CHAPTER ONE
INTRODUCTION

1.1 Background to the Study

In a challenging and changing world, people in general and students in particular are required to develop advanced thinking skills that would enable them take rational decisions to solve their problems. Critical Thinking seems to have been universally accepted in all fields as an important concept to enhance teaching and learning in and out of school because it involves intellectual capability, self-evaluation, logical reasoning and making rational decisions. In other words, Critical Thinking would enable one to analyze, evaluate, explain, and restructure thinking strategies which would help reduce the risk of accepting untrue statements. With Critical Thinking, one can easily solve equations sequentially using different methods to calculate figures and arrived at a valid answer.

The art of calculation is as old as man and Critical Thinking has been one of the tools used in solving problems in this regard. Critical Thinking requires one's ability to collect, interpret, analyze and evaluate data for the purpose of arriving at a reliable and valid conclusion (Paul,1997). In teaching Mathematics in schools therefore, Critical Thinking needs to be integrated and emphasized in the curriculum so that students can learn the skills and apply it to improve their performance and reasoning ability. In this context, if teachers are supposed to infuse Critical Thinking Skills to teach Mathematics in their classrooms, then teacher education programme should also allocate more courses for Critical Thinking so that prospective teachers would become models of thinking strategies who in turn will make the subject easier.

Critical Thinking may also involve logical reasoning and ability to separate facts from opinion, examine information critically with evidence before accepting or rejecting ideas and questions in relation to the issue at hand. In other words, it makes individuals to think, question issues, challenge ideas, generate solutions to problems and take intelligent decisions when faced with challenges. In applying Critical Thinking in school settings, it is necessary to develop thinking skills because people with Critical Thinking Skills would be able to understand the logical connections between ideas, construct and evaluate arguments, detect common mistakes in reasoning and solving problems systematically.

Facione (1990) identified six cognitive skills as central to the concept of Critical Thinking. These were: Interpretation, Analysis, Explanation, Evaluation, Self-regulation and Inference. Critical Thinking Skills therefore, are skills that enable one to analyse and synthesize information to solve problems in broad range of areas (Facione, 1990). Critical Thinking could also be seen as making rational and sound arguments because it requires the collection of facts, analysis of the facts and evaluation of such facts in such a way that would lead to reliable, valid, logical and trustworthy conclusions (Schafersman, 1991). In ensuring sound arguments, facts will be collected, analyzed , evaluated and conclusion will be drawn based on available facts. This fact could also be assessed by two or more people to have a critical view of assessing information and to enable one draw valid conclusion.

In improving learning in Mathematics, the use of Peer Assessment may also help the students to do better because it is a means of evaluating students and it adds a valuable dimension to learning and teaching. As a result, students might have the opportunity to talk, discuss, explain and challenge one another thus enabling them to achieve beyond what they can learn unaided. The participation in these activities might help in developing students' autonomy, maturity and critical abilities to solve Mathematical problems which would enable them to do well in their examinations.

Peer Assessment therefore is a process whereby students assess assignments or tests of their peers based on teacher's benchmark or Instructional Rubrics (Sadler & Eddie, 2006). The practice is employed to save teacher's time and improve students' understanding of course materials as students would be exposed to each other's work, learn different steps in presenting, analyzing, evaluating, and solving mathematical problems. These processes could be achieved through Instructional Rubrics.

The Instructional Rubrics are information feedback or scoring guides that are used by teachers and participants in assessing and evaluating students' work based on sets of criteria ranging from poor to excellent performance. These activities also encourage cooperative learning by making lower achievers learn from higher achievers thereby removing their phobia for Mathematics. Critical Thinking (CT) and Peer Assessment (PA) therefore can be used to enhance academic performance in Mathematics because both help in presenting mathematical solutions in logical and systematic manners. In applying CT and PA in school to enhance performance, gender and age differences are considered to be predictors of success in Mathematics.

Gender is one of the properties that distinguish organisms on the basis of their reproductive roles. In thought, one could argue that some female students may produce exactly the same scientific knowledge as their male counterparts. However, in reality it appears that some female students out-perform their male students on computational tasks but do less in problem solving tasks while male students outperform female students on spatial tasks (Clearly & Heng-Kietisak, 1989). The differences in performance depend not only on the ability of the students to solve Mathematical problems but also in their ability to understand what the problem is asking and what information is provided in the problem statement.

Gender differences in science have been discussed for years. These differences, if any, can be grouped into two main categories: differences in science ability and differences in attitude towards science. For example, researchers have found significant differences between male and female students in science achievement. The findings from (DeBaz; 1994; Lee and Burkam, 1996) showed that male students did better than female students in physical sciences while female students did better in life sciences Also, Hedges and Newell (1995) found that in science, boys outperform girls, but in reading and writing girls have the advantage.

In contrary, certain studies indicated that gender differences generally are small or non-existent. There are researchers who did a study on gender and achievement and found out that girls perform better in school than boys in all major subjects and that they graduate from high school with higher grade point averages (GPAs) than their male peers (Wong,, Lam, & Ho, 2002; Perkins, Kleiner, Roey & Brown, 2004). However, some researchers were of the view that there is no significant gender difference in academic achievement of students in Mathematics (Abubaka & Adegboyega, 2012; Ding, Song & Richardson; 2007). The interaction of male and female students in the class could also enhance performance in Mathematics as a result of brainstorming and competition. In considering the performance of students in Mathematics, age of students could also be used as predictors to academic achievement.

Age of students is very important in school especially at entry age to school because it helps in determining what a students can assimilate at different ages, hence it serves as predictors to success or failure of students in life. Different tasks are given to different individuals based on their ages to determine the level of adjustment toward the tasks. It seems that as students become older, they are able to do many tasks in Mathematics than when they were younger because their mental structure would have been developed to cope with challenges in Mathematics. However, as students become older, the correlation between age and school achievement in Mathematics might diminish because some of the older students tend to forget what they have learnt (Coleman, Campbell, Hobson, McParland, Mood, Weinfield & York 1966).

Mathematics as an important subject in modern society is useful in schools, workplaces, businesses and for personal decision-making. Mathematics is seen to be a language for everyday use whether in the market place, schools or even at home. The usefulness of Mathematics can even be observed in the application of numbers to measure length, volume, weight, density, temperature, speed, and acceleration. Mathematics is fundamental to national prosperity by providing tools for understanding Science, Engineering, Technology and Economics (Kulbir, 2006). The importance of this subject may have led the Nigerian government to make it a compulsory subject in basic education and senior secondary schools as well as a prerequisite for admission to tertiary institutions. In Nigeria, Mathematics is one of the core subjects taught at all levels but students seem to shy away from the subject for many reasons, some of which include phobia, teachers' attitude towards the teaching of Mathematics and students' negative attitude from the assumption that Mathematics is generally a difficult subject to study (Okereke, 2006).

Despite the importance placed on Mathematics by the society, Maduabum and Odili (2006) observed that some students lack interest in the subject and perform poorly in it. For example, the West African Senior Secondary School Certificate Examination (WASSCE) May/June results from 2006 to 2012 show that, on the average, less than 50% of the students passed Mathematics at credit level. This may be attributed to many factors traceable to the students, teachers, policy makers, content of the curriculum and the quality of the examination (Asun, 1986; Bajah, 1996). It appears that some teachers hardly teach well and cover the syllabus in Mathematics especially the difficult topics such as Bearing, Circle Geometric and Trigonometry (WAEC Chief Examiner’s report, 2012). Other teachers hardly motivate or encourage the students by not introducing relevant instructional materials to teach in the class room. More importantly, some of the teachers hardly emphasized on the importance of Mathematics in the class while some students have negative attitude towards Mathematics and therefore avoid Mathematics classes. One effect of this is the fact that many students are not able to make up to credit level pass in Mathematics.

Evidence from the WAEC Chief examiner’s report as presented in Table 1 below shows that in the years 2006 to 2012, the percentage of credit level passes in Mathematics was below 50% of all candidates who registered for the May/ June SSCE. The best result was recorded in 2009 when 47.04 of the registered candidates passed at credit level, the worst results was recorded in 2012 with 38.82%.

Table 1: Summary of May/June (SSCE) Mathematics Results (Nigeria) from 2006 – 2012

Year	Total Entry	Total Passes at credit level and above	(%)Passed
2006	1,149,277	472,674	41.12
2007	1,249,028	584,024	46.75
2008	1,369,142	188,394	43.50
2009	1,373,009	356981	47.04
2010	1,351,557	534841	41.95
2011	1,540,250	587,630	40.35
2012	1,672,224	649,156	38.81

Source: The West African Examination Council, Research Division Annual Reports (2006-2012).

The drop in performance in Mathematics in 2012 to less than 40% was also attributed to shallow knowledge of the subject matter, disregard for rubrics, incorrect interpretation of questions (WAEC chief examiner’s report, 2012) and inability of the teachers to diversify his or her teaching-learning strategies. These strategies could be integration of Critical Thinking

Skills and Peer Assessments in their lesson notes which are students centred approaches as opposed to traditional method which is teacher centred approach.

Ukeje (1986) observed that Mathematics is one of the poorly taught, widely hated and misunderstood subjects in secondary schools; therefore students, particularly girls, had negative attitude towards the subject and performed poorly than their male counterparts.
Osarenren and Asiedu (2007) also submitted that the reason for the continued poor performance of students in Mathematics could, among others, be attributed to the students' inability to think critically and analyze Mathematical concepts systematically. This further shows that Critical Thinking is an essential skill that is required to enhance performance in any subject especially in Mathematics.

1.2 Statement of the Problem

In Nigeria, students' academic performance in Mathematics in the Senior Secondary School Certificate Examinations (SSCE) has not been encouraging, despite the various approaches to learning and instruction made available to students. These approaches include the use of instructional materials, study habit, and recruitment of trained teachers. Mathematics teaching and learning at the senior secondary school level in Nigeria seem to have been plagued with poor results, giving teachers, parents, curriculum experts, counsellors and evaluators serious concern.

It is therefore of paramount importance to seek better strategies of teaching Mathematics with the aim of improving students understanding and performance in Mathematics examinations. Researchers (Barry, Ada & Jenny, 2003; Schafer, Ben and Newbery, 2001; Sadler & Eddie, 2006; Andrade & Du, 2005) were of the view that lack of Critical Thinking Skills and Peer Assessment using Instructional Rubrics in teaching Mathematics were some of the major factors that contribute to poor performance in Mathematics among secondary school students. Moreover, most of the assessment techniques and mode of teaching in schools were teacher-centered approaches which might not give students the opportunity to be creative in solving Mathematical problems and to assess one another in terms of their strengths and weaknesses;

unlike Peer Assessment which enable students to give each other valuable feedback so that they can learn from and support each other in Mathematics.

