that pupils understand a problem, the teacher has to discuss it by asking pupils to say what it means and by asking questions about the problem as a whole or about its parts. Discussion of the problem leads to the second stage, that is, planning its solution. The teacher should guide them to relate what is given to what is required. He may use diagram to clarify ideas. After making a plan, the teacher would then request pupils to try to solve the problem. Looking back on the solution is a habit in solving problems which should be developed and fostered. Any error in the method used for the solution can usually be identified when the solution is read over.

Earp (1970) suggested an approach similar to Polya's although his emphasis was on the role of reading. He emphasizes getting pupils to read and re-read a problem in order to get the specific facts and understand the vocabularies and concepts before planning its solution.

Dahmus (1970) gave steps to be followed in teaching verbal problems in the DPPC method. The four letters stand for the words "direct", "pure", "piecemeal" and "complete". The words describe the kind of translation involved. Dahmus emphasized analysing a problem into bits and pieces before it is solved.

Cherkas (1978) discussed qualities and techniques which he has found helpful in creating a positive classroom environment for teaching problem solving. He was of the opinion that free-hand drawing should be encouraged because he feels drawing free-hand serves to promote a greater degree of concentration, commitment and self-reliance in attacking problems. He also lays emphasis on teachers' explanation of some mathematical terms that have different meaning from the everyday English Language.

Goldberg (1981) revealed that co-operative problem solving in small groups of four or five members is helpful to both students and teachers. In his study, each group produces proofs or solutions to problems by working co-operatively together in class or out of class. The groups then submit written assignment after all the members of a group would have agreed that it is correct. The fact that the teacher could call on any student to explain a point on the paper submitted and also each group member receives the group grade makes the students work co-operatively together. Since students learn by working at a comfortable space, group problem solving will help students discover new strategies and solutions from their peers. This group problem solving could also be used to improve teachers' problem solving ability.

Gibney and Meiring (1983) reported that it was obvious that many teachers are apprehensive about their personal problem solving skills and are anxious about others finding them to be wanting in this area. They however suggested that these feelings could be overcome by encouraging group problem solving and using problems that yield to a variety of strategies.

From the foregoing, teachers have an important role to play in assisting pupils become better problem solvers. To help students improve in problem-solving ability, the teacher should create a climate in the class where students are free to ask questions. The teacher could also ask many questions that require thinking and then give students opportunity to think.

2.6 SUMMARY

In this chapter the researcher has reviewed literature related to the nature of problem solving in mathematics, concepts learning in mathematics, the relationship between reading ability, mathematical language,computational ability, teachers' method of teaching, exposure to solving problems, lengths of problems, and giftedness with problem-solving ability. The review also featured methods of teaching concepts and suggestions by researchers and educators on useful methods of developing problem solving skills.

The survey of available research reveals a number of findings. These are:

(i) There is an overwelming evidence showing the significant relationship between reading ability and problem solving;

(ii) Comprehension of mathematical language and computational ability have a significant effect on problem solving ability;

(iii) Teachers' method of teaching affects problem solving ability;

(iv) There is a moderate relationship between exposure to solving problems and problem-solving ability;

(v) It is not clear the extent to which lengths of a problem affect problem-solving ability;

(vi) There is a relationship between giftedness and problem solving ability although the sample used in the study reviewed seems too small.

Although, studies from other parts of the world show there exists a relationship between computational ability, reading ability and problem solving, research efforts in these areas have not received a significant attention in Nigeria. It is also not known whether there is any relationship between knowledge of concepts and problem solving ability in mathematics.

Even though a few methods of teaching problem solving skills have been described and tried out on pupils, they may be unsuitable for all the different aspects of mathematics. Apart from these, to the knowledge of this researcher, no study in Nigeria has investigated whether a method of teaching concepts would have any significant effect on the improvement of students' problem solving ability. Hence the researcher assumes the appropriateness of this study to investigate the relationship between knowledge of mathematics concepts and problem-solving ability.

CHAPTER THREE

DESIGN AND PROCEDURE

3.1 INTRODUCTION

This chapter describes the research design and the procedure adopted in the conduct of the study.

The chapter is divided into five major sections. Firstly, the aims of the study are summarized and the methods of achieving these aims are discussed. Secondly, the population and methods of subject selection (sample selection) are described. The third section deals with the development and procedure of constructing and validating the data gathering instruments. Fourthly the procedure of the pilot study is outlined. It describes the administration of the tests of computation, knowledge of concepts, comprehension of mathematics language and problem-solving in mathematics. The administration of the pre-and post-tests, methods of scoring and statistical analysis of the data were also treated. The last section discussed the procedures adopted for the final study.

3.2 THE AIMS AND THE DESIGN OF THE RESEARCH

Aims

As already discussed in chapter one, the aims of the study are:

1. To investigate the relationship between the knowledge of concepts and mathematics problem-solving ability.

2. To investigate the relationship between mathematics problem-solving ability and knowledge of concepts in mathematics while controlling for computational ability and comprehension of mathematical language.

3. To develop a method of teaching concepts in mathematics and to test its effectiveness on the improvement of problem-solving ability of three groups of students of different abilities.

Design of Research:

The ex post facto design was used in the first part of the study. It involves the collection of four sets of data from a group of subjects with the attempt to determine the relationship between those sets of data. This type of approach could be diagrammed as shown:

$$0_1 \quad 0_2 \quad 0_3 \quad 0_4$$

The second part of the study is an Experimental Design. Specifically, it is a Pretest - Posttest Control Group design. It involves three groups of subjects: two experimental groups and one control group. This can be diagrammed as follows:

R_I	0_I	X	0_2
R_{II}	0_3	Y	0_4
R_{II}	0_5	Z	0_6

As can be seen from the diagram, three groups were employed in this design, the experimental group I (R_1) received a treatment X (a concept-teaching approach combined with practice in problem-solving), the experimental group II (R_{11}) received treatment Y, (Practice in problem-solving with the researcher) while the control group (R_{111}) received treatment Z (practice exercises only).

3.3 POPULATION AND SAMPLING

This study was carried out in Lagos State of Nigeria. From the 15 Schools Management Committee in Lagos State, 10 SMC's were randomly selected.

For the Pilot study, four mixed secondary schools from one School Management Committee were randomly selected. Two schools each were used for the first and second parts of the study. Eighty-one students were used for the first phase while seventy-seven students participated in the second phase.

The remaining nine Schools Management Committees formed the sample of the main study. From each of the 9 SMC's, two schools were selected by stratified ramdom sampling from all the schools. Thus a total of 18 secondary schools which comprised 6 male schools, 6 female schools and 6 mixed schools were selected for the study. Apart from these, the three Federal Government Colleges in Lagos State were included in the conduct of the study. Altogether, 14 single sexed schools and 11 mixed schools were utilized in the conduct of this study.

For the first part of the main study, 12 (6 males and 6 females) Lagos State secondary schools and the 3 Federal Government Colleges in Lagos State were involved. The remaining 6 mixed schools formed the sample of the second part of the study.

For the first part of the study, seven hundred and five students took one or more of the four tests. However, only five hundred and eighty eight students who took all the four tests were found useful during the analysis.

For the seoond part of the study, two hundred and forty students were involved. Table 3 shows the distribution of students in the six schools.

TABLE 3

DISTRIBUTION OF STUDENTS IN THE SIX SCHOOLS

SCHOOLS	NUMBER OF STUDENTS
School 1	35
School II	42
School III	45
School IV	40
School V	40
School VI	38
Total	240

The population used for this study consisted of class four students in secondary schools. The secondary class four students were chosen for the study because they were expected to have known the essential and fundamental concepts for secondary school work by the end of their third year.

3.4 DATA GATHERING INSTRUMENTS:

The research tools used in this study were

1. Test on knowledge of Mathematical Concepts:

   This is a test of knowledge of concept. It consists of ninety four multiple choice items.

2. Test on Computation:

   This is a test on ability to compute. It consists of twenty-two multiple choice items.

3. Test on Comprehension of Mathematical Language:

   This is a test of comprehension of mathematical language. It consists of seventy-eight multiple choice items.

4. Test on Mathematics Problem Solving:

   This test consists of nineteen worded problems.

5. Pretest:

   It consists of sixty objective questions in mathematics. It was used to ascertain the equality of the groups and to group students into ability levels.

6. Post-test:

   It is based on the difficult concepts identified during the first part of the study. It consists of eleven worded problems based on these concepts.

3.5 CONSTRUCTION OF THE RESEARCH INSTRUMENTS:

3.5.1 Test on Knowledge of Mathematical Concepts:

The test on Knowledge of Mathematical Concepts was constructed by first listing concepts in the Junior Secondary School curriculum in Algebra and Geometry. On the whole, forty-seven concepts were listed. This is shown in Table 4

TABLE 4

LIST OF MATHEMATICS CONCEPTS

1. Acute Angle
2. Adjacent Angle
3. Alternate Angle
4. Area
5. Circle
6. Complementary Angle
7. Congruency
8. Convex polygon
9. Corresponding Angle
10. Cube
11. Cuboid
12. Cylinder
13. Cone
14. Equilateral Triangle
15. Factorisation
16. Inequality
17. Isosceles Triangle

18. Kite
19. Line of Symmetry
20. Obtuse Angle
21. Parallel Lines
22. Parallelogram
23. Pentagon
24. Perpendicular lines
25. Perimeter
26. Plane Figure
27. Pyramid
28. Pythagoras Theorem
29. Quadratic Equation
30. Rectangle
31. Regular Polygon
32. Rhombus
33. Right Angle
34. Reflex Angle
35. Scalene Triangle
36. Similar Figures
37. Simultaneous Linear Equation
38. Square
39. Supplementary Angle
40. Solid Figure
41. Trapezium
42. Triangle

43. Triangular Prism
44. Trigonometric Ratios
45 Variation
46 Vertically Opposite Angles
47 Volume

The test on knowledge of mathematical concepts seeks to determine the extent to which students can define, give examples, non-examples and attributes of concepts. In order to find out the extent to which a student understands a concept, two questions were set on each concept which made a total of 94 questions. The test blue print on knowledge of Mathematical Concepts is shown in Table 5.

TABLE 5

TEST BLUE PRINT ON KNOWLEDGE OF MATHEMATICAL CONCEPTS

CONTENT	BEHAVIOUR				
	DEFINITION	EXAMPLE	NON-EXAMPLE	ATTRIBUTES	TOTAL
Algebra					
Linear Equations		1		1	2
Linear Inequalities				2	2
Factorisation	1	1			2
Variation	1	1			2
Geometry					
3D Figures	2	1	-	13	16
Plane Figure	17	2	3	16	38
Similar Figures	1	-	-	1	2
Angles/Lines	12	4	-	12	28
Trigonometric Ratios	-	2	-	-	2
Total	34	12	3	45	94

3.5.2 Test on Computation:

Test on computation consists of twenty two items. There were four items on knowledge of specific facts and eighteen items on ability to carry out algorithms. The test blue print is shown in Table 6.

TABLE 6

TEST BLUE-PRINT FOR TEST ON COMPUTATIONAL ABILITY

	BEHAVIOURS		
CONTENT	Knowledge of Specific Facts	Ability to Carry Out Algorithms	Total
ALGEBRA:			
Simplification of algebraic Expressions		3	3
Linear Equations		5	5
Linear Inequalities		1	1
Expansion/Factorisation		3	3
Change of Subject/Variation		2	2
GEOMETRY:			
3D Figures	1		1
Plane figures	3	4	7
Similar shapes			
Angles/Lines			
Bisection/Construction			
Trigonometric Ratios			
Total	4	18	22

3.5.3 Test on Comprehension of Mathematical Language:

The test on comprehension of mathematical language was constructed by first listing words and symbols in the junior Secondary School Mathematics Curriculum in Algebra and Geometry with mathematical connotations.

On the whole, 71 words and 7 symbols were listed. This is shown in Table 7

TABLE 7

LIST OF WORDS AND SYMBOLS WITH MATHEMATICAL CONNOTATIONS:

1. Acute Angles
2. Adjacent Angles
3. Alternate Angles
4. Altitude
5. Algebraic Expression
6. Angle of Elevation
7. Angle of Depression
8. Area
9. Arc
10. Bearing
11. Bisection
12. Centre
13. Circle
14. Circumference
15. Change of Subject
16. Complementary Angles

17. Congruency
18. Constant
19. Corresponding Angle
20. Cosine of Angles
21. Cone
22. Cube
23. Cuboid
24 Cylinder
25. Diagonal
26. Diameter
27. Edges
28. Exterior Angles
29. Equilateral Triangle
30. Faces
31. Factorisation
32. Graph
33. Hypotenuse
34. Inequality
35. Isosceles Triangle
36. Kite
37. Line of Symmetry
38. Median
39. Obtuse Angle
40. Parallel Lines
41. Parallelogram

42. Pentagon
43. Perimeter
44. Perpendicular
45. Plane Figure
46. Polygon
47. Pyramid
48. Pythagoras Rule
49. Quadrilateral
50. Radius
51. Rectangle
52. Regular Polygon
53. Reflex Angle
54. Rhombus
55. Right Angle
56. Scale Factor
57. Scalene Triangle
58. Similar shapes
59. Simultaneous Linear Equations
60. Sine of Angle
61. Solid Figure
62. Supplementary Angles
63. Square
64. Tangent of Angles
65. Trapezium
66. Triangle

67. Triangular Prism

68. Variation

69. Vertex

70. Vertically Opposite Angle

71. Volume

72. $<$

73. $>$

74. $\leq$

75. $\geq$

76. $=$

77. $\propto$

78. $\equiv$

The test on comprehension of mathematical language seeks to determine students' ability in identification and interpretation of mathematical diagrams, knowledge of principles and rules, knowledge of terms and ability to change from one mode to another.

The test blue print is shown in Table 8

TABLE 8

TEST BLUE PRINT ON TEST OF COMPREHENSION OF MATHEMATICAL LANGUAGE

CONTENT	BEHAVIOUR: Identification and Interpretation of Maths Diagram	Knowledge of Principles and Rules	Knowledge of Terms	Ability to Change From one Mode to Another	TOTAL
ALGEBRA					
Linear Equation	-	-	1	3	4
Linear Inequalities	-	1	-	4	5
Factorisation	-	-	1	-	1
Variation	-	1	2	1	4
GEOMETRY					
3D Figures	7	4	-	-	11
Plane Figures	10	10	10	1	31
Similar Shapes	-	-	2	-	2
Angles/Lines	12	1	3	-	16
Bisection/Construction	-	1	-	-	1
Trigonometric Ratios	-	3	-	-	3
TOTAL	29	21	19	9	78

3.5.4 Test on Mathematics Problem Solving Ability:

The test on mathematics problem-solving ability consists of nineteen worded problems based on the concepts identified in the test of knowledge of mathematics concepts. A question was set on one or two or more of the identified concepts. The maximum number of concepts in a problem was five.

The first problem was based on the concept of simultaneous linear equations. It requires the students to find the cost of a sharpner and an eraser.

The second problem was on factorisation, students were required to factorise a quadratic expression.

The third problem was based on the concept of line of symmetry and equilateral triangle. Students were asked how many lines of symmetry in an equilateral triangle. They were also required to draw them.

The fourth problem was on the concepts of plane figure, parallelogram, trapezium, rhombus and Kite. Students were asked to say whether trapezium, rhombus and Kite are parallelograms with reasons.

The fifth problem was on the concepts of trigonometric ratio, quadrilateral and area. Students were asked to calculate the area of a quadrilateral.

The sixth problem was based on concept of angles (adjacent, alternate, corresponding, vertically opposite) and parallel lines. Students were asked to calculate a marked angle in the given figure.

The seventh problem was on the concepts of supplementary and complementary angles. Students were asked to find the value of a marked angle in a figure.

The eighth problem was on the concepts of acute, obtuse and reflex angles. Students were asked to find the value of an obtuse angle in a given diagram.

The ninth problem was on the concepts of cylinder, volume, circle and cone. Students were asked to calculate the volume of a cylinder.

The tenth problem was based on the concepts of perimeter, rectangle and square. Students were asked to calculate the perimeter of a rectangle.

The eleventh problem was on solid figure, cube and cuboid. Students were asked to calculate the height of a box.

The twelfth problem was based on the concepts of pyramid and triangular prism. Students were asked to calcualte the volume of a prism.

The thirteenth problem was based on the concepts of Pythagoras rule, right angle and perpendicular lines. Students were asked to calculate the length of a perpendicular in a triangle.

The fourteenth problem was based on the concepts of plane figure, isosceles and scalene triangles. Students were asked to name some shapes in a given figure.

The fifteenth problem was based on the concepts of convex polygon and pentagon. Students were required to calculate the angles of a pentagon.

The sixteenth problem was based on the concepts of similar shapes. Students were required to calculate the height of a smaller bowl of rice given the height of the bigger bowl.

The seventeenth problem was on the concepts of regular polygon and congruency. Students were required to prove that two triangles were congruent.

The eighteenth problem was on the concept of variation. Students were required to find the cost of a car service.

The nineteenth problem was on the concept of inequality. Students were required to find the range of values of a variable X.

3.5.5 The Pretest:

The pretest is an objective test based on the concepts of algebra and geometry of the Junior Secondary School Curriculum. The pretest seeks to find out students' knowledge of specific facts, concepts, principles and rules and their ability to carry out algorithms, to transfer problem from one mode to another, read and interprete problems, solve routine problems and solve verbal problems. The test-blue print is shown in Table 9.

TABLE 9

TEST BLUE PRINT ON THE PRETEST

CONTENT	BEHAVIOURS								
	Computat-ational		Comprehension				Application		
	Knowledge of Specific facts	Ability to Carry out Algorithms	Knowledge of Concepts	Knowledge of Principles and Rules	Ability to transfer problem from one mode to another	Ability to read and Interprete problems	Ability to solve routine problems	Ability to solve verbal problems	Total
ALGEBRA									
Linear Equations/		2				1	1	1	5
Linear Inequalities		1			2			1	4
Simplification/Factorisation	2	2	1	1					6
Change of Subject/Variation	1		1	2	1				5
GEOMETRY:									
3 Dimensional Shapes	1		3			1	1	2	8
Plane figures		1	2				7	1	11
Similar Figures			1				2	1	4
Angles/Lines			4	4					8
Trigonometric Ratios			2			1		2	5
Construction	2			2					4
TOTAL	6	6	14	9	3	3	11	8	60

3.5.6 The Post-Test:

The post-test was based on the difficult concepts identified from students' response to the test on mathematics concepts. A question was set on one or two or more of the concepts and eleven problems were set on the whole. The maximum number of concepts in a problem was five.

The first problem was based on the concept of simultaneous linear equations. Students were required to calculate Dupe's age and Eze's age.

The second problem was based on the concept of trigonometric ratio. Students were required to calculate how far up the wall a ladder reaches.

The third problem was on the concept of variation. Students were asked to caculate the effort necessary to carry certain load.

The fourth problem was on the concept of angles (supplementary, vertically opposite, adjacent and corresponding). Students were asked to calculate the marked angles in a given figure.

The fifth problem was on the concept of complementary angles. Students were required to calculate the value of a marked angle.

The sixth problem was on the concepts of pentagon and convex polygon. Students were asked to calculate the remaining angles of a pentagon.

The seventh problem was on the concepts of square, rhombus, and trapezium. Students were asked to say the difference between a square, a rhombus, and a trapezium. They were then asked to say whether a square is a rhombus.

The eighth problem was on the concepts of line of symmetry, Kite and trapezium. Students were required to say the number of lines of symmetry in some plane shapes with illustrations.

The ninth problem, was on the concept of similar figures, cube and cuboid. Students were required to calculate the length and height of a box.

The tenth problem was on the concept of congruency. Students were required to prove that two given triangles were congruent.

The eleventh problem was on the concepts of solids, pyramid, and triangular prism . Students were required to calculate the height of a pyramid on a triangular base.

Apart from these six instruments, during the conduct of the second part of the study, the researcher taught the experimental group using a concept teaching approach. A concept teaching approach is based on the work of Billings (1979), Dunn (1983), Tennyson & Cocchiarella (1986) and Charles (1980).

It is described below:

A CONCEPT TEACHING APPROACH:

I. Teacher asks the students to say what they know about each concept, that is, identification of the attributes of the concepts.

2. Teacher leads the students in the identification of other attributes of each concept which are not mentioned by the students.

3. Teacher asks the students to give examples of the concept under discussion.

4. Teacher asks the students to give non-examples of the concept.

5. Teacher asks the students to define the concept.

6. Teacher corrects the students and defines the concept properly.

3.6 VALIDATION OF THE RESEARCH TOOLS:

The test on comprehension of mathematical language contains seventy eight items, the test on computation twenty two questions while the knowledge of concepts contains ninety four items.

Test on mathematics problem solving ability contains nineteen worded problems, pre-test sixty objective questions while the post-test contains eleven worded problems. The instruments were validated first by two secondary school mathematics teachers. They made useful suggestions which enabled the researcher to make necessary modifications. The modified forms were then evaluated by four secondary school mathematics teachers, a mathematics educator and the researcher's supervisor. The teachers wrote reports confirming their agreement on the tests on algebra and geometry content of the junior secondary schools mathematics syllabus. The teachers also gave their approval that the tests were quite suitable for class four students. Some questions were however reworded based on the recommendations of these teachers and some other mathematics educators. These tests were prepared for administration for the pilot study.

3.7 PILOT STUDY:

After constructing the tests, it was dedided that a pilot study should be conducted. The purpose of the pilot study were:

1. To make ammendments in the tests for the final form of the administration of the data-gathering instruments. This will enable the investigator to find out any possible problem that can effect its final administration.

2 To determine the correlation between knowledge of concepts and problem - solving ability

3. To find the effect of computational ability, comprehension of mathematics language and knowledge of concepts on problem-solving ability.

4. To find the effect of a concept teaching approach on the problem-solving ability of an experimental group of students.

5. To determine the reliability of the mathematical language test, computation test, knowledge of concepts test and pretest by using Kuder Richardson formula 21.

6. To determine the reliability of mathematics problem - solving test, and the posttest by using **cronbach alpha method in** Ebel **1972.**

7. To analyse the objective test items for difficulty level and discriminating power.

3.7.1 Administration of the Research Tools for Pilot Study:

For the first part of the study, two classes of form four students from two mixed schools took part in the pilot study. Eighty one students participated in this phase. For the second part of the study, seventy-seven students from two different mixed secondary schools participated.

3.7.1.1 Administration and Scoring of the Test on Computation

The test on computation was administered to the two schools on the same day.

The computation test was scored by giving one point to each correct item and zero to a wrong item. Since the items were twenty-two in all, the marks obtained were converted to percentage for easy comparison. The scores of the computation test was out of hundred points.

3.7.1.2 Administration And Scoring of the Test on Knowledge of Mathematical Concepts:

The test on knowledge of mathematical concepts was administered on the second day in both schools.

The knowledge of concepts test was scored by giving one point to each correct item and zero to a wrong item. Since the items were ninety-four in all, (2 questions each on 47 concepts), the marks obtained were converted to percentage for easy comparison. Therefore, the scores of the knowledge of concepts were out of hundred. As it was said earlier, two questions were set on each concept. In order to find out the concepts that were not well understood by the majority of the students, average difficulties of the two questions set on each concept were calculated. and concepts with average difficulty less than 50% were identified. The difficult concepts identified numbered twenty-two. The concepts were

Adjacent angle

Complementary angle

Convex Polygon

Congruency

Corresponding angle

Cube

Cuboid

Kite

Line of Symmetry

Pentagon

Pyramid

Rhombus

Similar Figure

Simultaneous Linear Equations

Solid Figure

Supplementary Angle

Square

Trapezium

Triangular Prism

Trigonometric Ratios

Variations

Vertically Opposite angle

These concepts were later taught during the second part of the study.

3.7.1.3 Administration and Scoring of the Test on Comprehension of Mathematical Language

The test on Comprehension of Mathematical language was administered on the third day in each school. Like the other tests, one point was given to each correct item and a zero to a wrong item. The marks obtained were also converted to percentages for easy comparison.

3.7.1.4 Administration and Scoring of the Test on Mathematics Problem Solving Ability

Since the mathematics problem solving test was a written test, a marking scheme was prepared and used by the researcher in scoring the subjects on the problem solving test.

Table 10 shows the marks allotted to each question.

TABLE 10

MARKS ALLOTTED TO EACH QUESTION ON THE TEST OF MATHEMATICS PROBLEM-SOLVING

Question Number	Marks
1	7
2	2
3	2
4	3
5	5
6	3
7	3
8	5
9	3
10	4
11	3
12	3
13	5
14	3
15	6
16	5
17	5
18	8
19	5
Total:	80

The marks obtained by each student was converted to percentage for easy comparison. The marking scheme is provided in Appendix 'J'.

3.7.1.5 Administration And Scoring of the Pre-test

The pre-test was administered to another set of form four students from two mixed schools. Like the other objective tests, one point was awarded to each correct item and a zero to a wrong item. The marks obtained were converted to percentages. The key is in Appendix 'K'.

3.7.1.6 TEACHING PROCEDURE:

The teaching procedure was based on concept teaching approach described earlier. It consists of encouraging students to say what they know about the concepts (i.e identifying attributes) giving examples, non-examples and defining the concepts. The researcher corrects students' definition when necessary.

As revealed in the literature review, (Gagne 1977, Skemp 1971) there are basically two types of concepts; concrete and abstract concepts. Concrete concepts can be observed while abstract concepts are often learned by definition. Based on this, the 22 concepts identified were grouped into concrete, abstract and those that belong to both groups.

TABLE 11

CATEGORIES OF CONCEPTS IDENTIFIED

CONCRETE	Abstract/Concrete	Abstract
Cuboid	Convex Polygon	Complementary angle
Cube	Square	Adjacent angle
triangular prism	Trapezium	Supplementary angle
pyramid	Rhombus	Corresponding angle
solid figure	Kite	Vertically opposite angle
	Pentagon	Congruency
	Line of Symmetry	Simultaneous linear equations
		Variation
		Trigonometric Ratios
		Similar Shapes

The concrete concepts were taught by bringing out the shape, cut out models on cardboard paper, network on straws and freehand drawings on the blackboard.

The concepts that were classified as both concrete and abstract were taught first as concrete then later by definition.

The abstract concepts were taught by definition and drawings on the blackboard.

Concrete Concepts: The researcher shows the concepts by bringing out real objects, cut-out models on cardboard paper or network models. When asked to say the attributes of the concepts, most of the students did not understand the researcher. The researcher then decided to ask the students questions such as

How many sides has it got?

How many faces has it got?

How many edges has it got?

Can the faces lie flat on the table?

Are the sides equal?

The researcher then asks the students to give examples and non-examples of the concepts under discussion.

Blackboard Summary:

Solids are objects that occupy space. Examples are cuboid cube cylinder, cone, pyramid and prism.

Non examples are rectangle,square, triangle, etc.

A cuboid is a solid with 6 plane faces.
Each face is in the shape of a rectangle.
It has 12 edges and 8 vertices.

A cube is a cuboid in which all six faces are squares.
It has twelve edges and eight vertices

A triangular prism has five faces, 3 rectangular and 2 triangular.
It has 9 edges and 6 vertices.

The names of pyramids come from the shape of their base faces.
A triangular-based pyramid has four faces in which all the faces are triangles.
A square-based pyramid has five faces in which all faces apart from its base are triangular in shape.

CONCRETE/ABSTRACT CONCEPTS:

The researcher shows the concepts by bringing out cut out models on cardboard paper or network models. Most of the students were able to say the properties of the shapes, give examples and non-examples of the concepts under discussion.

When asked to define the concepts, many students were able to define Square, Trapezium, Rhombus and Pentagon. They were however unable to define Kite, Convex Polygon and line of symmetry correctly. The researcher then defined Kite, Convex Polygon and line of symmetry for the students.

Blackboard Summary

A polygon is any plane closed figure bounded by straight lines. A convex polygon is one in which no interior angle is greater than 180°. A square is a quadrilateral with four equal sides. Each angle is a right angle.

A line of symmetry is a line of fold that divides a plane figure into two equal parts.

A trapezium is a quadrilateral with a pair of sides parallel. It has two diagonals.

A rhombus is a parallelogram with a pair of adjacent sides equal .

All the sides of a rhombus are equal.

A kite is a quadrilateral in which one diagonal is a line of symmetry.

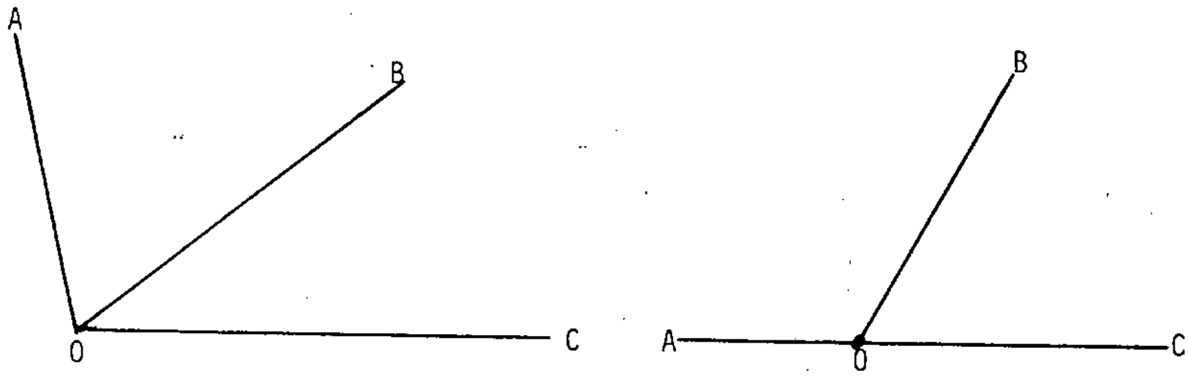
ABSTRACT CONCEPTS

With the abstract concepts, students were asked to define the concepts. The researcher corrects them when necessary. After the definition students were asked to give examples of the concepts.

BLACKBOARD SUMMARY

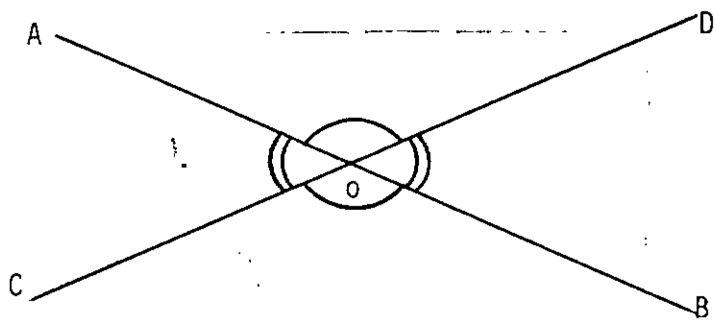
Two angles whose sum is 90° are called complementary. Examples are 40° and 50°, 60° and 30°. Complementary angles form a right angle. Two angles whose sum is 180° are called supplementary angles. E.g., 90° and 90°, 80° and 100°.

Adjacent angles are angles that lie beside each other and have a common vertex



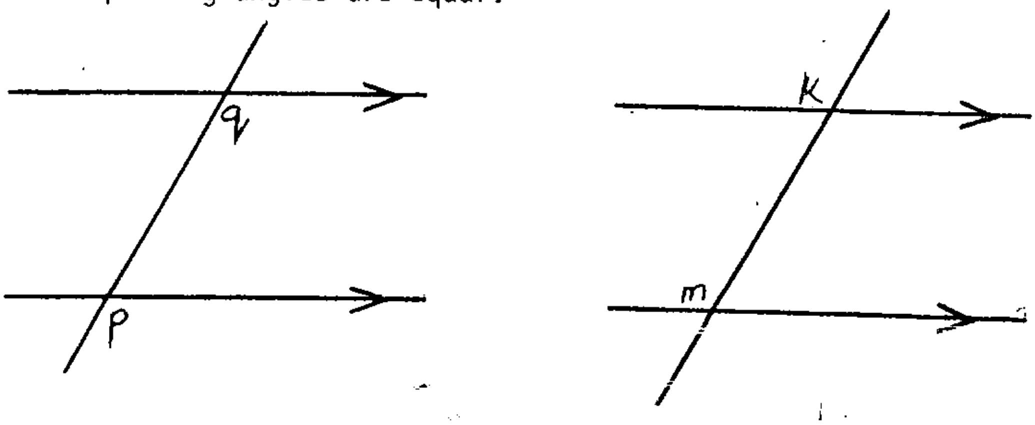
$A\hat{O}B$ is adjacent to $B\hat{O}C$

Vertically opposite angles are two angles opposite to each other which are formed when two straight line intersect.



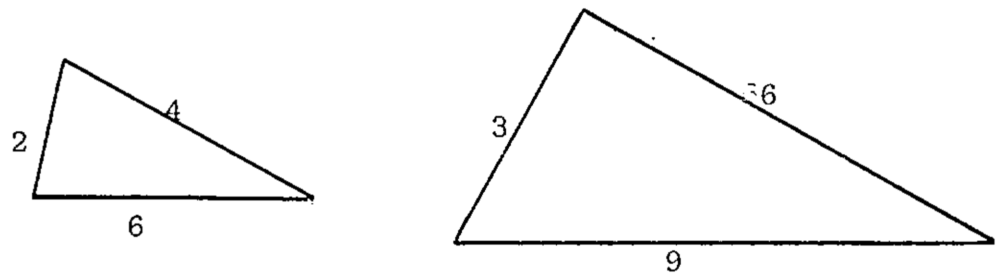
Vertically opposite angles are equal: $A\hat{O}C = B\hat{O}D$; $A\hat{O}D = B\hat{O}C$

Corresponding angles are in the same or corresponding positions at the two intersections of a pair of parallel line and a transversal. Corresponding angles are equal.



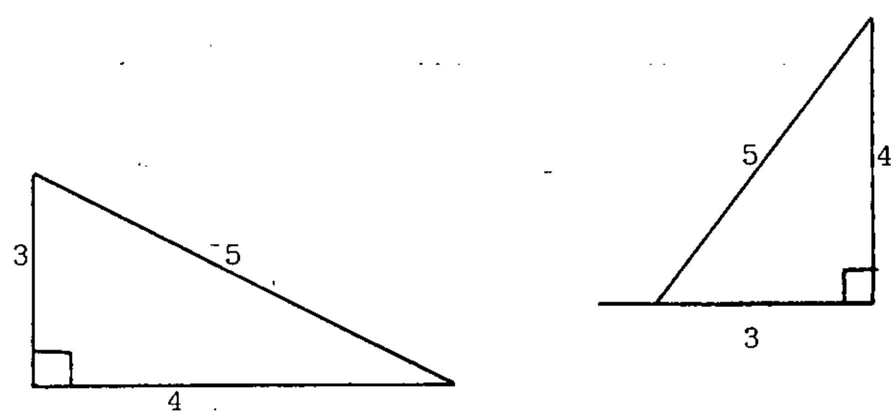
Similar figures are shapes whose corresponding sides are in the same ratio.

For example



Congruency - This is the term used in describing triangles that are equal in all respect.

Example



The conditions for congruency are side, side, side(SSS) side angle side (SAS), Angle, angle, side (AAS) and Right angle, hypotenuse side (RHS); Angle, side side, ASS is not a condition for congruency.

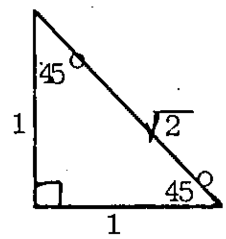
Simultaneous equations are a pair of linear equations involving the same unknowns.

E.g. $X + y = 4$

$2x + 3y = 11$

A pair of solution satisfies both equations.

Trigonometric Ratio is the appropriate ratio of angles in a right angled triangle.



$\sin 45^0 = \frac{1}{\sqrt{2}}$

$\cos 30^0 = \frac{\sqrt{3}}{2}$ etc

Variation: This is a mathematical statement which tells us the relationship between two or three variables.

Example - The distance covered in a journey varies directly as the amount of petrol used.

3.7.1.7 Administration and Scoring of the Posttest:

After teaching the experimental group for three weeks, the post-test was administered to the two classes of form four students. The question papers were collected from students and returned when the posttest had been administered to the two classes who took part in this phase. Students' papers were collected for analysis. The posttest was scored by the researcher. Table 12 shows the marks allotted to each question.

TABLE 12

MARKS ALLOTTED TO EACH QUESTION ON THE POST-TEST

QUESTION NUMBER	MARKS
1	6
2	4
3	8
4	3
5	3
6	5
7	4
8	4
9	4
10	4
11	5
Total	50

The post-test marking scheme is provided in Appendix "M" Test papers were collected, marked and scores recorded.

3.7.2 Analysis of Data

After scoring was completed, the analysis of the data of the pilot study was carried out.

The correlation between the scores of the knowledge of mathematical concepts and the mathematics problem-solving test was calculated to be 0.94 using pearson product-moment correlation coefficient. This is a relatively high correlation which indicates that there is a positive relationship between knowledge of concepts and mathematics problem-solving ability. Also high correlation of 0.70.and 0.78 were found to exist between computational ability and problem-solving and between comprehension of mathematical language and problem-solving ability respectively. The correlations are shown in Table 13.

TABLE 13

INTERCORRELATION AMONG THE TESTS

	Problem Solving	Knowledge of Concepts	Computation	Mathematical Language
Problem Solving	1.0	-	-	-
Knowledge of Concepts	0.94	1.0	-	-
Computation	0.70	0.65	1.0	-
Mathematical Language	0.78	0.81	0.73	1.0

To determine the effect of computational ability, comprehension of mathematics language and knowledge of mathematics concepts on problem-solving ability, the technique of multiple regression was used using the computer. The result is as shown below:

TABLE 14

MULTIPLE REGRESSION OF MATHEMATICAL LANGUAGE, KNOWLEDGE OF CONCEPTS AND COMPUTATIONAL ABILITY ON PROBLEM SOLVING

VARIABLE	MULTIPLE R	SIMPLE R	R SQUARE	B	BETA	F
Concepts	0.95	0.94	0.90	0.82	0.88	220.19(s)
Computation	0.95	0.70	0.50	0.13	0.17	11.47(s)
Maths. Language	0.78	0.78	0.62	0.62	0.04	-0.05(ns)
CONSTANT			-9.39			

(s) Significant

(ns) not significant

From the multiple regression analysis, it was clear that knowledge of concepts and computational ability contributed to the prediction of students' problem-solving ability while mathematical language did not.

To determine whether there is any difference in ability between the experimental and control groups, the t-test was used. The results showed that there was no significant difference between the means of the two groups as shown in table 15.

TABLE 15

THE t-VALUE OBTAINED IN COMPARING THE EXPERIMENTAL AND CONTROL GROUPS ON THE PRETEST:

GROUPS	NUMBER	MEANS	S	t VALUE
Experimental	41	27.20	11.68	0.41*
CONTROL	36	26.08	12.14	

* less than 0.05 level (not significant)

From the results of the pretest, the students were divided into higher achievers (H) average achievers (A) and lower achievers (L). The higher 25% were the higher achievers, the middle 50% the average achievers and the lower 25% were the lower achievers. The distribution of the students according to their ability groups is as shown in Table 16.

TABLE 16.

STUDENTS' DISTRIBUTION WITH RESPECT TO THE THREE ABILITY GROUPS

	H	A	L	TOTAL
Experimental	10	21	10	41
Control	9	18	9	36
Total	19	39	19	77

To determine the effect of a concept teaching approach on the experimental group,(higher achievers, average achievers and lower achievers) analysis of variance was used. The results are as shown in Tables 17 - 20 The results indicate that students in the experimental group, the higher achievers, average achievers and lower achievers who were taught with a concept teaching approach performed significantly better than their counterparts in the control group.

TABLE 17

ANALYSIS OF VARIANCE OF THE WHOLE DATA

Source of Variance	Sum of Squares	df	Variance Estimate	F
Between Group	3516.7	1	3516.7	19.1*
Within Group	13810.39	75	184.14	
Total	17327.09			

* Significant at 0.05 level.

TABLE 18

ANALYSIS OF VARIANCE OF HIGHER ACHIEVERS' PERFORMANCE:

SOURCE OF VARIANCE	SUM OF SQUARES	df	VARIANCE ESTIMATE	F
Between Group	672.04	1	672.04	4.45*
Within Group	1024.5	17	60.26	
Total	1696.53			

* Significant at 0.05 level.

TABLE 19

ANALYSIS OF VARIANCE OF AVERAGE ACHIEVERS' PERFORMANCE:

SOURCE OF VARIANCE	SUM OF SQUARES	df	VARIANCE ESTIMATE	F
Between Group	2148.58	1	2148.58	35.76*
Within Group	2223.11	37	60.08	
Total	4371.69			

* Significant at 0.05 level.

TABLE 20

ANALYSIS OF VARIANCE OF LOWER ACHIEVERS' PERFORMANCE

Source of Variance	Sum of Squares	df	Variance Estimate	F
Between Group	1280.92	1	1280.92	15.42*
Within Group	1412.23	17	83.07	
Total	2693.16			

* Significant at 0.05 level

The mean scores and standard deviations of each of the six tests were also calculated. The reliability coefficients of the objective tests were calculated using Kuder Richardson Formula 21 in Ferguson (1981). The reliability coefficient for the essay tests were calculated using Cronbach alpha's method in Ebel (1972).

In order to show whether analysis of variance or analysis of convariance should be used the correlation between pre-test and post-test had to be obtained. This was calculated as 0.71.

Since the correlation is less than 0.80, it shows blocking by ability level and that analysis of variance is the appropriate technique.

Table 21 shows the means, standard deviations and reliability co-efficients of the six tests.

TABLE 21

MEAN, STANDARD DEVIATION AND RELIABILITIES OF THE SIX TESTS

TESTS	MEAN	STANDARD DEVIATION	RELIABILITY COEFFICIENT
Computation	47.95	18.37	0.97
Mathematical concepts	47.81	15.16	0.90
Mathematical Language	54.95	15.07	0.90
Mathematical problem-solving (1st Phase)	33.47	14.14	0.90
Pre-test (2nd Phase)	26.68	11.83	0.83
Post-test (2nd Phase)	32.55	14.16	0.65

SUMMARY

The findings of this preliminary study show that:

1. there is a positive linear relationship between scores of knowledge of mathematics concepts and on problem-solving;

2. Knowledge of concepts contributed significantly to the prediction of problem-solving performance in the presence of computational ability and mathematical language.
3. there is a positive linear relationship between computational ability, comprehension of mathematical language and problem-solving performance.
4. students in the experimental group who were taught with a concept-teaching approach performed significantly better than their counterparts in the control group on a problem solving test.

3.8 SLIGHT ALTERNATIONS IN THE INTRUMENTS FOR THE MAIN STUDY

To put the data gathering instruments into the final form for the main study, the analysis of test items for the four objective tests were carried out. With the Computation test, two items had low difficulty level while one of them also has a negative discrimination. The two items were eliminated. Table 22 shows the difficulty levels and discriminating powers of the 22 items

TABLE 22

DIFFICULTY AND DISCRIMINATING POWER OF ITEMS ON COMPUTATIONAL ABILITY TEST

ITEM	DIFFICULTY	DISCRIMINATING POWER
1	76	0.33
2	74	0.52
3	48	0.37
4	78	0.37
5	50	0.41
6	46	0.56
7	54	0.85
8	65	0.48
9	80	0.22
10	26	0.37
11	48	0.74
12	50	0.63
13	13*	0.18
14	20	0.26
15	26	0.22

Table 22 Cont'd

ITEM	DIFFICULTY	DISCRIMINATING POWER
16	33	0.37
17	55	0.67
18	50	0.56
19	15*	-0.15
20	33	0.27
21	68	0.41
22	65	0.41

* items deleted

With the Comprehension of Mathematical language test, two items discriminated negatively, four items did not discriminate at all, they had zero discriminating powers, seven items had very low discriminating powers ranging from 0.04 to 0.19. Some of these items also had low difficulties. Six items had very high difficulty level while five items had very low difficulty level.

Hence the fourteen items were eliminated leaving sixty four items for the test. Table 23 shows the difficulty levels and discriminating powers of the seventy eight items.

TABLE 23

DIFFICULTY AND DISCRIMINATING POWER OF ITEMS ON COMPREHENSION OF MATHEMATICAL LANGUAGE

ITEM	DIFFICULTY	DISCRIMINATING POWER
1	98*	0.04
2	88*	0.15
3	55	0.59
4	66	0.59
5	100*	0
6	77	0.37
7	77	0.37

Table 23 Cont'd:

ITEM	DIFFICULTY	DISCRIMINATING POWER
8	79	0.33
9	68	0.59
10	96*	0.07
11	52	0*
12	68	0.33
13	80	0.37
14	77	0.44
15	91*	0.19
16	57	0.20
17	72	0.26
18	63	0.22
19	41	-0.07*
20	48	0.44
21	46	0.21*
22	33	0.67
23	33	0.37
24	63	0.52
25	41	0.74
26	46	0.33
27	41	0.37
28	65	0.63
29	83	0.33

Table 23 Continued

ITEM	DIFFICULT	DISCRIMINATING POWER
30	48	0.59
31	50	0.55
32	28	0.26
33	18*	0.30
34	15*	0
35	59	0.30
36	31	0.55
37	39	0.33
38	33	0.30
39	31	0.20
40	46	0.33
41	26	0.27
42	17*	0.19
43	65	0.48
44	67	0.52
45	54	0.63
46	13*	0.04
47	39	0.55
48	44	0.67
49	35	0.63
50	41	0.74
51	43	0.55
52	91*	0.19

Table 23 Cont'd:

ITEM	DIFFICULTY	DISCRIMINATING POWER
53	59	0.44
54	61	0.41
55	63	0.44
56	55	0.44
57	65	0.48
58	22	0.22
59	63	0.59
60	37	0.52
61	63	0.37
62	70	0.52
63	70	0.59
64	79	0.20
65	74	0.44
66	37	0.25
67	33	0.21
68	07*	-0.07
69	63	0.30
70	55	0.67
71	57	0.55
72	50	0.33
73	33	0.30
74	72	0.48

Table 23 Cont'd

ITEM	DIFFICULTY	DISCRIMINATING POWER
75	74	0.44
76	67	0.55
77	72	0.48
78	63	0*

*Items deleted.

Table 24 shows the difficulty levels and discriminating powers of test of knowledge of mathematics concepts

TABLE 24

DIFFICULTY AND DISCRIMINATING POWER OF ITEMS ON TEST ON KNOWLEDGE OF MATHEMATICAL CONCEPTS.

ITEMS	DIFFICULTY	DISCRIMINATING POWER
1	59	0.37
2	65	0.26
3	44	0.37
4	85	0.22
5	42	0.22
6	52	0.37

Table 2 4 Cont'd

ITEMS	DIFFICULTY	DISCRIMINATING POWER
7	80	0.33
8	72	0.33
9	69	0.26
10	57	0.63
11	28	0.48
12	33	0.22
13	20	0.26
14	56	0.33
15	63	0.44
16	59	0.74
17	55	0.52
18	63	0.44
19	13*	-0.03*
20	37	0.44
21	49	-0.07*
22	52	0.22
23	51	0.59
24	57	0.44
25	65	0.55
26	31	-0.03
27	59	0.44
28	39	-0.03

Table 24 Cont'd

ITEMS	DIFFICULTY	DISCRIMINATING POWER
29	45	0.41
30	11*	-0.07
31	24	0.26
32	52	0.30
33	24	0.33
34	38	0.59
35	57	0.04
36	48	0.22
37	17*	0.19
38	35	0.48
39	43	0.55
40	30	0.52
41	9*	-0.19
42	55	0.37
43	67	0.59
44	70	0.30
45	63	0.37
46	30	0.44
47	43	0.26
48	63	0.37
49	52	0.44

Table 24 Cont'd:

ITEMS	DIFFICULTY	DISCRIMINATING POWER
50	59	0.59
51	65	0.55
52	32	0.19
53	55	0.37
54	70	0.52
55	63	0.26
56	61	0.70
57	59	0.59
58	76	0.41
59	94	0.11
60	39	0.26
61	68	0.44
62	50	0.55
63	50	0.26
64	28	0.26
65	65	0.33
66	61	0.33
67	38	0.11
68	60	0.19
69	33	0.30
70	59	0.48

Table 2.4 Cont'd:

ITEMS	DIFFICULTY	DISCRIMINATING INDEX
71	43	0.30
72	70	0.15
73	31	0.19
74	52	0.59
75	35	0.19
76	63	0.30
77	37	0.15
78	70	0.37
79	20	0.15
80	13*	0.04
81	27	0.19
82	67	0.59
83	52	0.30
84	20	0
85	26	0.22
86	44	0.44
87	39	0.33
88	48	0.52
89	33	0.30
90	52	0.44

Table 24 Cont'd

ITEMS	DIFFICULTY	DISCRIMINATING INDEX
91	52	0.44
92	44	0.37
93	35	0.63
94	44	0.59

* Items reworded for the main study

With the knowledge of mathematical concepts, six items discriminated negatively while one item has zero discrimination. Out of these seven items, six of them are part of the twenty two concepts in which students' average difficulty was less than 50%. This researcher therefore believes that students did not understand these concepts and hence majority of them guessed which caused the negative discrimination. Some of the options to the items were however reworded for the main study.

Table 25 shows the average difficulty of knowledge of concepts test items.

TABLE 25

AVERAGE DIFFICULTY OF THE KNOWLEDGE OF CONCEPTS TEST ITEMS

ITEMS	DIFFICULTIES		AVERAGE DIFFICULTY
1,48	59	63	61
2,49	65	52	58.5
3,50	44	59	51.5
4,51	85	65	75
5,52	42	32	36*
6,53	52	55	53.5

Table 25 Cont'd

ITEMS	DIFFICULTIES		AVERAGE DIFFICULTY
7, 54	80	70	75
8, 55	72	63	67.5
9, 56	69	61	65
10, 57	57	59	58
11, 58	o 28	76	52
12, 59	o 33	94	63.5
13, 60	20	39	29.5*
14, 61	56	68	62
15, 62	63	50	56.5
16, 63	59	50	54.5
17, 64	o 55	28	41.5*
18, 65	63	65	64
19, 66	o 13	61	37*
20, 67	37	38	37.5*
21, 68	49	60	54.5
22, 69	52	33	42.5*
23, 70	51	59	55
24, 71	57	43	50
25, 72	65	70	67.5
26, 73	31	31	31*
27, 74	59	52	55.5

Table 2.5 Cont'd:

ITEMS	DIFFICULTIES		AVERAGE DIFFICULTY
28, 75	39	35	37*
29, 76	o 35	63	49*
30, 77	o 11	37	24*
31, 78	o 24	70	47*
32, 79	o 52	20	36.0*
33, 80	24	13	18.5*
34, 81	38	27	32.5*
35, 82	57	67	62
36, 83	48	52	50
37, 84	17	20	18.5*
38, 85	35	26	30.5*
39, 86	43	44	43.5*
40, 87	30	39	34.5*
41, 88	o 9	40	28.5*
42, 89	50	38	44*
43, 90	67	52	59.5
44, 91	70	52	61
45, 92	63	44	53.5
46, 93	30	35	32.5*
47, 94	43	44	43.5*

*The concepts taught during the second phase of the study. i.e concepts with average difficulty less than 50%.

o items in which the diffference in their difficulties were more than 20.

With some items, the difference in their difficulties were more than 20. A critical look at the item revealed that students did not know the definitions of some concepts but were able to give examples of the concepts. Apart from this, in some items the distractors were too close to the correct option, hence they were reworded.

With the pre-test, five items had low difficulty level. Two of these items discriminated negatively while one had zero discriminating power. One item had a high difficulty level. The six items were therefore eliminated leaving fifty-four questions for the test. Table 26 shows the difficulty level and discriminating powers of the sixty items.

TABLE 26

DIFFICULTY AND DISCRIMINATING POWER OF ITEMS ON PRETEST

Item	Difficulty	Discriminating Power
1.	52	0.30
2.	27	0.20
3.	23	0.37
4.	80	0.20
5.	70	0.47
6.	27	0.20
7.	27	0.27
8.	73	0.60
9.	45	0.57
10.	57	0.33
11.	03*	-0.4*
12.	20	0.13
13.	63	0.60
14.	50	0.60
15.	70	0.47
16.	83	0.23
17.	85	0.66
18.	37	0.27
19.	27	0.51
20.	43	0.47
21.	70	0.33
22.	23	0.20

TABLE 26 Cont'd:

ITEM	DIFFICULTY	DISCRIMINATING POWER
23	30	0.53
24	20	0.27
25	30	0.23
26	20	0.47
27	07*	-0.40*
28	63	0.47
29	37	0.58
30	23	0.36
31	40	0.53
32	23	0.61
33	27	0.37
34	23	0.71
35	24	0.59
36	20	0.23
37	33	0.60
38	27	0.53
39	60	0.33
40	33	0.53
41	90*	0.06
42	66	0.40
43	81	0.20
44	76	0.33

Table 26 Cont'd

ITEM	DIFFICULTY	DISCRIMINATING POWER
45	74	0.53
46	20	0.61
47	70	0.20
48	27	0.20
49	20	0.27
50	80	0.40
51	53	0.67
52	70	0.33
53	17*	0.39
54	30	0.53
55	06*	0.27
56	23	0.60
57	37	0.45
58	09*	0.60
59	77	0.20
60	60	0.57

These final tests were prepared for use in the main study. These tests are in Appendices A - F.

Apart from the alterations in the tests, the number of groups used in the second phase of the study was changed from two to three in order to cater for more methods used by classroom teachers. The groups consist of Experimental groups I, II and the control group.

3.9 MAIN STUDY

ADMINISTRATION AND SCORING OF INSTRUMENTS

During the first part of the study the test on computation and test on knowledge of concepts were administered on the same day in each school while the test on comprehension of mathematical language and problem-solving were administered the following day. The administration of the four tests in the fifteen schools was completed within a month. The three objective tests were scored by awarding one point to a correct item and zero to a wrong item. Students' scores in each of the test were converted to percentages for easy comparison. For the test on mathematics problem solving, a marking scheme was used in scoring.

During the second part of the study the pre-test was administered to all students in the six schools selected for this phase within one week.

Like the other objective tests, the pre-test was scored by awarding one point to each correct item and a zero to a wrong item. Students' scores were converted to percentages.

Two classes in two schools were grouped into experimental I, II and control groups. Table 27 shows the distribution of students in each group.

TABLE 27

DISTRIBUTION OF STUDENTS IN THE THREE GROUPS:

GROUPS	SCHOOLS	NUMBER OF STUDENTS
Experimental I	1 and 3	80
Experimental II	4 and 5	80
Control	2 and 6	80
Total		240

After grouping the students into three groups, students in each group were grouped into higher achievers, average achievers and lower achievers.

The highest 25% formed the higher achievers, the middle 50% the average achievers and the lower 25% the lower achievers. Table 28 shows the distribution of students according to their ability levels.

TABLE 28

DISTRIBUTION OF STUDENTS' ABILITY IN THE THREE GROUPS

TREATMENT GROUPS	ABILITY			
	HIGH	AVERAGE	LOW	TOTAL
Experimental I	20	40	20	80
Experimental II	20	40	20	80
Control	20	40	20	80
Total	60	120	60	240

The three groups were subjected to treatments.

Table 29 shows the treatment administered on each group.

TABLE 29

THE GROUPS TESTED AND THE TREATMENT ADMINISTERED

Groups	Treatment		
Experimental 1	Pre-test	Concept-teaching approach Combined with practice in problem solving	Post-test
Experimental II	Pre-test	Practice in problem-solving with the researcher	Post-test
Control	Pre-test	Practice in problem-Solving on their own	Post-test

As shown in table 29, the experimental group I was taught using a concept-teaching approach and practice in solving problems while students in the experimental group II only practised solving problems. The control group was not taught but were given the same practice exercises administered to the experimental groups.

The twenty-two difficult concepts identified during the first part of the study were grouped under eight topics. The topics were

1. Solid Figure - Cube, Cuboid, Triangular prism, Pyramid
2. Plane Shapes - Square, Trapezium, Rhombus, Kite, Pentagon
3. Convex Polygon and line of Symmetry
3. Angles - Complementary, Supplementary, Adjacent, Corresponding and Vertically Opposite.
4. Congruency
5. Simultaneous Linear Equations
6. Similar Shapes
7. Variation
8. Trigonometric Ratio

Pre-treatment Meeting

Before the treatment sessions commenced, the researcher made an initial visit to each of the schools, and with the help of the class teachers tried to establish a rapport between herself and the students. During the administration of the pre-test, the students were made to understand that the test had nothing to do with their class evaluation. The class teachers urged the students to co-operate with the researcher. The researcher told the students the aim of the study in simple terms - to gather information on the problems encountered in learning mathematics and to suggest possible solutions to the authorities concerned.

After the pretest, the researcher thanked the students for their co-operation and informed them of her subsequent weekly visits.

Since most of the concepts that are being taught have been treated with the students before now and due to the scheme of work designed by the School Management Committee, the researcher was made aware that the teaching may not be done throughout all mathematics periods for the week. After discussions with the head of mathematics and class teachers, the researcher arrived at some regular visits to the schools which included free period and periods of teachers who were on maternity leave.

The researcher taught the experimental groups four periods a week for five weeks.

The Experimental Sessions

Experimental Group I

As discussed earlier, the twenty two concepts were categorised into concrete, abstract and those that belong to the two groups (i.e., concrete/abstract). With the experimental group I, solid figure was taught as a concrete concept while plane shapes were taught as both concrete and abstract concepts. Angles, congruency, simultaneous linear equations, similar shapes, variations and trigonometric ratios were taught as abstract concept. After each topic, students were given practice exercises which were later solved in class. The practice exercises and their solutions are in Appendix 'L'.

Experimental Group II

Students in experimental group II were asked to solve the same practice exercises given to the experimental group I. The researcher later led the students in solving the exercises in the class. While solving the problems with this group, the researcher asked questions, made sure students understood the problem and how to solve each problem. When solving the problems on the blackboard, students were allowed to ask questions on the problem being solved.

The researcher practised solving problems with experimental group II for four periods per week for a duration of five weeks. The practice exercises and their solutions are in appendix "L".

The Control Group:

The control group was not taught. They were however given the same problems treated with the experimental groups to practise with on their own.

Post Treatment Assessment:

At the end of the training sessions both experimental groups I and II and the control group were given the post-test. With the post-test, a marking scheme was used in scoring students' papers.

Control of Extraneous Variables:-

There were different measures taken to control for extraneous variables which might affect the results obtained in this study. The schools used were selected among many schools within Lagos State. The subjects in the study consisted of a cluster sample made up of form four secondary school classes. This implies that they were already equated on a number of variables such as age, intellectual ability and other social skills capable of affecting their performance.

There were equal representation of subjects in the three groups in the study (80 each). Teacher effect was adequately controlled for as the researcher taught both experimental groups.

CHAPTER FOUR

PRESENTATION OF RESULTS

4.1 INTRODUCTION:

In view of the overwhelming evidence pointing to the fact that students perform poorly in problem-solving and which impairs achievement, this research sets out to examine the relationship between knowledge of concepts and problem solving and to test the effectiveness of a concept teaching approach on the improvement of students' problem solving performance. Accordingly, the following hypotheses were formulated.

1. There will be no significant relationship between students' knowledge of mathematical concepts and their problem solving ability
2. There will be no significant relationship between students' knowledge of mathematical concepts and performance in solving mathematical problems in the presence of the following factors, namely: computational ability; and comprehension of mathematical language.
3. There will be no significant difference in the mathematical problem-solving performance of students in the experimental and control groups.
4. There will be no significant difference in the mathematical problem-solving performance of higher achievers in the experimental and control groups.

5. There will be no significant difference in the mathematical problem-solving performance of average achievers in the experimental and control groups.

6. There will be no significant difference in the mathematical problem-solving performance of lower achievers in the experimental and control groups.

7. There will be no significant interaction between students' ability levels and the instructional approach they are exposed to.

In order to examine these stated hypotheses, six research tools were designed and used. These were, test on computation, test on knowledge of mathematical concepts, test on comprehension of mathematical language, test on mathematics problem-solving ability, pre-test to ascertain equality of groups and post-test to find out the effect of the teaching approaches. Apart from these, a concept-teaching approach combined with practice in solving mathematical problems was used in teaching the experimental group I students. This chapter therefore presents the results of the statistical analysis of the data obtained as they relate to the stated hypotheses. Relevant conclusions were drawn on the basis of acceptance or rejection of the hypotheses. Graphical illustrations were used to further clarify data shown in tables. The chapter is divided into the following sections:-

(i) Performance of subjects and bivariate correlations among variables in the study.

(ii) Multivariate analysis of the relationship between problem-solving performance and three other variables operating jointly.

(iii) Performance of students in the pre-test.

(iv) Results of the performance of experimental and control groups on the post-test.

(v) Results of the performance of higher achievers in the experimental and control groups on the post-test.

(vi) Results of the performance of average achievers in the experimental and control groups on the post-test.

(vii) Results of the performance of lower achievers in the experimental and control groups on the post-test.

(viii) Interaction between ability and treatment.

(ix) Summary of results.

4.2 Performance of Subjects and Bivariate Correlations Among Variables in the Study

The variables which featured in the first phase of this study and their labels are tabulated below:

TABLE 30

VARIABLE LIST IN THE MAIN STUDY

Variable	Variable Label
Knowledge of Mathematical Concepts	Var 01
Computation Ability	Var 02
Comprehension of Mathematical Language	Var 03
Mathematics Problem-Solving	Var 04

The mean scores and standard deviations of subjects on the variables 01 to 04 are presented in Table 31. There were 588 cases.

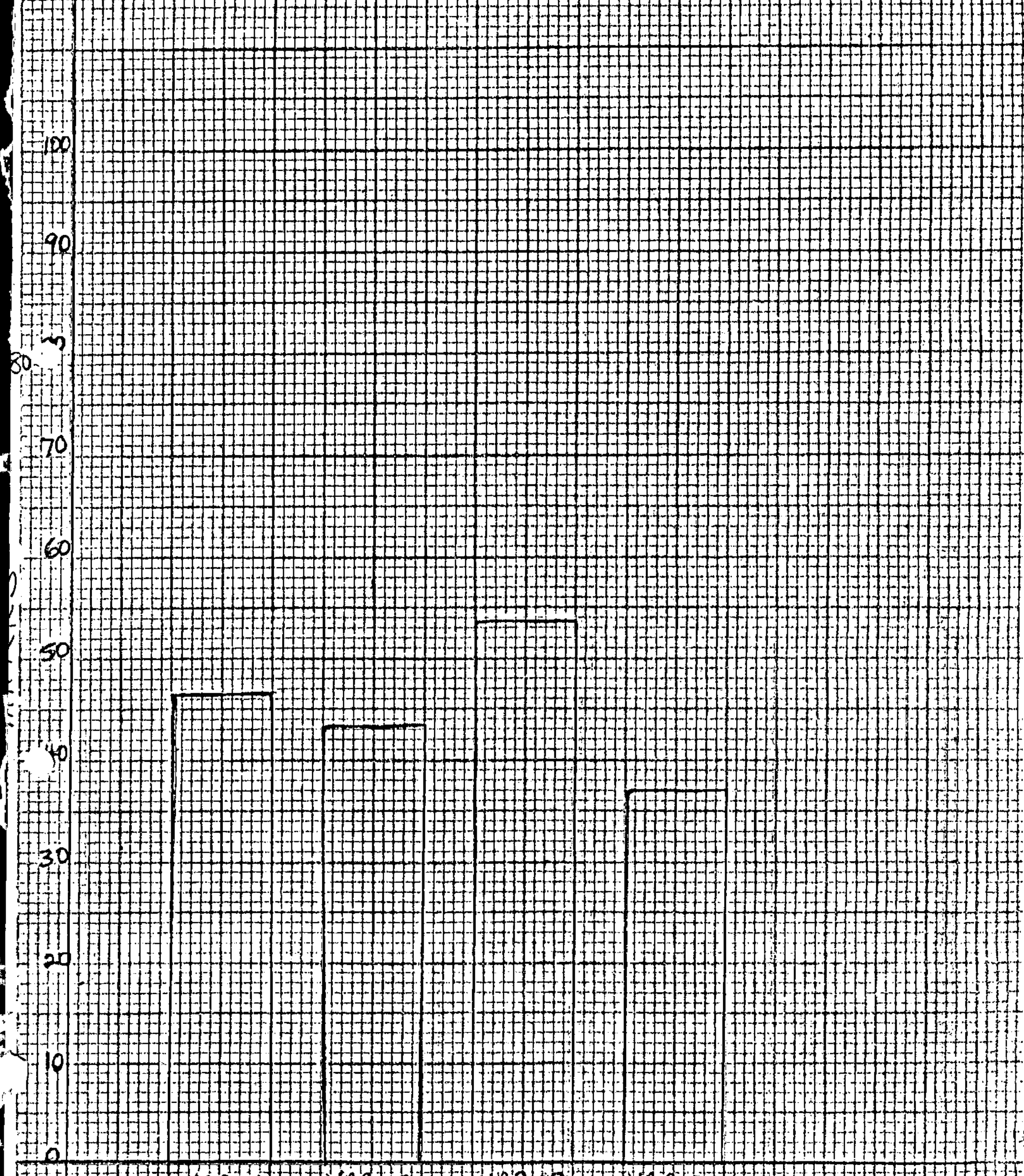
TABLE 31

MEANS, STANDARD DEVIATIONS OF VARIABLES 01 TO 04

Variable	Mean	Standard Deviation
Var 01	46.95	15.89
Var 02	43.53	18.00
Var 03	53.88	15.78
Var 04	37.00	15.97

Figure 2 shows a graphical presentation of the mean scores in percentages of all subjects on the four tests that is, test on knowledge of mathematical concepts, test on computation, test on comprehension of mathematical language and test on mathematics problem-solving ability.

MEAN SCORES IN PERCENTAGES OF STUDENTS ON TKMC, TC, TCM



TESTS

FIG 11

The intercorrelations among the variables 01 to 04 are shown in Table 32. Results are rounded off to two decimal places.

TABLE 32

INTERCORRELATIONS AMONG THE FOUR VARIABLES

VARIABLE	01 Knowledge of mathematical concepts	02 computational ability	03 comprehension of mathematical language	04 problem-solving ability
01	1.0			
02	0.73	1.0		
03	0.81	0.74	1.0	
04	0.91	0.68	0.77	1.0

Tables33 to 35 show selected statistics obtained for Bivariate Regression analysis involving variable 04 and each of the variables 01, 02 and 03. The statistics were generated by subprogram regression.

TABLE 33

SELECTED STATISTICS FOR BIVARIATE REGRESSION OF PROBLEM-SOLVING WITH KNOWLEDGE OF CONCEPTS

			DF	SS	MS	F
Multiple R	0.91	Analysis of Variance				
R Squared	0.82					
		Regression	1	123001.36	123001.36	2697.90
Standard error	6.74	Residual	586	26716.63	45.59	
B	0.97					
Constant	-5.76					

TABLE 34

SELECTED STATISTICS FOR BIVARIATE REGRESSION OF PROBLEM-SOLVING WITH COMPUTATIONAL ABILITY

			DF	SS	MS	F
Multiple R	0.68	Analysis of Variance				
R. Squared	0.46					
		Regression	1	69241.19	68241.19	504.19
Standard error	11.70	Residual	586	80476.81	137.33	
B	0.60					
CONSTANT	10.74					

TABLE 35

SELECTED STATISTICS FOR BIVARIATE REGRESSION OF PROBLEM-SOLVING WITH COMPREHENSION OF MATHEMATICAL LANGUAGE

			DF	SS	MS	F
Multiple R	0.77	Analysis of Variance				
R Squared	0.59					
		Regression	1	89026.41	89026.41	859.58
Standard error	10.16	Residual	586	60691.58	103.57	
B	0.78					
CONSTANT	-5.04					

The bivariate correlation between problem-solving ability and knowledge of mathematical concepts is significant at less than 1% level. The first null hypothesis of the study which stated that there will be no significant linear relationship between students' knowledge of mathematical concepts and their problem solving ability is therefore rejected. Therefore, there is a positive significant linear relationship between problem-solving ability and knowledge of concepts. Also the bivariate correlations between problem-solving ability and computational ability and comprehension of mathematical language were significant at less than 1% level.

The results in tables 33 and 35 show that the highest correlation is between problem-solving ability and knowledge of concepts. The correlation between problem-solving and mathematical language is the next highest.

Knowledge of concepts accounts for about 82% of the performance score in problem solving while comprehension of mathematical language accounts for about 59% independently. The least correlation is between computational ability and problem-solving performance. Computational ability explains for about 46% of the variation in problem-solving scores.

4.3 MULTIVARIATE ANALYSIS OF THE RELATIONSHIP BETWEEN PROBLEM SOLVING ABILITY AND THREE OTHER VARIABLES OPERATING JOINTLY

The variable 04, mathematics problem-solving ability is the dependent variable. The variables 01, 02 and 03 are the independent variables in this study. Table 36 shows the result of multivariate analysis carried out on the data using the SPSS program regression for 588 cases.

TABLE 36

SELECTED STATISTICS FROM MULTIPLE REGRESSION

			DF	SS	MS	F
Multiple R	0.91	Analysis of Variance				
R^2	0.83	Regression	3	123694.99	41231.66	925.31
Standard Error	6.68	Residual	584	26023.00	44.56	

The multiple correlation coefficient, 0.91 is significant at less than 0.01 level. The second null hypothesis of the study is therefore rejected. The multiple correlation coefficient between performance in problem solving and the independent variables (computational ability, knowledge of mathematical concepts and comprehension of mathematical language) is therefore significantly different from zero.

The three independent variables operating jointly explain for about 83% of the variation in problem-solving scores.

Kim et al (1975) suggested that the partial regression coefficients may be "used as measures of the influence of each independent variable upon tolerance with adjustments made for all other independent variables".

The Beta value in Table 37 could therefore be used as a measure of the effect of an independent variable when the effects of all other variables are statistically controlled.

TABLE 37

VARIABLES IN THE MULTIPLE REGRESSION EQUATION

Independent Variables	B	Beta	F
Var 01 Knowledge of Concepts	0.814	0.81	686.44 (S)
Var 02 Computation	0.0095	0.01	0.16 (NS)
Var 03 Comprehension of mathematics language	.111	0.11	12.25 (S)
CONSTANT	-7.62		

The F ratio opposite each Beta is used to test the significance of Beta

Table 37 shows that the Beta for computation is not significant at 5% level. The Betas for knowledge of mathematical concept and comprehension of mathematical language are significant at less than 0.01 level. The result in Table 37 also shows the order of the influence of each variable in problem-solving ability. It shows that knowledge of concepts has a greater influence on problem solving than comprehension of mathematical language.

4.4 PERFORMANCE OF STUDENTS IN THE PRE-TEST

Prior to the administration of the treatment, the pre-test was administered to the three groups to ascertain the equality of the groups. Table 38 shows the F ratio obtained in the comparisons

TABLE 38

STUDENTS' PERFORMANCE IN THE PRE-TEST

Source of Variance	SS	DF	MS	F
Between Groups	128.91	2	64.45	0.27*
Within Group	55888.89	237	235.82	
Total	56017.80	239		

*Not Significant

Furthermore, to find out whether or not there is any difference between the means of each ability group, analysis of variance was done. Tables 39 to 41 show that there is no significant difference between the means of the three ability groups.

TABLE 39

HIGHER ACHIEVERS' PERFORMANCE IN THE PRE-TEST

Source	SS	DF	MS	F
Between Groups	118.53	2	59.27	1.44*
Within Group	2346.20	57	41.16	
Total	2464.73	59		

*Not Significant.

TABLE 40

AVERAGE ACHIEVERS' PERFORMANCE IN THE PRETEST

Source	SS	DF	MS	F
Between Groups	217.115	2	108.56	1.77*
Within Group	7179.48	117	61.36	
Total	7396.59	119		

* Not Significant

TABLE 41

LOWER ACHIEVERS' PERFORMANCE IN THE PRE-TEST

Source	SS	DF	MS	F
Between Group	0.53333	2	0.267	0.02*
Within Group	793.79	57	13.93	
Total	794.33	59		

* Not Significant

Table 42 shows the means and standard deviations of the three ability groups in the pretest.