More importantly, the traditional way of teaching Mathematics in school sometimes involves repetition and memorization of previously taught materials which fill the students' minds with knowledge of Mathematics without explaining in detail the processes of analyzing, evaluating and arriving at conclusions. In addition, these processes may not make the students to be critical in thinking because some of them might find it difficult to apply the knowledge acquired to solve Mathematical problems in a new situation.

The National Policy on Education NPE, (2008) provides that the educational system in Nigeria should enable students to imbibe the ability to make rational decisions and acquire skills and competencies that would enable them to function well and solve problems in the society. Critical Thinking Skill has been adopted in Nigeria as one of the nation's educational goals to make students creative (Owolabi, 2003). However, it has not been fully incorporated and given wide recognition as one of the major concepts in the school curriculum. Owolabi (2003) further explained that the failure rate might be attributed among other reasons to lack of interpreting, explaining, analyzing and evaluating questions. This may be one of the reasons why the student's poor performance seen in Table 1 still persists. As a result, it has created a wide gap in performance as lack of integration of Critical Thinking Skills and Peer Assessment in the teaching of Mathematics may lead to students' poor performance. Therefore, this study examined the impact of Critical Thinking Skills and Peer Assessment on senior secondary school students' performance in Mathematics in Delta State, Nigeria

1.3 Theoretical Framework

The Theoretical frameworks for this study were:

Constructivist Theory

Cognitive-field Theory

Realistic Mathematics Education Theory

Infusion Theory

Constructivist Theory: **(Brunner, 1966)**

This theory states that a learner could create or construct new ideas and concepts of solving problems by using his past and current knowledge. Brunner (1966) asserted that the fulcrum of constructivism is that people construct their own understanding and knowledge of the world, by experiencing things and reflecting on those experiences. Piaget (1967) also suggested that through processes of accommodation and assimilation, individuals construct new knowledge from their experiences. In the teaching-learning process, the teacher makes sure he understands the students' pre-existing conceptions, and guides the activity to address them and then builds on them. Put succinctly, the teacher's role is to support active process through exploration and dialogue.

According to Bruner (1966), a theory of instruction should address four major aspects: (1) predisposition towards learning, (2) structuring a body of knowledge so that it can be most readily grasped by the learner, (3) introducing the most effective sequences in which to present materials, and (4) recognizing the nature and the spacing of rewards and punishments. In making sure that students go through the processes of independent learning, teachers encourage the students to solve problems posed to them as proof that they understood it. By these activities the students are encouraged to conceive themselves as problem-solvers, design and perform relevant experiments. This theory is relevant to this study because it encourages students to be creative and independent in solving problems, and indeed all mathematical problems.

Gestalt Cognitive-field Theory (**Wertheimer, 1959**)

This theory states that learning process involves constant organization, restructuring, and organization of stimuli elements into a meaningful whole or pattern resulting from many interacting influences in the environment of the learner. The intervening variables are significant cognitive processes by which "mental maps" are constructed for guiding the individual's performance (Ilogu, 2005). Gestalt theory claimed that we experience the world in meaningful patterns or as an organised whole; thus, knowledge is organized to solve a problem. Therefore, we should view learning from the perspective of problem solving.

Wertheimer (1959) postulates that knowledge is grouped into elements according to the following principles: proximity, similarity/differentiation, closure and simplicity. These principles are called the laws of organization and are used in the context of explaining perception and problem-solving. In order to teach the learners well, the teachers must understand the mental models that students use to perceive the world and the assumptions they make to support those models. This theory would be useful to this study because effective learning of Mathematics would be achieved as a result of learners' perception or survey of the problem through the cognitive processes.

Realistic Mathematics Education Theory (RMET): **(Freudenthal, 1991)**

This theory states that Mathematics is a human activity that must be connected to reality (Freudenthal, 1991). In other words, Mathematics must be close to the learner and be relevant to everyday life situation to enable the learner think, reason and apply what he has learnt to a real or practical situation. The word "reality", refers not just to connection with the real world but also to problem situations which are real in students' mind.

The following characteristics were considered useful for the theory:

- Use of real life contexts as a starting point for learning.
- Use of models as a bridge between abstract and real objects that help students learn Mathematics at different levels of abstractions.
- Use of students' own production or strategy as a result of their doing Mathematics.
- Interaction which is essential for learning of Mathematics between teacher and students and among students.
- Connection to other disciplines and to meaningful problems in the real world.

The role of students in RMET is mostly to make students work collaboratively for their answers or for direction towards a standard solution and make contributions in class. This theory is relevant to this study because it advocates interactive learning which will make students become creative in selecting real life context that will simplify learning and solve Mathematical problems.

Infusion Theory (Swartz and Perkins, 1994)

The infusion theory states that traditional curriculum materials should be restructured to integrate teaching for thinking into subject areas so that students would be aware of the skills, understand it, practise and apply it in other contexts. In other words, if the teaching of thinking is explicit in a subject area it would have a great impact on the students' performance. Furthermore, the more the classroom instruction incorporates an atmosphere of thoughtfulness, the more the students would be able to value good thinking.

Moreover, the more the teaching of thinking is integrated into content instruction, the more students would think about what they are learning. Swarzts and Perkins (1994) also identified various ingredients in Infusion Theory as reliability, causal explanation, argument analysis and the use of evidence for inferences. This theory is relevant to this study because it makes the students understand and learn the content and subject better, interpret meaning of concepts, understand logical structure, detect fallacious arguments and improve academic performance in Mathematics.

1.4 Purpose of the Study

The purpose of the study is to investigate the impact of Critical Thinking Skills and Peer Assessment on senior secondary school students' performance in Mathematics in Delta State. Specifically, this study therefore is designed to meet the following objectives:

1. To find the difference in Mathematics Performance Tests among students exposed to training instruction on Critical Thinking Skills, Peer Assessment, and those in the control group.

2. To examine difference in Mathematics Performance Tests among participants due to gender.

3. To determine difference in Mathematics Performance tests due age

4. To investigate the interaction effect of treatments and gender on students performance in Mathematics.

5. To determine the interaction effect of treatments and age on students performance in Mathematics.

6. To examine the interaction effect of age and gender on students performance in Mathematics.

7. To examine the interaction effect of treatments, age and gender on students performance in Mathematics.

8. To determine the linear relationship between Mathematics Performance Test scores and a set of variables such as Critical Thinking, Peer Assessment and Gender

1.5 Research Questions

The following research questions guided the study:

1. Would there be any difference in Mathematics performance test among students exposed to training instructions on Critical Thinking Skills, Peer Assessment and those in the control group?

2. Will there be any difference on students' performance in Mathematics due to gender?

3. Would there be any difference on students' performance in Mathematics due to age?

4. What is the interaction effect of treatments and gender on students' performance in Mathematics?

5. What is the interaction effect of treatments and age on students' performance in Mathematics.

6. Would there be any interaction effect of age and gender on students' performance in Mathematics?

7. What is the interaction effect of treatments, age and gender on students' performance in Mathematics?

8. Is there any linear relationship between Mathematics Performance Test scores and a set of variables such as Critical Thinking, Peer Assessment and Gender?

1.6 Research Hypotheses:

The following five research hypotheses are formulated and tested:

1. There is no significant difference in Mathematics performance test among students exposed to Critical Thinking Skills, Peer Assessment and those in the control group.

2. There is no significant main effect of gender on students' performance in Mathematics.

3. There is no significant main effect of age on students performance in Mathematics.

4. There is no significant interaction effect of treatment and gender on students' performance in Mathematics.

5. There is no significant interaction effect of treatment and age on students performance in Mathematics.
6. There is no significant interaction effect of gender and age on students performance in Mathematics.
7. There is no significant interaction effect of treatments, gender and age on students' performance in Mathematics.
8. There is no significant linear relationship between Mathematics Performance Test scores and a set of variables such as Critical Thinking, Peer Assessment and Gender.

1.7 Significance of the Study

This study would be relevant to policy makers and curriculum developers in the education sector because it will help them to review the present educational policy and curriculum with the aim of introducing Critical Thinking Skills as a core component of higher thinking skills development in Mathematics curriculum so as to improve students' learning abilities.

The information gathered in this study would help institutions and researchers in education to have better understanding of Critical Thinking Skills and Peer Assessment, so as to be able to evaluate students' academic performance in secondary schools more appropriately.

The information gathered in this study would help tertiary institutions, teachers and researchers in education to review the entry age of students to adjust to challenges as some younger students in terms ability can compete favourably with older students irrespective of gender.

The study would help secondary school students to be critical in solving problems, evaluate new ideas, select the best ideas and modify them, if necessary.

Finally, it would be relevant to teachers and students to encourage peer interaction and peer-assessment in Mathematics among students.

1.8 Scope and Delimitation of the Study

The study covered Senior Secondary School Two (SSII) classes from four secondary schools (single sex and co-educational) in Delta state. The study also covered students' performance in Mathematics, Critical Thinking, Critical Thinking Skills, Peer Assessment, Instructional Rubrics, Gender and Age.

Operational Definition of Terms

Academic Performance: This is regarded as students' scores in different subjects in school after exposure to teaching or training. In this study, it refers to students' scores in Mathematics.

Assessment: Assessment is a systematic process of gathering information regarding performance of students in Mathematics and organizing the information into interpretative form so as to make useful decisions that will improve students' learning. In this study, it refers to systematic process of collecting information and making a judgement on students test in Mathematics.

Critical Thinking: Critical Thinking is a process in which a person uses his or her mind to interpret, analyze, evaluate and solve problems systematically, detect inconsistent reasoning, study information carefully and engage in reflective and independent thinking in solving Mathematical problems.

Critical Thinking Skills: These are skills acquired through instruction or training to enable students to interpret , analyze, evaluate and solve mathematics problems In this study Critical Thinking Skills are skills participants acquired through instruction to enable them improve their performance in Mathematics. These skills can be elaborated to include: interpretation, explanation, analysis, evaluation, self-regulation and inference.

Infusion technique: This refers to the process of integrating thinking skills into the teaching of Mathematics concepts because it makes students learn the content easier and better.

Instructional Rubrics: These are scoring guides used to assess performance so as to assign marks along specific set of criteria that describe what poor, fair and good performance is.

Mathematics Performance: Mathematics Performance refers to the proficiency in the learning of mathematical concepts determined by the students' performance in terms of scores obtained in the achievement tests in some selected concepts in Mathematics. In this study, it refers to the performance of the students in the fifty five items (fifty multiple choice and five theory questions) in Mathematics Performance Test (MPT).