TABLE 42

MEANS AND STANDARD DEVIATIONS OF THE ABILITY GROUPS IN THE PRE-TEST.

	Number	Means	Standard Deviation
HIGHER ACHIEVERS			
Experimental I	20	51.8	6.99
Experimental II	20	48.5	5.02
Control	20	51.0	7.03
AVERAGE ACHIEVERS			
Experimental I	40	26.28	8.15
Experimental II	40	27.6	7.96
Control	40	29.55	7.37
LOWER ACHIEVERS			
Experimental I	20	11.9	3.58
Experimental II	20	11.7	3.39
Control	20	11.9	4.18
The Whole Group	240	29.47	15.35

4.5 Performance of Experimental and Control Groups on the Post Test

One of the concerns of the research was to examine whether the use of a concept-teaching approach with practice in problem-solving will facilitate the problem-solving abilities of form four secondary school students. To examine this, a pre-test was designed and administered to the experimental and control groups used in the study and scores were recorded. Thereafter, a five week teaching session was held with experimental group I who were taught using a concept teaching approach combined with practice in problem-solving. The experimental group II only practiced solving the same problems treated with experimental group I. The control group did not receive any teaching. However, they were asked to practice solving the same problems treated with the experimental groups. The same post-test was administered to the three groups after the teaching sessions. It was hypothesised that there will be no significant difference in the mathematical problem-solving performance of students in the experimental and control groups. From this hypothesis, the means of the three groups were not to be significantly different.

Table 43 shows the means and standard deviations for the experimental and control groups on the post-test while Table 44 shows the result of the analysis of variance.

TABLE 43

MEANS AND STANDARD DEVIATIONS FOR THE EXPERIMENTAL AND CONTROL GROUPS ON THE POST-TEST

Groups	Number	Mean	Standard Deviation
EXPERIMENTAL I	80	46	14.90
EXPERIMENTAL II	80	40.85	15.16
EXPERIMENTAL I	80	46	14.90
CONTROL	80	28.3	16.79
EXPERIMENTAL II	80	40.85	15.16
CONTROL	80	28.3	16.79
WHOLE GROUP	240	38.38	17.26

TABLE 44

ANALYSIS OF VARIANCE OF STUDENTS' PERFORMANCE

Source	Sum of Squares	D.F	Mean Square	F
Between	13261.73	2	6630.87	26.77
Within	58699.18	237	247.68	
Total	71960.90	239		

The result shows that F-ratio is significant at 0.05 level. The null hypothesis is therefore rejected. Hence, there is a difference in the mathematical problem-solving performance of students in the experimental and control groups. However, a significant F does not tell us which means differ significantly, but that at least one is reliably different from some others (Garrett and Wood worth, 1960). The normal belief is that t-tests performed on all possible pairs of means involved in the F-test would reveal the significant differences. This Glass and stanley (1970) categorically stated is an unacceptable methodology.

The T-test according to them is meant to be properly assigned to two random samples and not two sample means selected from a collection of N means. The Tukey and Scheffe multiple comparison procedures are the most useful procedures to employ. However, Scheffe method is used for unequal sample sizes. The Tukey method was employed for this study since sample sizes are equal and the method is regarded as superior to the Scheffe method as regards detection of significant differences. A greater number of significant differences between means are produced by the Tukey method with shorter confidence intervals.

Table 45 presents the differences obtained between the means of the three groups used in the study.

TABLE 45

DIFFERENCE IN MEANS OF EXPERIMENTAL AND CONTROL GROUPS ON POST-TEST

		Difference		
	Mean	Control	Experimental II	Experimental I
Control	28.3	0	12.55	17.7
Experimental II	40.85	- 12.55	0	5.15
Experimental I	46	- 17.7	- 5.15	0

To conclude if a significant difference occurs between the means of the groups, the confidence intervals around the differences between the three pairs are constructed as follows:

$$(\overline{xi} - \overline{xI\cdot}) \pm 1 - \alpha q_{1}, \text{N-IJ} \sqrt{MSw/n/J}$$

where $1 - q_1$, N-IJ is the 100(1 - x) percentile point in the studentized range distribution with 1 and N-IJ degrees of freedom. (1, J are number of levels of each factor).

MSw is the mean-square within factor levels from the one-way ANOVA

n is the number of observations in any one group.

The results obtained using the above construction is outlined on Table 46.

TABLE 46

SIMULTANEOUS CONFIDENCE INTERVALS AROUND PAIRED MEAN DIFFERENCES BY TUKEY METHOD ($I = 3$ $\alpha = .05$)

Difference between Means	$.95\ q_{3},237\sqrt{MSw/n}$	Confidence Interval
$\overline{xI} - \overline{xII} =$ 5.15	3.31 (1.76) = 5.82	5.15 ± 5.82 = (0.67, 10.97) NS
$\overline{xI} - \overline{xc} =$ 17.7	5.82	17.7 ± 5.82 = (11.88, 23.52) S
$\overline{xII} - \overline{xc} =$ 12.55	5.82	12.55 ± 5.82 = (6.73, 18.37) S

Confidence interval on $\overline{xI} - \overline{xc}$ and $\overline{xII} - \overline{xc}$ do not include zero, hence the conclusion that a significant difference lies between the means of experimental group I and control and between experimental group II and control is reached. The confidence interval on the means of experimental groups I and II include zero, therefore no evidence of difference in their means exists. The results showed that the two experimental groups performed significantly better than the control although there is no significant difference between the experimental groups.

The confidence given by the Tukey method is 95% that these three conclusions are simultaneously correct.

The students` scores are presented in the graph shown in Figure 111. It is obvious that the three groups performed differently in the post-test. The experimental groups tend towards higher scores on the post-test than the control group. This confirms that the experimental groups performed better in the post-test than the control group.

STUDENTS' PERFORMANCE IN THE POST-TEST.

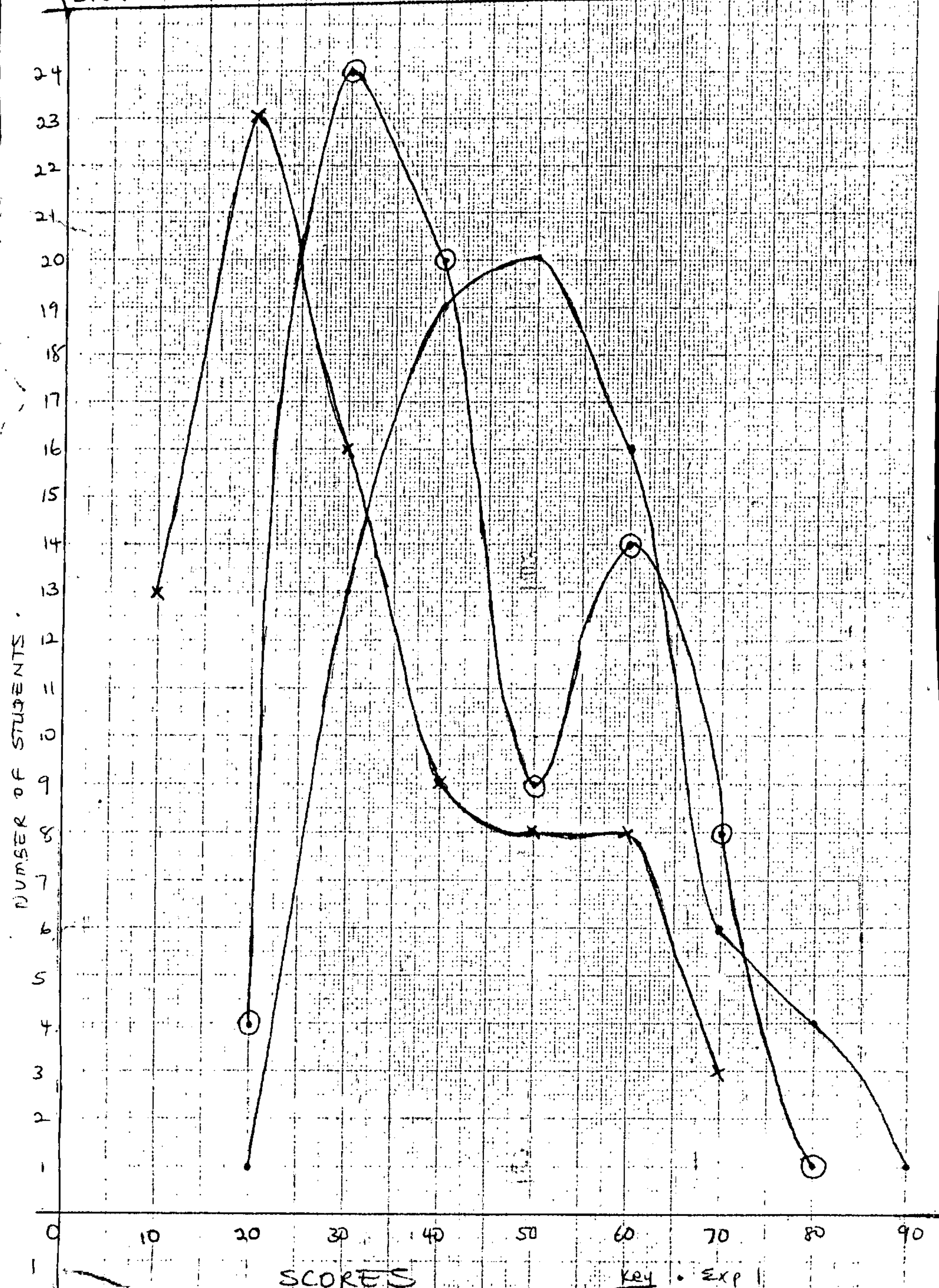


Fig III

4.5.1 Performance of Higher Achievers in the Experimental and Control Groups on the Post-test.

Hypothesis four stated that there will be no significant difference in the mathematical problem-solving performance of higher achievers in the experimental and control groups.

To test the hypothesis, higher achievers' scores in the post-test were analysed using analysis of variance. The result is presented in Table 47.

TABLE 47

ANALYSIS OF VARIANCE OF HIGHER ACHIEVERS' PERFORMANCE IN THE POST-TEST

Source	SS	DF	MS	F
Between Groups	1923.73	2	961.87	14.31*
Within Group	3831.20	57	67.21	
TOTAL	5754.93	59		

*Significant at 0.05 level

The result shows that the F-ratio obtained for the post-test problem solving ability of higher achievers is significant at 0.05 level. The null hypothesis which stated that there will be no significant difference between higher achievers' performance in the post-test is therefore rejected.

In order to determine the significance of the differences in the mean scores of the groups the confidence intervals around the differences between the three groups are constructed.

Table 48 shows the means and standard deviations of higher achievers' performance in the post-test while Table 49 shows the confidence intervals by Tukey method.

TABLE 48

MEANS AND STANDARD DEVIATIONS FOR HIGHER ACHIEVERS ON THE POST-TEST

Groups	Number	Means	Standard Deviation
Experimental I	20	65.2	8.86
Experimental II	20	61.6	8.22
Experimental I	20	65.2	8.86
Control	20	51.8	9.19
Experimental II	20	61.6	8.22
Control	20	51.8	9.19

TABLE 49

SIMULTANEOUS CONFIDENCE INTERVALS AROUND PAIRED MEAN DIFFERENCE FOR HIGHER ACHIEVERS

Difference between Means	$.95\ q_{3,57}\sqrt{MSw/n}$	Confidence Interval
$\bar{x}_I - \bar{x}_{II} = 3.6$	3.4 (1.83) = 6.22	$3.6 \pm 6.22 = (-2.62, 9.82)$ NS
$\bar{x}_I - \bar{x}_c = 13.4$	6.22	$13.4 \pm 6.22 = (7.18, 19.62)$ S
$\bar{x}_{II} - \bar{x}_c = 9.8$	6.22	$9.8 \pm 6.22 = (3.58, 16.02)$ S

The results showed that the two experimental groups performed significantly better than the control group. However, there is no significant difference between the performance of higher achievers in the experimental groups I and II.

The ratings of the three groups are further presented in the graph shown in Figure IV. From the graph, it can be seen that the post-test scores of higher achievers in the experimental groups I, II and control group range from 60-90, 50-80 and 40-70 respectively. As the scores increases, the number of students reduces.

4.5.2 Performance of Average Achievers in the Experimental and Control groups on the Post-Test

It was stated in hypothesis five that there will be no significant difference in the mathematical problem-solving performance of average achievers in the experimental and control groups. To test the hypothesis, average achievers' performance in the post-test were analysed.

Table 50 shows the analysis of variance of average achievers' performance in the post-test.

TABLE 50

ANALYSIS OF VARIANCE OF AVERAGE ACHIEVERS' PERFORMANCE IN THE POST-TEST

SOURCE	SS_e	DF	MS	F_1
Between Groups	5889.29	2	2944.65	29.12*
Within Group	11830.48	117	101.12	
Total	17719.77	119		

*Significant at 0.05 level

The F ratio obtained is significant at 0.05 level. The null hypothesis is therefore rejected. Hence, there is a significant difference in the mathematical problem solving performance of students in the experimental and control groups.

Since the F-test refutes the null hypotheses, further analysis is carried out to evaluate mean differences. Table 51 shows the mean scores and standard deviations of average achievers on the post-test.

TABLE 51

MEANS AND STANDARD DEVIATIONS FOR AVERAGE ACHIEVERS ON THE POST-TEST

Groups	Number	Means	Standard Deviation
Experimental I	40	41.53	10.38
Experimental II	40	36.25	10.31
Experimental I	40	41.53	10.38
Control	40	24.75	9.04
Experimental II	40	36.25	10.31
Control	40	24.75	9.04

Table 52 shows the multiple comparison method using Tukey's approach.

TABLE 52

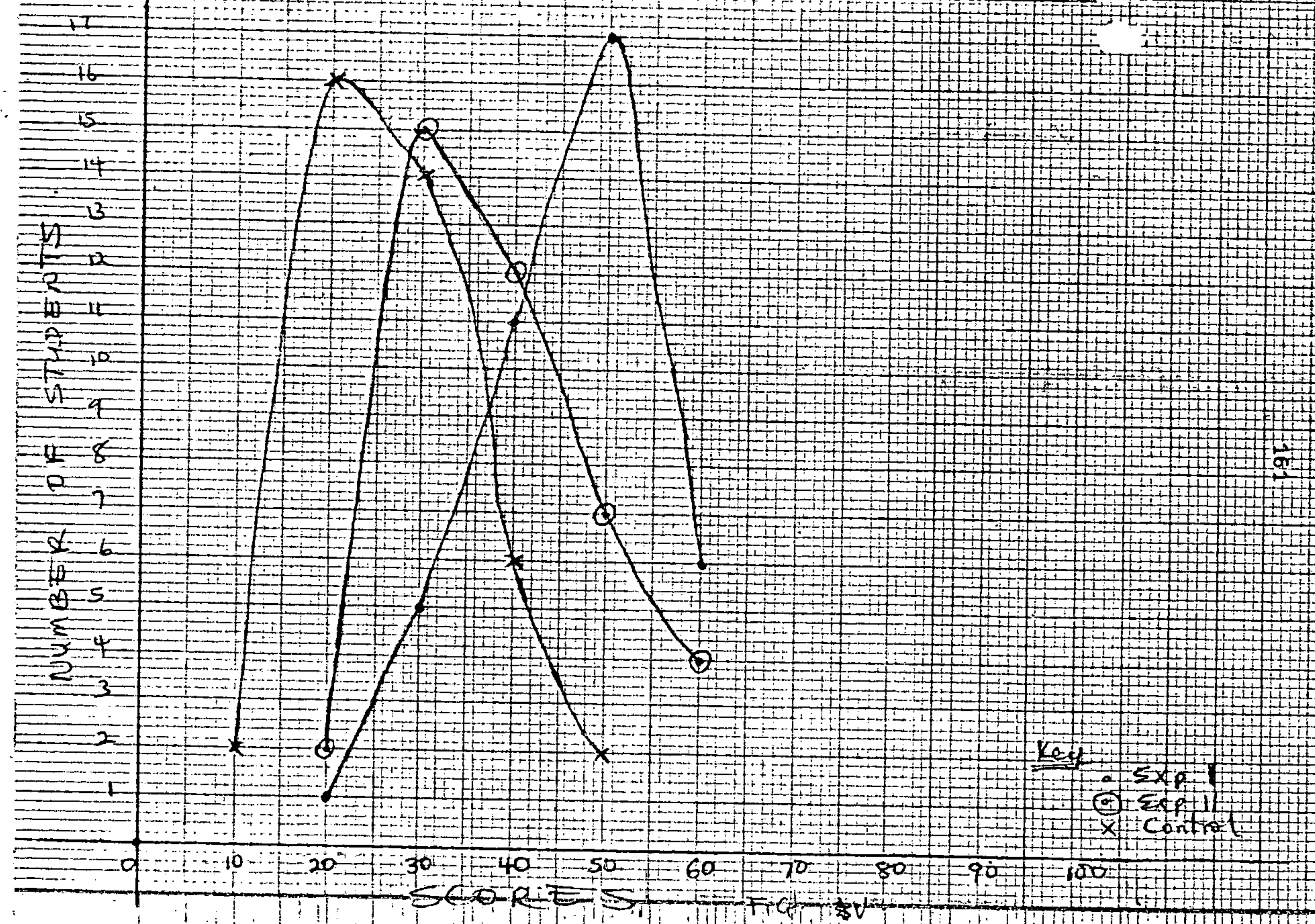
SIMULTANEOUS CONFIDENCE INTERVALS AROUND PAIRED MEAN DIFFERENCE FOR AVERAGE ACHIEVERS

Difference between Means	$.95\, q_{3,117}\sqrt{MSw/n}$	Confidence Interval
$\overline{xi} - \overline{xii} = 5.28$	3.36 (1.59) = 5.34	5.28 ± 5.34 = (0.06, 10.62) NS
$\overline{xi} - \overline{xc} = 16.78$	5.34	16.78 ± 5.34 = (11.44,22.12) S
$\overline{xii} - \overline{xc} = 11.50$	5.34	11.50 ± 5.34 = (6.16, 16.84) S

The results above show that students in experimental group I performed better then their counterparts in experimental group II though not significant. Apart from this, the groups who were taught, performed significantly better than the control group.

The performance of the students is graphically presented in Figure V. Students in experimental group I tend to have higher scores on the post-test than those in the experimental group II who in turn had higher scores than their counterparts in the control group.

AVERAGE ACHIEVERS' PERFORMANCE IN THE POST-TEST



161

4.5.3 Performance of Lower Achievers in the Experimental and Control Groups on the Post-Test.

Hypothesis six stated that there will be no significant difference in the mathemtical problem-solving performance of lower achievers in the experimental and control groups.

To test the hypothesis, lower achievers' scores in the post-test were analysed used analysis of variance. The result is presented in Table .53.

TABLE 53

ANALYSIS OF VARIANCE OF LOWER ACHIEVERS' PERFORMANCE IN THE POST-TEST

Source	SS	DF	MS	F
Between Groups	6060.43	2	3030.33	50.08*
Within Group	3449.3	57	60.51	
Total	9509.73	59		

*Significant at 0.05 Level

The result shows that the F-ratio obtained for the post-test problem solving ability of the lower achievers is significant at 0.05 level. The null hypothesis which stated that there will be no significant difference between lower achievers' performance in the post-test is therefore rejected.

In order to determine the significance of the differences in mean scores of the group, multiple comparison analysis was carried out. Table 54 shows the mean scores and standard deviations of lower achievers on the post-test.

TABLE 54

MEANS AND STANDARD DEVIATION FOR LOWER ACHIEVERS ON THE POST-TEST

Groups	Number	Mean	Standard Deviation
Experimental I	20	35.75	9.11
Experimental II	20	29.3	7.78
Experimental I	20	35.75	9.11
Control	20	11.9	5.38
Experimental II	20	29.3	7.78
Control	20	11.9	5.38

Table 55 shows the multiple comparison method using Tukey's approach.

TABLE 55

SIMULTANEOUS CONFIDENCE INTERVALS AROUND PAIRED MEAN DIFFERENCE FOR LOWER ACHIEVERS

Difference between Means	$.95\, q_{3,57} \sqrt{\frac{MSw}{n}}$	Confidence Interval
$\overline{xI} - \overline{xII} = 6.45$	3.4 (1.74) = 5.92	6.45 $\pm$ 5.92(0.53, 12.37) S
$\overline{xI} - \overline{xc} = 23.83$	5.92	23.83 $\pm$ 5.92(17.91, 29.75) S
$\overline{xII} - \overline{xc} = 17.4$	5.92	17.4 $\pm$ 5.92 (11.48, 23.32) S

The result above revealed that the two groups who were taught performed significantly better than the control group. Also lower achievers in experimental group I who were taught using a concept teaching approach combined with practice in problem-solving performed significantly better than students in experimental group II who only received practice in problem solving. To highlight the performance of lower achievers in the post-test, the graph in figure VI is presented. The range of scores for the control group is 10-30 while that of experimental groups I and II are 30 - 60 and 20 - 50 respectively.

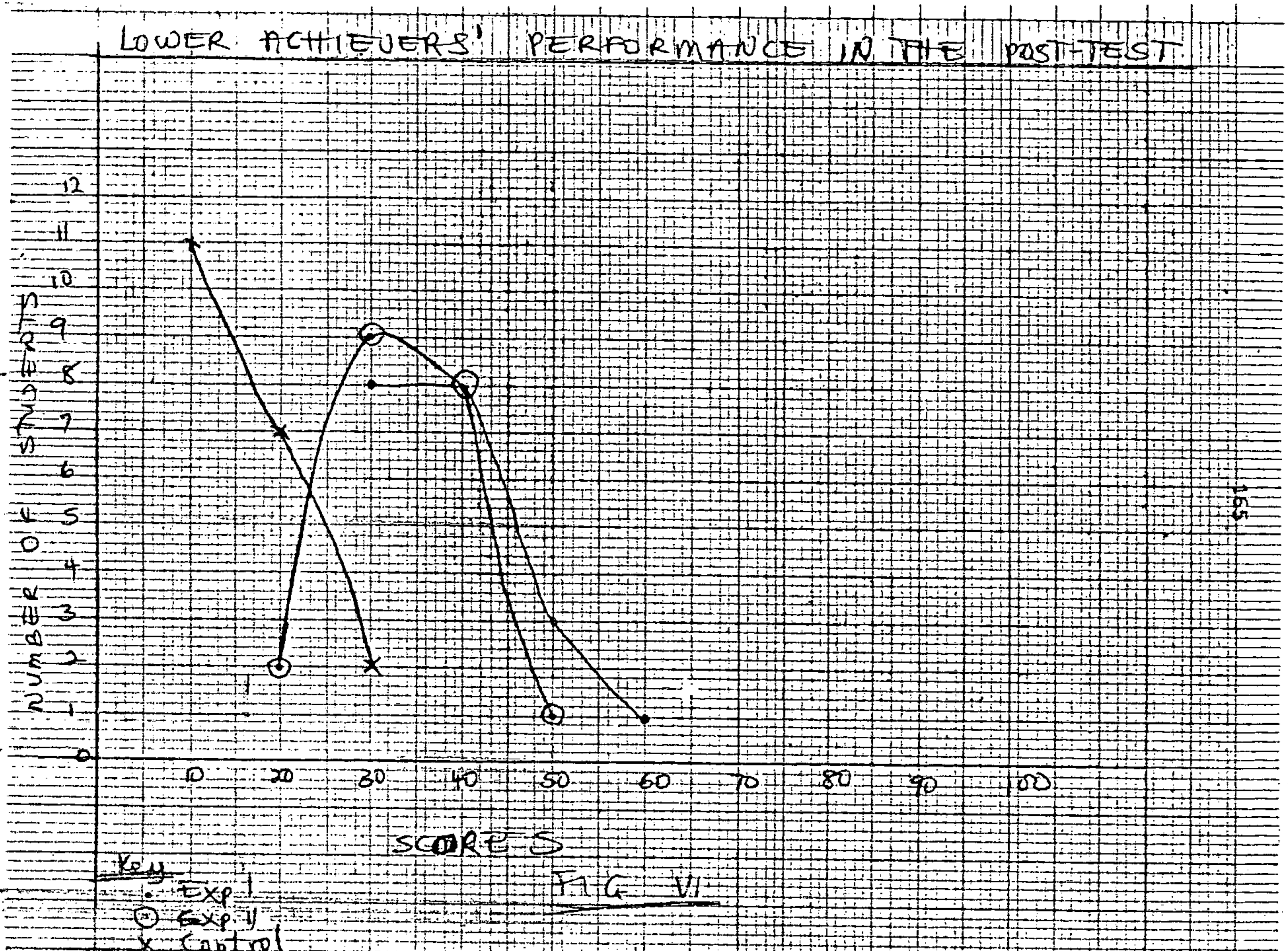

LOWER ACHIEVERS' PERFORMANCE IN THE POST-TEST
NUMBER OF STUDENTS
0
1
2
3
4
5
6
7
8
9
10
11
12
10
20
30
40
50
60
70
80
90
100
SCORES
Key
. Exp I
⊙ Exp II
x Control

FIG VI

4.6 Interaction between Ability and Treatment

To test hypothesis seven which states that there will be no significant interaction between students' ability levels and the instructional approach they are exposed to, a two-way analysis of variance was applied and the result is presented on Table 56.

TABLE 56

TWO WAY ANALYSIS OF VARIANCE OF STUDENTS' PERFORMANCE

Source of Variance	Sum of Squares	df	Variance Estimate	F
Ability	38691.142	2	19345.57	239.99*
Treatment	13264.78	2	6632.39	82.28*
Interaction	636.144	4	159.036	1.97
Error	18621.32	231	80.61	

* Significant

The result above shows that the F-ratio obtained for students' abilities and treatment were significant at 0.05 level. This confirms our previous findings that significant differences exist among the three groups and between the ability levels.

At 0.05 level, the interaction effect is not significant. Hence the seventh hypothesis is accepted. This implies that differences in effect of methods is constant for all levels of ability or differences in effect of ability is constant for all levels of method.

4.7 SUMMARY OF RESULT:

The result showed that there were significant positive linear correlations between problem solving scores and scores on the following variables: students' knowledge of concept, computational ability and ability to comprehend mathematical language. The highest positive correlation of 0.91 was obtained between problem-solving scores and scores on knowledge of concepts. The next highest correlation occurred between scores on problem-solving and scores on comprehension of mathematical language. The least correlation is between scores on computation and problem-solving.

The relationship between problem solving and each independent variable was altered when the other variables were statistically controlled. Consequently, computation had no significant influence on problem-solving when all the other variables were operating jointly.

However, test of knowledge of concepts and test of comprehension of mathematics language had significant influences when all other variables were controlled.

The size of the B value (Table 37) was used as a measure of the influence of each variable and this revealed the following order of importance. The knowledge of concepts was the most important variable for problem-solving. The comprehension of mathematical language was next in order of importance, while the ability to calculate is the least important of the three ability variables.

As revealed in Table 36, the multiple correlation coefficient between problem-solving scores and the three independent vairables - knowledge of concepts, computational ability and comprehension of mathematical language was 0.91 and was found to be significant at less than 0.01 level. All the three variables operating jointly accounted for about 83% of the variation in problem-solving scores.

Analysis of variance and multiple comparison were used to test the significance of the difference between mean scores of the experimental and control groups in the three ability groups -(higher achievers, average achievers, and lower achievers)- which learnt problem-solving in mathematics by using different methods. It was found that there were significant differences in the performance of the experimental and control groups on the post-test. The higher, average and lower achievers in the experimental groups performed significantly better than their counterparts in the control group. Also, lower achievers in experimental group I who were taught using a concept teaching approach combined with practice in problem-solving performed significantly better than those in the experimental group II who only received practice in problem-solving.

There was however no significant difference between higher and average achievers performance in the two experimental groups.

A concept teaching approach combined with practice in problem-solving or practice in problem-solving alone is therefore more effective than just asking students to practise solving problems on their own.

CHAPTER 5

DISCUSSION OF RESULTS

5.1 INTRODUCTION:

The purpose of this study was to analyse the relationship between form four students' knowledge of mathematics concepts and their performance in problem-solving. It was also the aim of the investigation to find out the extent to which knowledge of concepts affects problem solving in the presence of computational ability and comprehension of mathematical language. The study also aims at formulating a concept teaching strategy in mathematics and testing its effectiveness on an experimental group of students of three different ability groups.

In chapter four, the findings of the present study were presented in four parts. The bivariate correlations between pairs of variables were displayed. Next, the statistical data on the multiple regression analysis with problem-solving ability as the dependent variable and three independent variables were also presented. The third part showed data on the performance of students in the pre-test. Lastly, the results on the comparison of the effectiveness of a concept teaching approach combined with practice in problem-solving; and teaching based on practice in problem-solving alone were presented.

This chapter contains a discussion of the findings in chapter four and an attempt to interpret them correctly. The first section contains some interpretation on the findings in the study.

In the second part, some of the implications of the study are discussed. Section three contains the limitations of the study, while suggestions for future research are set out in section four.

5.2 FINDINGS IN THE STUDY AND THEIR INTERPRETATION

5.2.1 The Findings and the Interpretations among the Variables in the Study.

Table 32 shows that all the three variables krowledge of concepts, computational ability and comprehension of mathematical language correlated significantly with problem-solving ability with correlation coefficients of 0.91, 0.68 and 0.77 respectively.

The variation in scores on problem-solving, which is attributable to knowledge of concepts was about 82%. The variation included not only that due to its direct influence on problem-solving, but also that portion due to its indirect influence through mathematical language and problem solving ability path, computational ability and problem-solving ability path, without adjusting for other variables. Similar interpretation can be made for each of the variables involved in the study.

Knowledge of concepts and mathematical language are very strongly related. They have a co-efficient of correlation of 0.81, with one of them accounting for about 65% of the variation in the other. This finding supports Vigotsky's(1950) claim that a high correlation exists between language and concept formation.

In general, there was no pair of variables which was both highly correlated with problem-solving and had low bivariate correlation with each other. The prediction in problem-solving scores is therefore not substantially improved when knowledge of concepts is used as the first predictor, and other ahievement variables are subsequently introduced.

The data in Tables 33 to 35 show that knowledge of mathematical concepts, comprehension of mathematical language and computation are important, in that order, in the solution of problems in mathematics. They explain for about 82%, 59% and 46% respectively of the variation in problem solving scores. The importance of these variable is confirmed both in the daily classroom work in mathematics and by previous research findings (Jackson and Phillips, 1983; Carpenter and Reys, 1980, Tade, 1982; Kalejaiye, 1981). Students who have not grasped the underlying concepts in a problem find it extremely difficult if not impossible to proceed with the solution of a mathematics problem because of their inability to comprehend what is read and hence plan a correct method of solving the problem. Also, inability to comprehend mathematical language or compute often poses serious difficulty for students attempting to solve mathematics problems.

5.2.2 The Findings on the Multiple Regression Analysis of the Problem-Solving Ability

All the three independent variables - computation, knowledge of concepts and comprehension of mathematics language explain for about 83% of the variations in problem-solving scores when operating jointly. Also the multiple correlation coefficient, R, is significantly different from zero at less than 0.01 level. This led to the rejection of the null hypothesis and acceptance of a significant relationship between knowledge of concepts and problem solving, in the presence of computational ability and comprehension of mathematical language.

The regression coefficients for knowledge of concepts and comprehension of mathematical language were significant. But the regression coefficient for computation was not significant.

The results show that computation can be deleted from the regression equation. The remaining two variables -knowledge of concepts and comprehension of mathematical language are sufficient to explain the variation in problem-solving ability among form four students. Although the computational ability assists students in obtaining accurate results and correct final answers to problems, form four secondary school students have had enough practice in this skill at the primary and junior secondary school levels. Hence the non significant nature of computation in the regression equation.

The influence of the knowledge of concepts becomes more substantial than the influence of comprehension of mathematics language on problem solving performance in the presence of computational ability. This result is consistent with classroom observation: students who understood mathematical concepts thoroughly cannot only define but give examples, non-examples, attributes and categorically state why certain objects were not examples of the concept under discussion.

Therefore, students who have comprehended mathematical concepts are better placed in solving mathematical problems than students who can only comprehend mathematical language.

The knowledge of concepts enables students to understand the problems which lead easily to solutions. The knowledge of concepts is therefore, the most important factor in solving problems in mathematics and will become increasingly important as the students progress from the secondary school to the university .

5.2.3 Findings on the Experimental Groups:

A concept teaching approach which provides students with opportunity to actually practise solving problems in class was more effective than just giving practice in solving problems or asking form four students to practise solving problems on their own.

These findings can be explained as follows: a concept teaching approach is designed to help students understand the underlying concepts by giving examples, non-examples and by defining and identifying attributes of concepts. Practice in problem-solving entails allowing students to read the problem and ask questions to clarify meaning of terms or phrases before they proceed to solve the problem. Apart from these, the researcher led the students in solving the practice problems on the blackboard during which students were free to ask questions. Solving the problem on the blackboard was a source of feedback to the students.

However, the control group who were asked to practise solving problems on their own received no source of motivation or feedback from the researcher.

The fact that there was no significant difference between higher achievers taught by a concept-teaching approach combined with practice in problem solving and those taught by practice in solving problems only is not surprising. This is because the higher achievers are mainly the brilliant students, hence the basic knowledge learnt from a concept teaching approach might have been achieved during the questions asked while the researcher led the students in solving the problems. Apart from this, the higher achievers are capable of understanding mathematical concepts with minimum assistance from a teacher.

Also, with the average achievers the fact that students who practised solving problems with the researcher were free to ask questions where explanations were given caused the no significance difference between them and those who were taught using a concept teaching approach.

This finding that practice aids performance in problem-solving is supported by Friendsen (1980) and Lemonye and Tremblay (1986). However, with the lower achievers, there was a significant difference in the performance of those taught by a concept teaching approach combined with practice in solving problems and those taught by practice in solving problems only. This shows that a concept teaching approach in which students named examples, non-examples, defined and identified the attributes of a concept were essential basic skills needed for the improvement of problem-solving in mathematics.

The findings also show that Nigerian Secondary School Students in form four especially lower achievers will find a concept teaching approach combined with practice in problem-solving or practice in problem-solving alone as used in this study useful teaching techniques in overcoming their problem-solving difficulties in mathematics.

5.3 SUMMARY OF FINDINGS:

The findings show that:

1. there is a high significant relationship between students' knowledge of mathematical concepts and problem-solving ability.

2. knowledge of concepts contributed significantly to the prediction of problem-solving performance in the presence of computational ability and comprehension of mathematical language;

3. the study comfirms the previous research findings on the existence of a significant relationship between computational ability and ability to comprehend mathematical language with problem-solving ability;

4. these three variables are required in problem solving; knowledge of concepts, computational ability and ability to comprehend mathematical language;

Knowledge of concepts is the most important of them, since it explains for the highest percentage of variation in mathematics problem-solving scores when all the other variables are controlled.

5. A concept teaching approach combined with practice in problem solving or practice in problem solving alone is more effective than asking students to practise solving problems on their own.

6. Lower achievers who were taught using a concept-teaching approach combined with practice in problem solving performed significantly better than their counter-parts who only practised solving problems in the classroom. However, there was no significant difference in the higher and average achievers' performance in the two experimental groups.

The findings of the present study carry some educational implications for the improvement of concepts acquisition and problem-solving performance in Mathematics education.

5.4 IMPLICATIONS OF THE STUDY AND RECOMMENDATIONS FOR TEACHING MATHEMATICS IN NIGERIAN SECONDARY SCHOOLS.

A major problem that initiated this research is the poor academic performance of students in mathematics. This study has found and established that there is a positive significant correlation between knowledge of concepts and problem-solving in mathematics.

Apart from the fact that this study will lead to further research it has some basic implications for educational or curriculum practice.