Peer Assessment: This is an interactive and dynamic process that involves learners in assessing, critiquing and making value judgment on the quality and standard of work of other learners, and providing feedback to peers to enable them to enhance their learning and performance. In this study, it refers to training acquired by participants on how to constructively assess each other's work using success criteria or Rubrics.

CHAPTER TWO

REVIEW OF LITERATURE

The review of literature was carried out using the following outlines:

- Concept and Nature of Mathematics
- The Concept of Critical Thinking, Skills, Process and Models
- Concept of Peer Assessment, Principles and Process
- Concept of Instructional Rubrics and its uses
- Critical Thinking Skills and Performance in Mathematics
- Peer Assessment and Performance in Mathematics
- Instructional Rubrics and Mathematics assessment
- Gender and Age in Mathematics Performance
- Gender and Age in Critical Thinking and Mathematics
- Gender and Age in Peer Assessment and Mathematics
- Conceptual Framework between Critical Thinking, Peer Assessment and Mathematics.
- Appraisal of Literature Review

2.1 Concepts and Nature of Mathematics

Mathematics is seen as vital tool for the understanding and application of science and technology. This discipline plays the vital role for national development, which has become an imperative in the developing nations of the world (Bassey, Joshua & Asim, 2008). In realization of the significant role of Mathematics to nation building, the government of the Federal Republic of Nigeria made the subject compulsory at the basic and secondary levels.

This was aimed at ensuring the inculcation of Mathematics literacy with logical and abstract thinking, needed for living, problem solving, and educational furtherance. Tobias & Ernest (1998) opined that many people regard Mathematics as Magicians possessed by supernatural powers. Although this is very flattering for successful Mathematics, it is yet very bad for those who for one reason or the other are attempting to learn the subject. Many students feel that they will never be able to understand Mathematics and they prefer not to attend Mathematics lesson at all because of their belief that it is a dreadful subject. It is extremely very bad for human beings to acquire the habit of cowardice in any field. The ideal mental health is to be ready to face any challenge which life may bring.

In today's high and ever increasing technology-world, it is important that students, right from childhood, develop ability to do Mathematics so that when they grow up they won't have any fears about the subject. This is particularly important because Mathematics has helped in solving problems, and has made students to be creative (Adebule 2002). The nature of Mathematics has been the focus of much writing over the last few decades (Begg, 1994, 2005; Dossey, 1992; Presmeg, 2002; Winter, 2001).

Dossey (1992) argues that different conceptions of Mathematics influence the ways in which society views Mathematics. This can influence the teaching of Mathematics, and communicate subtle messages to children about the nature of Mathematics that "affect the way they grow to view Mathematics and its role in the world". Similarly, Presmeg (2002) argued that beliefs about the nature of Mathematics either enable or constrain "the bridging process between everyday practices and school Mathematics".

Different dichotomies have been used to highlight the contrasting ways in which Mathematics is viewed. Dossey (1992) distinguished between external conceptions of Mathematics held by those who believe that Mathematics is a fixed body of knowledge that is presented to students, and internal conceptions that view Mathematics as personally constructed, internal knowledge. Begg (1994, 2005) contrasted Mathematical content (knowledge and procedures) with Mathematical processes (reasoning, problem-solving, communicating and making connections).

The view of winter (2001) was about a tension between a mechanistic view of Mathematics (as in the development of skills and knowledge), and Mathematics as a means towards fostering citizenship and responsibility within society (the development of personal, spiritual, moral, social and cultural dimensions). A distinction has been made between Mathematical activity carried out for its own sake and Mathematical activity that is useful for something else (Huckstep, 2000). In order to distinguish between the aims and purposes of Mathematics education, Huckstep (2000) asks: “What are we trying to do in Mathematics education?” and “What are we trying to do it for?”

This particular dichotomy is closely related to the debate about what is Mathematics and what is numeracy (Hogan, 2002; Stoessiger, 2002). Definition of numeracy emphasizes the practical or everyday uses of Mathematics in contexts such as homes, workplaces and communities (Stoessiger, 2002). Odili (1990) defines Mathematics as a science precise in method and faultless in logic and that if a person is exposed to it systematically for a sufficient time usually through a course of study, he is bound to be influenced by its contents, method, logic and procedures.

Umoinyang (2007) viewed Mathematics as the foundation of science and technology without wish a nation can never be prosperous and economically independent. He further noted that competence in Mathematics provide many opportunities for career choice and production of highly defined personnel required by industry, technology and science. Watson (2005) also revealed that the relevance of Mathematics in the development of manpower in social science cannot be overstressed because it is needed in econometrics for the establishment of economics models as a powerful statistical techniques that could be used by business analysts. Furthermore, Armstrong (2006) found that Mathematics concepts such as sets, inequalities, matrix functions, series, progression statistics and calculus are useful in other areas of social science such as business.

This view was supported by Olayinka and Omoegun (1998), who opined that Mathematics is the core of all science subjects, for there is nothing we do that does not have, at least, an element of Mathematics. It is applicable to everyday living whether in the market, schools or at home. It also helps to develop the deductive thinking of a student, thereby encouraging curiosity and innovation. Abe (1998) has reported that Mathematics books might be influential in shaping pupils’ attitude toward the learning, understanding and better performance in Mathematics.

Ojerinde (1999), on his part, defined Mathematics as the communication system for those concepts of space, shapes, sizes, quantity, and others used to describe diverse phenomena, both in physical and economic situations. Saxe (1991) and Scribner (1984) in their study found out that an individual's Mathematics ability is significantly influenced by his participation in encompassing cultural practices such as going shopping, completing worksheets in class, and selling lemonade in the street. Crawford (1980), in his study found out that student's lack of success in Mathematics might be caused by poor Mathematics instruction, insufficient number of Mathematics courses, unintelligible textbooks and misinformation about what Mathematics was and what it was not.

2.1.2 Mathematics Curriculum in Nigeria

There are new trends and developments in Mathematics education relevant to our educational efforts in this period of rapid expansion. Some of these are partly responsible for the philosophy behind the Mathematics curricular. In a nutshell, the curricular were based on, the objective of Mathematics education set out in the National Policy on Education, the categories of students that will be utilizing Mathematics for various purposes (utilitarian values) and other values such as cultural and disciplinary issues.

Kalejaiye (1985) felt that the content of the primary school syllabus is so wide that an average child cannot cover it in six years. He advocated for narrowing down of the syllabus to ensure that the Primary school pupils developed favorable interest towards Mathematics. He attributed failure in Secondary schools to the following reasons: The content of the Secondary syllabus is a bit wide and cannot be covered by an average Mathematics teacher teaching average students, Poor teaching of Mathematics sometimes due to poor knowledge on the part of the Mathematics teacher and unfavorable attitude displayed by students caused by environmental influences.

2.1.3 The Usefulness of Mathematics in Secondary School Curriculum

The education of children and youths always takes place in a particular society because it serves the need of the society. It is evident that any consideration of acceptance of any subject in school curriculum should take cognizance of the goals of education in that society. The Nigerian National Curriculum Conference (1969) had observed that the socio-economic development of the previous decade indicated a general progress but secondary education must remain a terminal education for majority. Mathematics is very useful in school curriculum with the following:

- ❖ **Mathematics as a Science of Numbers and Measurement**

The concept of Mathematics (number) is as old as man himself, but numeric symbol developed when it became necessary for man to keep records of the numbers of this belongings, or of objects around him or to solve some daily problems (Sule and Aiyedun, 2006).

Examples:

- The set of counting or natural numbers, N= {E, 2, 3, …} developed to enable the ancient Indians and the Arabs to count and keep records of their sheep and other objects.
- The set of whole numbers, W = {0,1,2,3,….} became necessary to enable the ancient Indians measure whole units
- The set of integers, Z ={… -3, -2, -1, 0, 1, 8, 3,…} developed to aid the Greeks solve problems of the form 1+
- the set of rational numbers (of the form p where q is non-zero) became necessary to enable Mathematics have a system in which division is always possible. Imagine a world without measurements. What a chaotic world that would be! There would be nothing like money, or time. The drugs we take will have no doses, speed will either be too much or too slow or not in existence at all. Historians will not be able to write or read history, magistrates cannot determine the numbers of years of imprisonments, lawyer cannot quote pages of law books, and administrators will not know the number of staff nor be able to plan for them. You cannot buy or sell. Life will come to a standstill".

- ❖ **Mathematics as Symbolic Universal Language**

According to Sule and Aiyedun (2006) Mathematics makes use of symbols to represent concrete objects, words or expressions, other symbols, abstract ideas or concept. For example, the concept of two is represented by the symbols for addition and Δ represents a triangle. The beauty of Mathematics as a symbolic language lies in the simplicity and brevity of the symbolic forms the impersonal character of the symbols devoid of emotions.

Examples: • The beauty of "Mr. Bex, the magistrate is on trial for armed robbery" we can write "p is on trial for armed robbery?

• Instead of “Mrs. May, the first lady is the defence witness number one”, we can write, “y is DW1”

• For the following long sentences: “A games master bought 5 rackets and 8 tennis balls from one shop for a total sum of N132. In the second shop he spent N156 to buy 7 rackets and 4 tennis balls. Find the unit prices of a racket and a tennis ball. We can simply write: Solve simultaneously $5r + 8b$ and $7r + 4d = 156$ where r is the price per racket and b is the price per tennis ball. From these examples, we see the Mathematical symbols are freely used in law courts. Other examples are “PH/2” which stands for “prosecution witness number 2”. Of course in keeping records to cases, of prisoners, fines terms of imprisonment, dates and case references, Mathematics concept are utilized.

In fact, in all science subjects; physics, chemistry, engineering, agriculture, biology, integrated science, geography and others Mathematical symbols are freely used. For example *H2O* in chemistry is the chemical symbol for water, while *NaCl* is the formula for common salt. This Mathematically shows that one molecule of sodium chemically combines with one molecule of chlorine. The mastery of the use of this symbolic language by Nigerian students will inculcate in them the habit of brevity, clarity and precision of expression and will bring them nearer to unity with other human beings in the world.

2.2 Concept of Critical Thinking

In our everyday lives, we are faced with decisions that require reasoning, understanding, interpreting, analysing and evaluating information. This process involves Critical Thinking because it would enable one to take complex decisions, act ethically, and be able to adapt to changes in any given environment. Critical Thinking is a complex concept that consists of multi-dimensional constructs involving cognitive skills and affective dispositions, so it is not surprising that many teachers are unsure about how to teach their students to think critically.

According to Paul (1997), Critical Thinking is a difficult concept to define and more importantly to use because it involves the systematic process of gathering information, interpreting, analyzing and evaluating the information to arrive at reliable and valid conclusion. Different scholars have given different kinds of definitions to Critical Thinking (Bloom, 1959; Dantas-Whitney, 2002; Facione, 2011; Lau, 2009). One of them is that Critical Thinking involves being purposeful, goal directed, making decisions based on evidence rather than guessing in a scientific problem-solving process (Nugent and Vitale, 2008).

Critical Thinking also involves deep reasoning and a consideration of what we received rather than a forthright acceptance of different ideas (Mansoor, 2012). This means that ideas and suggestions from people about a phenomenon cannot be fully accepted if it does not go through the systematic and logical process of finding the truth. Huitt (1998) also viewed Critical Thinking as a disciplined mental activity of evaluating arguments or propositions and making judgments that can guide the development of beliefs and taking action. Critical Thinking further involves reflective types of thinking; that is thinking about the activities we do (Dantas-Whitney, 2002). In addition, the ability to make sound arguments that is very logical is one of the parts of being able to think critically. However, the arguments we make should be rational, sound in nature and be valid (Lau, 2009).

Moreover, Wood (2002) believes that ideal critical thinkers are those who are open minded; ready and eager to explore all ideas and all points of view, including those alien or those opposed to their own. The ideas and statement of the Critical Thinker are not threatened by opposing views, because they are looking for the truth that stand any scrutiny. Critical Thinking also includes both cognitive and personal competencies. Cognitive competencies include having the ability to dissect, modify, analyse, interpret, examine, correlate, synthesize, summarise, understand, and make inferences and generalizations (Nugent and Vitale, 2008). Personal competencies on the other hand, include being tolerant of ambiguity, thinking independently, having perseverance, being self-confident, inquisitive, motivated, a risk taker, reflective, creative, and curious (Nugent and Vitale, 2008).

The literature on Critical Thinking has its root in two primary academic disciplines: Philosophy and Psychology (Lewis & Smith, 1993). Sternberg (1986) has also noted a third Critical Thinking strand within the field of education. These separate academic strands have developed different approaches to defining Critical Thinking that reflect their respective concerns. These are as follows: Philosophical approach, Cognitive Psychological approach and Educational approach.

The Philosophical Approach: The writings of Socrates, Plato, and Aristotle exemplify the philosophical approach. This approach focuses on the hypothetical critical thinker, enumerating the qualities of a person rather than the behaviours or actions the critical thinker can perform (Lewis & Smith, 1993; Thayer-Bacon, 2000). Sternberg (1986) has noted that this school of thought approaches the critical thinker as an ideal type, focusing on what people are capable of doing under the best of circumstances. Accordingly, Paul (1992) discusses Critical Thinking in the context of "perfections of thought".

Facione (1990) views an ideal critical thinker as someone who is inquisitive in nature, open-minded, flexible, fair-minded, has a desire to be well-informed, understands diverse viewpoints, and is willing to suspend judgment and consider other perspectives. Those working within the philosophical tradition emphasized qualities or standards of thought. For example, Bailin (2002) defines Critical Thinking as thinking of a particular quality, essentially good thinking that meets specified criteria or standards of adequacy and accuracy. Furthermore, the philosophical approach has traditionally focused on the application of formal rules of logic (Lewis & Smith, 1993; Sternberg, 1986).
The definitions of Critical Thinking emerging from the philosophical tradition include:

- "the ability, propensity and skills to engage in an activity with reflective skepticism" (McPeck, 1991,1994). This involves appropriate use of a well-thought out skepticism which serves to establish the true reasons on which various beliefs are based. These reasons are related to the epistemology of each discipline. In his view, Critical Thinking in general is an inconceivable concept, since a person is always thinking of something and the quality of this thinking always depend on the manner in which the criteria of the specific discipline were learned.

- "reflective and reasonable thinking that is focused on deciding what to believe or do" (Ennis, 1993). The reflective thinking refers to the awareness that is manifested in the search for, or the use of valid reasons while the term 'focused' implies a non-accidental intellectual activity. In other words, an activity based on reasons and consciously focused on a goal and phrase regarding what is to be believed or accomplished indicates that Critical Thinking can evaluate statements and beliefs as well as actions (Norris & Ennis, 1998).

- “skillful, responsible thinking that facilitates good judgment because it (1) relies upon criteria, (2) is self-correcting, and (3) is sensitive to context” (Lipman, 1988). To him, individuals need Critical Thinking to help them distinguish, from among all the information they receive, the most relevant according to objectives they pursue, so Critical Thinking is a tool for countering unconsidered actions and thoughts.

- “purposeful, self-regulatory judgment which results in interpretation, analysis, evaluation, and inference, as well as explanation of the evidential, conceptual, methodological, criteriological, or conceptual considerations upon which that judgment is based” (American Philosophical Association, 1990). This definition was buttressed by Delphi panel, that an ideal critical thinker is habitually inquisitive, well informed, trustful of reason, open-minded, flexible, fair-minded in evaluation, honest in facing personal biases, prudent in making judgements, willing to reconsider, clear about issues, orderly in complex matters, diligent in seeking relevant information, reasonable in the selection of criteria, focused in inquiry, and persistent in seeking results which are as precise as the subject and the circumstances of inquiry.

- “disciplined, self-directed thinking that exemplifies the perfections of thinking appropriate to a particular mode or domain of thought” (Paul, 1992).

- thinking that is goal-directed and purposive, “thinking aimed at forming a judgment,” where the thinking itself meets standards of adequacy and accuracy (Bailin et al., 1999).
- “judging in a reflective way what to do or what to believe” (Facione, 2000,).
- “ability to think clearly and rationally in understanding the logical connections between ideas and constructs, evaluate arguments, detect inconsistent reasoning, solve problems systematically, identify the relevance of ideas, and engage in reflective and independent thinking (Fisher, 2005).

The Cognitive Psychological Approach: The cognitive psychological approach contrasts with the philosophical perspective in two ways. First, cognitive psychologists, particularly those who believed in the behaviorist tradition and the experimental research paradigm, tend to focus on how people actually think versus how they could or should think under ideal conditions

(Sternberg, 1986). Second, rather than defining Critical Thinking by pointing to characteristics of the ideal critical thinker or enumerating criteria or standards of "good" thought, those working in cognitive psychology tend to define Critical Thinking by the types of actions or behavior of critical thinkers. Typically, this approach to defining Critical Thinking includes a list of skills or procedures performed by critical thinkers (Lewis & Smith, 1993).

Philosophers have often criticized this latter aspect of the cognitive psychological approach as being reductionist—reducing a complex orchestration of knowledge and skills into a collection of disconnected steps or procedures (Sternberg, 1986). For example, Bailin (2002) argues that it is a fundamental misconception to view Critical Thinking as a series of discrete steps or skills, and that this misconception stems from the behaviourist's need to define constructs in ways that are directly observable.

According to this argument, the actual process of thought is unobservable, cognitive psychologists therefore tend to focus on the products of such thought behaviour or overt skills (e.g., analysis, interpretation, formulating good questions). Other philosophers have also cautioned against confusing the activity of Critical Thinking with its component skills (Facione, 1990), arguing that Critical Thinking is more than simply the sum of its parts (Van Gelder, 2005).

The definitions of Critical Thinking that have emerged from the cognitive psychological approach include:

- "the mental processes and strategies used in solve problems, make decisions, and learn new concepts" (Sternberg, 1986).
- "intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing and evaluating information generated through observation, experience, reflection, reasoning and communication which serves as a guide to belief and actions (Paul, 1997).
- "the use of those cognitive skills or strategies that increase the probability of a desirable outcome" (Halpern, 1998). The desirable outcome is defined by the individual, as making good career choices or wise financial investments." She says Critical Thinking

is purposeful, reasoned, and goal directed. She goes on to say that Critical Thinking is the type of thinking used in problem solving, determining probable outcomes, formulating inferences, and making decisions.

- "seeing both sides of an issue, being open to new evidence that disconfirms your ideas, reasoning dispassionately, demanding that claims be backed by evidence, deducing and inferring conclusions from available facts, solving problems, and so forth" (Willingham, 2007).

The Educational Approach: Finally, those working in the field of education have also participated in discussions about Critical Thinking. Bloom (1956) and his associates are included in this category. Their taxonomy for information processing skills is one of the most widely cited sources for educational practitioners when it comes to teaching and assessing higher-order thinking skills. Bloom's taxonomy is hierarchical, with "comprehension" at the bottom and "evaluation" at the top. The three highest levels (analysis, synthesis and evaluation) are frequently said to represent Critical Thinking (Kennedy, Fisher, & Ennis, 1991).

The benefit of the educational approach is that it is based on years of classroom experience and observations of student learning in the classroom, unlike the philosophical and the psychological traditions which are not base on that (Sternberg, 1986).

2.2.1 Differences between Critical Thinking and Thinking

The difference between Critical Thinking and Thinking are illustrated below:

Table 2: Critical Thinking is different from thinking with the following examples

	Thinking	Critical Thinking
Focus	On information: data, facts, examples. On ideas: opinions, positions.	On ideas: assumptions, biases, flaws in reasoning, point of view, context, implications.
Activity	Organising and making connections between pieces of information or ideas, sometimes making basic inferences.	Deeply and broadly questioning and testing the ways in which an idea is formed as well as how you have been interpreting and examining the ideas. Thinking about your own thinking while you are thinking about thinking of others.
Goal	To form an opinion about what you are thinking	To apply criteria in forming a conclusion or evaluation about what you have been thinking about and how you have been thinking about it.

Paul and Linda, 2008

A well cultivated critical thinker always:

• raises vital questions and problems, formulating them clearly and precisely.

• gathers and assesses relevant information, using abstract ideas to interpret it effectively.

• comes to well-reasoned conclusions and solutions, testing them against relevant criteria and standards.

• thinks open mindedly within alternative systems of thought, recognizing and assessing information as need be, their assumptions, implications and practical consequences.

• communicates effectively with others in figuring out solutions to complex problems.

Critical Thinking in essence is self-directed, self-disciplined, self-monitored and self-corrective thinking. In other words, it requires rigorous standards of excellence and mindful command of thought which enables one to take useful decision in solving real life problems.