This study has implications for authors of mathematics textbooks. Green and Schulman (1982) asserted that the elementary school mathematics textbooks begin formal development of problem solving at the third grade level (primary class III) through traditional story problem which requires students to choose an operation (addition, substraction, multiplication or division) to answer the question. However, observation shows that the mathematical problems presented in the secondary school mathematics curriculum in Algebra and Geometry are not solved by simply choosing the correct operation. Students in secondary schools are required to read, think, visualize, sketch, understand, plan a solution, carry out the plan and crosscheck the reasonableness of the answer. As a consequence of the lack of prior experience with this type of problem, the secondary students' grasp of the subject is impeded. In order to improve problem-solving performance of secondary students in mathematics provision should be made for introducing similar problems into the experience of upper elementary classes.

Bearing in mind that true problem-solving is a complex process which requires a wide repertoire of knowledge, not only of particular skills and concepts but also of the relationships among them and the fundamental principles that unify them, authors of mathematics textbooks should endeavour to provide different types of mathematical problems for students to practise with in class.

Apart from these, the contexts of many textbooks problems are contrived and uninteresting, making it difficult to generate interest. Problem solving abilities of students could be aroused it the task has a relevant and interesting context.

Another important factor in respect of concept development and problem solving is the curriculum. In this respect, curriculum developers must remember that within a class of students, there will be a considerable range in the extent of conceptual growth due to differences in natural ability, experiences, home environment and perhaps, in previous teaching. Moreover, a student's development may be uneven across the concepts. With this in mind, the curriculum developers must stress the necessity for individual and small work groups, giving to each student or group the materials which is suitable for them at their particular stage of understanding and development. The teaching of mathematics to a whole class at once should be discouraged.

For curriculum materials to promote concept development, appropriate concept testing and reinforcing activities should be included. Each should require students not only to identify new examples but also to cite the presence (or absence) of the concept characteristic.

Curriculum experts should also be charged with the responsibility of pressing for changes in teacher training colleges and faculties of education in Universities from the standpoint of students' intellectual and conceptual growth. Also, since problem-solving is a complex activity, a course specifically on problem-solving where various types of mathematics problems and the possible strategies of solving them will be discussed should be incorporated into the curriculum of teachers in training.

Students rely on memorizing rules, procedures, imitating examples in the text and those solved by the teacher when solving problems. Researchers have interpreted this to mean that there is inadequate development of the fundamental concepts (Confrey and Lanier, 1980; Confrey, 1981). Poor understanding of concepts is also accompanied by weak strategies for approaching mathematical problems or ideas. These weaknesses show up in various descriptions of mathematical processes (Kruteskii, 1976) and problem solving skills (Lesh et al, 1980). This implies that teachers should emphasize concepts and processes, provide non routine problems and concrete examples.

A concept teaching approach combined with practice in problem solving or practice in problem solving alone used in this study provides a ready solution to students' problems in mathematics. A concept teaching approach consists of identifying attributes, giving examples, non-examples and defining the concepts. In addition to these, students were assisted to understand the problem by explaining key terms, asking them questions and allowing them to try their hands on the problem first and then give a possible reason for the approach embarked upon before actually solving the problem on the blackboard for the students.

The teacher-student discussion that ensued during the solution of practice exercises are useful teaching techniques in solving mathematical problems. These and similar activities are therefore recommended as powerful instructional strategy during problem solving sessions in the classroom.

In addition to knowledge of concepts students should be able to comprehend mathematical language.

From the research conducted by Othenburn and Nicholson (1976) it was shown that students do not understand some mathematical words which mathematics teachers and examiners assume were comprehensible. This study also shows that comprehension of mathematical language aids problem solving ability. It is necessary therefore, that before questions are solved the important mathematical language should be explained and defined to give them the correct meanings such that the students know exactly what to do.

The teaching of mathematical language should also start from the primary school where the basic mathematics language is laid If this continues up to the secondary level, there will be a high chance of improving students' problem-solving ability.

Lack of understanding of questions leads to guessing of answers. Students need to be encouraged not to guess answers but to understand what the question is asking for. This will help students to be able to restate the question in their own words so that the working will lead unmistakingly to the answer. Apart from these many operations are involved in solving mathematics problems.

One, two or combination of many operations may be required for solving a particular problem. If certain operations are therefore used, reasons must be given for their use and why they are the correct operations.

The fact that computational ability does not enter the regression equation after controlling for knowledge of concepts and comprehension of mathematics language shows that, to some extent form four secondary school students are competent with the basic mathematical operations or manipulations. Hence the need to introduce calculators in Nigeria senior secondary schools.

With the introduction of calculators in secondary schools, more students will enjoy mathematics better than before since the calculator eliminates the least pleasant aspects of doing mathematics. Also, computational difficulties involved in solving statistical problems like mean and variance of grouped data will be virtually eliminated. With time consuming computations out of the way, it will be possible to give more attention to basic ideas and concepts which are fundamental to the learning of mathematics.

5.5 CONCLUSION

This study has established a significant positive relationship between knowledge of concepts and problem-solving ability in mathematics. Also the study identified basic skills which might foster problem-solving ability among form four secondary school students. These are:

1. Knowledge of concepts
2. Comprehension of mathematical language and
3. Computation

In order to improve students' performance in problem solving, these skills should be taught intensively at the secondary school level. It was also shown that teaching concepts with examples, non examples, identification of attributes and definition with practice in problem solving will assist students in solving problems.

A concept-teaching approach which consists of naming examples, non examples, identifying attributes and defining concepts could be classified under polya's (1967, 1970, 1981) first stage of his four phase problem-solving model of assisting students to understand a problem before proceeding to solve it.

This approach combined with practice in problem solving will assist students in solving mathematical problems.

Therefore teaching strategies involving these kinds of illustrations should continue to be employed in teaching students to solve mathematics problems.

5.6 Limitations of the study.

The first part of the study employed an ex-post-facto research design which is not a true experimental design. Therefore, the relationship between the dependent and independent variables were simply correlational and could not be attributed to causal factors.

The relative contributions of the independent variables were obtained from the values of B and beta weights. These only provided rough interpretations since the independent variables are highly correlated.

5.7 SUGGESTIONS FOR FURTHER RESEARCH

The limitations and the implications of this study should necessitate further research. The present study has focussed on algebra and geometry components of the mathematics curriculum. The researcher however realises that there are other areas of mathematics which require attention. It is therefore suggested that further research is needed on such aspects as number and numeration and statistics.

It was pointed out that this study was limited to schools in Lagos State. The findings of the study cannot be generalised to all schools in the country. Further research is therefore needed to find out if the same results will be obtained from the other states of the Federation. Apart from this, the sample used in the study consisted of students who live in an urban environment. Hence, further research is needed to find out if the same results will be obtained in a similar research involving a sample of students from a rural environment.

In the second part of the study a concept-teaching approach combined with practice in problem-solving was used with the experimental group I for only five weeks. Further research is needed in order to determine its cumulative effect over a longer period of time.

REFERENCES

Adeagbo A. O. (1987): Winning more students for Mathematics: Some strategies. Paper presented at the 28th Annual Conference of Science Teachers Association of Nigeria (STAN) held at Bauchi 17th - 22nd August, 1987.

Adeyegbe S. O. and O. A. Olamousi (1987): "Pattern of Enrolment and Achievement in WAEC Examinations in Science Mathematics and Technology subjects for the period 1980 - 1985" Paper presented at the Commonwealth African Regional Workshop on Gender Stereotyping in Education held at Accra, Ghana. 12th - 16 January, 1987.

Afemikhe O. A. (1988): The effect of Formative Tests on Problem Solving Habits and Mathematics Confidence. Journal of Research in Curriculum. Vol. 6, No. 1. Pp 139 - 148.

Aiken L. R. (Jr) (1972): Language Factors in Learning Mathematics. Review of Education Research. Vol. 42, Pp. 359 - 380.

Aiken L. R. and Dreger (1961): The effect of attitude on Performance in Mathematics. Journal of Educational Psychology. Vol. 52, Pp. 19 - 24.

Aiyedun J. O. (1981): The effect of Nursery School Education on attainments in Mathematics among Primary School children. An unpublished M. Ed dissertation. University of Ibadan, Nigeria.

Ale S. O. (1980c): Mathematics Education - Nigeria. Mathematical Association of Nigeria

Ale S. O. (1981): Difficulties facing Mathematics Teachers in Developing Countries - A case study: Educational Studies in Mathematics. Vol. 12, Pp. 479 - 489.

Balow, I. H. (1964) Reading and Computational Ability as Determinants of Problem-Solving. Arithmetic Teacher. Vol. 11, No. 1. Pp. 18 - 22

Bassler O. C., M.I. Beers and L. I. Richardson (1975): Comparison of Two Instructional Strategies for Teaching the Solution of Verbal Problems. Journal for Research in Mathematics Education. Vol. 6, No. 3, Pp. 170 - 177.

Bell E. S. and R, N. Bell (1985): Writing and Mathematical Problem-Solving: Arguments in Favour of Synthesis. School Science and Mathematics. Vol.85, No. 3. Pp. 210 - 219.

Benhaim D., G. Lappan and R. T. Houang (1985): Visualizing Rectangualar Solids made of small cubes: Analyzing and Effecting Students Performance. Educational Studies in Mathematics. Vol. 16 No. 4 pp. 389 - 407.

Bidwell J. K. (1983): Calculation Skills Vs Problem Solving: Scotland Vs Michigan. School Science and Mathematics Vol. 85, No. 8. pp. 682-693.

Billings G (1979): Do verbal labels help children acquire science concepts? School Science and Mathematics. Vol. LXXV, pp. 287 - 293.

Borasi R. (1986): On the Nature of Problems. Educational Studies in Mathematics. Vol. 17, pp. 125 - 141.

Brenman A. D. H. & W. P. Dunlop (1978): What are the Prime Factors of Reading Mathematics? Reading Improvement. Vol. 22, No. 2, pp. 152 - 159.

Brown E & T Abel (1965): Research in the teaching of elementary School Mathematics. Arithmetic Teacher, Vol. 12, pp. 547 - 549.

Brown M. (1978): Cognitive Development and the Learning of Mathematics in Cognitive Development in the School years. Edited by Ann Floyd Croom Helm, London in Association with the Open University Press.

Brueckner L. J., F. E. Grossnickleand J. Reckzeh (1957): Developing Mathematical Understanding in the Upper Grades Winston, 1957.

Bruner, J. S. (1956): Towards a Theory of Instruction.

Burton L. (1980): The Teaching of Mathematics to Young Children Using a Problem Solving Approach. Educational Studies in Mathematics. Vol. 11, pp. 43 - 58.

Carpenter, T. P. and Reys (1980): Problem Solving in Mathematics: National Assessment. Educational Leadership, Vol. 37, No. 7, pp 562 - 563.

Casey D. P. (1978): "Failing Students: A Strategy of Error Analysis in P. Costello (ed): Aspects of Motivation Melnourne, Mathematical Association of Victoria. 295 - 306.

Chacko, I. (1976): Use of Audio-Visual in Mathematics Teaching Within Benin City and its Environs. Unpublished PGDE Project, University of Nigeria, Nsukka.

Chapman T. R. (1966): The Teaching of Problem Solving in Arithmetic West African Journal of Education. Vol. 10, No. 3, pp. 139-142.

Charles R. I. (1985): The Role of Problem Solving. Arithmetic Teacher, Vol. 32, No. 6, pp. 48 - 50.

Charles R. I. (1980): Exemplification and Characterization moves in the Classroom Teaching of Geometry Concepts. Journal for Research in Mathematics Education. Vol 11, pp. 10 - 20.

Cherkas, B. M. (1978): The Teacher as a Role Model in Problem Solving. School Science and Mathematics. Vol. 78, No. 3, pp. 193 - 196.

Clement, M. A. (1980): Analyzing children's errors in Written Mathematical Tasks. Educational Studies in Mathematics. Vol. 11, No. 1 pp. 1 - 2.

Clement, J. (1982): Algebra - Word problem solution: Thought process underlying a common misconception. Journal for Research in Mathematics Education. Vol. 13: No. 1, pp. 16 - 30.

Clifford and Richard (1971): Introduction to Psychology International Student Edition. Kogakusha Co. Limited, Japan.

Confrey J. and Lanier P.(1980): Students Mathematical Abilities: A Focus for the Improvement of Teaching General Mathematics. School Science and Mathematics. Vol. 80, No. 7, November, 1980.

Confrey, J. (1981): Concepts, Processes and Mathematics Instruction For the Learning of Mathematics. Vol. 1, No. 5, July, 1981.

Dahmus, M. E. (1970): How to Teach Verbal Problems School Science and Mathematics. Vol. 70. No. 2, pp. 121 - 158.

Degman J. (1967): General Anxiety and Attitudes Towards Math. Graduate Research in Education and Related Discipline. Vol. 3, pp. 49 - 62.

Devault M. V. (1981): Doing Mathematics is Problem-Solving. Arithmetic Teacher. Vol. 28, No. 8, pp. 40 - 43.

Dunn C. S. (1983): The Influence of Instructional Methods on Concept Learning. Science Education. Vol. 67, pp. 647 - 656.

Earp N. W. (1970): Procedures for Teaching Reading in Mathematics. Arithmetic Teacher. Vol. 17, No. 7, pp. 575 - 579.

Eggen, P. D., D. P. Kauchak and R. J. Harder (1979): Strategies for Teachers. Information Processing Models in the Classroom. Prentice Hall Inc., Englewood Cliffs, (USA).

Ehrenberg S. D. (1981): Concept Learning: How to make it Happen in the Classroom. Educational Leadership. Vol. 39, No. 1, pp. 36 - 43.

Ekenstam A. A. & K. Greger (1983): Some Aspects of Children's Ability to Solve Mathematical Problems. Educational Studies in Mathematics. Vol. 14, No. 4, pp. 369 - 384.

Fajemidagba O. (1986): Mathematical Word Problem Solving: An analysis of Errors Committed by Students. The Nigerian Journal of Guidance and Counselling. Vol. 2, No. 1, pp. 23 - 30.

Fakuade R. A. (1983): An Aspect of Inability to do Mathematics Successfully among Nigerian Secondary School Students: A pilot study. Journal of Science Teachers Association of Nigeria Vol. 22, No. 1, pp. 58 - 65.

Ferguson G. A. (1981): Statistical Analysis in Psychology and Education. Mc. Graw-hill International Book Company. Japan.

Flener (1979): Teacher Assistance in Problem Solving. Educational Research Quarterly. Vol. 4, No. 2, pp. 43 - 51.

Friensen C. D. (1980): Meeting the Needs of Mathematically Gifted Students. School Science and Mathematics. Vol. 80, No. 2, pp. 127-130.

Gagne R. M. (1965): Conditions of Learning. 1st Edition. New York, Holt, Rinehart and Winston.

Gagne, R. M. (1977): Conditions of Learning. Third Edition. Holt, Rinehart and Winston.

Garrett H. E. and R. S. Woodworth (1970): Statistics in Psychology and Education. Longmans, Green and Co. New York. London.

Gibney T. C. and S. P. Meiring (1983): Problem Solving: A success Story. School Science and Mathematics. Vol. 83, No. 3, pp. 194 - 203.

Goldberg D. J. (1981): Problem Solving in Small Groups. International Journal of Mathematical Education in Science and Technology. Vol. 12, No. 2, pp. 211 - 217.

Greenes C. E. and Schulman (1982): Developing Problem Solving Ability with Multiple Condition Problems. Arithmetic Teacher. Vol. 30, No. 2, pp. 18 - 21.

Hollands R. (1983): A Dictionary of Mathematics. Sheck Wah Tong Printing Press Limited, Hong-Kong.

Jackson M. B. & E. R. Phillips (1983): Vocabulary Instruction in Ratio and Proportion for Seventh Graders. Journal for Research in Mathematics Education. Vol. 14, No. 4, pp. 337-343.

Jerman M (1973): Problem Length as a Structural Variable in Verbal Arithmetic Problems. Education Studies in Mathematics. Vol. 5, pp. 109 - 123.

Johnson H. C. (1944): Effects of Instruction in Mathematical Vocabulary Upon Problem Solving in Arithmetics. Journal of Educational Research. Vol. 38.

Johnson D. M. and R. P. Stratton (1966): Evaluation of Five Methods of Teaching Concepts. Journal of Educational Psychology. Vol. 57, No. 1, pp. 48 - 53.

Johnson D. A. and G. R. Rising (1967): Guidelines for Teaching Mathematics. Wadsworth Publishing Company Inc. Belmont, California, U. S.A.

Johnson D. C. & R. D. Harding (1979): University Level Computing and Mathematical Problem-Solving Ability. Journal for Research in Mathematics Education. Vol. 10, No. 1, pp. 37 - 55.

Kalejaiye A. O. (1979): An Analysis of the Performance of Bilingual Children in Solving Verbal Problems in Mathematics. A Doctoral Dissertation, University of Ibadan.

Kalejaiye, A. O. (1981): A Comparison of the Effectiveness of Two Methods of Teaching and Verbal Problems to Bilingual Nigerian Primary School Pupils of Three Different Levels of Ability. Alvan Ikoku School of Education Journal. Vol. 1, No. 2, pp. 120 - 127.

Kantowski M. A. (1977): Processes Involved in Mathematical Problem-Solving. Journal for Research in Mathematics. Vol. 8, pp. 163 - 180.

Kantowski M. G. (1981): "Problem Solving" In Fennema E. (ed). Mathematics Education Research: Implications for the 80's Reston Va pp 111 - 126.

Kerlinger, F. N. and E. J. Pedhazur (1973): Multiple Regression in Behavioural Research. Holt, Rinehart and Winston, Inc.

Kilpatric J. (1969): Problem-Solving in Mathematics. Review of Educational Research. Vol. 39, No. 4, pp. 523 - 534.

Kim Jae-On and F. J. Kohout (1975): Multiple Regression Analysis: Subprogram Regression. In Statistical Package for the Social Sciences. By Nie NH, CH, Hull, J. G. Jenkins, K. Steinbrenner and D. H. Bont Second Edition. Mc Graw-Hill Book Company, 1975.

Klausmeier, H. J. & F. H. Hooper (1971): Conceptual Development and Instruction: In Review of Research in Education Vol. 2, Editted by F. N. Kerlinger Peacock Publishers.

Knifong J. D. and B. Holtan (1976): An Analysis of Children's Written Solutions to Word Problems. Journal for Research in Mathematics Education, Vol. 7, pp. 106 - 112.

Koliagin, Y. M. (1981): Teaching Mathematics through Problem-Solving and Teaching the Solving of Problems in the Secondary Schools in Anglo-Soviet Seminar Mathematics Education. St. Anthony's College, Oxford (1981).

Koyejo, M. O. (1987): A Survey of Teaching Materials in Primary School Mathematics. A B.Sc Project of the Faculty of Education, University of Lagos.

Krulik, J. (1977): Problem-Solving - Some Considerations. Arithmetic Teacher. Vol. 25, No. 3, pp. 51 - 52.

Krutetskii, V. A. (1976): The Psychology of Mathematical Abilities in School Children. Chicago: University of Chicago Press.

Le Blan J. F. (1977): You can Teach Problem Solving Arithmetic Teacher, Vol. 25, No. 2, pp. 16 - 20.

Lemonyne, G. and C. Tremblay (1986): Addition and Multiplication: Problem-Solving and Interpretation of relevant data. Educational Studies in Mathematics. Vol. 17, pp. 97 - 123.

Lesh, R; Landau, M.; and E. Hamilton (1980): Rational Number Ideas and the Role of Representational Systems. Proceedings of the Fourth International Conference for the Psychology of Mathematics Education, 50 - 59, Berkeley, Califonia.

Linville, W. J. (1976): Syntax, Vocabulary and the Verbal Arithmetic Problems. School Science and Mathematics. Vol. 76, pp. 152 - 158.

Lovell, K. (1971): The Growth of Basic Mathematics and Scientific Concepts in Children University of London Press Limited.

Lovelace, T. L. and C. K. McKnight (1980): The Effects of Reading Instruction on Calculus Students' Problem-Solving. Journal of Reading. Vol. 23, No. 4, pp. 305 - 308.

Lucas J. F. (1974): The Teaching of Heuristic Problem Solving Strategies in Elementary Calculus. Journal for Research in Mathematics Education. Vol. 5, pp. 36 - 45.

McDermott, L. (1966): A Study of factors that cause fear and dislike of Mathematics. Ann. Arbor Mich. University. Micro Nos 2358 -2362.

Moore, J. W. and J. A. Schaut (1977): Increasing Instructional Effectiveness through the Use of a Problem-Solving Approach to the Design of Instructional System. Journal of Experimental Education. Vol. 47, No. 2, pp. 151-161.

Munro, J. (1979): Language Abilities and Mathematics Performance Reading Teacher. Vol. 32, No. 8.

National Council of Supervisors of Mathematics (1977): Arithmetic Teacher Vol. 25, No. 1 pp. 19 - 22.

National Council of Teachers of Mathematics (1980): An Agenda for Action Reston, Va.: The Council.

Neale, D. (1969): The Role of Attitudes in Learning Mathematics. Arithmetic Teacher Vol. 16, pp. 631 - 640.

Newman, M. W. (1977): An Analysis of Sixth grade pupils' errors on written mathematical tasks in M. A. Clements and J. Foyster (Ed). Research in Mathematics Education in Australia Education in Australia 1977. Melbourne. Vol. 1, 239 - 258.

Nigeria Educational Research Council (1977): A Study of the Problems and Issues of Mathematics Teaching in Nigeria. A report of the NERC national task-force on Mathematics

Obioma, G. O. and R. O. Ohuche (1985): The Effects of An Instructional Treatment on Two Mathematical Ability Groups in Number Sentence Problem. Journal of Science Teachers Association of Nigeria. Vol. 23, No. 1 & 2, pp. 179-185.

Obioma, G. O. and M. O. Odibe (1987): The Effect of Small Group Instructional Technique on Mathematics Achievement of Junior-Secondary Slow Learners. Journal of Science Teachers Association of Nigeria. Vol. 25, No. 2, pp. 92 - 96.

Otterburn, M. K. and A. R. Nicholson (1976): The Language of (CSE) Mathematics, Mathematics in School. Vol. 5, No. 5, pp. 18 - 20.

Park, O. (1984): Example Comparison Strategy Versus Attribute Identification Strategy in Concept Learning. American Educational Research Journal. Vol. 21, pp. 145-162.

Piaget, J. (1964): Development and Learning. Journal of Research in Science Teaching. Vol. 2, No. 3, pp. 176 - 185.

Polya, G. (1967): How to Solve it. Wiley & Sons, Inc. New York, London.

Polya, G. (1970): Mathematical Discovery: On Understanding, Learning and Teaching Problem Solving. First Edition, Wiley, New York.

Polya, G. (1981): Mathematical Discovery: On Understanding, Learning and Teaching Problem Solving, Wiley, New York.

Pottenger, M. J. & Leth (1969): Problem Solving. Arithmetic Teacher Vol. 16. No. 1, pp. 21 - 24.

Riedesel, C. A. (1969): Verbal Problem Solving: Suggestions for improving Instruction. Arithmetic Teacher. Vol. 11, pp. 312 - 316.

Schoelfeld, A. H. (1979): Explicit Heuristic Training as a Variable in Problem Solving Performance. Journal of Research in Mathematics Education. Vol. 10, No. 3, pp. 173 - 183.

Schoelfeld, A. H. (1982): Measures of Problem Solving Performance and of Problem-Solving Instructions. Journal for Research in Mathematics Education. Vol. 13, No. 1, pp. 31 - 49.

Skemp, R. (1971): The Psychology of Learning Mathematics. Penguin Books.

Span, P and Overtoom-Corsmit (1986): Information Processing by Intellectually Gifted Pupils Solving Mathematical Problems. Educational Studies in Mathematics Vol. 17, pp. 273 - 295

Tade, G. M. (1982): The Relationship Between the Understanding of Mathematical Language and Mathematics Problem-Solving Ability of Class Four Students in Lagos State Secondary Schools. M.Ed Thesis, University of Lagos.

Taschow, H. G. (1969): Reading Improvement in Mathematics. Reading Improvement. Vol. 6, No. 3, pp. 62 - 67.

Tate, M. W. and B. Stauler (1964): Errors in Judgement of Good and Poor Problem Solvers. Journal of Experimental Education. 32, pp. 371 - 376.

Tennyson R. D.; Chao, J. N. and T. Youngers (1981): Concepts Learning effectiveness using prototype and skill development presentation forms. Journal of Educational Psychology. Vol. 73, pp. 326 - 334.

Tennyson, R. D.; Youngers J.; and Seubsonthi, P (1983): Concept Learning by Children Using Instructional presentation Forms for prototype formation and Classification Skill Development. Journal of Education Psychology. Vol. 75. No. 2, pp. 280 - 291.

Tennyson, R. D. and M. J. Cocchiarela (1986): An Emperically Based Instructional Design Theory for Teaching Concepts Review of Educational Research. Vol. 56, No. 1, pp. 40 - 71.

Vigotsky, L. O. (1950): Thoughts and Language. Boston, MIT Press, Texas, USA.

Watson, I. (1980): Investigating Errors of Beginning Mathematics. Educational Studies in Mathematics, Vol. 11, No. 3, pp. 319-329.

West African Examination Council (1983): Chief Examiners Report in Mathematics.

Wheatley, C. L. (1980): Calculator use and Problem-Solving Performance. Journal for Research in Mathematics Education. Vol. 11, No. 5, pp. 323 - 335.

Wilson, John (1966): Thinking with Concepts. Cambridge University Printing House, 1966.

Zammarelli, J and N. Bolton, (1977): The Effects of Playon Mathematical Concept formation. British journal of Educational Psychology. Vol. 47, pp. 155 - 161.

APPENDIX "A"

TEST ON KNOWLEDGE OF MATHEMATICS CONCEPTS

Name: --

School: ---

Sex:- Male: ▱ Female : ▱ Class:------

Please answer the following questions by putting a circle round the letter indicating the correct answer.

1. An acute angle is
   (A) More than 180^0
   (B) Less than 180^0
   (C) More than 90^0
   (D) Less than 90^0
   (E) Equal to 360^0

2. An obtuse angle is
   (A) An angle more than 90^0 but less than 180^0
   (B) An angle more than 180^0 but less than 270^0
   (C) An angle more than 270^0 but less than 360^0
   (D) An angle less than 90^0
   (E) An angle equal to 360^0

3. A reflex angle is
   (A) An angle equal to 360^0
   (B) An angle equal to 180^0
   (C) An angle less than 180^0

(D) An angle more than 180^0 but less than 360^0

(E) An angle more than 90^0 but less than 180^0

4. A right angle is

(A) An angle of 180^0

(B) An angle of 90^0

(C) An angle of 270^0

(D) An angle more than 180^0

(E) An angle more than 180^0

5. Suppose A and B are two acute angles of P^0 and q^0 respectively. A and B are complementary angles if and only if

(A) $p-q = 0$

(B) $p + q = 90^0$

(C) $p + q = 180^0$

(D) $0 \angle p + q \angle 90$

(E) $90 \angle p + q \angle 180$.

6. A scalane triangle is one in which

(A) All sides are equal

(B) Two sides are equal

(C) All angles are equal

(D) No two sides are equal

(E) Three sides are equal

7. An equilateral triangle is one with
   (A) No two sides equal
   (B) Two sides equal
   (C) Only two angles equal
   (D) No sides equal
   (E) All sides equal

8. An isosceles triangle is one in which
   (A) All sides are equal
   (B) Two sides are equal
   (C) No two sides are equal
   (D) All sides are different
   (E) Three sides are equal

9. A rectangle is
   (A) A triangle
   (B) A parallelogram in which all angles are 90^0 each
   (C) A kite
   (D) A rhombus
   (E) A trapezium in which all angles are 90^0 each.

10. A quadrilateral is
    (A) An isosceles Triangle
    (B) A curved figure
    (C) A solid figure
    (D) A five sided figure
    (E) A plane figure

11. What is the locus of points in a plane at a given distance from a given point?

(A) A line

(B) Two parallel lines

(C) Two perpendicular lines

(D) A circle

(E) Two points

12. A triangle is a

(A) Solid figure

(B) Curved figure

(C) A four sided figure

(D) Plane figure

(E) Heptagon

13. Which of the following is an example of supplementary angles?

(A) 80^0, 100^0

(B) 40^0, 50^0

(C) 200^0, 70^0

(D) 200^0, 160^0

(E) 90^0

14. Parallel lines are

(A) Lines which face different directions

(B) Lines which are 90^0 to each other

(C) Lines marked with arrows pointing to the same direction

(D) Lines which are 45^0 to each other

(E) Marked with a line

15. Perpendicular lines form

(A) An x shape

(B) An L shape

(C) A Z shape

(D) A P shape

(E) An O shape

16. In a parallelogram

(A) Opposite sides are not equal

(B) Opposite angles are not equal

(C) The diagonals do not bisect each other

(D) The diagonals bisect each other

(E) Each angle is 100^0.

17. A pentagon is

(A) A triangle

(B) A quadrilateral

(C) A rectangle

(D) Square

(E) A polygon

18. In a regular polygon

(A) All sides are parallel

(B) All angles are equal

(C) All angles are not equal

(D) All sides are equal

(E) The exterior angles are not equal.

19. In a solid figure

(A) All the faces are the same

(B) The curved face is more than the plane face

(C) All faces cannot lie flat on a plahe surface

(D) All faces can lie flat on a plane surface

(E) The curved face is less than the plane face.

20. A simultaneous linear equations are

(A) Two equations with two pairs of solutions

(B) Quadratic equations

(C) Equations with two or more solutions

(D) Two equations with one pair of solutions

(E) Equations without unique answers.

21. Which of the following cannot be a plane figure?

(A) Circle

(B) Quadrilateral

(C) Tetrahedron

(D) Trapezium

(E) Pentagon

22. A line of symmetry is
    (A) An altitude
    (B) A perpendicular bisector
    (C) A line that divides a plane figure into two equal halves.
    (D) A perpendicular line
    (E) A curved line

23. The perimeter of an object is
    (A) The area of the object
    (B) The length multiplied by its breath
    (C) The distance round the object
    (D) The volume of the object
    (E) Halve the area of the object

24. The area of an object is
    (A) The amount of space occupied by the object
    (B) The Perimeter of the object
    (C) The volume of the object
    (D) The distance round the object
    (E) Amount of quantity that the object can hold.

25. The volume of any container is
    (A) The length of the container
    (B) The breadth of the container
    (C) The length multiplied by the breadth

(D) One third of the quantity that the container can hold

(E) Amount of quantity that the container can hold

26. Adjacent angles are

(A) Complementary angles

(B) Supplementary angles

(C) Acute angles

(D) Obtuse angles

(E) Reflex angles

27. In this figure alternate angles are

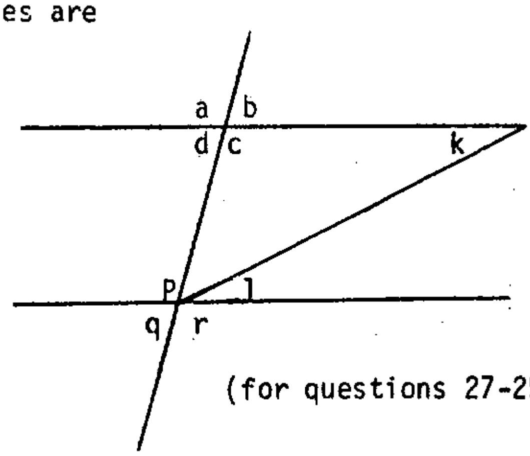
(A) Equal

(B) Similar

(C) Different

(D) Supplementary

(E) Complementary



(for questions 27-29)

28. Vertically opposite angles are

(A) Equal

(B) Similar

(C) Adjacent

(D) Supplementary

(E) Complementary

29. Corresponding angles are

(A) Supllementary

(B) Equal

(C) Complementary

(D) Similar

(E) Obtuse

30. A convex polygon is

(A) A curved figure

(B) A plane figure bounded by straight lines

(C) An open figure

(D) A plane figure bounded by curved lines

(E) A plane, open figure

31. A square is a

(A) Rectangle

(B) Triangle

(C) Pentagon

(D) A rectangle whose adjacent sides are equal

(E) An hexagon.

32. If the three sides of one triangle are respectively equal to the three sides of another triangle, then the triangles are said to be

(A) Similar

(B) Equal

(C) Equilateral

(D) Regular Polygons

(E) Congruent

33. A cube has

(A) 8 vertices

(B) 6 vertices

(C) 4 vertices

(D) 12 vertices

(E) 1 vertex

34. A cuboid has

(A) 4 faces

(B) 6 faces

(C) 8 faces

(D) 10 faces

(E) 2 faces

35. A cylinder has

(A) 4 faces

(B) 1 plane face and 1 curved face

(C) 1 plane face and 2 curved faces

(D) 6 faces

(E) 2 plane faces and 1 curved face.

36. A cone has

    (A) A plane face only

    (B) A curved face only

    (C) A plane face and a curved face

    (D) Two plane faces

    (E) Two curved faces

37. A triangular prism has

    (A) 1 face

    (B) 2 faces

    (C) 3 faces

    (D) 4 faces

    (E) 5 faces

38. A pyramid on a square base has

    (A) 2 faces

    (B) 5 faces

    (C) 3 faces

    (D) 4 faces

    (E) 6 faces

39. A trapezium is

    (A) A quadrilateral with a pair of opposite sides parallel

    (B) A quadrilateral with a pair of opposite angles complementary

    (C) A quadrilateral with a pair of opposite angles equal

(D) A rhombus

(E) A kite

40. A rhombus is a

(A) A parallelogram with all its sides equal

(B) Kite

(C) Pentagon

(D) Rectangle

(E) Triangle

41. A kite has

(A) No line of symmetry

(B) 4 lines of symmetry

(C) 2 lines of symmetry

(D) 3 lines of symmetry

(E) 1 line of symmetry

42. Similar figures are

(A) Figures which have the same shape

(B) Figures which are equal

(C) Figures that are congruent

(D) Triangles

(E) Rectangles

43. Pythagoras rules states that in a right angled triangle

(A) The square on the hypotenuse is equal to the sum of the squares on the other two sides.

(B) The square on the hypotenuse is equal to the difference of the squares on the other two sides.

(C) The square on the opposite side is equal to the sum of the squares on the other side.

(D) The square on the hypotenues is equal to the square of the opposite side.

(E) The square on the hypotenuse is equal to the square on the adjacent sides.

44. Factorisation means

(A) Subtracting

(B) Multiplying

(C) Adding

(D) Expressing an expression in multiples of two

(E) Expressing an expression in terms of its factors.

45. Put the correct symbol

2 ———— 5

(A) $>$

(B) $\leq$

(C) $\angle$

(D) $\geq$

(E) $=$

46. If the ratio of y to x is always constant, then

(A) y is said to be equal to x

(B) y is proportional to $^1/x$

(C) y is said to vary indirectly as x

(D) y is proportional to x^2

(E) y is said to vary directly as x.