2.2.2 Concept of Critical Thinking Skills

Critical Thinking Skills are skills one acquire through instruction or training to be able to interpret, analysed, evaluate and solved problems. A sound student is good at interpreting information, comprehending and expressing meaning about a wide variety of experiences, beliefs, procedures and rules. A competent critical thinker using Analysis would be good at identifying the relationship between statements, questions, concepts or descriptions to express beliefs, judgments or reasons.

Students excelling in Evaluation are competent at assessing credibility of statements and representations of others and assessing the logical strength of statements, descriptions or questions. Proficient students in Inference skill have the ability to draw reasonable conclusions and/or hypotheses based on facts, judgments, beliefs, principles, concepts or other forms of representation. Explanation experts are good at stating and justifying the results of one's reasoning using each of the aforementioned abilities. Self-regulation the last skill, alludes to the ability of an individual to monitor their own personal cognitive activities to make sure that they are engaged in Critical Thinking.

Facione (1990) who conducted a national Delphi study identified six cognitive skills as central to the concept of Critical Thinking Skills. These are:

Interpretation: This simply means to comprehend and express the meaning or significance of a wide variety of experiences, situations, data, events, judgments, conventions, beliefs, rules, procedures, and criteria. Interpretation includes the sub-skills of categorization, decoding significance, and clarifying meaning.

Analysis Skills: This skills enable participants to break down data, information, questions, rules and mathematics equations into simpler form for better understanding of the concepts. This could be achieved through examining ideas and detecting arguments.

Evaluation Skills: This skills enable participants to collect data in order to make value judgement on data, information, and methods of arriving at a solution in solving mathematics problems**.** This could be achieved through assessing claims and assessing arguments.

Inference: This simply means to identify and secure elements needed to draw reasonable conclusions; to form conjectures and hypotheses; to consider relevant information and to deduce the consequences flowing from data, statements, principles, evidence, judgments, beliefs, opinions, concepts, descriptions, questions, or other forms of representation. The sub-skills of inference are querying evidence, conjecturing alternatives, and drawing conclusions.

Explanation Skills: This also means being able to present in a cogent and coherent way the results of one's reasoning. In order words, it means to state and justify that reasoning in terms of evidential, conceptual, methodological, and contextual considerations upon which one's results were based; and to present one's reasoning in the form of cogent arguments. This could be achieved through stating results, justifying procedures and presenting the arguments.

Self-regulation: This means to self-consciously monitor one's cognitive activities and re-assessing one's work. This can also be achieved through self-examination and self-correction.

According to Ennis (1989), empirical inquiry is needed to determine how certain aspects of Critical Thinking apply to a particular content area. He supported the need for contextual domain or subject specific to Critical Thinking based on three observations: background knowledge is essential for making justified Critical Thinking judgments; Critical Thinking varies from field to field; a full understanding of a field requires the ability to think critically in the field (Ennis,1990). Kintsch 1994; Tindal and Nolet 1995; Cheak 1999; and Halliday 2000 have tested Ennis's claims. They discovered that, indeed Critical Thinking is subject matter specific. They also agreed with Ennis that Critical Thinking should be evaluated with a discipline specific measure of Critical Thinking. This means that mathematics education students should be developing Critical Thinking Skills in Science and Technological development areas.

The following are the Critical Thinking Skills described by (Sheffer and Rubenfeld, 2000).

1. Analyzing	• Separating or breaking a whole into parts to discover their nature, functional and relationships. • "I studied it piece by piece" • "I sorted things out"
2. Applying Standards	• Judging according to established personal, professional, or social rules or criteria. • "I judged it according to..."
3. Discriminating	• Recognizing differences and similarities among things or situations and distinguishing carefully as to category or rank. • "I rank ordered the various..." • "I grouped things together"
4. Information Seeking	• Searching for evidence, facts, or knowledge by identifying relevant sources and gathering objective, subjective, historical, and current data from those sources • "I knew I needed to lookup/study..."

	• "I kept searching for data."
5. Logical Reasoning	• Drawing inferences or conclusions that are supported in or justified by evidence • "I deduced from the information that..." • "My rationale for the conclusion was..."
6. Predicting	• Envisioning a plan and its consequences • "I envisioned the outcome would be..." • "I was prepared for..."
7. Transforming Knowledge	• Changing or converting the condition, nature, form, or function of concepts among contexts • "I improved on the basics by..." • "I wondered if that would fit the situation of ..."

Although measures such as California Critical Thinking Disposition Index (CCTDI) and the California Critical Thinking Skills Test (CCTST) are widely used for assessment unlike Watson-Glaser instrument that only tests for Critical Thinking Skills for both secondary school students and other cooperate bodies. The Critical Thinking skill is based on 5 subscales which assess specific areas: analysis, evaluation, inference, deductive reasoning and inductive reasoning:

- The analysis subscale measures whether someone can comprehend and express the meaning in a wide variety of data, experiences, and judgments. It includes the skills of categorizing, determining significance, and clarifying meaning.

- The evaluation subscale measures an individual’s ability to assess information and the strength of actual or inferential relationships. It also relates to the ability to state the results of one’s reasoning.

- The inference subscale measures one’s ability to identify and secure information needed to draw conclusions. For example, can the person form conjectures and hypotheses, consider relevant information and come up with potential consequences.

- The deductive reasoning subscale measures the subject's ability to begin with a premise, and by assuming it is true, conclude that the findings are also true (as with algebraic, geometric and mathematical proofs).

- The inductive reasoning subscale measures a person's ability to begin with a premise and by applying related knowledge and experience, reach a conclusion that is likely to be true. Statistical inferences, use of similar experiences and relevant cases (as in legal reasoning) are examples.

Good Critical Thinking Skills bring numerous benefits such as:

- Skills in Critical Thinking brings precision to the way you think and work.
- Practice in Critical Thinking helps one to be more accurate and specific in noting what is relevant and what is not.
- It improves attention and observation.
- Improve ability to identify the key points in text or other message rather than becoming distracted by less important material.
- It improves ability to respond to the appropriate points in a message.
- It makes one get knowledge to one's point easily and accurately.
- It makes one apply analysis skills in a variety of situations (Cottrell, 2005).

2.2.3 Critical Thinking Process

Critical Thinking Process may involve systematic use of knowledge and intelligence to effectively arrive at the most reasonable and justifiable positions on issues that identify and overcome the numerous hindrances to rational thinking (Caroll,2000).

According to Caroll, (2000), Critical Thinkers underwent the following five processes

Step 1: Adopt the Attitude of a Critical Thinker

Step 2: Recognize and Avoid Critical Thinking Hindrances

Step 3: Identify and Characterize Arguments

Step 4: Evaluate Information Sources

Step 5: Evaluate Arguments

Each of these steps is described separately below.

Step 1: Adopt the Attitude of a Critical Thinker

The first step is developing proper attitude. Such attitude embodies the following characteristics:

- Open-mindedness
- Intellectual humility
- Free thinking
- High motivation

Open-mindedness and skepticism*:* This means seeking the facts, information sources and reasoning to support issues we intend to judge; examining issues from as many sides as possible; rationally looking for the good and bad points of the various sides examined; accepting the fact that we may be in error ourselves and maintaining the goal of getting at the *truth* (or as close to the truth as possible), rather than trying to please others or find fault with their views. Too much skepticism will lead one to doubt everything and commit oneself to nothing, while too little will lead one to gullibility and credulousness.

Intellectual humility: This means adhering tentatively to recently acquired opinions; being prepared to examine new evidence and arguments even if such examination leads one to discover flaws in one's own cherished beliefs; to stop thinking that complex issues can be reduced to matters of 'right & wrong' or 'black & white' and to begin thinking in terms of 'degrees of certainty' or 'shades of grey'. Sometimes 'I don't know' can be the wisest position to take on an issue. As Socrates noted, 'Arrogance does not befit the critical thinker'.

Free Thinking: A critical thinker must also have an independent mind, i.e. be a free thinker. To think freely, one must restrain one's desire to believe because of social pressures to conform. One must be willing to ask if conformity is motivating one's belief or opinion and if so, one must have the strength and courage to at least temporarily abandon one's position until one can complete a more objective and thorough evaluation.

High Motivation: Finally, a critical thinker must have a natural curiosity to further one's understanding and be highly motivated to put in the necessary work sufficient to evaluate the multiple sides of issues. This may require the critical thinker to ask many questions, which can be unsettling to those asked to respond. A critical thinker cannot be lazy.

Step 2: Recognize & Avoid Critical Thinking Hindrances

A Critical Thinker should understand how to recognize and avoid hindrances that characterize everyday life. Some of these hindrances result from unintentional and natural human limitations while others are clearly calculated and manipulative. Some are obvious, but most are subtle or insidious. In every day life, people are exposed to things that hinder their ability to think clearly, accurately and fairly. These hindrances can be divided into four categories namely: Basic Human Limitations, Use of Language, Faulty Logic or Perception and Psychological and Sociological Pitfalls.

Step 3: Identify & Characterize Arguments

At the heart of Critical Thinking is the ability to recognize, construct and evaluate arguments. In the context of Critical Thinking, an argument means the presentation of a reason(s) to support a conclusion(s), or: there must be one or more reason statements and one or more conclusion statements in every argument (Caroll, 2000). Depending on usage and context, reasons are synonymous with: premises, evidence, data, propositions, proofs, and verification. Again, depending on usage and context, conclusions are synonymous with: claims, actions, verdicts, propositions, and opinions.

Argument Example:

Argument = Reason + Conclusion

Don't Trust John	because	he's a politician.
Conclusion	**Indicator**	**Reason**

A critical thinker must learn to pick out arguments from verbal or written communication. Sometimes arguments will have indicators such as 'since', 'because', 'for', 'for the reason that' and 'as indicated by' to separate the conclusion statement(s) from the reason statement(s) that follows (see above example). At other times, arguments will have indicators such as 'therefore', 'thus', 'so', 'hence', and 'it follows that' to separate the reason statement(s) from the conclusion statement(s) that follows. In some cases there will be no indicator words at all; the context alone will indicate if a statement is intended as a reason, a conclusion, or neither.

Formal logic divides arguments into inductive and deductive arguments. While Critical Thinking is an informal application of logic, the critical thinker should at least understand the fundamental differences between the two forms. If one thing follows necessarily from another, this implies a deductive argument. In other words, a deductive argument exists when 'B' may be logically and necessarily inferred from 'A.' For example, if one makes the statement "All bachelors are unmarried ('A')" and "John is a bachelor ('B')", then one can deductively reach the conclusion that John must be unmarried.

However, most arguments that one encounters in daily life are inductive. Unlike deductive arguments, inductive arguments are not 'black and white', because they do not prove their conclusions with necessity. Instead, they are based on reasonable grounds for their conclusion. A critical thinker should understand that no matter how strong the evidence in support of an inductive argument, it will never prove its conclusion by following with necessity or with absolute certainty. Instead, a deductive argument provides only proof to a degree of probability or certainty.

Arguments presented by courtroom attorneys are good examples of inductive arguments, whereupon a defendant must be found guilty beyond a reasonable doubt (equivalent to reasonable grounds). It is always possible that an inductive argument that has sound reasons will have an erroneous conclusion. For example, even though a jury finds a defendant guilty beyond a reasonable doubt, there is always a possibility (even if remote) that the defendant had not committed the crime. The critical thinker should assess the cogency of inductive arguments. An argument is cogent if, when the premises are all true then the conclusion is probably true. That is, one should assess an inductive argument in terms of degrees of probability rather than absolute 'right & wrong' or 'black & white'. This applies even if a 'yes/no' or 'either/or' decision must be made or judgment must be rendered on the argument.

Step 4: Evaluate Information Sources

Most arguments reference facts to support conclusions. But an argument is only as strong as its weakest link. If the facts supporting an argument are erroneous, so will be the argument. A critical thinker must have a sound approach for evaluating the validity of facts. Aside from one's personal experiences, facts are usually acquired from information sources such as eyewitness testimony or people claiming to be experts. These sources are typically cited in the media or published in reference books.

In a society where entertainment and amusement have become lifelong goals, it is often difficult to find unbiased and objective information on a subject. For example, the mass media has found "what if" journalism sells very well: What if the President did some horrible thing; What if the Secretary was motivated by some criminal behaviour, etc. It is common to see reputable journalists reporting on inflammatory speculation as if it was an important news event. How can we expect to cut through the advertising, hype, spin, innuendos, speculation, distortions, and misinformation overloads on TV, radio, newspapers, magazines and the internet, in order to ascertain what is factually correct? Even some reputable publishers seem to be more interested in selling books or periodicals than confirming the truth of what they publish. So how are we to know which information sources to trust?

While there is no simple answer, a critical thinker should look for information sources which are credible, unbiased and accurate. This will depend on such things as the source's qualifications, integrity and reputation. In order to assess these conditions, Caroll (2005) seek answers to the following types of questions:

1. Does the information source have the necessary qualifications or level of understanding to make the claim (conclusion)?
2. Does the source have a reputation for accuracy?
3. Does the source have a motive for being inaccurate or overly biased?
4. Are there any reasons for questioning the honesty or integrity of the source?

If any of the answers are "no" to the first two questions or "yes" to the last two, the critical thinker should be hesitant about accepting arguments which rely on such sources for factual information. This may require additional investigation to seek out more reliable information sources. Information sources often cite survey numbers and statistics which are then used to support arguments. It is extremely easy to fool people with numbers. Since the correct application of numbers to support arguments is necessary but not sufficient, therefore, it is important that a critical thinker become educated in the fundamental principles of probability and statistics before believing the statistical information supporting an argument. One does not need to be a mathematics major to understand these principles. There are a few right ways and many wrong ways to sample populations, perform calculations, and report the results. If a source is biased because of self-interest in the outcome, it more often than not used one of the wrong ways. Perhaps the most important question the critical thinker should ask of any statistical result is: Were the samples taken representative of (a good cross section of) the entire target population?

Step 5: Evaluate Arguments

The last step to Critical Thinking is evaluating arguments. It has three-step process to assess whether: (1) assumptions are warranted; (2) reasoning is relevant and sufficient, and (3) relevant information has been omitted. Each step is described below.

Assumptions. Assumptions are essentially reasons implied in an argument that are taken for granted to be true. Using our earlier argument example, “Don’t trust John because he’s a politician”, the implied assumption is that politicians cannot be trusted. The first step to evaluating arguments is to determine if there are any assumptions and whether such assumptions are warranted or unwarranted. A warranted assumption is one that is either:

1) Known to be true; or
2) Is reasonable to accept without requiring another argument to support it.

An assumption is unwarranted if it fails to meet either of the two above criteria.

Regarding the first criterion, it may be necessary for the critical thinker to perform independent research to verify what is “known to be true.” If the critical thinker, despite such research, is unable to make a determination, he or she should not arbitrarily assume that the assumption is unwarranted.

Regarding the second criterion, a critical thinker normally evaluates the reasons for assumptions in relation to three factors: (a) one’s own knowledge and experience; (b) the information source for the assumption and (c) the kind of claim being made.

If an argument has an unwarranted assumption, and if this assumption is needed to validate the argument’s conclusion, the critical thinker has good cause to question the validity of the entire argument

Reasoning: The second step to evaluating arguments is to assess the relevance and sufficiency of the reasoning (or evidence) in support of the argument’s conclusion. It is helpful to think of “relevance” as the quality of the reasoning, and “sufficiency” as the quantity of the reasoning. Good arguments should have both quality (be relevant) and quantity (be sufficient). It is generally easier (although not always) to pick out reasoning that is relevant (i.e., on the subject or logically related) than it is to determine if the reasoning is sufficient (i.e., enough to validate the argument). So how can one evaluate the sufficiency of reasoning (evidence) to support a conclusion? The term reasonable doubt, as used in a court of law, is considered a good guideline. But how does one go about determining reasonable doubt? Unfortunately, there is no easy answer, but here are some criteria. First, it is important to maintain the attitude of a critical thinker (from Step 1) and be aware of Critical Thinking hindrances (from Step 2). Second, ask yourself the purpose or consequences of the argument being made. This will sometimes determine how much (sufficiency) evidence is required.

Third, the thinker should become aware of contemporary standards of evidence for the subject. For example, you could not judge the sufficiency of evidence for a scientific claim unless you were knowledgeable of the methods and standards for testing similar scientific claims. Finally, the sufficiency of evidence should be in proportion to the strength to which the conclusion is being asserted. Thus, evidence that is not sufficient to support a strong conclusion may be sufficient to support a weaker conclusion.

When evaluating multiple pieces of evidence, both pro and con, how does one weigh the evidence to determine if, the argument is cogent? Again, there is no hard and fast rule. All things being equal, the more reliable the source, the more weight should be given to the evidence. Additionally, more weight should generally be given to superior evidence in terms of its relevance and sufficiency to validate the argument, all things being equal.

Omissions: A cogent argument is one that is complete in that it presents all relevant reasoning (evidence), not just evidence that supports the argument. Arguments that omit relevant evidence can appear to be stronger than they really are. Thus, the final step to evaluating arguments is attempting to determine if important evidence has been omitted or suppressed. Sometimes this happens unintentionally by carelessness or ignorance, but too often it is an intentional act. Since it is usually unproductive to confront arguers and ask them to disclose their omissions, the critical thinker's best course of action is to seek opposing arguments on the subject, which could hopefully reveal such omissions. It is a rare arguer who actively seeks out opposing views and treats them seriously, yet that is precisely what a critical thinker must do when developing his or her own arguments.

Five Steps to Critical-Thinking, Problem-Solving, and Decision Process (Guffey, 1998)

Guffey (1998) developed a five- step plan for thinkers to make the best decisions and to become valuable students and workers. The steps are:

1. Identify and clarify the problem.

The first task is recognizing that a problem exists. Some problems are big and unmistakable, such as failure of an air-freight delivery service to get packages to customers on time. Other problems may be continuing annoyances, such as regularly running out of toner for an office copy machine. The first step in reaching a solution is pinpointing the problem area.

2. Gather information

Learn more about the problem situation. Look for possible causes and solutions. This step may mean checking files, calling suppliers or brainstorming with fellow students/ workers. For example, the air-freight delivery service would investigate the tracking systems of the commercial airlines carrying its packages to determine what went wrong.

3. Evaluate the evidence

Where did the information come from? Does it represent various points of view? What biases could be expected from each source? How accurate is the information gathered? Is it fact or opinion? For example, it is a fact that packages are missing; it is an opinion that they are merely lost and will turn up eventually.

4. Consider alternatives and implications

Draw conclusions from the gathered evidence and pose solutions. Then, weigh the advantages and disadvantages of each alternative. What are the costs, benefits, and consequences? What are the obstacles, and how can they be handled? Most important, what solution best serves your goals and those of your organization? Here's where your creativity is very important.

5. Choose and implement the best alternative

Select an alternative and put it into action. Then, follow through on your decision by monitoring the results of implementing your plan. The freight company decided to give its unhappy customers free delivery service to make up for the lost packages and downtime. On the job you would want to continue observing and adjusting the the solution to ensure its effectiveness over time.

2.2.4 Critical Thinking Model

The Critical Thinking Model is a systematic model used by individuals to arrive at a reliable and valid conclusion. This model was developed by Paul and Elder (2008). They believed that critical thinkers routinely apply the intellectual standards to the elements of reasoning in order to develop intellectual traits.

Paul and Linda's Critical Thinking model

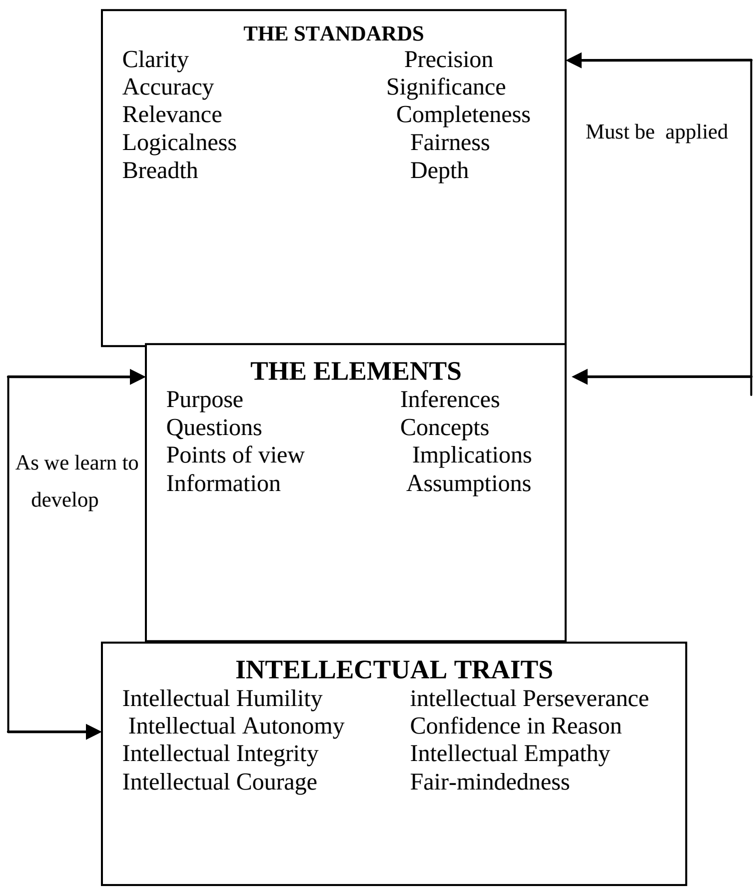


Fig1: Paul & Linda 2008

1. Universal Intellectual Standard and Questions that can be used to apply them

Universal intellectual standards are standards which should be applied to thinking to ensured his quality. For them to be learned, they must be taught explicitly. The ultimate goal, then, is for these standards to become infused in the thinking of students, thereby forming part of their inner voice, guiding them to reason better.