47. In the figure below Sin 30^0 =

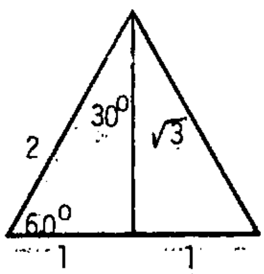
(A) $\sqrt{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) 30^0

(D) 2

(E) $\frac{1}{2}$



48. Which of the following is a measure of an acute angle?

(A) 45^0

(B) 90^0

(C) 135^0

(D) 180^0

(E) 225^0

49. Which of the following is a measure of an obtuse angle?

(A) 45^0

(B) 90^0

(C) 135^0

(D) 180^0

(E) 360^0

50. Which of the following is a measure of a reflex angle?

(A) 360^0

(B) 180^0

(C) 250^0

(D) 90^0

(E) 370^0

51. The angle formed by drawing a perpendicular to a line is

(A) Acute angle

(B) Obtuse angle

(C) Reflex angle

(D) Right angle

(E) Zero

52. If two angle are complementary, then both are

(A) Acute

(B) Obtuse

(C) Right

(D) Congruent

(E) Similar

53. In a scalene triangle

(A) All angles are equal

(B) Two angles are equal

(C) All angles are different

(D) Three angles are equal

(E) One angle is equal

54. Equilateral triangle has

(A) One angle not equal to others

(B) Two angles equal

(C) All angles equal

(D) No two angles equal

(E) All the angles different

55. In an isosceles triangle,

(A) No two angles are equal

(B) Three angles are equal

(C) All angles are different

(D) Two angles are equal

(E) One angle is 180^0

56. In a rectangle

(A) All angles add up to 360^0

(B) All angles add up to 180^0

(C) All sides are equal

(D) One side is longer than the rest

(E) All angles add up to 270^0

57. A quadrileteral has

(A) 3 sides

(B) 4 sides

(C) 5 sides

(D) 6 sides

(E) 7 sides

58. In a circle, the radius is

(A) Distance round the outer part

(B) Distance from the centre to any part of the circumference

(C) Distance from a point on the circumference to another point in the circle

(D) The origin

(E) Diameter

59. A triangle has

(A) 3 sides

(B) 4 sides

(C) 5 sides

(D) 6 sides

(E) 7 sides

60. Supplementary angles are

(A) Angles that add up to 90^0

(B) Angles that add up to 180^0

(C) Angles that add up to 270^0

(D) Angles that add up to 360^0

(E) Angles less than 90^0

61. Parallel lines are

(A) Perpendicular lines

(B) Two lines from a point

(C) Three lines from the original

(D) Lines that never meet however far they are produced

(E) Lines that meet at a point

62. Perpendicular lines are

(A) Lines that meet at an angle of 90^0

(B) Lines that meet at an angle of 270^0

(C) Lines that never meet

(D) Lines that are parallel

(E) Two lines in a plane

63. A parallelogram is a

(A) Solid figure

(B) Quadrileteral with a pair of sides parallel

(C) Pentagon

(D) Scalene triangle

(E) Quadrilateral with its opposite sides parallel

64. A pentagon is

(A) A curved figure

(B) A six sided closed plane figure

(C) A five sided closed plane figure

(D) A four sided figure

(E) An open figure

65. In a regular polygon

(A) All sides are parallel

(B) All angles are not equal

(C) All sides are not equal

(D) The exterior angles are not equal

(E) All sides are equal

66. Which of the following is a solid figure?

(A) Circle

(B) Cylinder

(C) Pentagon

(D) Parallelogram

(E) Rhombus

67. Which of the following is a simultaneous linear equation?

(A) $x + y = 6$

$2x = 3y + 7$

(B) $x - y - 2 = 0$

$z = 2$

(C) $p + q = r$

$5 + q = k$

(D) $1 + k = 1$

$z + c = D$

(E) $k - 2x + 3y = 5$

$k + 2x - 3y = 2$

68. A plane figure has a

(A) A curved surface

(B) A flat surface

(C) A T shape

(D) An oblong surface

(E) Two or three faces

69. How many lines of symmetry does a rectangle have?

(A) 1

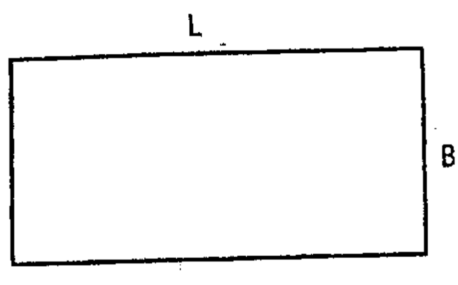
(B) 2

(C) 3

(D) 4

(E) No line of symmetry

70. The perimeter of this figure is



(A) $2L + B$

(B) $L \times B$

(C) $L \div B$

(D) $\frac{L \times B}{2}$

(E) $2(L + B)$

71. The area of the figure below is

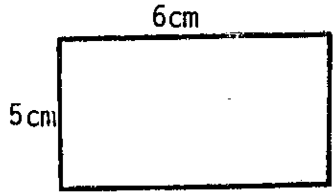
(A) (6 + 5)cm

(B) (6 x 5)cm

(C) 6cm x 5cm

(D) 2(6 + 5)cm

(E) 2(6 ÷ 5)cm



72. In calculating the volume of an object, one needs

(A) Knowledge of breadth

(B) Knowledge of length

(C) Knowledge of height

(D) Knowledge of length and breadth

(E) Knowledge of length, breadth and height.

73. If two lines AOB and OX meet at O. The angles AOX and XOB are called

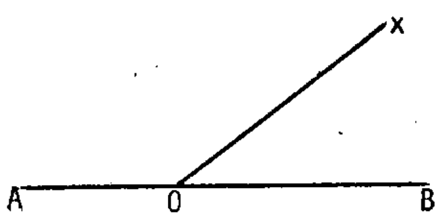
(A) Alternate angles

(B) Acute angles

(C) Right angles

(D) Adjacent angles

(E) Corresponding Angles

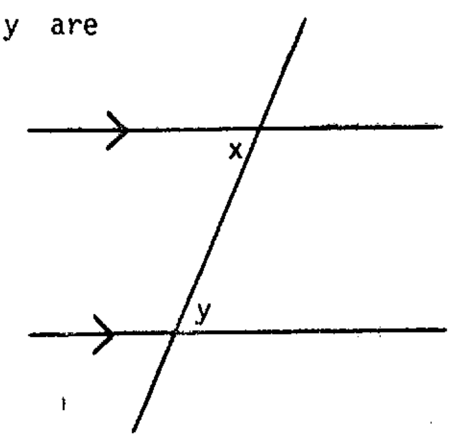


74. In the figure below, angles x and y are

(A) Obtuse angles

(B) Adjacent angles

(C) Vertically opposite angles



(D) Alternate angles

(E) Similar angles

75. If two straight lines AOB and XOY meet at O, the opposite angles formed by the pair are said to be

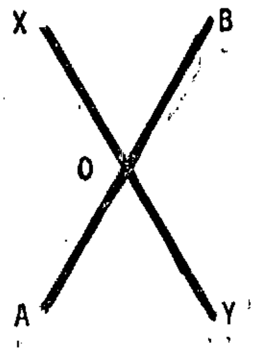
(A) Alternate angles

(B) Reflex angles

(C) Vertically opposite angles

(D) Alternately opposite angles

(E) Acute angles



76. In the figure below angles p and q are

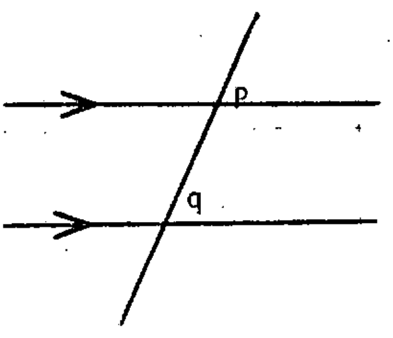
(A) Alternate angles

(B) Corresponding Angles

(C) Similar angles

(D) Right angles

(E) Obtuse angles



77. In a convex polygon

(A) All sides are equal

(B) All angles are equal

(C) All angles are 90^0

(D) The sides are not equal

(E) All sides are parallel

78. A square has

(A) Three sides

(B) Four equal sides

(C) Fóur equal sides

(C) Four or more equal sides

(D) Five sides

(E) Six sides

79. Pick the one that is not a sufficient condition for congruency

(A) SSS

(B) RHS

(C) ASS

(D) AAS

(E) SAS

80. A cube has

(A) 8 edges

(B) 6 edges

(C) 4 edges

(D) 12 edges

(E) 1 edge

81. A cuboid has

(A) 8 edges

(B) 6 edges

(C) 4 edges

(D) 12 edges

(E) 1 edge

82. In a cylinder,

(A) The four plane faces are equal

(B) The curved face is equal to the plane face

(C) The four plane faces are not equal

(D) The two plane faces are not equal

(E) The two plane faces are equal

83. A cone has

(A) 1 vertex

(B) 2 vertices

(C) 3 vertices

(D) 4 vertices

(E) No vertices

84. A triangular prism has

(A) 2 vertices

(B) 4 vertices

(C) 6 vertices

(D) 8 vertices

(E) 10 vertices

85. A pyramid on a square base has

(A) 2 vertices

(B) 3 vertices

(C) 4 vertices

(D) 5 vertices

(E) 6 vertices

86. In a trapezium

(A) All angles are equal

(B) All sides are equal

(C) The angles are not necessarily equal

(D) Each angle is 90^0

(E) Each angle is 45^0

87. A rhombus has

(A) No line of symmetry

(B) 1 line of symmetry

(C) 2 lines of symmetry

(D) 3 lines of symmetry

(E) 4 lines of symmetry

88. A kite is a

(A) Triangle

(B) A rectangle

(C) A pentagon

(D) A quadrilateral

(E) A parallelogram

89. In similar figures

(A) Corresponding sides are equal

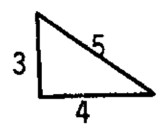
(B) All sides are equal

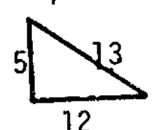
(C) Two angles are equal

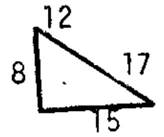
(D) Corresponding sides are in the same ratio

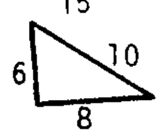
(E) All angles are different from each other.

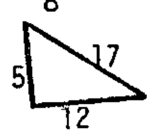
90. Which of the following is not true?

(A) 

(B) 

(C) 

(D) 

(E) 

91. Factorise $2ax^2 - 2a^2x$ completely

(A) $2(ax^2 - a^2x)$

(B) $2ax^2(1 - a)$

(C) $2ax(x - a)$

(D) $2a(x^2 - ax)$

(E) $2a(x^2 - x)$

92. Put the correct symbol

-2 ———— -5

(A) $>$

(B) $\leq$

(C) $\geq$

(D) $<$

(E) $=$

93. y varies inversely as x^2 implies

(A) $y = x^2$

(B) $y = \frac{1}{x^2}$

(C) $y = 1/x$

(D) $y = Kx^2$

(E) $y = \frac{k}{x^2}$

94. In this figure

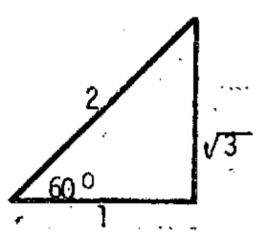
$\cos 60^0 =$

(A) $\frac{1}{2}$

(B) 2

(C) $\sqrt{3}$

(D) $\frac{1}{\sqrt{3}}$



(E) None of the above.

APPENDIX "B"

TEST ON COMPUTATION

Name: --

School: --

Sex: Male [ ] Female [ ] Class: ------------

Please answer the following questions by putting a circle round the letter indicating the correct answer.

1. Multiply $(x + 3)$ by $(2x - 5)$

   (A) $2x^2 - x - 15$

   (B) $2x^2 + x - 15$

   (C) $2x^2 - x + 15$

   (D) $15 - x - 2x^2$

   (E) $15 - x^2 + 2x$

2. Which of the following is equivalent to $-2x(3x - 1)$?

   (A) $6x - 2x$

   (B) $-6x^2 - 2x$

   (C) $-6x^2 + 2x$

   (D) $-6x + 2$

   (E) $6x^2 - 2x$

3. Factorise $5h^2 + 10gh - 20g^2h$

(A) $5h^2(1 + 2g - 4g^2)$

(B) $5(h^2 - 2gh + 4g^2h)$

(C) $5h(h + 2g - 4g^2)$

(D) $5h(h - 2g + 4g^2)$

(E) $5h^2(1 + 2g - 4g^2)$

4. Calculate AC in the diagram below

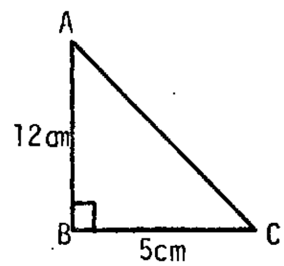
(A) 17

(B) 12.5

(C) 13

(D) 8

(E) 169



5. The volume of a spherical cap of height hcm is given by the relation $V = \pi h^2 (r - \frac{h}{3})$ where r cm is the radius of the sphere. Express r in terms of v, h and π.

(A) $r = v\pi h^2 + h/_3$

(B) $r = \frac{v}{\pi h^2} + \frac{h^2}{3}$

(C) $V\pi h^2 - \frac{h}{3}$

(D) $r = \frac{v}{\pi h^2} - \frac{h}{3}$

(E) $r = \frac{v}{\pi h^2} + \frac{h}{3}$

6. Given that $x = -2$, $y = \frac{1}{2}$ evaluate $xy^2 - x^2y$

(A) $-2\frac{1}{2}$

(B) -10

(C) $2\frac{1}{2}$

(D) 6

(E) 8

7. Given that the area of a triangle ($.\frac{1}{2}$ base x height) is $15cm^2$. Calculate the height of the triangle if the length of the base is 6cm.

(A) $2\frac{1}{2}$cm

(B) 4cm

(C) 5cm

(D) 15cm

(E) 45cm

8. The formular for the area of a circle is

(A) πr^2

(B) $2\pi r$

(C) $\frac{\pi^r}{r}$

(D) $\frac{2\pi}{r}$

(E) $2\pi r^2$

9. Which is the longest side in a right angled triangle ABC?

(A) Adjacent

(B) Opposite

(C) The side facing the acute angle

(D) The Hypotenuse

(E) The side facing the obtuse angle

10. Simplify $\frac{3x + 2}{3} - \frac{x - 1}{4} - \frac{5}{12}$

(A) $\frac{5x + 2}{4}$

(B) $\frac{12x + 15}{12}$

(C) $\frac{3x - 4}{12}$

(D) $\frac{4x + 1}{4}$

(E) $\frac{3x + 2}{4}$

11. If $E = \frac{1}{2}cm (v^2 - U^2)$ find m, When $E = 270$, $V = 10$ and $U = 8$

(A) 164

(B) 36

(C) 15

(D) 75

(E) 0.13

12. Simplify $4x - (2y - 3) + x$

(A) $5x - 2y - 3$

(B) $5x - 2y + 3$

(C) $5x + 2y - 3$

(D) $3x + 3 - 2y$

(E) $5x + 3 + 2y$

13. In the figure below, what is the value of x?

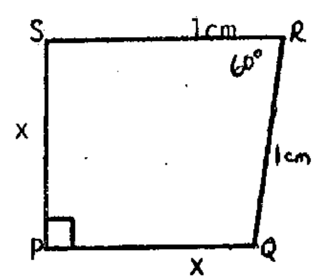
(A) 2

(B) $2\sqrt{2}$

(C) $4\sqrt{2}$

(D) $\sqrt{\frac{1}{2}}$

(E) $3\sqrt{2}$



14. If in the SLE, $x - y = 2$ and $x + 2y = 1$, find the value of x.

(A) $x = 1$

(B) $x = 1\frac{1}{3}$

(C) $x = \frac{1}{3}$

(D) $x = 1\frac{2}{3}$

(E) $x = -\frac{1}{3}$

15. If $4(2 - x) < 3(3 - 2x)$ which of the following is true?

(A) $x > \frac{1}{2}$

(B) $x < \frac{1}{2}$

(C) $x > \frac{3}{2}$

(D) $x < 1$

(E) $x > 1$

16. Find the value of P if $2(P - 4) = 2$

(A) 3

(B) 4

(C) 5

(D) 6

(E) 8

17. Solve the equation $\frac{2}{t} = \frac{3}{t + 1}$

(A) 5

(B) 4

(C) 3

(D) 2

(E) 1

18. If x varies as the square of y and x = 4 when y = 6, find the value of y when x = 16

(A) $\frac{1}{9}$

(B) $\frac{2}{3}$

(C) ± 12

(D) ± 36

(E) 48

19. The formula for the circumference of a circle with a radius r is

(A) $C = \pi R^2$

(B) $C = 4\pi r$

(C) $C = \pi r$

(D) $C = 2\pi$

(E) $C = 2\pi r$

20. The formular for the volume of a cuboid is

(A) Length

(B) Lenght and Breadth

(C) Length X Breadth X Height

(D) Length x Height

(E) Length X Breadth X Length

APPENDIX "C"

TEST ON COMPREHENSION OF MATHEMATICS LANGUAGE

Please answer the following questions by putting a Circle round the Letter indicating the correct answer:

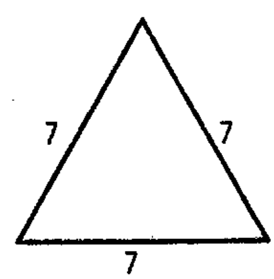
Name: --

School: ---

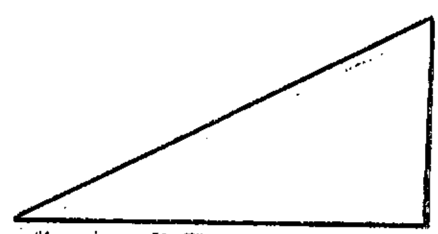
Class: --------------------------- Sex: ------------------

Identify the following figures in Nos 1 - 9

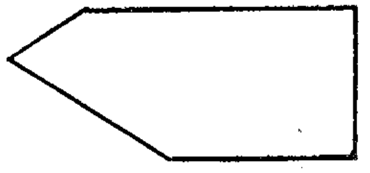
1. (A) Isosceles Triangle
   (B) Equilateral Triangle
   (C) Scalene Triangle
   (D) Right angled Triangle
   (E) Obtuse angled Triangle



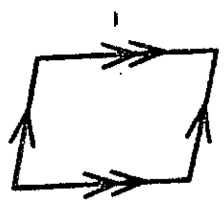
2. (A) Scalene Triangle
   (B) Isoceles Triangle
   (C) Equilateral Triangle
   (D) Right-angled Triangle
   (E) Circle



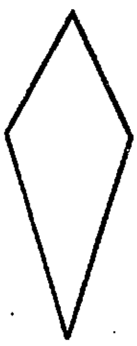
3. (A) Hexagon
   (B) Regular Polygon
   (C) Pentagon
   (D) Scalene
   (E) Parallelogram



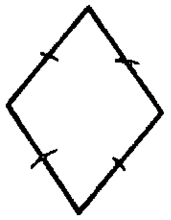
4. (A) Square
   (B) Parallelogram
   (C) Kite
   (D) Rectangle
   (E) Trapezium



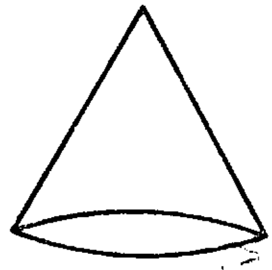
5. (A) Rhombus
   (B) Parallelogram
   (C) Kite
   (D) Triangle
   (E) Trapezium



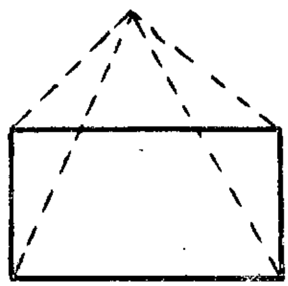
6. (A) Rhombus
   (B) Rectangle
   (C) Triangle
   (D) Kite
   (E) Trapezium



7. (A) Cube
(B) Prism
(C) Cylinder
(D) Pyramid
(E) Cone

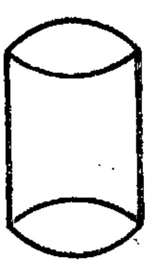


8. (A) Cube
(B) Cone
(C) Prism
(D) Pyramid
(E) Cylinder

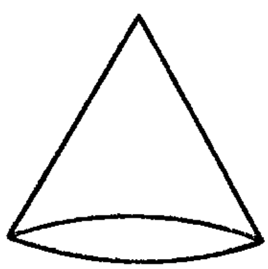


9. (A) Triangle
(B) Cone
(C) Prism
(D) Square
(E) Pyramid

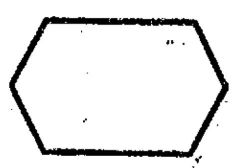
10. Which is a plane figure?



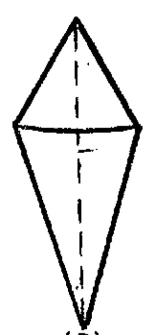
(A)



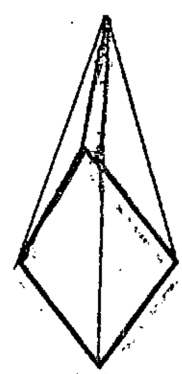
(B)



(C)

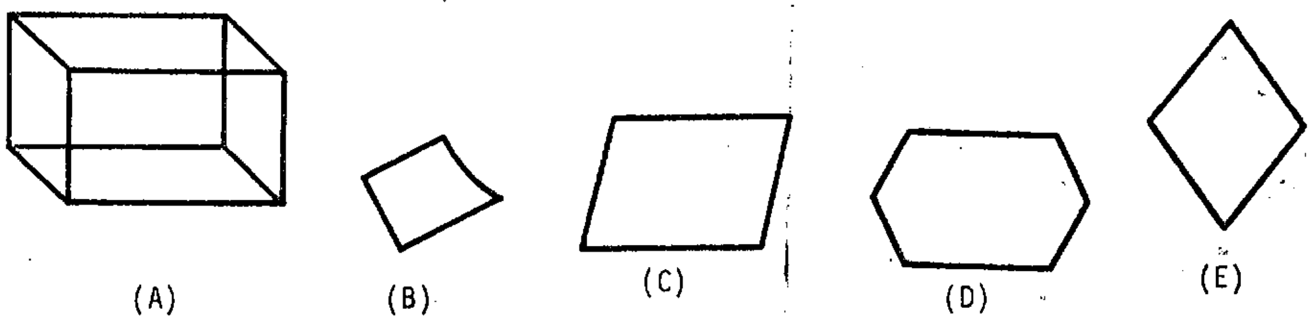


(D)



(E)

11. Which is a solid figure?



12. Identify the angles in the diagram below, which is an obtuse angle?

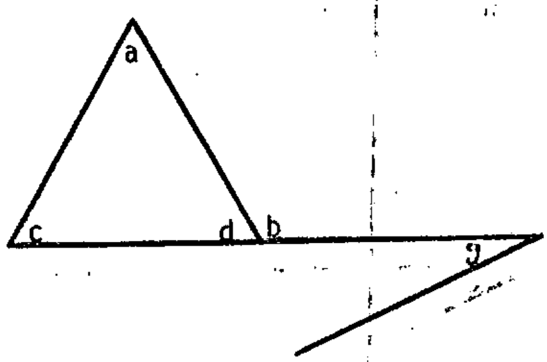
(A) C

(B) A

(C) G

(D) B

(E) D



13. Which angles are vertically opposite in the diagram below?

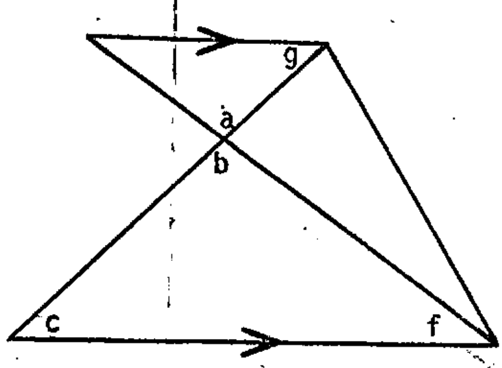
(A) A and C

(B) A + B and C

(C) C and F

(D) A and B

(E) G only



14. Which are corresponding angles from the diagram below?

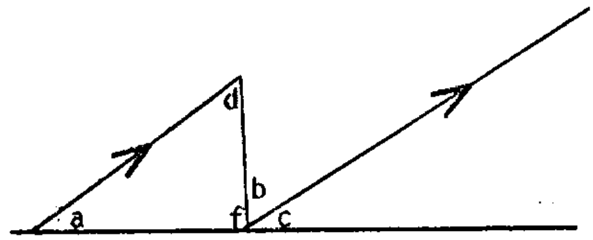
(A) a and d

(B) a and f

(C) a and c

(D) b and d

(E) b and f



15. Which are alternate angles from the diagram below?

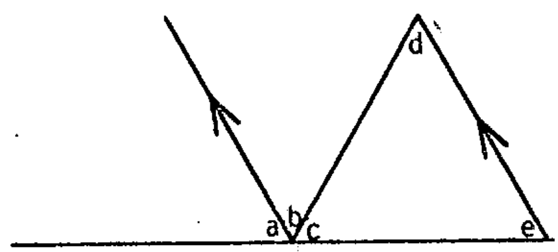
(A) a and b

(B) a and C

(C) b and d

(D) b and e

(E) d and e



16. Which are adjacent angles in the figure below?

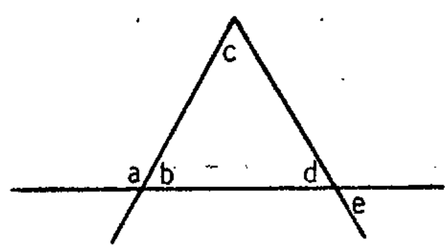
(A) a

(B) b

(C) c and d

(D) a and b

(E) d and e



17. Which is a right angle?

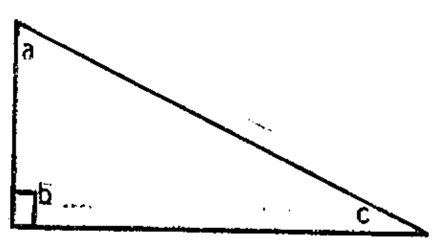
(A) a

(B) b

(C) C

(D) a + b

(E) b + c



18. Which is an acute angle?

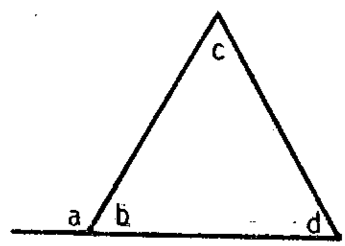
(A) a

(B) b

(C) c + a

(D) c + d

(E) b + c



19. Which angles are supplementary?

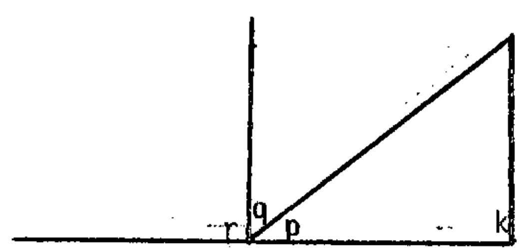
(A) p and q

(B) a

(C) k

(D) r, p and q

(E) r + p



20. Which angles are complementary?

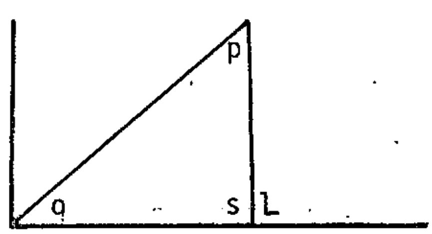
(A) p and q

(B) p and s

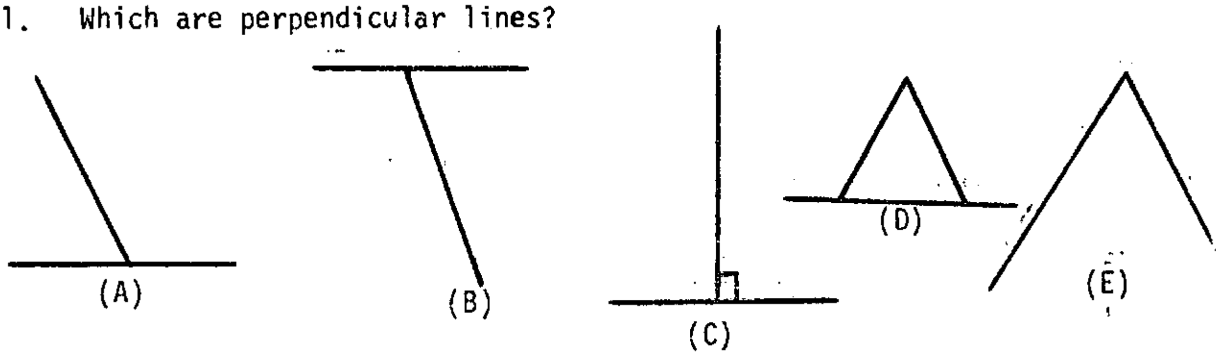
(C) p and L

(D) q

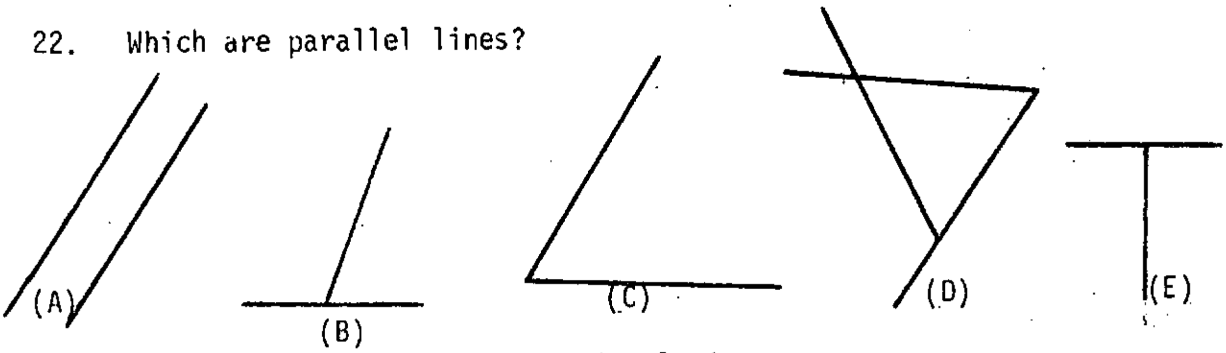
(E) p



21. Which are perpendicular lines?



22. Which are parallel lines?



23. The sum of the angles of a triangle is

(A) Between 90^0 and 180^0

(B) Between 180^0 and 360^0

(C) 360^0

(D) 270^0

(E) 180^0

24. The sum of angles of a quadrileteral is

(A) Between 90^0 and 180^0

(B) Between 180^0 and 360^0

(C) 180^0

(D) 360^0

(E) 270^0

25. The exterior angle of a triangle equals

(A) the sum of interior angles

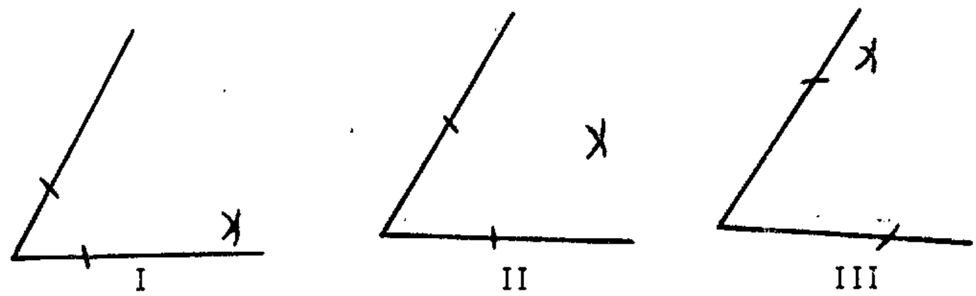
(B) the sum of exterior angles

(C) the difference of the non adjacent exterior angle?

(D) difference of the non-adjacent interior angles

(E) the sum of the two interior opposite angles

26. Which of the following illustrates the arcs used in bisecting an angle?



(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II and III

27. Which of the following represents and inverse variation between x and y.

(A) $y = -2x$

(B) $y = \frac{x}{2}$

(C) $y = \frac{4}{x}$

(D) $y = 3x^2$

(E) $x = 2y^2$

28. In a regular polygon, sum of exterior angles equals

(A) 90^0

(B) 180^0

(C) 270^0

(D) 360^0

(E) 450^0

29. In a convex polygon of n sides, the sum of interior angles is

(A) 360^0

(B) 180^0

(C) (2n - 4) right Ls

(D) 2(N-4) right Ls

(E) 2n right L's

30. How many faces does a cuboid has?

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

31. How many vertic es do we have in a cube?

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

32. How many edges does a cuboid has?

(A) 12

(B) 10

(C) 8

(D) 6

(E) 4

33. The area of the figure below is

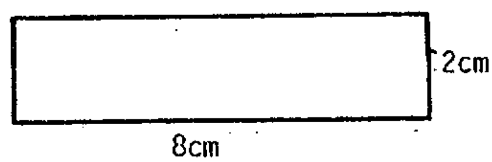
(A) 8cm + 2cm

(B) 8cm - 2cm

(C) 2(8cm x 2cm)

(D) 8cm ÷ 2cm

(E) 8cm x 2cm



34. The volume of a Box of dimensions 4cm by 6cm by 8cm is

(A) 4cm + 6cm + 8cm

(B) 4cm x 6cm x 8cm

(C) 8cm ÷ (4cm x 6cm)

(D) 8cm - 6cm x 4cm

(E) $\frac{8cm \times 6cm}{4}$

35. The pythagoras rule states that in a right angled triangle KLM

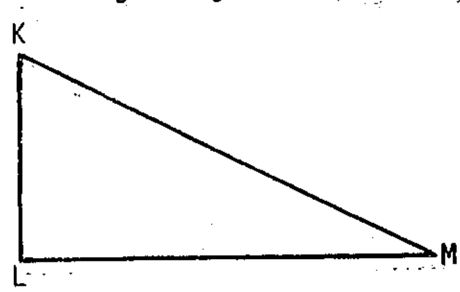
(A) $KL^2 = KM^2 + ML^2$

(B) $LM^2 = LK^2 + KM^2$

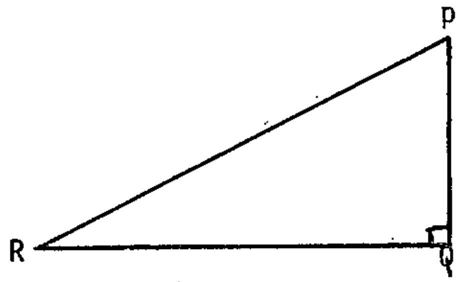
(C) $KM^2 = KL^2 + LM^2$

(D) $KM^2 = (KL + LM)^2$

(E) $KL^2 = (KM + ML)^2$



Use the right angled triangle PQR for Nos 36 - 38



36. Sine P equals

(A) $\frac{RQ}{RP}$

(B) $\frac{RQ}{PQ}$

(C) $\frac{PQ}{RQ}$

(D) $\frac{RP}{RQ}$

(E) RP

37. Cosine R equals

(A) $\frac{RQ}{PQ}$

(B) $\frac{PQ}{RQ}$

(C) $\frac{RP}{RQ}$

(D) $\frac{RQ}{RP}$

(E) RQ

38. Tangent P equals

(A) $\frac{PQ}{RP}$

(B) $\frac{RQ}{PQ}$

(C) $\frac{PQ}{RQ}$

(D) $\frac{RQ}{RP}$

(E) RP

39. $-x < 4$ implies

(A) $X < -4$

(B) $X > 4$

(C) $X < 4$

(D) $X = -4$

(E) $X > -4$

40. $\cong$ means

(A) equal to

(B) less than

(C) congruent to

(D) divided by

(E) none of the above

41. $\geq$ means

(A) greater than or equal to

(B) less than or equal to

(C) greater than

(D) equal to

(E) divided by

42. $\leq$ means

(A) greater than or equal to

(B) Less than

(C) Less than or equal to

(D) Equal to

(E) greater than

43. $<$ means

(A) greater than

(B) greater than or equal to

(C) less than

(D) less than or equal to

(E) equal to

44. $>$ means

(A) Equal to

(B) less than or equal to

(C) Less than

(D) greater than

(E) greater than or equal to

45. $P \alpha q$ means

(A) p equals to q

(B) P varies partly as q

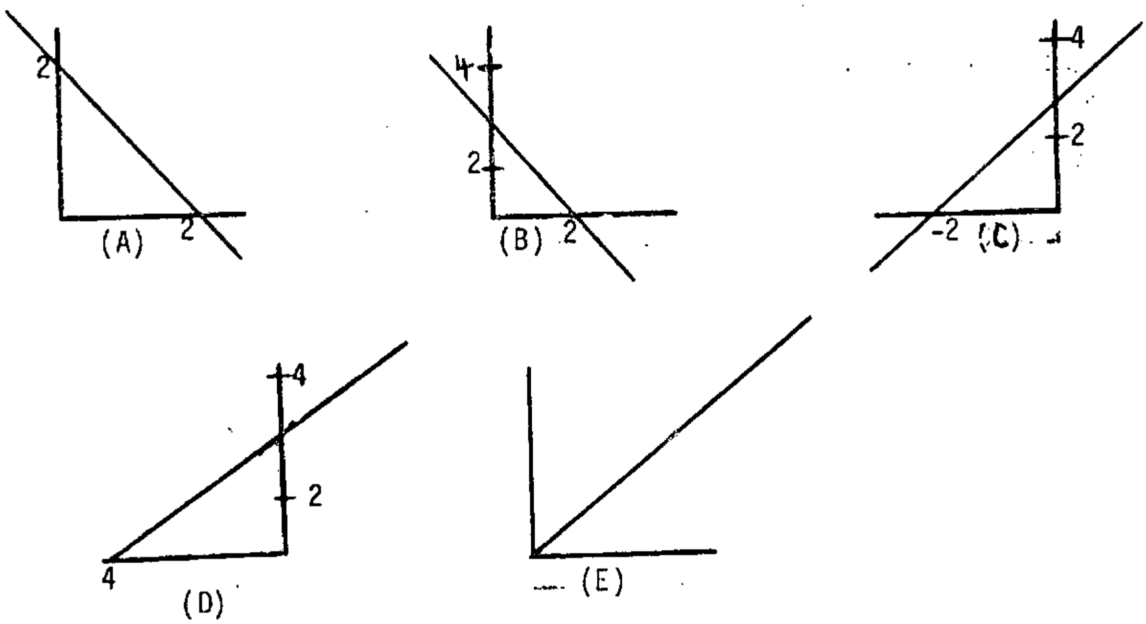
(C) P varies jointly as q

(D) P varies directly as q

(E) P varies inversely as q

46. Which of the following is a shetch graph of a relation

$y = \frac{6 - 3x}{2}$?