Clarity: The word clarity is viewed as understanding things as they are, and—most importantly—as they could and should be. Clear thinkers are those who know how to reach clarity in their field or discipline. We use the terms clear thinker and clear thinking instead of critical thinker or Critical Thinking in order to emphasize the more comprehensive mindset needed to achieve clarity, which includes the ability to make emotional connections and to persuade others. The following questions are raised to prove clarity: Could you elaborate further on that point? Could you express that point in another way? Could you give me an illustration? Could you give me an example? Clarity is a gateway standard. If a statement is unclear, we cannot determine whether it is accurate or relevant. In fact, we cannot tell anything about it because we don't yet know what it is saying. For example, the question "What can be done about the education system in Nigeria is unclear?". In order to adequately address the question, we would need to have a clearer understanding of what the person asking the question is considering the "problem" to be. A clearer question might be "What can educators do to ensure that students learn the skills and abilities which help them function successfully on the job and in their daily decision-making?"

Accuracy: Accurate thinking could be seen as using our minds, not our emotions, to correctly understand, evaluate and react to events. Accurate thinking will place any event in a proper perspective and is perfected by the ability to spot thinking errors and replacing them with accurately structured thoughts. The following questions determine how to verify accurate statement, questions and statement. Is that really true? How could we check that? How could we find out if that is true? A statement can be clear but not accurate, as in "Most dogs are over 300 pounds in weight."

Precision: Precision is similar to accuracy. It defines the possible or actual deviation from the exact answer. For example, the precision of a numerical value depends on how many significant digits are used, and the possible deviation in measurement from the smallest value given. The following, expanciate how precision questions were raised. Could you give me more details? Could you be more specific? A statement can be both clear and accurate, but not precise, as in “Chibuzor is overweight.” (We don’t know how overweight Chibuzor is, one pound or 500 pounds.)

Relevance: Relevance can be described as something that is of importance. Relevance is a description of items, person or anything that one can think of as important since it helps to give someone a clear perspective of a particular thing that could have happened long ago or recently. The following relevant questions were raised to show how important and related the information or concept is to thinking. How is that connected to the question? How does that bear on the issue? A statement can be clear, accurate, and precise, but not relevant to the question at issue. For example, students often think that the amount of effort they put into a course should be used in raising their grade in a course. Often, however, “effort” does not measure the quality of students’ learning, and when that is so, effort is irrelevant to their appropriate grade.

Depth: Depth in thinking is the quality of having a lot of knowledge, information, understanding or experience about a concept or things. The following questions were raised: How does your answer address the complexities in the question? How are you taking into account the problems in the question? Are you dealing with the most significant factors? A statement can be clear, accurate, precise, and relevant, but superficial (that is, lack depth). For example, the statement “Just Say No”, which is often used to discourage children and teens from using drugs, is clear, accurate, precise, and relevant. Nevertheless, it lacks depth because it treats an extremely complex issue, the pervasive problem of drug use among young people, superficially. It fails to deal with the complexities of the issue.

Breadth: Breadth could be seen as wide range or scope of knowledge, information and experience about a thing, concept or things. The following questions were raised to determine the scope of information one ask. Do we need to consider another point of view? Is there another way to look at this question? What would this look like from a conservative standpoint? What would this look like from the point of view of a lay man or experts? A line of reasoning may be clear, accurate, precise, relevant, and deep, but lack breadth (as in an argument from either the conservative or liberal standpoints which gets deeply into an issue, but only recognizes the insights of one side of the question).

Logic: Logic is the science of how to evaluate arguments and reasoning. Critical Thinking is a process of evaluation which uses logic to separate truth from falsehood, reasonable from unreasonable beliefs. If you want to better evaluate the various claims, ideas, and arguments you encounter, you need a better understanding of basic logic and the process of Critical Thinking. These are not trivially pursuits; they are essential to making good decisions and forming sound beliefs about our world. The following questions were raised to examine or prove the relationship between concepts: Does this really make sense? Does that follow from what you said? How does that follow? I don't see how both can be true. When we think, we bring a variety of thoughts together into some order. When the combination of thoughts are mutually supporting and make sense in combination, the thinking is "logical." When the combination is not mutually supporting, is contradictory in some sense, or does not "make sense," the combination is "not logical."

Fairness: Fairness is seen to be unbiased in taking decisions and judgement about a concept or things. The following questions were raised to determine fairness. Are we considering all relevant viewpoints in good faith? Are we distorting some information to maintain our biased perspective? Are we more concerned about our vested interests than the common good? We naturally think from our own perspective, from a point of view which tends to privilege our position. Fairness implies the treating of all relevant viewpoints alike without reference to one's own feelings or interests. Because we tend to be biased in favor of our own viewpoint, it is important to keep the standard of fairness at the forefront of our thinking. This is especially important when the situation may call on us to see things we don't want to see, or give something up that we want to hold onto.

2. Element of Thought

There are 8 various elements of thought one considers while thinking critically because whenever we think, it is done for a purpose, within a point of view based on assumptions leading to implications and consequences. We use concepts, ideas, and theories to interpret data, facts, and experiences in order to answer questions, solve problems, and resolve issues. (The Miniature Guide to Critical Thinking Concepts and Tools 2008). These 8 elements are as follows:

(1) All reasoning has a PURPOSE

Your purpose is your goal, objectives or what you are trying to accomplish. You should be clear about your purpose and your purpose should be justifiable. The following are statements raised for the purpose of thinking critically before analyzing concepts or issues.

- State your purpose clearly.
- Distinguish your purpose from related purposes.
- Check periodically to be sure you are still on target.
- Choose significant and realistic purposes.

Questions which target purpose are as follows:

- What is your, my, their purpose?
- What is the objective of this assignment (task, job, experiment, policy, strategy, etc.)?
- should we question, refine, modify our purpose (goal, objective, etc.)?
- What is the purpose of this meeting (chapter, relationship, action)?
- What is your central aim in this line of thought?
- What is the purpose of education?
- Why did you say…?

(2) All reasoning is an attempt to figure something out, to settle some QUESTION, solve some PROBLEM.

The question lays out the problems or issues and guides our thinking. When the question is vague, our thinking will lack clarity and distinctiveness.

- State the question at issue clearly and precisely.
- Express the question in several ways to clarify its meaning and scope.
- Break the question into sub-questions.
- Distinguish questions that have definitive answers from those that are a matter of opinion and from those that require consideration of multiple viewpoints.

Questions which target the question

- What is the question I am trying to answer?
- What important questions are embedded in the issue?
- Is there a better way to put the question?
- Is this question clear? Is it complex?
- I am not sure exactly what question you are asking. Could you please explain it?
- The question in my mind is this: How do you see the question?
- What kind of question is this? Historical? Scientific? Ethical? Political? Economic? Or…?
- What would we have to do to settle this question?

(3) All reasoning is based on ASSUMPTIONS. Assumptions are beliefs one take for granted. They usually operate at the subconscious or unconscious level of taught. Make sure you are clear about the assumptions and they are justified by sound evidence.

• Clearly identify your assumptions and determine whether they are justifiable.

• Consider how your assumptions are shaping your point of view.

Questions you can ask about assumptions

- What am I assuming or taking for granted?
- Am I assuming something I shouldn't?
- What assumption is leading me to this conclusion?
- What is… (this policy, strategy, explanation) assuming?
- What exactly do sociologists (historians, mathematicians, etc.) take for granted?
- What is being presupposed in this theory?
- What are some important assumptions I make about my roommate, my friends, my parents, my instructors, my country?

(4) All reasoning is done from some POINT OF VIEW

Point of view is literally " the place" from which one view something. It includes what you are looking at and the way that you fully consider other relevant viewpoints.

• Identify your point of view.

• Seek other points of view and identify their strengths as well as weaknesses.

• Strive to be fair-minded in evaluating all points of view.

Questions to check your point of view

- How am I looking at this situation? Is there another way to look at it that I should consider?
- What exactly am I focused on? And how am I seeing it?
- Is my view the only reasonable view? What does my point of view ignore?
- Have you ever considered the way ____(Japanese, Muslims, Nigerians, etc.) view this?
- Which of these possible viewpoints makes the most sense given the situation?
- Am I having difficulty looking at this situation from a viewpoint with which I disagree?
- What is the point of view of the author of this story?
- Do I study viewpoints that challenge my personal beliefs?

(5) All reasoning is based on DATA, INFORMATION and EVIDENCE

Information includes the fact, data, evidence, or experiences we use to figure things out. It does not necessary imply accuracy or correctness. The information you use should be accurate and relevant to the question or issue you are addressing.

• Restrict your claims to those supported by the data you have.
• Search for information that opposes your position as well as information that supports it.
• Make sure that all information used is clear, accurate, and relevant to the question at issue.
• Make sure you have gathered sufficient information.

Questions which target information

- What information do I need to answer this question?
- What data are relevant to this problem?
- Do we need to gather more information?
- Is this information relevant to our purpose or goal?
- On what information are you basing that comment?
- What experience convinced you of this? Could your experience be distorted?
- How do we know this information (data, testimony) is accurate?
- Have we left out any important information that we need to consider?

(6) All reasoning is expressed through, and shaped by CONCEPTS and IDEAS

Concepts are ideas, theories, laws, principles, or hypotheses we use in thinking to make sense of things. Be clear about the concepts you are using and the use them justifiably.

• Identify key concepts and explain them clearly.

• Consider alternative concepts or alternative definitions of concepts.

• Make sure you are using concepts with care and precision.

Questions you can ask about concepts

- What idea am I using in my thinking? Is this idea causing problems for me or for others?
- I think this is a good theory, but could you explain it more fully?
- What is the main hypothesis you are using in your reasoning?
- Are you using this term in keeping with established usage?
- What main distinctions should we draw in reasoning through this problem?
- What idea is this author using in his or her thinking? Is there a problem with it?