47. Form an algebraic expression from the statement below

"I think of a number, if I add 20 to it, it will give me twice the number"

(A) $x - 20 = 2x$

(B) $x - 2x = 20$

(C) $x - 20 = \frac{x}{2}$

(D) $x + 20 = 2x$

(E) $x + 20 = \frac{x}{2}$

48. If $P = M + N$, then which of the following will be true?

I $N = P - M$

II $P - N = M$

III $N + M = P$

(A) I only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II and III

49. An hypotenuse is

(A) a right angled triangle

(B) the shortest side a right angled

(C) the longest side of a triangle

(D) the longest side in a right angled triangle

(E) the side with the smallest angle

50. The distance from the centre of the circle to any part of the circumference is called

(A) Origin

(B) Centre

(C) Chord

(D) Diameter

(E) Radius

51. A line through the centre which touches two points on the circum ference of the circle is called.

(A) Diameter

(B) Chord

(C) Circum ference

(D) Centre

(E) Radius

52. The length round a circle is called

    (A) circle

    (B) length

    (C) diameter

    (D) Circum ference

    (E) Circumcircle

53. Part of circumsference of a circle is called

    (A) Origin

    (B) Centre

    (C) Arc

    (D) Circle

    (E) Quadrant

54. An altitude is

    (A) a line which meets another line at an angle of 45^0

    (B) a line which touches another line

    (C) a line which meets another line at an angle

    (D) a line which meets another line at an angle of 90^0

    (E) a line which divides another line into three parts.

55. A median is

    (A) An angular bisector

    (B) A straight line

    (C) A line that touches another line

(D) a line that divides another line into 3 equal parts

(E) a line which divides another line into two equal parts

56. A line joining two opposite corners of a plane figure is called

(A) Straight line

(B) Length

(C) Breadth

(D) Vertex

(E) Diagonal

57. Two similar pots which are cylindrical in shape have diameters 18cm and 10cm respectively. What is the scale factor of the bigger pot\to the smaller one.

(A) 10cm

(B) 18cm

(C) $180cm^2$

(D) 5:9

(E) 9:5

58. The fixed point inside a circle is called the

(A) Origin

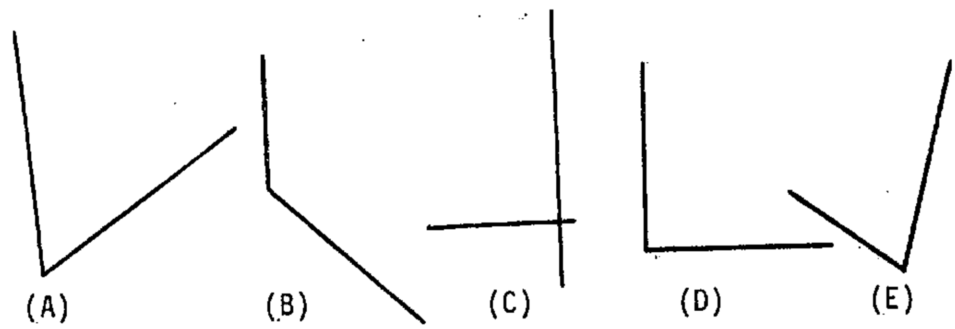
(B) Centre

(C) Chord

(D) Radius

(E) Diameter

59. A bearing of 60^0 can be drawn as



60. A line of fold which derives a figure into two equal halves is

(A) a straight line

(B) a parallel line

(C) a perpendicular line

(D) a dividing line

(E) a line of symmetry

61. The angle formed when an observer looks at an object higher than him is called

(A) Angle of elevation

(B) Alternate angles

(C) angle of depression

(D) adjacent angles

(E) interior angles

62. The angle formed when an observer looks at an object below his eye level is called?

    (A) angle of elevation

    (B) angle of depression

    (C) angle of turn

    (D) angle below eye level

    (E) opposite angles

63. Shapes that look alike but are not necessarily of the same size are referred to as

    (A) similar shapes

    (B) same shapes

    (C) equal shapes

    (D) ratio shapes

    (E) congruent shapes

64. The process of breaking down an expression into factors is called.

    (A) almighty formular

    (B) Expression

    (C) Completing the square

    (D) Factorisation

    (E) Algebraic Expression

APPENDIX "D"

TEST ON MATHEMATICS PROBLEM SOLVING ABILITY

Name: ---

School: ---

Class:------------------------- Sex: ------------------------

---

(Please Answer All Questions)

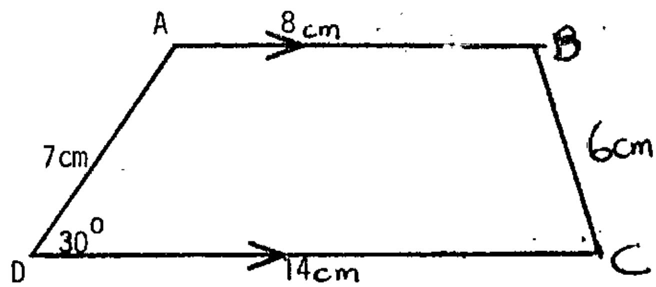
1. Two sharpners and three erasers cost 65k. Three sharpners and five erasers cost ₦1.05k. Find the cost of each.

2. Factorise $2a^2 - 15ab + 18b^2$

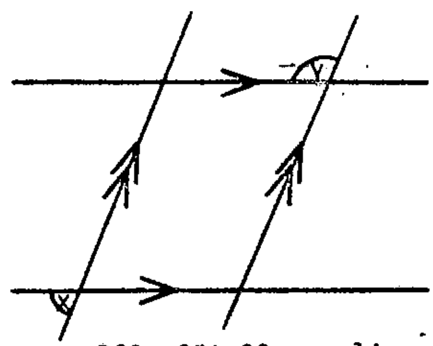
3. How many lines of symmetry in an equilateral triangle? (Draw them)

4. Which of the following plane figures is a parallelogram?
   - (i) trapezium
   - (ii) rhombus
   - (iii) Kite (give reasons)

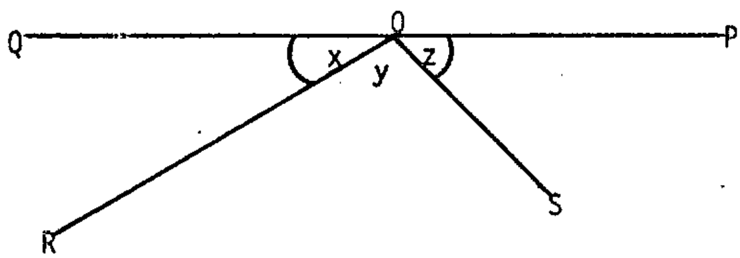
5. The quadilateral ABCD below has /AB/ = 8cm, /BC/ = 6cm, /DC/ = 14cm, /AD/ = 7cm and ADC = 30^0, calculate the area of the quadrilateral



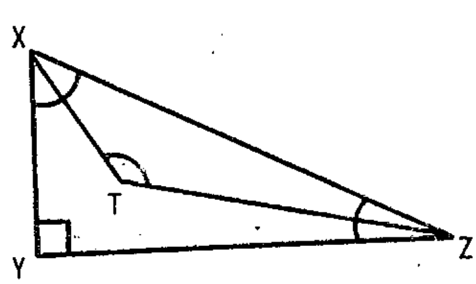
6. Find the value of y when x = 30^0



7. In the diagram POQ, OR, OS are lines. Find the value of y if x + z = y



8. The angle Y of the triangle XYZ is 90^0. Find the value of the obtuse angle XTZ if XT and TZ bisect angles YXZ and YZX respectively



9. What is the volume of a cylinder of base radius 4cm and height 28cm?

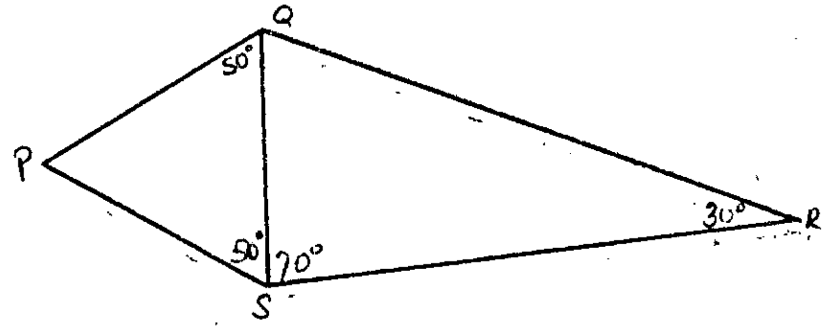
10. The perimeter of a rectangle is 24cm. If one of the side is 4cm, find the area of the rectangle.

11. A box volume $90cm^3$ has a base of area $20cm^2$. Calculate the height of the box.

12. A prism has a base 5m by 12m by 13m and a height of 10m. Calculate its volume.

13. A triangle ABC has /AB/ = 3cm, /AC/ = 4cm and the angle A a right angle. Find the length of the perpendicular from A to BC.

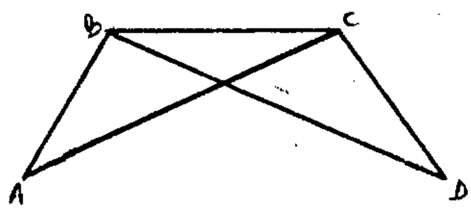
14. In the plane figure below, the line QS divides the shape into two different shapes. What are the names of these shapes?



15. If two of the angles of a pentagon are equal and each of the remaining angle is 20^0 greater than each of the first two, calculate the angles in the pentagon.

16. Two heaps of rice are of similar shape and contain 128kg and 250kg of rice respectively. If the height of the bigger heap is 70cm. Calculate the height of the smaller one.

17. ABCD is part of a regular polygon. Prove that triangles ABC, BCD are congruent and that /AC/ = /BD/



18. The cost of a car service is partly constant and partly varies with the time it takes to do the work. It costs ₦35 for a $5\frac{1}{2}$ hour service and ₦29 for a 4 hour service. Find the cost of a $7\frac{1}{2}$ hour service.

19. A boy scored 5x marks in the first two examinations papers and (x + 10) marks in the second. He came second in the examinations, the first boy scoring a total of 118 marks. From an inequality in x and solve it to find the range of values of x.

APPENDIX "E"

PRE-TEST

Name:--

School:---

Class:----------------------------Sex:----------

Please answer the following questions by writing down the letter indicating the correct answer.

1. If $\frac{1}{2} - x = x - {}^{1}/_{3}$. What is the value of x?

   (A) $\frac{1}{12}$

   (B) $\frac{1}{6}$

   (C) $\frac{5}{12}$

   (D) $\frac{2}{3}$

   (E) $\frac{5}{6}$

2. If y varies inversely as x^2 and y = 8 when x = 3 the value of y when x = 4 is

   (A) 2

   (B) 4

(C) 16

(D) 8

(E) 32

3. Simplify $\frac{1}{x-1} - \frac{3}{1-x}$

(A) $\frac{4}{x-1}$

(B) $\frac{2}{x-1}$

(C) $\frac{2}{1-x}$

(D) $\frac{4}{1-x}$

(E) $\frac{3}{1-x}$

4. If $2x + 3y = 7$ and $3x + 4y = 10$, the value of $5x + 7y$ is

(A) -3

(B) $15\frac{2}{3}$

(C) 17

(D) 32

(E) $8\frac{1}{3}$

5. A boy is 10 years old. In six years time, he will be twice as old as his sister is now. How old will his sister then be?

(A) 8

(B) 11

(C) 14

(D) 26

(E) 30

6. Solve for X, $\frac{1}{x} = \frac{1}{4} + \frac{1}{5}$

(A) $^{9}/_{20}$

(B) $2^{2}/_{9}$

(C) $4\frac{1}{2}$

(D) 9

(E) 7

7. Make x the subject of the formula in the equation $ax = b + cx$

(A) $X = \frac{c}{a-b}$

(B) $X = \frac{b}{a - c}$

(C) $X = \frac{b}{c - a}$

(D) $X = \frac{c}{b - c}$

(E) $X = \frac{c}{b - a}$

8. If $23^2 - 17^2 = 6x$, find the value of x.

(A) 1

(B) 6

(C) 30

(D) 35

(E) 40

9. The volume of a retangular solid on a square base is $576cm^3$. If its height is 9cm, calculate the length of the side of the base.

(A) 8

(B) 16

(C) 32

(D) 64

(E) 40

10. The circumference of a circle is 22cm, calculate the radius given $\pi = 3\frac{1}{7}$.

(A) $1\frac{3}{4}$

(B) $3\frac{1}{2}$

(C) 7

(D) $\sqrt{7}$

(E) $\frac{5}{6}$

11. The area of a square is equal to the area of a rectangle of sides 9cm by 16cm. Calculate the length of the side of the square.

(A) 11cm

(B) 12cm

(C) $12\frac{1}{2}$cm

(D) 14cm

(E) 21cm

12. Find the value of y when x = 50^0

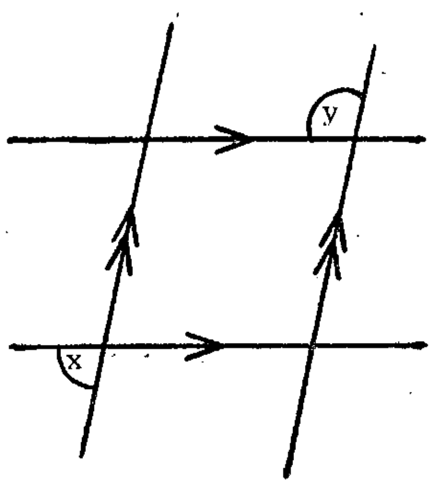
(A) 50^0

(B) 100^0

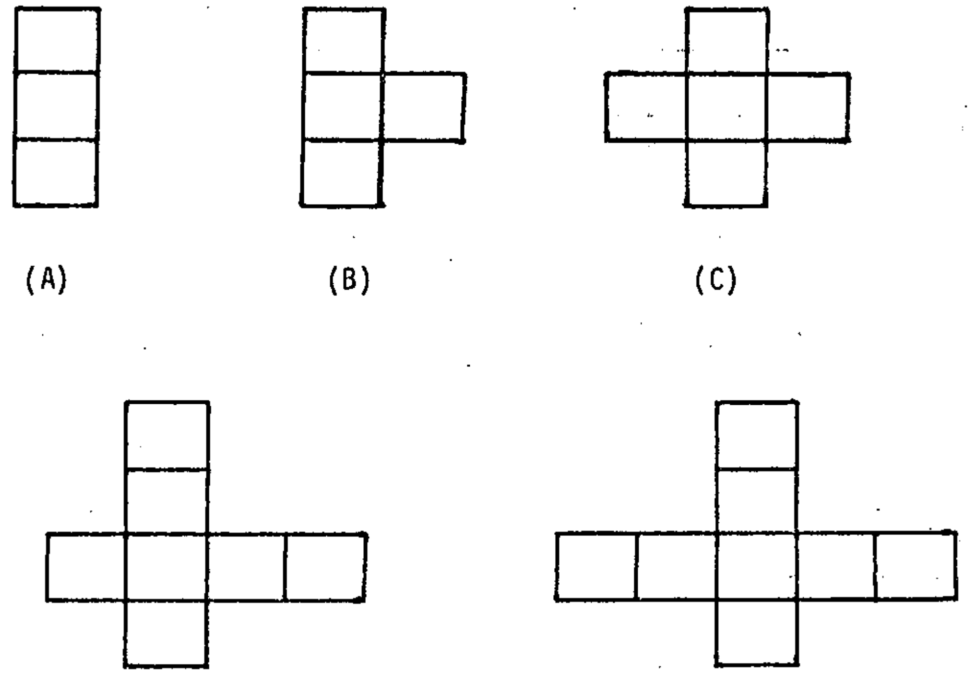
(C) 40^0

(D) 130^0

(E) 75^0



13. Which of the following is the net of a cube?



(A) (B) (C) (D) (E)

14. How many faces has a triangular prism?

(A) 2

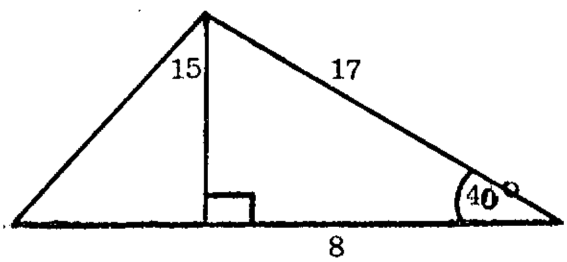
(B) 3

(C) 4

(D) 5

(E) 6

For Questions 15 and 16.



15. What is the value of Sin 40^0

(A) $\frac{15}{17}$

(B) $\frac{8}{17}$

(C) $\frac{8}{15}$

(D) $\frac{15}{8}$

(E) 15

16. Calculate Cos 50^0

(A) $\frac{8}{17}$

(B) $\frac{8}{15}$

(C) $\frac{15}{17}$

(D) $\frac{15}{8}$

(E) $\frac{17}{15}$

17. Each angle of a regular polygon is 162^0. How many sides has it?

(A) 20

(B) 16

(C) 12

(D) 8

(E) 4

18. A solid which have no vertex is

(A) Cone

(B) Prism

(C) Cuboid

(D) Cube

(E) Cylinder

19. Three angles of a hexagon are each x^0. The other three are $2x^0$ each. Calculate c.

(A) 70^0

(B) 75^0

(C) 80^0

(D) 85^0

(E) 90^0

20. Find the value of x in the figure below.

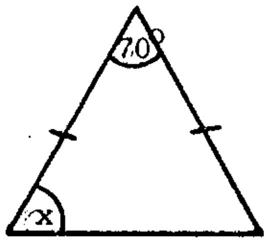
(A) 48^0

(B) 52^0

(C) 40^0

(D) 55^0

(E) 70^0



21. Each face of a cuboid is in the shape of a

(A) Triangle

(B) Rectangle

(C) Square

(D) Hexagon

(E) Circle

22. Two pots similar in shapes are respectively 21cm and 14cm high. If the smaller pot holds 4 litres, calculate the capacity of the larger one.

(A) 3.5 litres

(B) 7 litres

(C) 10 litres

(D) 12.5 litres

(E) 13.5 litres

23. Find y in the figure below.

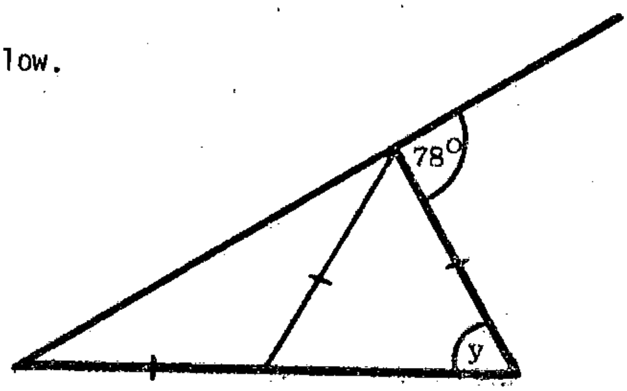
(A) 25^0

(B) 52^0

(C) 40^0

(D) 61^0

(E) 78^0



24. The sides of a triangle are 6cm, 7cm and 8cm. The shortest side of a similar triangle is 2cm. Find the lengths of the longest side of the similar triangle.

(A) 4cm

(B) $\frac{8}{3}$ cm

(C) $\frac{4}{3}$ cm

(D) $\frac{8}{5}$ cm

(E) $\frac{7}{2}$ cm

25. The ratio of the areas of two circles is $\frac{4}{9}$. Find the ratio of their radii.

(A) $\frac{9}{4}$

(B) $\frac{3}{2}$

(C) 2:3

(D) 3:4

(E) 2:5

26. In the diagram K, M, N, are parallel lines. What is the value of the angle marked x?

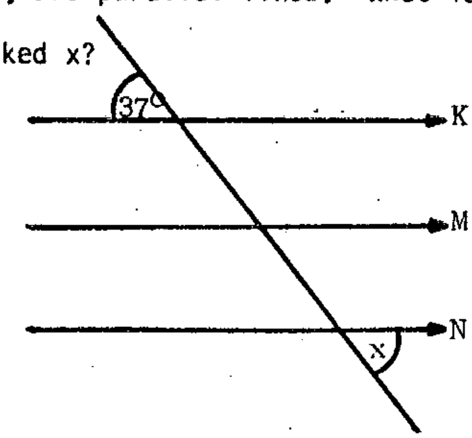
(A) 143^0

(B) 107^0

(C) 53^0

(D) 37^0

(E) 27^0



27. The length of a rectangle is 3 times its width. If the perimeter is 72cm, calculate the width of the rectangle.

(A) 4cm

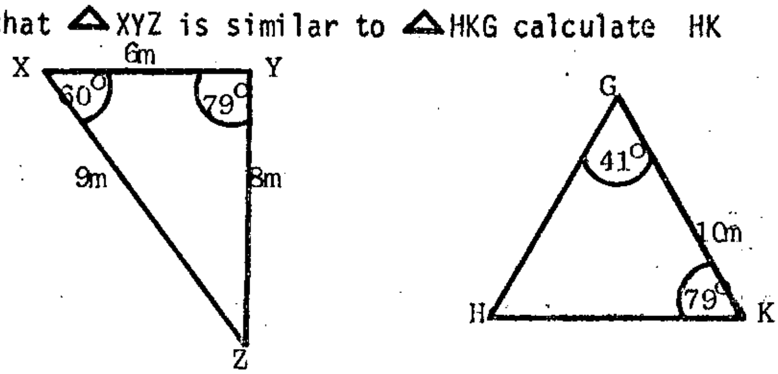
(B) 27cm

(C) 12cm

(D) 18cm

(E) 9cm

28. Given that △XYZ is similar to △HKG calculate HK



(A) 10.25m

(B) 8.5m

(C) 7.5m

(D) 9.5m

(E) 8.25m

29. Factorise $a^2 - 2ab - 15b^2$

(A) $(a + 3b)(a - 5b)$

(B) $(a + 3b)(a - 6b)$

(C) $(a - 5b)(a - 3b)$

(D) $(a + 5b)(a + 3b)$

(E) $(a - 3b)(a + 5b)$

30. Find the range of values of x for which $2x + 6 < 5(x - 3)$

(A) $x > -7$

(B) $x < 7$

(C) $x > 7$

(D) $x < -7$

(E) $x \geq 7$

31. The angle of depression of a boat at sea from the top of the Cliff is 72^0. What is the angle of elevation of the top of the Cliff from the boat?

(A) 18^0

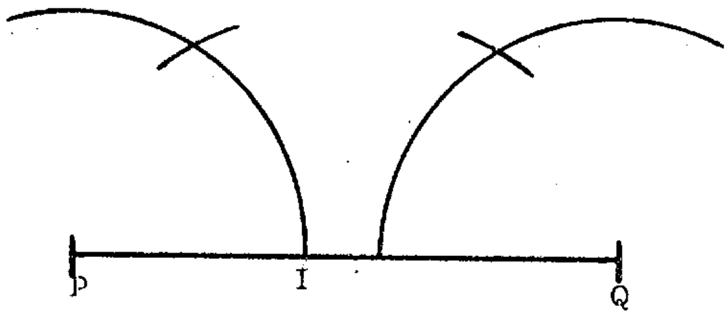
(B) 36^0

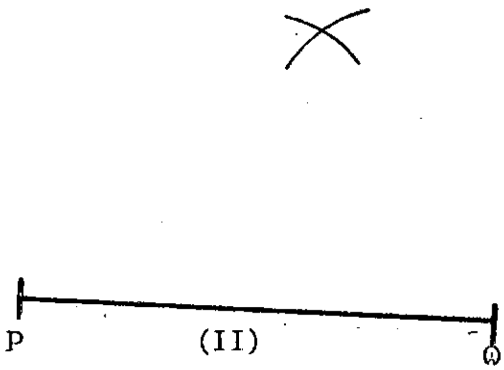
(C) 72^0

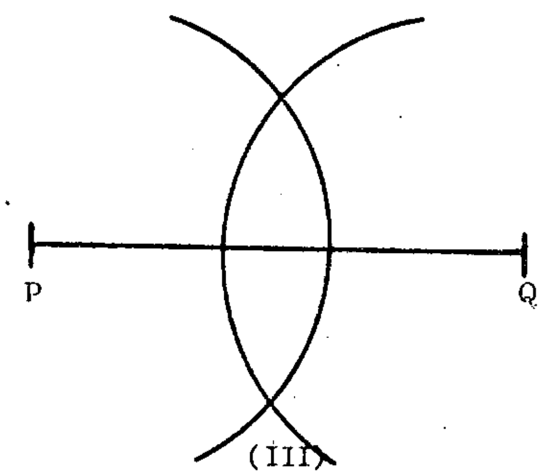
(D) 90^0

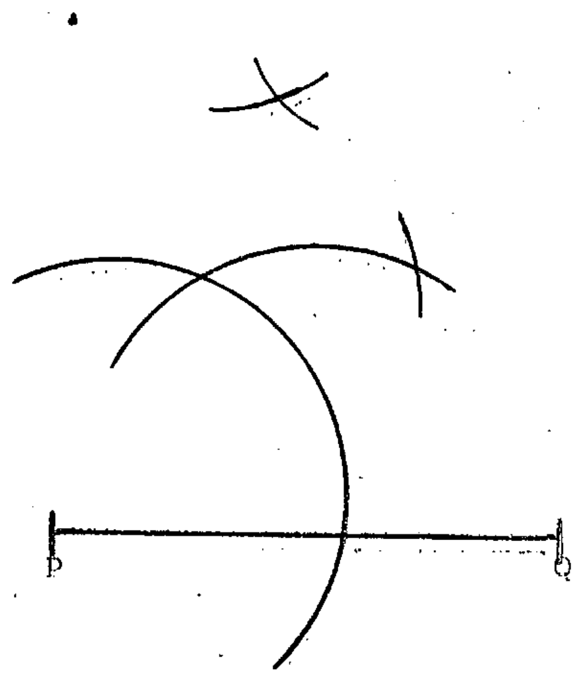
(E) 108^0

32 Which of the following constructions shows how to bisect $\overline{PQ}$?

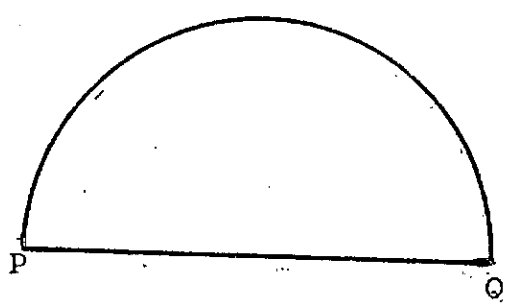








(IV)



X
(V)

(A) I

(B) II

(C) III

(D) IV

(E) V

33. In the figure below, what is the value of the angle marked m?

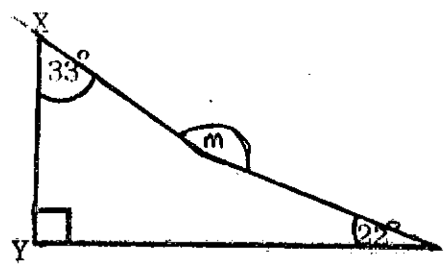
(A) 111^0

(B) 118^0

(C) 129^0

(D) 145^0

(E) 167^0



34. If $^2/_3 - 3x > 2(1 - x)$ which of the following is true?

(A) $x > {^4/_3}$

(B) $x < {^4/_3}$

(C) $x > -\frac{3}{4}$

(D) $x < -{^4/_3}$

(E) $x > -{^4/_3}$

35. The diagonals of a rhombus are 6cm and 8cm long. What is the length of the sides of the rhombus?

(A) 5cm

(B) 6cm

(C) 7cm

(D) 10cm

(E) 14cm

For Nos 36 and 37.

A ladder 20m long rests against a vertical wall so that the distance between the foot of the ladder and the wall is 9m. Use the above information to answer questions 36 and 37.

36. Find, correct to the nearest degree the angle the ladder makes with the wall.

(A) 24^o

(B) 27^o

(C) 68^o

(D) 66^o

(E) 90^o

37. Find, correct to one decimal place, the height above the ground at which the upper end of the ladder touches the wall.

(A) 9m

(B) 11m

(C) 69.4m

(D) 20m

(E) 17.9m

38. If $h(m + n) = m(h + r)$ find h in terms of m, n and r.

(A) $h = \frac{mr}{2m + n}$

(B) $h = \frac{mr}{n - m}$

(C) $h = \frac{m + r}{r}$

(D) $h = \frac{m + n}{m}$

(E) $h = \frac{mr}{n}$

39. The variables X and Y are connected by the relation Y varies inversely as X. The following table shows the values of Y for some selected values of X.

X	10	20	30	40
Y	12	6	?	3

What is the missing value of Y?

(A) 1

(B) 2

(C) 4

(D) 5

(E) 8

40. $(n + 5)$ is One Of the factors of $n^2 - 8n - 65$. What is the other factor?

(A) $n - 15$

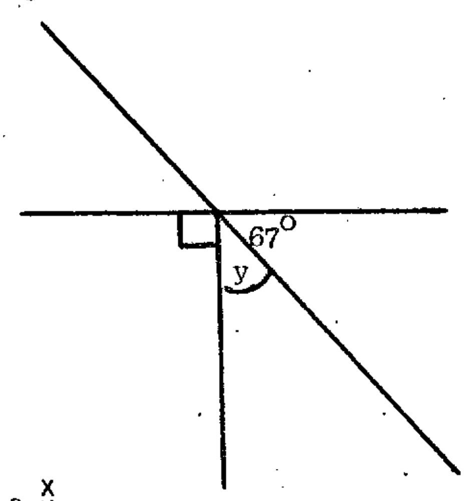
(B) $n + 13$

(C) $n - 13$

(D) $n - 60$

(E) $n + 60$

41. Find the size of the angle marked by the letter y in the diagram.



(A) 67°

(B) 60°

(C) 37°

(D) 30°

(E) 23°

42. Simplify $(y + x)^2 - 2y^x$

(A) $y^2 + x^2 + 4xy$

(B) $y^2 + x^2$

(C) $y^2 - x^2$

(D) $y^2 + x^2 - 2xy$

(E) $y^2 + x^2 - 4xy$

43. Find the slant height of a right circular cone of height 12cm, if the diameter of the base is 10cm?

(A) 120cm

(B) 17cm

(C) 13cm

(D) 2cm

(E) 1.2cm

44. Evaluate $(3.6)^2\pi - (3.5)^2\pi$ in terms of π

(A) 0.01π

(B) 0.1π

(C) 71π

(D) 7.1π

(E) 0.71π

45. In the trapezium PQRS, PQ//SR, $Q\hat{R}S = 90^0$, /PQ/ = 6cm, /RS/ = 9cm and /PS/ = 5cm. Calculate the perimeter of the trapezium.

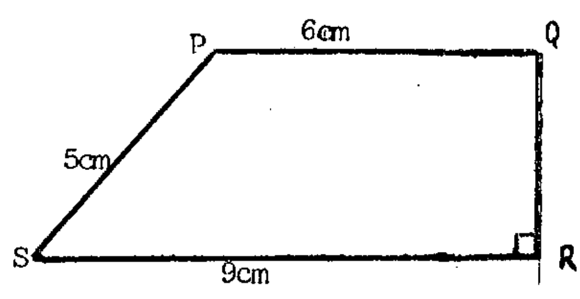
(A) 23cm

(B) 30cm

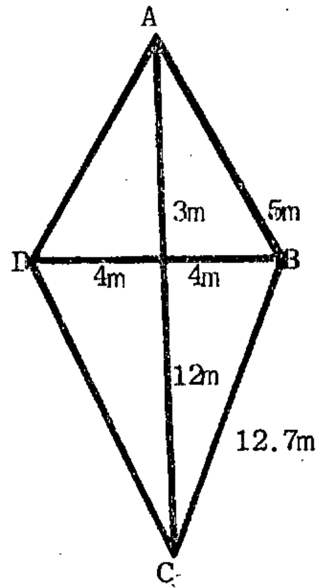
(C) 25cm

(D) 24cm

(E) 60cm



46. What area of paper is needed to make the Kite in the diagram?



(A) $60m^2$

(B) $64m^2$

(C) $75m^2$

(D) $96m^2$

(E) $120m^2$

47. Which of the following represents a direct variation between x and y?

(A) $y = 2x$
(B) $y = x^2/3$
(C) $y = 4/x$
(D) $y = 3x^2$
(E) $x = 2y^2$

48. Find the values of x for which $\frac{1 - 2x}{5} > \frac{5 - x}{3}$

(A) $X > 22$
(B) $X < 28$
(C) $X > 12$
(D) $X > -28$
(E) $X > -22$

49. Which of the following will lead to the correct construction of an angle of 75^0

I. Construct an angle of 45^0, then construct an adjacent angle of 60^0 and bisect it.

II. Construct two adjacent angles of 60^0, bisect one of them to get 30^0, then bisect the middle angle of 30^0 to get 15^0.

III. Construct an angle of 90^0 and bisect to get 45^0, then bisect one of the angles of 45^0.

A. I & II only

B. II & III only

C. I & III only

D. I, II & III

E. None.

50. Which of the following correctly interprets the equation

$2y + 8 = \frac{y}{3} - 2$

A. Twice y and 8 is equal to the difference between y and two third

B. The sum of 8 and twice y is two less than one third of y.

C. Twice the sum of y and 8 is equal to one third of y.

D. Twice the sum of y and 8 is equal to one third the difference between y and 1

E. The sum of 8 and twice y is one third less than y.

51. In the figure PQ//RS, $\angle$ QPT = 60^0, $\angle$ PTR = 40^0 and $\angle$ TRS = x. Calculate the value of x.

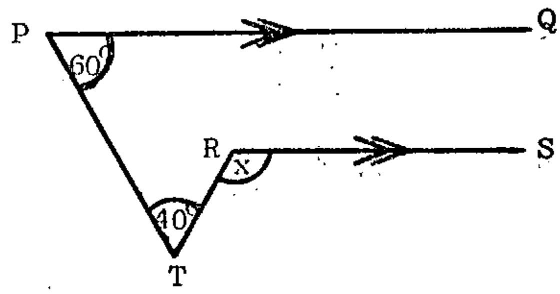
A. 20^0

B. 60^0

C. 80^0

D. 100^0

E. 140^0



52. The diagram shows the arcs used in constructing an angle PQR. What is the size of the angle?

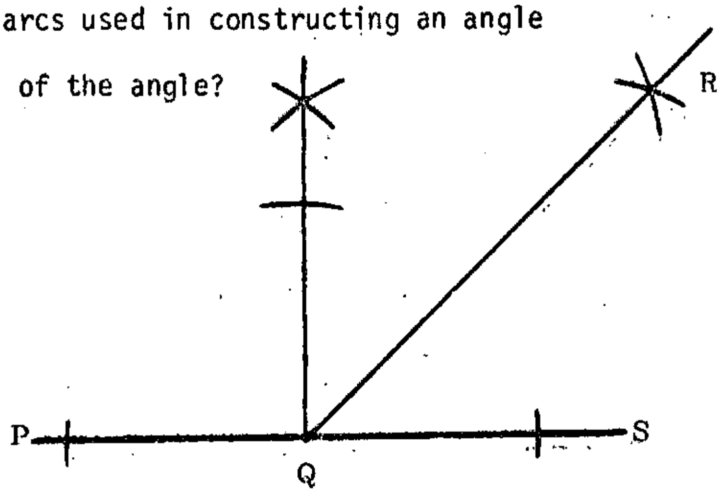
A. 45^0

B. 60^0

C. 120^0

D. 135^0

E. 150^0

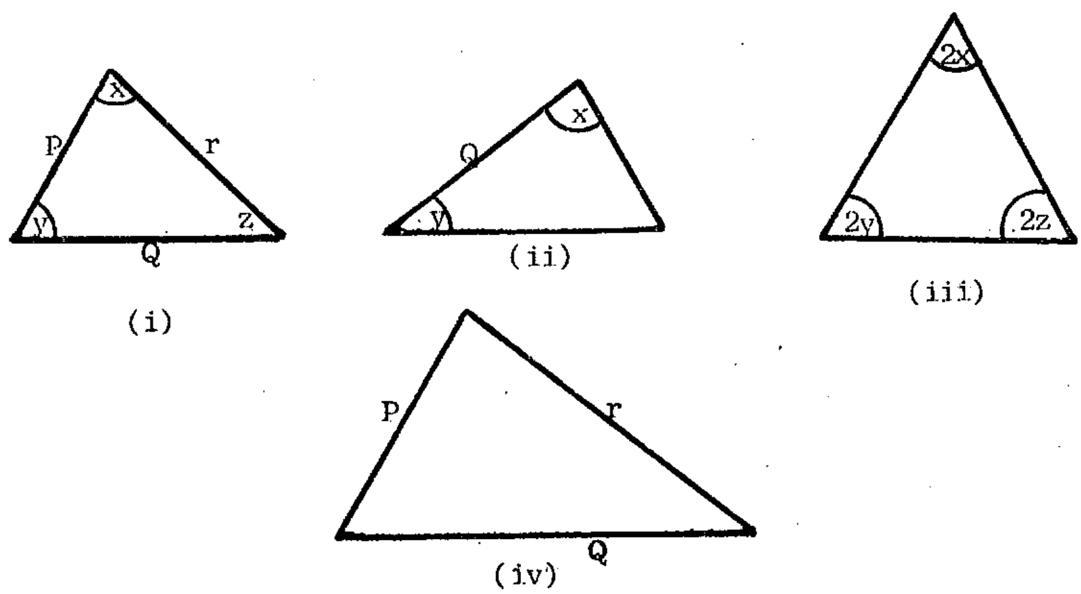


53. Which of the following statements is true of the triangle below?

I. (i) is congruent to (iv)

II. (ii) is congruent to (iii)

III. No two triangles are congruent



A. I only

B. II only

C. III only

D. I and II only

E. II and III only.

54. A woman buys x bananas at 10k each and (x + 6) cakes at 30k each. If she wishes to have some change from a ₦5.00 note, form an inequality in x and solve for x.

A. $x = 4$

B. $x = 8$

C. $X < 8$

D. $X < 4$

E. $X \leq 6$

APPENDIX "F"

POST-TEST

Name:--

School:--

Class:--------------------------------Sex :------------------

Please answer all questions

---

1. Dupe's age and Eze's age add up to 25 years. Eight years ago, Dupe was twice as old as Eze. How old are they now?

2. A ladder 5m long leans against a wall so that it makes an angle of 60^{0} with the horizontal ground. Calculate how far up the wall the ladder reaches.

3. The effort required to raise a load is partly constant and partly proportional to the load. The effort necessary for a load of 8 newtons is 6 newtons and for a load of 12 newtons is 8 newtons. Find the effort necessary for a load of 20 newtons.

8. How many lines of symmetry are in a

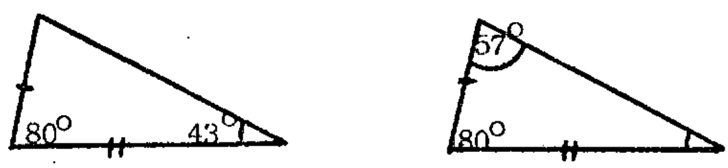
   (a) Kite

   (b) Trapezium

   Illustrate with diagrams.

9. A matchbox is in the shape of a cuboid 8cm long, 4cm wide and 3cm high. Matchboxes are packed in a similar box, a cuboid 48cm wide. Calculate the length and height of the box.

10. What do you understand by the term congruency? Are these triangles congruent?



11. A pyramid on a triangular base of dimensions 3cm by 4cm by 5cm has a volume of 30cm^3. Calculate the height of the pyramid.

APPENDIX "G"

ANSWERS TO:-

TEST ON KNOWLEDGE OF MATHEMATICAL CONCEPTS

1. D
2. A
3. D
4. B
5. B
6. D
7. E
8. B
9. B
10. E
11. D
12 D
13. A
14. C
15. B
16. D
17. E
18. B
19. C
20. D
21. C

22.	C
23.	C
24.	A
25.	E
26.	B
27.	A
28.	A
29.	B
30.	B
31.	D
32.	E
33.	A
34.	B
35.	E
36.	C
37.	E
38.	B
39.	A
40.	A
41.	E
42.	A
43.	A
44.	E
45.	C
46.	E

47.	E
48.	A
49.	C
50.	C
51.	D
52.	A
53.	C
54.	C
55.	D
56.	A
57.	B
58.	B
59.	A
60.	B
61.	D
62.	A
63.	E
64.	C
65.	E
66.	B
67.	A
68.	B
69.	B
70.	E
71.	C

72.	E
73.	D
74.	D
75.	C
76.	B
77.	D
78.	B
79.	C
80.	D
81.	D
82.	E
83.	A
84.	C
85.	D
86.	C
87.	C
88.	D
89.	D
90.	E
91.	C
92.	A
93.	B
94.	A

APPENDIX "H"

ANSWERS TO TEST ON COMPUTATION

1.	B
2.	C
3.	C
4.	C
5.	E
6.	A
7.	C
8.	A
9.	D
10.	E
11.	C
12.	B
13.	D
14.	D
15.	B
16.	C
17.	D
18.	C
19.	E
20.	C

APPENDIX "I"

ANSWERS TO TEST ON COMPREHENSION OF MATHEMATICAL LANGUAGE

1.	B
2.	A
3.	C
4.	B
5.	C
6.	A
7.	E
8.	D
9.	B
10.	C
11.	A
12.	B
13.	D
14.	C
15.	C
16.	D
17.	B
18.	B
19.	D
20.	A
21.	C
22.	A
23.	E

24.	D
25.	E
26.	B
27.	C
28.	D
29.	C
30.	E
31.	E
32.	A
33.	E
34.	B
35.	C
36.	A
37.	D
38.	B
39.	E
40.	C
41.	A
42.	C
43.	C
44.	D
45.	D
46.	B
47.	D
48.	E

49.	D
50.	E
51.	A
52.	E
53.	C
54.	D
55.	A
56.	E
57.	E
58.	B
59.	A
60.	E
61.	A
62.	B
63.	A
64.	D

APPENDIX "J"

MARKING SCHEME OF TEST ON MATHEMATICS PROBLEM SOLVING

1. Let the cost of the sharpner be x K (1)

   Let the cost of the eraser by yK

   $2x + 3y = 65$ -------- (i)

   $3x + 5y = 105$ -------- (ii) (1)

   (i) x 3 $6x + 9y = 195$ ---------(iii)

   (ii) x 2 $6x + 10y = 210$ --------- (iv) (1)

(iv) - (iii)

$y = 15$ (1)

Substitute $y = 15$ in eq. (i)

$2x + 3 \times 15 = 65$ (1)

$2x = 65 - 45$

$= 20$

$x = 10$ (1)

CHECK

$2 \times 10 + 3 \times 15 = 20 + 45 = 65$

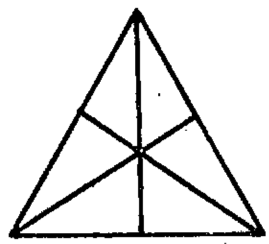
$3 \times 10 + 5 \times 15 = 30 + 75 = 105$ (1)

2. $2a^2 - 15ab + 18b^2$

   $2a^2 - 12ab - 3ab + 18b^2$

   $2a(a - 6b) - 3b(a - b)$ (1)

   $(a - 6b)(2a - 3b)$ (1)

3. 

(1)

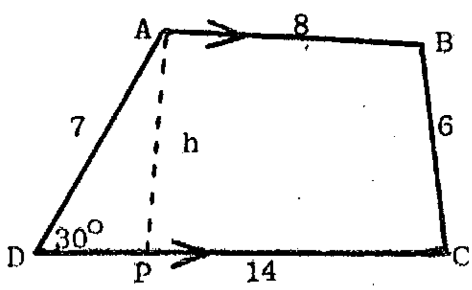
There are 3 lines of symmetry in an equilateral triangle (1)

4. Only a rhombus is a parallelogram (1)

It has two pairs of opposite sides equal and parallel (1)

With trapezium only a pair is parallel and may not be equal (1)

With a Kite the, the opposite sides are not equal

5. 

In ADP, $\sin 30^0 = {}^h/_7$ (1)

$\therefore\ h = 7 \sin 30^0 = {}^7/_2$ (1)

Area of a quadrilateral = Area of trapezium

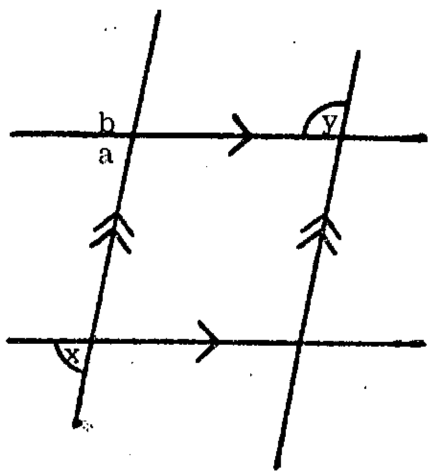
$= \frac{1}{2}$ (sum of ∥ lines) x height (1)

$= \frac{1}{2}(8 + 14) \times \frac{7}{2}$ (1)

$= \frac{1}{2} \times \cancel{22}^{11} \times \frac{7}{\cancel{2}_1} = \frac{77}{2}$

$= 38.5\text{cm}^2$ (1)

6.



$\angle x = \angle a = 30^0$ (Corresponding $\angle$) (1)

$\angle a + \angle b = 180^0$ ($\angle s$ on a straight line)

$\therefore \angle b = 180^0 - 30^0$

$= 150^0$ (1)

But $\angle b = \angle y$ (Corresponding angles)

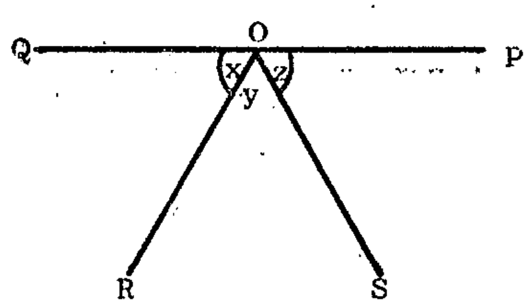
$\therefore y = 150^0$ (1)

7. Given $x + z = y$

But $x + y + z = 180^0$

(Angles on a Straight line) (1)

$x + z + y = 180$



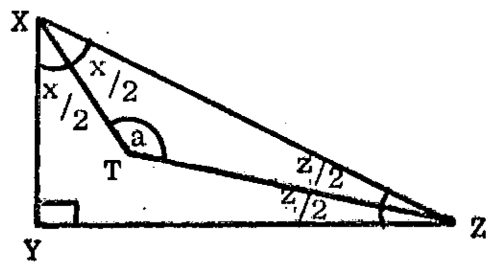
Since $x + z = y$;

$y + y = 180^0$ (1)

$2y = 180^0$

$y = 90^0$ (1)

8.



Let $\angle$ x TZ = a

Since XT and TZ bisect angles YXZ and YZX, each of the bisected angles equals $^x/_2$ and $^z/_2$ respectively (2)

$\therefore \frac{x}{2} + {}^z/_2 + a = 180^0$ (sum of angle in a triangle) (1)

$x + z + 2a = 360^0$

But $x + z = 90^0$

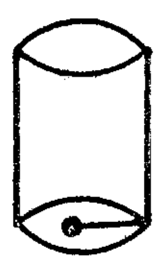
$\therefore 90^0 + 2a = 360^0$ (1)

$2a = 270^0$

$a = 135^0$

Hence $\angle$ XTZ $= 135^0$ (1)

9.

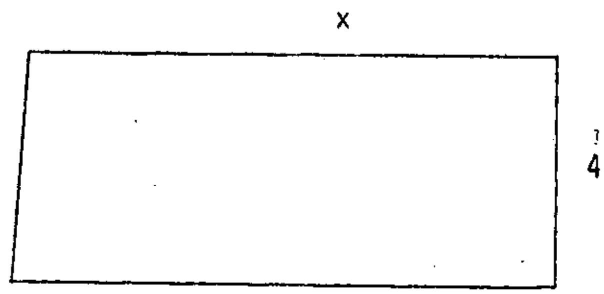


Volume of a cylinder $= \pi r^2 h$

$= \frac{22}{\cancel{7}_1} \times 4 \times 4 \times \cancel{28}^4$ (1)

$= 22 \times 4 \times 4 \times 4 = 1408cm^3$ (2)

10.



Let one side be x cm and the other is 4cm (1)

$\therefore$ Perimeter $= 2(x + 4)$ (1)

$24 = 2(x + 4)$

$\therefore x + 4 = 12$

$x = 8$ (1)

$\therefore$ the length is 8cm

Area = length x breadth

= 8cm x 4cm

= $32cm^2$ (1)

11. Volume of a box = Area of base x height

$= 20 \times h$ (1)

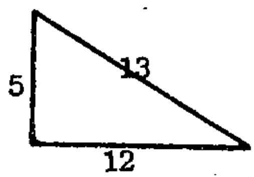
$90 = 20h$

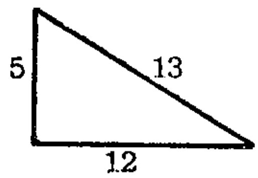
$h = \dfrac{90cm^3}{20cm^2}$ (1)

$= 4.5cm$

$\therefore$ height of box is 4.5cm (1)

12. Volume of a prism = Area of base x height

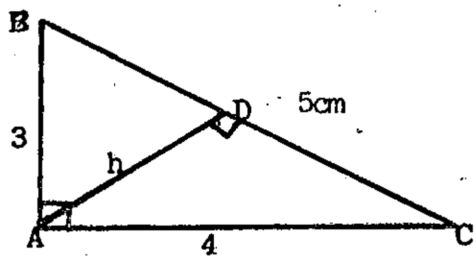
= 

Area of 

$\frac{1}{2} \times 12 \times 5 = 30cm^2$ (2)

$\therefore$ Volume of prism $= 30cm^2 \times 10cm$

$= 300cm^3$ (1)

13. 

Since $\triangle ABC$ is right angled

$BC^2 = AB^2 + AC^2$

$= 3^2 + 4^2 = 25$

$\therefore BC = 5cm$ (1)

Area of $\triangle ABC = \frac{1}{2} \times 4 \times 3 = 6cm^2$ (1)

Using BC as base, area of $\triangle ABC = \frac{1}{2} \times 5 \times h = 6cm^2$ (1)

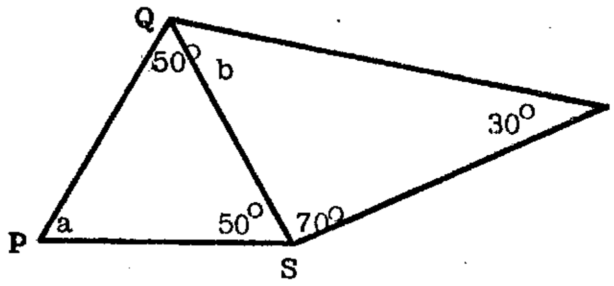
$\frac{5h}{2} = 6cm^2$

$h = \frac{6 \times 2}{5} = {}^{12}/_{5}$ (1)

$= 2.4cm$

$\therefore$ The length of the perpendicular from A to BC is 2.4cm (1)

14.



In $\triangle$PQS, $a = 180^0 - 100^0$

$= 80^0$ (1)

In $\triangle$QSR, $b = 180^0 - 100$

$= 80^0$ (1)

$\therefore$ $\triangle$PQS is an isosceles while $\triangle$QSR is scalene (1)

15. A pentagon is a five sided figure

Let two equal angles be x^0 each

$\therefore$ the other three angles be $(x + 20)^0$ each (1)

The sum of interior angles of a n- sided polygon is

$(2n - 4)$ right angles.

Since $n = 5$

Sum of angles of a pentagon $= (2 \times 5 - 4)\ 90^0$ (1)

$= (10 - 4)\ 90^0$

$= 6 \times 90^0$

$= 540^0$ (1)

$\therefore$ $x + x + (x + 20)^0 + (x + 20)^0 + (x + 20)^0 = 540^0$ (1)

$$5x + 60^0 = 540^0$$

$$5x = 480^0$$

$$x = 96^0 \qquad (1)$$

$\therefore$ the angles are 96^0, 96^0, 116^0, 116^0, 116^0 (1)

16. Ratio of masses = Ratio of Volume

$$= \frac{128}{250} \qquad (1)$$

$$= \frac{64}{125}$$

$$= (^4/_5)^3 \qquad (1)$$

$\therefore$ Ratio of length $= \frac{4}{5}$ (1)

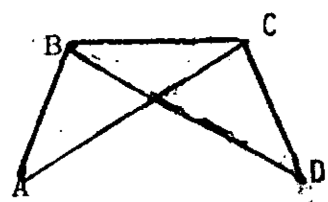
Height of bigger heap is 70cm,

$\therefore$ Height of smaller one is $\frac{4}{5} \times \overset{14}{\cancel{70}}$ cm (1)

$$= 56\text{cm}$$

Height of smaller heap is 56cm (1)

17.



Given:- ABCD, a part of a regular polygon

To prove :- (i) $\triangle ABC \equiv \triangle BCD$

(ii) $/AC/ = /BD/$

Proof:- I $\triangle$s, ABC and BCD

AB = CD (part of a regular polygon) (1)

BC = BC (Common) (1)

angle ABC = angle DCB (angles in a regular polygon) (1)

$\therefore$ $\triangle ABC \equiv \triangle BCD$ SAS (1)

Since the $\triangle$s are congruent, the third sides must be equal.

Hence $/AC/ = /BD/$ (1)

18. Let cost be represented by N C

Let time be represented by Th (1)

$C \propto K + T$ (1)

$C = K_1 + K_2 T$ ----- (i)

$35 = K_1 + {}^{1}/_{2} \; K_2$ --------(ii)

$29 = K_1 + 4 K_2$ --------- (iii) (1)

(iii) - (ii) $\quad 6 = {}^{3}/_{2} \; K_2$ (1)

$\therefore \; K_2 = \frac{6 \times 2}{3}$

$= 4$ (1)

Substitute $K_2 = 4$ in eq. (ii)

$29 = K_1 + 4 \times 4$

$= 16 + k_1$

$\therefore K_1 = 29 - 16$

$\therefore \; K_1 = 13$ (1)

$\therefore \; C = 13 + 4T$

When $t = 7\frac{1}{2}$,

$C = 13 + 4 \times {}^{15}/_{2}$ (1)

$= 13 + 30$

$= 43$

$\therefore$ the cost of a $7\frac{1}{2}$ hour service is ₦43. (1)

19. In the first exam paper he scored (5x) marks

In the second exam paper he scored (x + 10) marks

Since he came second with the boy who was first

scoring 118, it means that he would score less than 118 (1)

$$\therefore \quad 5x + x + 10 < 118 \qquad (1)$$

$$6x + 10 < 118$$

$$6x < 108$$

$$x < 18 \qquad (1)$$

Since x is a score it could be zero or greater than 0. (1)

$\therefore$ The range of values of x is $0 \leq x < 18$ (1)

APPENDIX "K"

ANSWERS TO PRE- TEST:

1.	C
2.	A
3.	A
4.	C
5.	A
6.	D
7.	B
8.	E
9.	A
10.	B
11.	B
12.	D
13.	D
14.	D
15.	A
16.	C
17.	A
18.	E
19.	C
20.	D
21.	B
22.	E

23.	B
24.	B
25.	C
26.	D
27.	E
28.	C
29.	E
30.	C
31.	C
32.	B
33.	D
34.	B
35.	A
36.	B
37.	E
38.	E
39.	C
40.	C
41.	E
42.	B
43.	C
44.	E
45.	D

46.	A
47.	A
48.	E
49.	A
50.	B
51.	D
52.	D
53.	A
54.	C

APPENDIX "L"

PRACTICE EXERCISES FOR THE EXPERIMENTAL GROUPS:

PRACTICE EXERCISE I

SOLID SHAPES

1. Write down four everyday objects which are cuboids.

2. How many square faces has a cube, draw the skeleton view of a cube?

3. A cuboid is made from wire so that it is 15cm long, 12cm wide and 8cm high. What length of wire has been used?

4. Draw a skeleton view of a triangular prism.

5. How many faces, edges and vertices does a triangular prism have?

6. What is the shape of the end faces of a triangular Prism? What is the shape of the other faces of a triangular Prism?

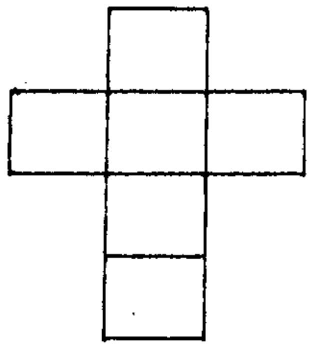
7. Find the volume of a pyramid with a square base of length 6cm, and perpendicular height 4cm.

8. A pyramid has a total of 6 triangular faces, How many faces does it have altogether? What is the shape of its other face?

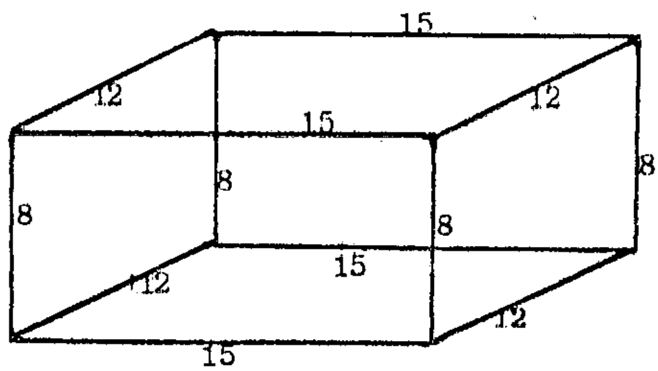
SOLUTION TO PRACTICE EXERCISE I:

1. Boxes, Room, Book, Block etc.

2. 6



3.



The Length of wire used is equal to the perimeter of the base, top and the length of the heights.

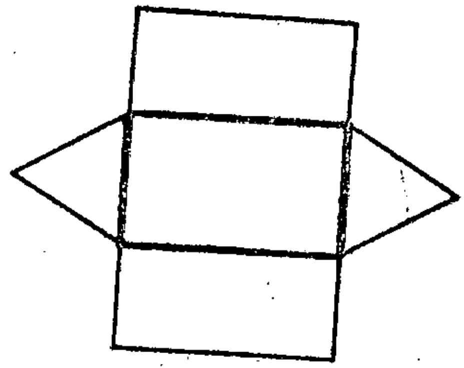
Perimeter of base + Perimeter of top + all heights

$= 2(15 + 12) + 2(15 + 12) + 4(8)$

$= 54 + 54 + 32$

$= 140\text{cm}$

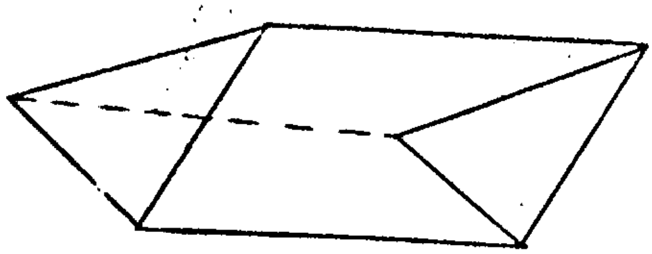
4.



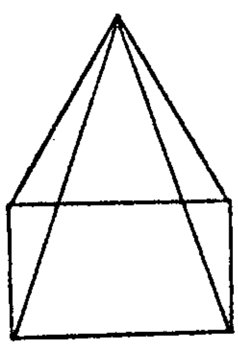
5. A triangular prism has five faces. It has 9 edges and 6 vertices.

6. Triangle

Rectangle



7. Volume of a pyramid $= \frac{1}{3}$ base area x height



$= \frac{1}{3} \times 6 \times 6 \times 4$

$= 48cm^3$

8. It has 7 faces. The other face is Hexagon.

PRACTICE EXERCISE II

PLANE SHAPES

1. Name 3 shapes which are special examples of trapezium.

2. The diagonals of a Kite cross at right angles. Name other shapes for which this is always true.

3. Write down three things that are different between a Kite and a rhombus. Is a rhombus a Kite?

4. In a Mathematical Workshop it was discovered that the cross section of a new apparatus is that of a trapezium whose area is 100 cm^2 and sum of its parallel sides 20cm, Calculate the distance between them?

5. The diagonals of a square are equal in length. Name another shape for which this is always true.

6. How many lines of symmetry has a rhombus?

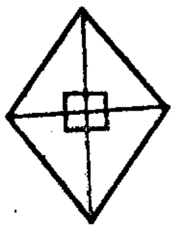
7. Calculate the interior angles of a regular hexagon

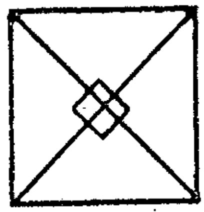
8. Four angles of a pentagon are 70^0, 80^0, 110^0 and 120^0. Find the fifth angle.

SOLUTIONS TO PRACTICE EXERCISE II

1. Parallelogram, retangle, rhombus or square.

2.



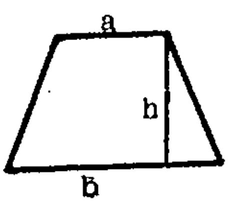


Square

Rhombus

3. In a rhombus, all four sides are equal in length. There are two pairs of opposite angles and there are two lines of symmetry. None of this true for a Kite. A rhombus is a special kind of kite.

4.



Let a and b be parallel sides

Area of a Trapezium

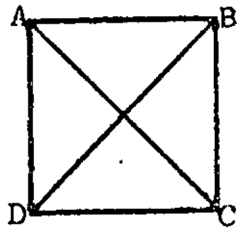
$= \frac{1}{2}$ (sum of parallel lines) x height

$100 = \frac{1}{2} \times 20 \times h$

$\therefore h = \frac{100 \times 2}{20}$

$= 10.$

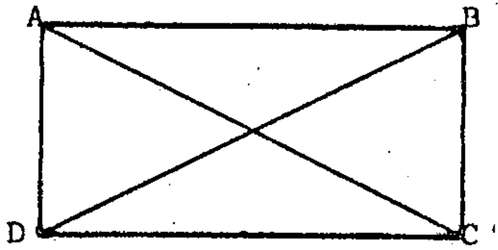
The distance between the parallel sides is 10cm.

5. 

SQUARE

AC = BD

Another shape in which its diagonal is equal in length is a rectangle



AC = BD

6. A rhombus has two lines of symmetry

7. A hexagon has 6 sides. Sum of interior angles of an n sided polygon is 2n - 4 right angles.

$\therefore$ Sum of interior angles of a hexagon is

$2 \times 6 - 4$ right angles

$=$ 8 right angles

$= 8 \times 90 = 920^0$

$\therefore$ Each interior angle is $\frac{720^0}{6} = 120^0$

8. Sum of interior angles of a pentagon

$$= 2 \times 5 - 4 \text{ right angles}$$
$$= 6 \times 90^0$$
$$= 540^0$$

Sum of four angles $= 70^0 + 80^0 + 110^0 + 120^0$
$$= 380^0$$

$\therefore$ The fifth angle $= 540 - 380$
$$= 160^0.$$

PRACTICE EXERCISE III

ANGLES

1. Write down the angles complementary to 28^0, 56^0, 39^0, $(20 + x)^0$, $(60 + y)^0$

2. Write down the angles supplementary to $(120 + z)^0$, $(60 + y)$, 45^0, 26^0, 121^0.

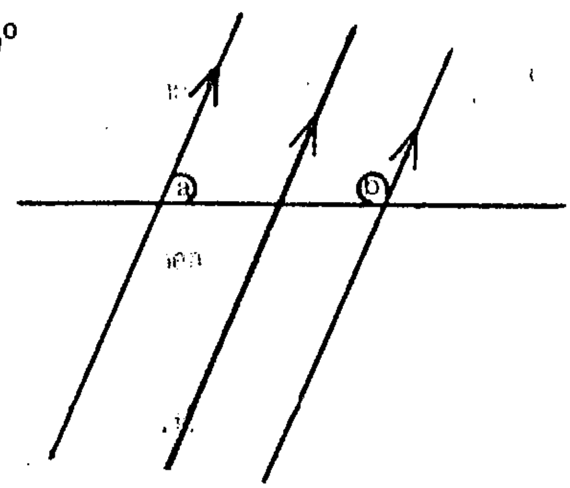
3. If two adjacent angles on a straight line are

   (i) z^0 and $3z^0$
   (ii) p^0 and $(p + 30)^0$
   (iii) r^0 and $4r^0$
   (iv) q^0 and $(q + 40)^0$

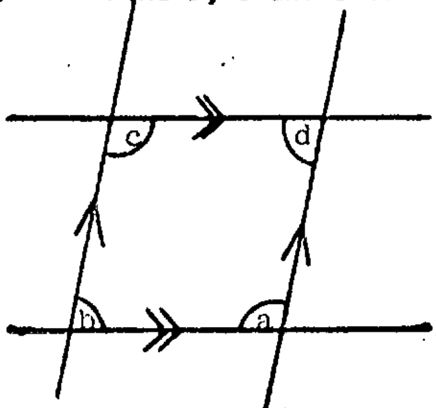
   Find the valuesof z, p, r and q.

4. Two lines AOB and XOY meet at O. If AOX = 120^0 write down the size of the angles XOB, BOY and YOA.

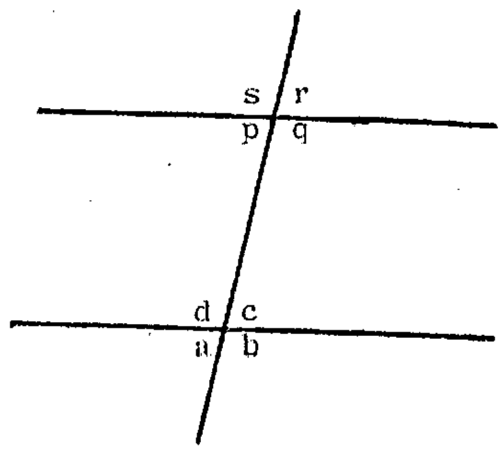
5. Find b if a = 70^0



6. Find B, C and D if $a = 130^0$



7. In the figure below



Find (i) b if $S = 120^0$

(ii) a if $q = 110^0$

SOLUTION TO PRACTICE EXERCISE III

1. (i) $90^0 - 28^0 = 62^0$ is complementary to 28^0

(ii) $(90 - 56)^0 = 34^0$ is complementary to 56^0

(iii) $(90 - 39)^0 = 51^0$ is complementary to 39^0

(iv) $/90 - (20 + X)/ = (70 - X)^0$ is complementary to $20 + X$

(v) $90^0 - (60 + y) = (30 - y)^0$ is complementary to $(60 + y)^0$

2. $180^0 - (120 + Z)^0 = (60 - z)^0$ is supplementary to $(120 + Z)^0$

$180^0 - (60 + y)^0 = (120 - y)^0$ is supplementary to $(60 + y)^0$

$180^0 - 45^0 = 135^0$ is supplementary to 45^0

$180^0 - 26^0 = 154^0$ is supplementary to 26^0

$180^0 - 121^0 = 59^0$ is supplementary to 121^0

3. (i) $z + 3z^0 = 180$

$4z = 180$

$z = 45^0$

(ii) $P + P + 30 = 180$

$2P = 150$

$P = 75^0$

(iii) $r + 4r = 180^0$

$5r = 180^0$

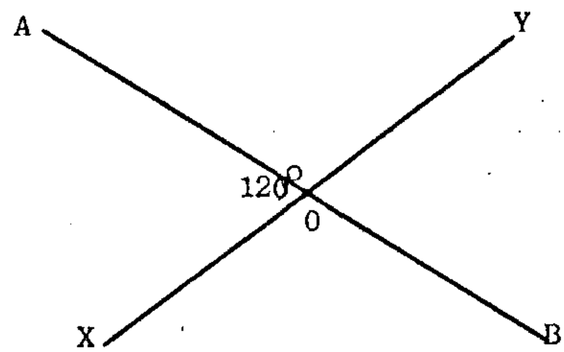
$r = 36^0$

(iv) $q + q + 40 = 180^0$

$2q = 140^0$

$q = 70^0.$

4\.



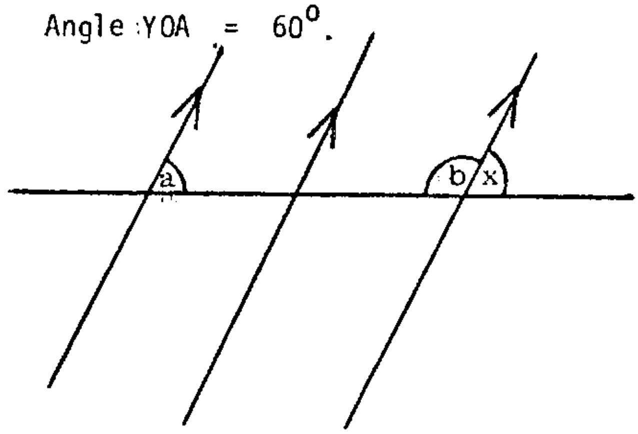
Angle XOB $= 180^0 - 120^0$ (Adjacent Ls on a Straight line)

$= 60^0$

Angle BOY $= 120^0$ (Vertically Opposite angles)

Angle YOA $= 60^0$.

5\.



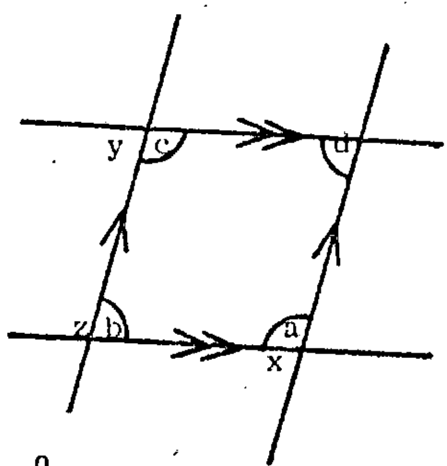
$X = 70^0$ (Corresponding angles) but

$b + x = 180^0$ (Angles on a Straight line)

$b + 70 = 180^0$

$b = 110^0$

6.



$a = 130^0$

$a + x = 180^0$ Adjacent $\angle$s on a Straight line

$130^0 + x = 180^0$

$x = 50^0$

But $x = d$ Corresponding $\angle$s

$\therefore d = 50^0$

Also $d = y = 50^0$ (Corresponding $\angle$s)

But $y + C = 180^0$ $\angle$s. on a straight line

$\therefore C = 180 - 50$

$= 130^0$

Also $a = z = 130^0$

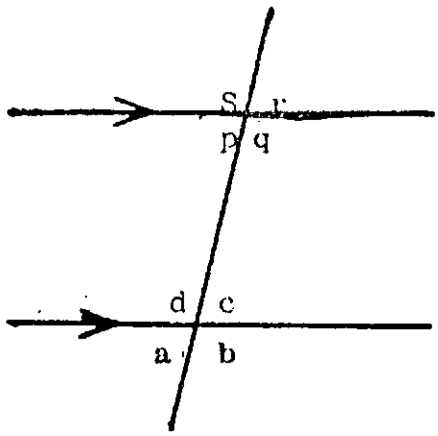
$\therefore b = 180 - 130^0 = 50^0$

$b = 50^0$

$C = 130^0,$

$d = 50^0$

7.



(i) $S = 120^0$

$d = S = 120^0$ (Corresponding Ls)

$d = b$ vertically opposite Ls

$\therefore\ b = 120^0$

(ii) $q = 110^0$

$b = q = 110^0$ (Corresponding angles)

But $a + b = 180^0$ (Adjacent Ls on a straight line)

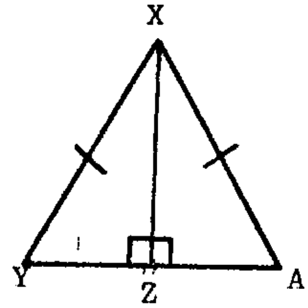
$\therefore\ a + 110 = 180^0$

$\therefore\ a = 70^0$

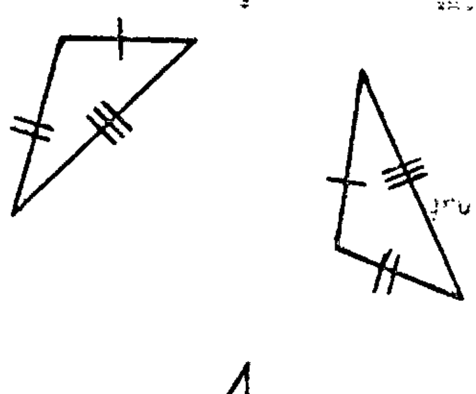
PRACTICE EXERCISE IV

CONGRUENCY

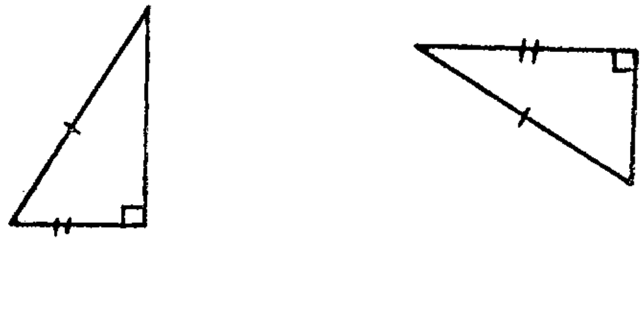
1. Name the triangle which is congruent to $\triangle XYZ$, and state the case of congruency?



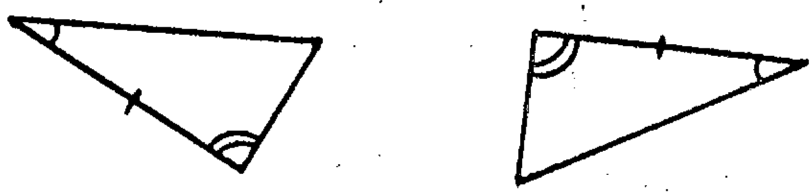
2. In the diagram below, pairs of triangles with equal sides or equal angles are shown with the same mark. State the case of congruency?



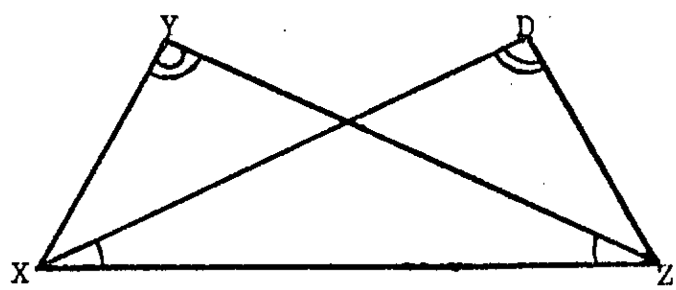
3.



4.



5. Name the triangle which is congruency to $\triangle$XYZ and state the case of congruency?



SOLUTIONS TO PRACTICE EXERCISE IV

1. $\triangle$XAZ (Right Angle Hypotenuse side) (RHS)

2. SSS

3. RHS

4. ASA

5. $\triangle$ZDX (AAS)

PRACTICE EXERCISE V

SIMULTANEOUS LINEAR EQUATIONS

1. Solve the equations $3x + 2y = 24$

   $x + y = 10$

2. Solve the equations $2x - 3y + 2 = x + 2y - 5$

   $= 3x + y$

3. A motorist travels for 30km at xKm/h and for 90km at yKm/h and takes $2\frac{1}{2}$hours for the journey, If the speeds are interchanged the journey takes $2\frac{1}{6}$hours. Find X and y.

4. 7 cups and 8 plates cost ₦6.30
   8 cups and 7 plates cost ₦6.45
   Calculate the cost of a cup and of a plate.

5. A man's age and his son's add to 45 years. Five years ago the man was 6 times as old as his son. How old was the the man when the son was born?

SOLUTION TO PRACTICE EXERCISE V

1. $3x + 2y = 24$ -----------(i)

$x + y = 10$ ------------ (ii)

(ii) x 2; $2x + 2y = 20$ ------------(iii)

(i) - (iii) $x = 4$

Substitute $x = 4$ in equation (ii)

$$x + y = 10$$

$$4 + y = 10$$

$$y = 6$$

$$(x = 4, \; y = 6)$$

CHECK $3 \times 4 + 2 \times 6 = 12 + 12 = 24$

$$4 + 6 = 10$$

2. $2x - 3y + 2 = x + 2y - 5 = 3x + y$

$2x - 3y + 2 = x + 2y - 5$ ----------- (i)

$2x - 3y + 2 = 3x + y$ --------------- (ii)

From (i) $x - 5y = -7$ ------------ (iii)

(ii) $-x - 4y = -2$ ------------ (iv)

(iii) + (iv) $-9y = -9$

$$y = 1$$

Substitute $y = 1$ in equation (iii)

$$x - 5y = -7$$

$$x - 5 = -7$$

$$x = -7 + 5$$

$$= -2$$

$$(x = -2, y = 1)$$

3. 1st Case:

distance = 30km,

Speed = xKm/h

$\therefore$ time taken = $(\frac{30}{x})$

distance = 90Km

Speed = yKm/h

time taken = $(\frac{90}{y})$ h

Total time taken = $2\frac{1}{2} = \frac{5}{2}$ h

$$\therefore \quad \frac{30}{x} + \frac{90}{y} = \frac{5}{2}$$

2nd Case:

when the speed are interchanged we have the time taken is $2^1/_6$

$$\frac{30}{y} + \frac{90}{x} = 2^1/_6 = \frac{13}{6}$$

$$\therefore \quad 2xy\left(\frac{30}{x} + \frac{90}{y} = \frac{5}{2}\right) = 60y + 180x = 5xy \text{ ------- (i)}$$

$$6xy\left(\frac{50}{y} + \frac{90}{x} = \frac{13}{6}\right) = 180x + 540y = 13xy \text{ ------(ii)}$$

$\therefore$ (ii) - (i)

$$480y = 8xy$$

$$\frac{480}{8} = x$$

$$x = 60$$

Substitute in (i)

$$10800 + 540y = 780y$$

$$240y = 10800$$

$$y = \frac{10800}{240} = 45$$

$$(x = 60, \; y = 45)$$

4. Let a cup cost xK and a plate yK

$$\therefore 7x + 8y = 630 \quad \text{-------(i)}$$

$$8x + 7y = 645 \quad \text{----- (ii)}$$

Multiply (i) by 7 and

Multiply (ii) by 8

$$49x + 56y = 4410$$

$$64x + 56y = 5160$$

$$15x = 750$$

$$\therefore x = 50$$

Substitute in (i)

$$350 + 8y = 630$$

$$8y = 280$$

$$y = 280 = 35$$

The cost of a cup is 50k while that of a plate is 35k.

5. Let the age of the son 5 years ageo be x, then his father was 6x years old. Their present ages are:

$(x + 5)$ and $(6x + 5)$ respectively

$$\therefore \quad (x + 5) + (6x + 5) = 45$$

$$7x + 10 = 45$$

$$7x = 35$$

$$x = \frac{35}{7} = 5$$

$\therefore$ the son was 5 years old 5 years ago

$\therefore$ Presently he is $(5+5) = 10$years old

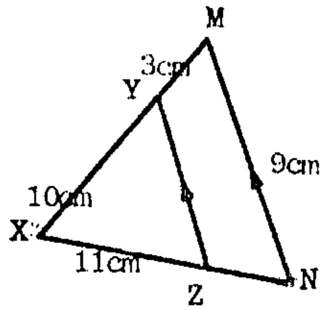
$\therefore$ the father is presently $(6 \times 5 + 5) = 35$ years old

$\therefore$ When the son was born 10years ago the father was $(35 - 10) = 25$ years old.

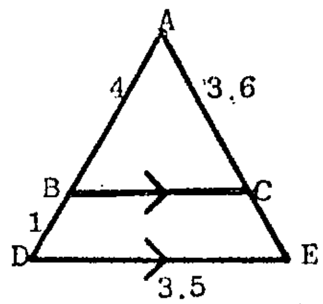
25years

PRACTICE EXERCISE VI
SIMILAR SHAPES

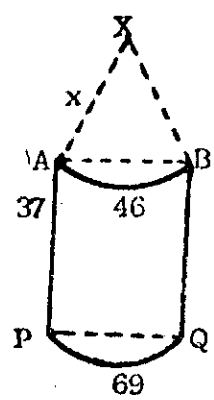
1. In the diagram calculate YZ and ZN



2. In the diagram, if AB = 4m. BD = 1m, AC = 3.6m and DE = 3.5m. Calculate BC and CE?



3. At a mathematical workshop recently held in Lagos a locally made equipment "Lag Sag" is in the form of part of the curved surface of a cone as in the diagram. If AP = 37cm, AB = 46cm, and PQ = 69cm find XP?



4. Find the height of a tree whose shadow is 42m long when the shadow of a man 1.8m tall is 2.4m long?

5. A cuboid is 4cm long, 7cm wide, and 10cm high. A similar cuboid is 25cm high. Calculate its length and width.

SOLUTION TO PRACTICE EXERCISE VI

1. $\dfrac{\triangle XMN}{\triangle XYZ}$; $\dfrac{XM}{XY} = \dfrac{XN}{XZ} = \dfrac{MN}{YZ}$

$$\frac{13}{10} = \frac{9}{YZ} \Rightarrow YZ = \frac{10 \times 9}{13} = 6.9\text{cm}$$

$$\frac{XM}{XY} = \frac{XN}{XZ} \;;\; \frac{13}{10} = \frac{XN}{11} \Rightarrow XN = \frac{13 \times 11}{10} = 14.3$$

$$\therefore ZN = 14.3 - 11 = 3.3\text{cm}$$

2. $\dfrac{\triangle ABC}{\triangle ADE}$; $\dfrac{AB}{AD} = \dfrac{AC}{AE} = \dfrac{BC}{DE}$

$$\frac{4}{5} = \frac{3.6}{AE} = \frac{BC}{3.5}$$

$$\therefore \frac{4}{5} = \frac{BC}{3.5} \Rightarrow BC = \frac{3.5 \times 4}{5} = 2.8\text{m}$$

$$\frac{4}{5} = \frac{3.6}{AE} \Rightarrow AE = \frac{3.6 \times 5}{4} = 4.5\text{m}$$

$$\therefore CE = 4.5 - 3.6 = \underline{\underline{0.9\text{m}}}$$

3. $\dfrac{\triangle XAB}{\triangle XPQ}$; $\dfrac{AX}{XP} = \dfrac{XB}{XQ} = \dfrac{AB}{PQ}$ Let $XA = d$

$$\frac{d}{37+d} = \frac{46}{69} \Longrightarrow 69d = 46 \times 37 + 46d$$

$$\therefore \quad 23d = 46 \times 37$$

$$\therefore \quad d = \frac{46 \times 37}{23} = 74$$

$$\therefore \quad XP = 74 + 37 = 111$$

4. Let the height of the tree be h

$$\therefore \quad \frac{h}{42} = \frac{1.8}{2.4}$$

$$\therefore \quad h = \frac{42 \times 1.8}{2.4} = \underline{\underline{31.5m}}$$

5. Let the length and width be x, y respectively

$$\therefore \quad \frac{4}{x} = \frac{7}{y} = \frac{10}{25}$$

$$\frac{4}{X} = \frac{10}{25} \Longrightarrow X = \frac{25 \times 4}{10} = \frac{100}{10} = 10cm$$

$$\frac{7}{y} = \frac{10}{25} \Longrightarrow y = \frac{25 \times 7}{10} = \frac{175}{10} = \underline{\underline{17.5cm}}$$

PRACTICE EXERCISE VII

VARIATION

1. If the mass, Mg, of a piece of metal varies directly with its volume V cm^3; Find the relationship between M and V.

2. The thickness of a book varies directly with the number of pages in the book. A book is 1.2cm thick and contains 300 pages. What is the thickness of the first 80pages of the book.

3. The mass of rice that each woman gets when sharing a sack varies inversely with the number of women, when there are 20 women, each /gets 6kg of rice. If there are 9 women, how much does each get?

4. At a mathematics/Science workshop held at Lagos, it was discovered in a locally made thermometer that the height hcm of mercury in the thermometer varies directly with the temperature T^{oC} of the mercury. When T = 45, h = 6.75. Find the relationship between h and T. Hence find h when T = 76.

5. X is partly constant and partly varies as y; When $y = 2$, $X = 30$ and when $y = 6$, $X = 50$.

Find X when $y = 3$

SOLUTION TO PRACTICE EXERCISE VII

(1) Let M representsmass of the metal in g

V represents Volume of the metal in cm^3

If the mass varies directly with volume, it means that

$$M \propto V$$

$$\Longrightarrow \underline{M = KV}$$

where K is the constant of proportionality.

(2) Let the thickness of the book be represented by 't' and the number of pages by 'n'

Since $t \propto n$

i.e $t = Kn$ -----------(i)

for $t = 1.2cm$ & $n = 300$

from (i)

We have $K = {}^{t}/_{n} = \frac{1.2}{300} = \frac{12}{3000}$

$\therefore$ For the first 80pages of the book, the thickness

$t = Kn = \frac{12}{3000} \times \frac{80}{1} = \frac{24}{75} = \underline{\underline{0.32cm}}$

(3) Let the mass of rice each women gets be 'M' and the number of women be represented by 'N' from the information given

$$M \propto {}^{1}/_{n}$$

$\Longrightarrow M = {}^{K}/_{n}$ ----------- (i) i.e $K = MN$

$\therefore$ For $n = 20$ & $M = 6$ $\Longrightarrow$ $K = 20 \times 6 = 120$

$\therefore$ For $n = 9$

$$m = {}^{k}/_{n} = \frac{120}{9} = 13\tfrac{1}{3}\text{Kg}$$

i.e. $\underline{\underline{M = 13\tfrac{1}{3}\ \text{Kg}}}$

(4) Let h represents height of mercury in the thermometer

T represents temperature of the thermometer

And h varies directly as T

i.e. $h \propto T$

or $h = KT$ ------------ (i)

Given $T = 45$ when $h = 6.75$

$$\therefore \quad K = {}^{h}/_{T} = \frac{6.75}{45}$$

$$= 0.15$$

From (i)

$\Longrightarrow$ $\underline{h = 0.15T}$

When $T = 76$

$h = KT = 0.15 \times 76$

$= \underline{\underline{11.4\text{cm}}}$

5. $X \propto K + y$

$X = K_1 + K_2 y$ ----------- (i)

When $y = 2$, $X = 30$; $30 = K_1 + 2K_2$ ------ (ii)

When $y = 6$, $X = 50$; $50 = K_1 + 6K_2$ --------- (iii)

Equation (iii) - (ii) $20 = 4K_2$

$K_2 = 5$

Substitute $K_2 = 5$ in equation (ii)

$30 = K_1 + 2 \times 5$

$K_1 = 30 - 10$

$= 20$

From equation (i) $X = 20 + 5y$

When $y = 3$,

$X = 20 + 5 \times 3$

$= 20 + 15$

$= 35$

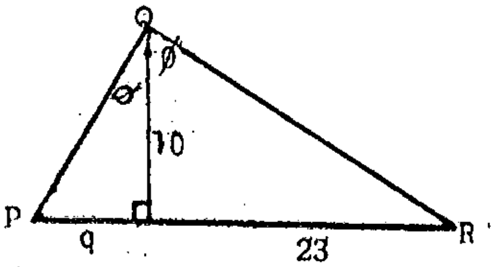
when $y = 3$, $X = 35$.

PRACTICE EXERCISE VIII

TRIGONOMETRIC RATIOS

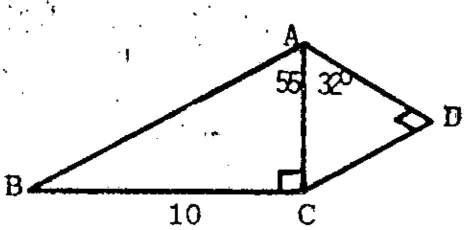
1. The ratio of the sides of an isosceles triangle is 7:6:7 Find the base angle to the nearest degree.

2. In the figure below, calculate θ and ϕ. Hence, find the size of angle PQR to the nearest degree



3. A triangle has sides 8cm and 5cm and an angle of 90^{0} between them. Calculate the smallest angle of the triangle.

4. In the figure below, calculate
   (i) /AC/
   (ii) /AD/

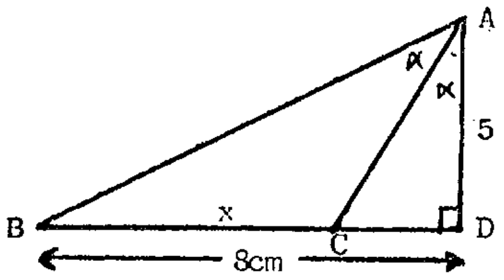


Give your answers correct to three significant figures.

5. In the figure below

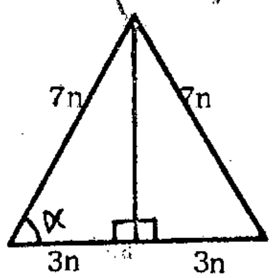
(a) Write down the value of tan 2x.

Hence calculate (b) α (c) x



SOLUTION TO PRACTICE EXERCISE VIII

1. Let the sides be 7n, 6n, 7n.



$\therefore \quad \text{Cos } \alpha = \dfrac{3n}{7n} = {}^{3}/_{7}$

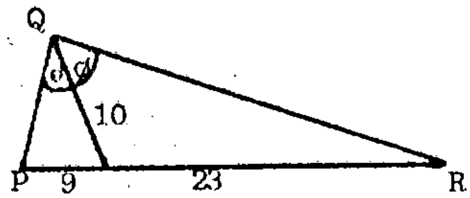
$\alpha = \text{Cos}^{-1}\ {}^{3}/_{7}$

$= \text{Cos}^{-1}\ 0.4285$

$= 64.62^{0}$

$\therefore$ base angle to nearest degree is 65^{0}.

2.



$\tan \theta = \dfrac{9}{10}$

$\theta = \tan^{-1} 0.9$

$= 41.99^{0}$

$\tan \phi = \dfrac{23}{10} = 2.3$

$\emptyset = \tan^{-1} 2.3$

$= 66.5^{0}$

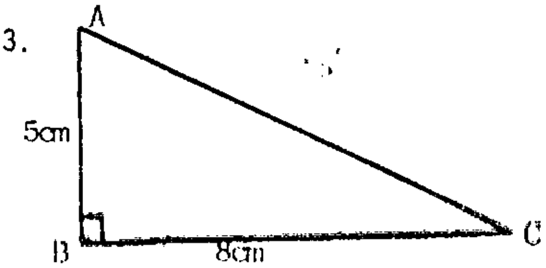
$\angle PQR = \theta^{0} + \emptyset^{0}$

$= 41.99 + 66.5^{0}$

$= 108.49$

$= 108^{0}$

3.



The smallest angle of the triangle will be the angle opposite the shortest side. Angle C is Opposite to the shortest side AB which is 5cm

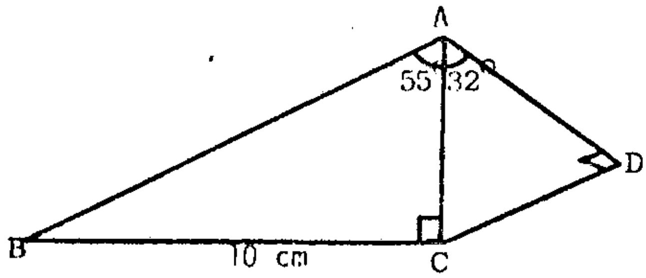
$\tan C = \frac{5}{8}$

$= 0.625$

$C = \tan^{-1} 0.625$

$= 32^{0}$

4.



$\tan 55^{0} = \frac{BC}{AC}$

$= \frac{10}{AC}$

$\therefore \quad AC = \frac{10}{\tan 55^0}$

$= 7.00cm$

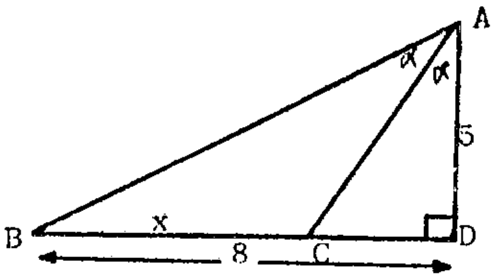
$\text{Cos } 32^0 = \frac{AD}{AC}$

$= \frac{AD}{7}$

$\therefore \quad AD = 7 \text{ Cos } 32^0$

$= 5.94cm.$

5.



(a) $\tan 2\alpha = \frac{8}{5} = 1.6$

(b) $2\alpha = \tan^{-1} 1.6$

$= 57.99^0$

$\therefore \quad \alpha = \frac{57.99}{2}$

$= 28.99^0$

$= 29^0$

(c) Since $BC = X\text{cm}$ and $BD = 8\text{cm}$

then $CD = 8 - X$

In $\triangle ACD$, $\tan \alpha = \frac{CD}{AD}$

$$= \frac{8 - x}{5}$$

But $\alpha = 29^0$, $\tan 29^0 = \frac{8 - x}{5}$

$$\therefore 8 - x = 5 \tan 29^0$$

$$x = 8 - 5 \tan 29^0$$

$$= 8 - 2.77$$

$$= 5.23\text{cm}$$

APPENDIX "M"

MARKING SCHEME FOR POST-TEST

1. Let Dupe's age be x years
   and Eze's age be y years $(\frac{1}{2})$

$$x + y = 25 \quad \text{-------} (i) \quad (\tfrac{1}{2})$$

Eight years ago, Dupe was x - 8 years
Eight years ago, Eze was y - 8 years $(\frac{1}{2})$

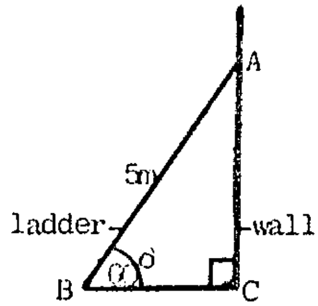
$$x - 8 = 2(y-8) \quad \text{--------} (ii) \quad (1)$$
$$x - 2y = -16 + 8$$
$$x - 2y = -8 \quad \text{--------} (iii) \quad (\tfrac{1}{2})$$

(i) - (iii) $3y = 33$
$$y = 11 \quad (1)$$

Substitute $y = 11$ in equation (i)
$$x + 11 = 25$$
$$x = 14 \quad (1)$$

$\therefore$ Dupe is 14years old and Eze is 11years old (1)

2. Required to calculate AC



(1)

$\text{Sin } 60 = \frac{AC}{5}$ (1)

$\therefore AC = 5 \text{ Sin } 60$ (1)

$= 4.33m$ (1)

$\therefore$ the ladder reached 4.33_m of the wall.

3. Let E represent effort and L represent Load (½)

$E \propto K_1 + L$ (1)

$E = K_1 + K_2 L$ --------- (i)

$6 = K_1 + 8 K_2$ ----------- (ii)

(1)

$8 = K_1 + 12 K_2$ ------------ (iii)

(iii) - (ii)

$2 = 4K_2$ (1)

$\therefore K_2 = \frac{2}{4} = \frac{1}{2}$ (1)

Substitute $K_2 = \frac{1}{2}$ in equation (ii)

$6 = K_1 + 8 \times \frac{1}{2}$ (½)

$6 = K_1 + 4$

$\therefore K_1 = 6 - 4$

$= 2$ (1)

From (i) $E = 2 + \frac{L}{2}$ $(\frac{1}{2})$

When $L = 20$

$E = 2 + \frac{20}{2}$

$= 2 + 10$

$= 12.$ (1)

$\therefore$ Effort necessary for a load of 20 newtons is 12 newtons. $(\frac{1}{2})$

4. $x = 20^0$ Alternate Ls $(\frac{1}{2})$

$m = 160^0$ Adj. angles on a straight line $(\frac{1}{2})$

$M = q = 160^0$ (corresponding Ls) $(\frac{1}{2})$

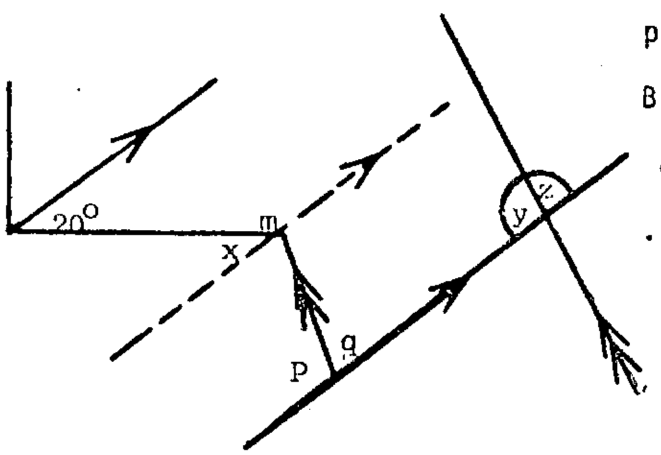
$p = x = 20^0$ (corresponding Ls) $(\frac{1}{2})$

But $q = z$ (Corresponding Ls)

and $P = y$ (corresponding Ls)

$\therefore y = 20^0$ $(\frac{1}{2})$

$z = 160^0$ $(\frac{1}{2})$



5. $x + 2x + 3x = 90^0$ (OP $\perp$ OQ) (1)

$6x = 90^0$ (1)

$x = 15^0$ (1)

6. A pentagon has 5 sides

Sum of Interior angles $= (2 \times 5 - 4)\ 90^0$ (1)

$= 6 \times 90^0$

$= 540^0$ (1)

One angle is 160^0

The remaining four angles $= 540^0 - 160^0$

$= 380^0$ (1)

$\therefore$ Each of the four angles $= \frac{380^0}{4}$ (1)

$= 95^0$ (1)

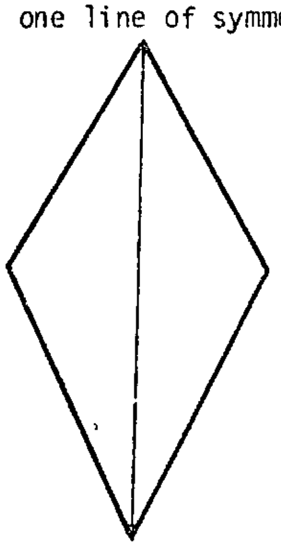
7. (i) Each angle of a square is a right angle while a rhombus is not (1)

(ii) A rhombus has two pairs of parallel sides while a trapezium has only one. (1)

The diagonals of a rhombus meet at the centre while trapezium does not. (1)

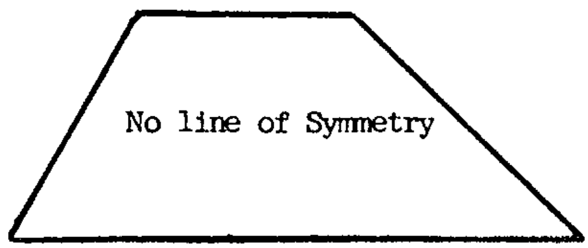
A square is a special kind of rhombus (1)

8. A Kite has one line of symmetry (1)



(1)

A trapezium has no line of symmetry (1)



(1)

9. Ratio of corresponding width $= \frac{48}{4}$ (1)

$= \frac{12}{1}$ (1)

$\therefore$ length of the other match box is 12 x 8

= 96cm (1)

$\therefore$ height of the other matchbox = 12 x 3

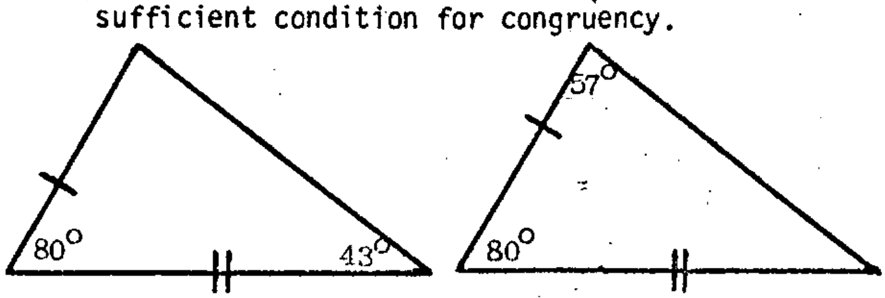
= 36cm (1)

10. Congruency means shapes that are equal in all respect. ($\frac{1}{2}$)

The conditions for congruency in (2)

triangles are SSS, SAS, AAS and RHS. Ass is not a

sufficient condition for congruency.

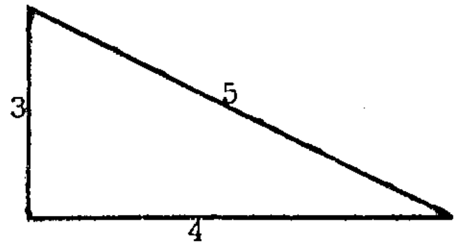


The triangles are congruent (SAS) ($\frac{1}{2}$)

(1)

11. The triangular base dimensions = 3cm by 4cm by 5cm.

This is a right angled triangle (1)



Area of base = $\frac{1}{2} \times 4 \times 3$

$= 6cm^2$ (1)

Volume of a pyramid a triangular base

$= \frac{1}{3}$ area of base x height

$30 = \frac{1}{3} \times 6 \times h$ (1)

$\therefore h = \frac{30}{2} = 15$ (1)

The height of the pyramid is 15cm (1)

APPENDIX 'N'

6th July, 1987.

The Principal,
St. Timothy College,
Onike.

Dear Sir,

TO WHOM IT MAY CONCERN

The bearer, Mrs. A. O. Adeagbo is an M.Phil/Ph.D. student in this Department. She would like to administer some tests to Form Four students in your school.

Kindly oblige.

Yours faithfully,

Prof. A. O. Kalejaiye.

UNIVERSITY OF LAGOS

Department of Curriculum Studies
FACULTY OF EDUCATION
LAGOS, NIGERIA

Cable and Telegrams: UNIEDUC
Telephone : 821904

OUR REF ________________

9th February, 1988.

Isolo, High School,
Isolo, Lagos.

Dear Sir,

The bearer, Mrs. A.O. Adeagbo is a Ph.D student in this Department. She would like to administer some test to form four students in your school.

Kindly oblige.

Yours faithfully,

Prof. A.O. Kalejaiye.

UNIVERSITY OF LAGOS
FACULTY OF EDUCATION
LAGOS NIGERIA
DEPARTMENT OF CURRICULUM STUDIES

Our Ref..................................
Your Ref..................................

THE UNIVERSITY OF LAGOS

CABLES & TELEGRAM: UNIEDUC
TELEPHONE: 842141-4 EXTS. 224 & 227

Date 7th March, 1988

Dear Sir,

The bearer, Mrs. A. O. Adeagbo is a Ph.D. student in this Department. She would like to teach some concepts in Mathematics to Form Four students in your school.

Kindly oblige.

Yours faithfully,

Prof. A. O. Kalejaiye

APPENDIX 'O'

1st July, 1987

Dear teacher

Please assess the suitability of these research instruments for form four students in terms of whether

(a) the items are readable

(b) the questions or stem are well constructed

(c) the answers (options) are equally plausible

(d) the test covers the Junior Secondary School Algebra and Geometry only) Mathematics contents.

Thank you.

Adeagbo A. O. (Mrs)

Given the standard of WAEC school certificate examination in Maths nowadays the test on computational ability is suitable for Form students

Mr. R. A. Adeoye
Maths Teacher
Okoka High Sch
10/7/87.

ASSESSMENT OF THE RESEARCH INSTRUMENT FOR FORM IV

The questions set out in the research question paper are of standard but covers just a portion of the School Certificate and JSS/SSS Syllabuses. The technical aspects of the Test on Comprehension of Mathematics Language is suitable and I am sure will easily betray the crackey and rickety foundation laid for the students while studying the subject in the lower classes.

Generally speaking, the items (questions) in the tests are readable, intelligently constructed while the answers (options) are as well reasonable. The tests are concise, complete and of quality as far as Algebra and Geometry is concerned.

Abiola Tunde Fafowora
St Timothy's College
Onike Lagos.

ALL-SCHOOLS

SIMPLE REGRESSION

ANALYSIS OF VARIANCE	DF	SS	MS	F
REGRESSION	1	123001.362	123001.362	2697.89993
RESIDUAL	586	26716.6314	45.5915211	
TOTAL	587	149717.994		

MULTIPLE R	.906395959
R-SQUARED	.821553633
STANDARD ERROR	6.74065635

VARIABLE	B &	BETA	STD ERROR B
CONCEPTS	.911019238	.906395959	
175393959			
CONSTANT A	-5.76492355		

]?CHR$(12)

ALL-SCHOOLS

SIMPLE REGRESSION

ANALYSIS OF VARIANCE	DF	SS	MS	F
REGRESSION	1	89026.414	89026.414	859.58344
RESIDUAL	586	60691.5817	103.56925	
TOTAL	587	149717.994		

MULTIPLE R	.77112
R-SQUARED	.59463
STANDARD ERROR	10.15958

VARIABLE	B	BETA	STD ERROR B
ML	.78039	.77112	.02662
CONSTANT A	-5.04747		

ALL-SCHOOLS

SIMPLE REGRESSION

ANALYSIS OF VARIANCE	DF	SS	MS	F
REGRESSION	1	69241.1883	69241.1885	504.19671
RESIDUAL	506	80476.8052	137.33243	
TOTAL	507	149717.994		

MULTIPLE R	.68004
R-SQUARED	.46248
STANDARD ERROR	11.69895

VARIABLE	B	BETA	STD ERROR B
LA	.60336	.68006	.02687
CONSTANT A	10.73391		

MULTIPLE REGRESSION

MATRIX OF DEVIATION, CROSS-PRODUCTS, CORRELATION COEFFS, AND SD OF DA

149717.994	135015.109	114759.939	114079.231
.906395957	148202.259	121949.979	118721.299
.680056907	.726350546	190202.449	122893.082
.771120835	.806592315	.737008052	146182.134
15.9704792	15.8894316	18.0006837	15.7807667

COEFF OF X	BETAS
.814060179	.809928951
9.5490966E-03	.010763
.111227211	.109905948

R-SQUARED .826186546

MULTIPLE REGRESSION EQUATION

Y = -7.62220884 + .814060179X1 + 9.5490966E-03X2 + .111227211X3

F = 2.71306767E-03

MEAN

PS	37.0034014
CONCEPTS	46.9455783
CA	43.5306123
ML	53.8843537

STANDARD DEVIATION

PS	15.9704792
CONCEPTS	15.8894316
CA	18.0006837
ML	15.7807667

MULTIPLE REGRESSION TABLE

ANALYSIS OF VARIANCE	DF	SS	MS	F
REGRESSION	3	123694.992	41231.6639	925.308041
RESIDUAL	584	26023.0006	44.5599325	

MULTIPLE R .908948044
R SQUARE .826186546 .826186544
STD ERROR 6.67532266

	SQ-MULT-CORR	STD ERROR	T
1	.688667041	.0310765144	26.1953502
2	.592961492	.0239908956	.398030017
3	.698945025	.0318820149	3.49549623