(7) All reasoning contains INFERENCES or INTERPRETATIONS by which we draw CONCLUSIONS and give meaning to data

Inferences should logically follow from the evidence. Infer no more or less than what is implied in the situation.

• Infer only what the evidence implies.

• Check inferences for their consistency with each other.

• Identify assumptions that lead to inferences.

Questions to check your inferences

- What conclusions am I coming to?
- Is my inference logical?
- Are there other conclusions I should consider?
- Does this interpretation make sense?
- Does our solution necessarily follow from our data?
- How did you reach that conclusion?
- What are you basing your reasoning on?
- Is there an alternative plausible conclusion?
- Given all the facts what is the best possible conclusion?
- How shall we interpret these data?

(8) All reasoning leads somewhere or has IMPLICATIONS and CONSEQUENCES

Implications are claims or truths that logically follows from other claims or truths. Implications follow from thoughts. Consequences follows from actions. Implications are inherent in your thoughts, whether you see them or not. The best thinkers think through the logical implications in a situation before acting.

• Trace the implications and consequences that follow from your reasoning.

• Search for negative as well as positive implications.

• Consider all possible consequences.

Questions you can ask about implications

- If I decide to do "X", what things might happen?
- If I decide not to do "X", what things might happen?
- What are you implying when you say that?
- What is likely to happen if we do this versus that?
- Are you implying that…?
- How significant are the implications of this decision?
- What, if anything, is implied by the fact that a much higher percentage of poor people are in jail than wealthy people?

3. Essential Intellectual Traits

Intellectual Humility: It means having a consciousness of the limits of one's knowledge, include a sensitivity to circumstances in which one's native egocentrism is likely to function has self-deceptiveness; sensitivity to bias, prejudice and limitations of one's viewpoint. Intellectual humility depends on recognizing that one should not claim more than one actually knows. It does not imply spinelessness or submissiveness. It implies the lack of intellectual pretentiousness, boastfulness, or conceit, combined with insight into the logical foundations, or lack of such foundations, of one's beliefs.

Intellectual Courage: This means having a consciousness of the need to face and fairly address ideas, beliefs or viewpoints toward which we have strong negative emotions and to which we have not given a serious hearing. This courage is connected with the recognition that ideas considered dangerous or absurd are sometimes rationally justified (in whole or in part) and that conclusions and beliefs inculcated in us are sometimes false or misleading. To determine for ourselves which is which, we must not passively and uncritically "accept" what we have "learned." Intellectual courage comes into play here, because inevitably we will come to see some truth in some ideas considered dangerous and absurd, and distortion or falsity in some ideas strongly held in our social group. We need courage to be true to our own thinking in such circumstances. The penalties for nonconformity can be severe.

Intellectual Empathy: This means having a consciousness of the need to imaginatively put oneself in the place of others in order to genuinely understand them, which requires the consciousness of our egocentric tendency to identify truth with our immediate perceptions of long-standing thought or belief. This trait correlates with the ability to reconstruct accurately the viewpoints and reasoning of others and to reason from premises, assumptions, and ideas other than our own. This trait also correlates with the willingness to remember occasions when we were wrong in the past despite an intense conviction that we were right, and with the ability to imagine our being similarly deceived in a case-at-hand.

Intellectual Autonomy: It means having rational control of one's beliefs, values, and inferences. The ideal of Critical Thinking is to learn to think for oneself, to gain command over one's thought processes. It entails a commitment to analyzing and evaluating beliefs on the basis of reason and evidence, to question when it is rational to question, to believe when it is rational to believe, and to conform when it is rational to conform.

Intellectual Integrity: It means the recognition of the need to be true to one's own thinking; to be consistent in the intellectual standards one applies; to hold one's self to the same rigorous standards of evidence and proof to which one holds one's antagonists; to practise what one advocates for others; and to honestly admit discrepancies and inconsistencies in one's own thought and action.

Intellectual Perseverance: This means having a consciousness of the need to use intellectual insights and truths in spite of difficulties, obstacles, and frustrations; firm adherence to rational principles despite the irrational opposition of others; a sense of the need to struggle with confusion and unsettled questions over an extended period of time to achieve deeper understanding or insight.

Confidence in Reason: Confidence that, in the long run, one's own higher interests and those of humankind at large will be best served by giving the freest play to reason, by encouraging people to come to their own conclusions by developing their own rational faculties; faith that, with proper encouragement and cultivation, people can learn to think for themselves, to form rational viewpoints, draw reasonable conclusions, think coherently and logically, persuade each other by reason and become reasonable persons, despite the deep-seated obstacles in the native character of the human mind and in society as we know it.

Fair-mindedness: It means having a consciousness of the need to treat all viewpoints alike, without reference to one's own feelings or vested interests, or the feelings or vested interests of one's friends, community or nation; it implies adherence to intellectual standards without reference to one's own advantage or the advantage of one's group.

Gerras Stephen's Critical Thinking Model

This model is a derivative of the element of reasoning presented by Paul and Elder. Although the model starts with the element CLARIFY CONCERN, the model is not necessary linear.

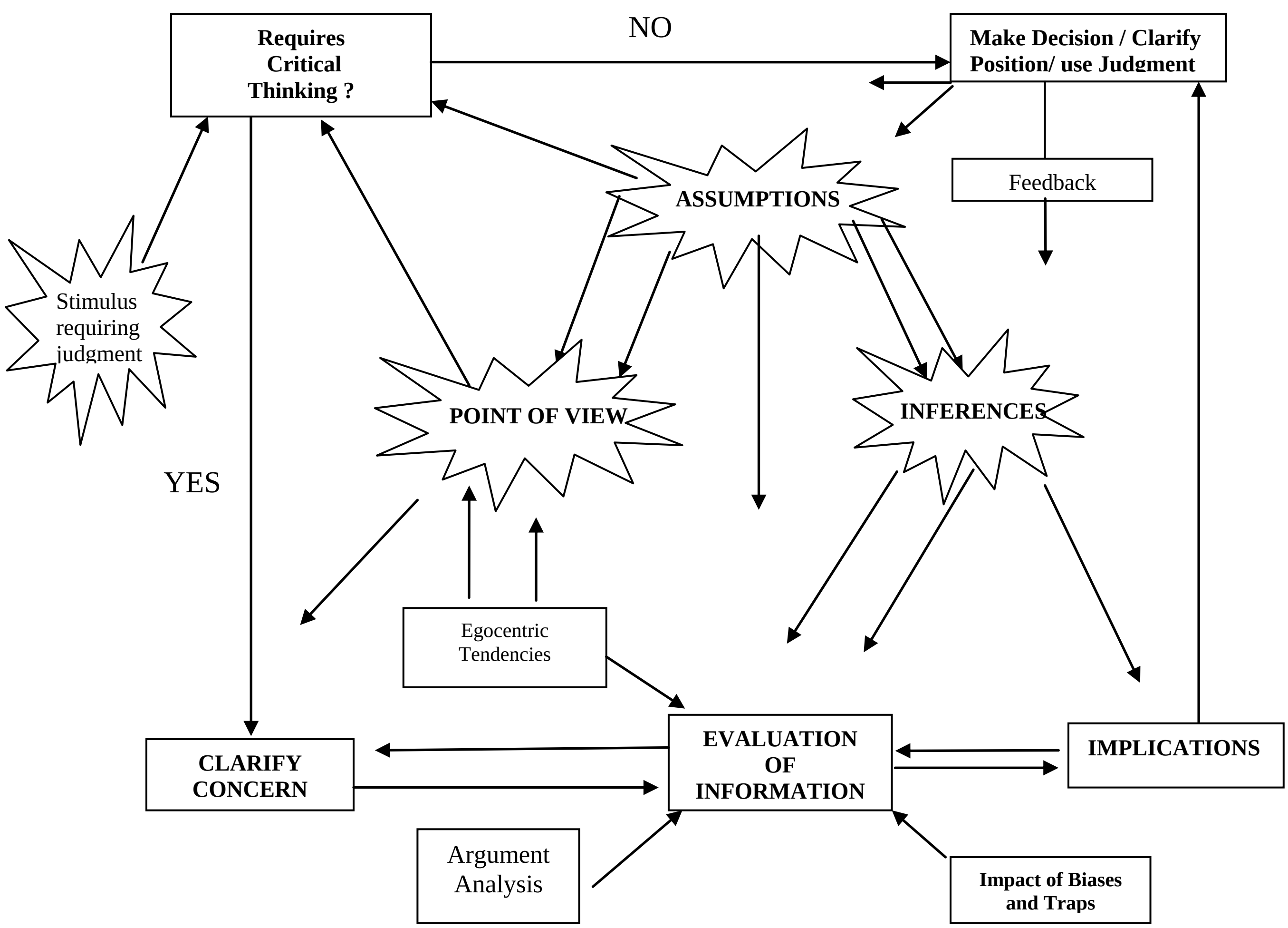


Figure: 2 Gerras' (2008) Critical Thinking Model

It is more important that critical thinkers process information and reason with the vocabulary of the model which is systematic in nature. First, the star in the centre, POINT OF VIEW, ASSUMPTIONS, and INFERENCE are meant to demonstrate that this is generally a non-linear model. Your ASSUMPTIONS, for instance, will impact how you define the boundaries of the issues. Although, there are arrows from CLARIFY CONCERN to EVALUATE INFORMATION (implying linearity), there is also a reciprocal arrow going in the reverse direction to suggest that as you are EVALUATING INFORMATION, you may end up redefining the concern. If for example, you are seeking to CLARIFY CONCERN regarding some inappropriate behaviour by your teenage son or daughter, the EVALUATION OF INFORMATION may indicate that the “real” issue has to do more with the nature of the relationship between you and your child than the actual behaviour prompting initial concern. The non-linear nature of the model will be more evident as read about the component.

This model starts with an individual perceiving some stimulus which is an “automatic response”. In most cases, the automatic mode is appropriate and the perceiver should proceed to make a decision. However, if the topic is complex, has important implications, or there is a chance that strong personal views on the issue might lead to biased reasoning, then thinking critically about the issue makes good sense.

The first step in Critical Thinking methodology is CLARIFY the CONCERN. The problem or issue need to be identified and clarify up front because a critical thinker must be proactive as well as reactive. This will give the critical thinker wide knowledge about the problem especially if they are complex and to identify the root causes. A critical thinker must ensure that the problem is not framed in a way that unduly limits response options.

Another element of the Critical Thinking is POINT OF VIEW. Critical thinkers should always strive to adopt a point of view that is fair to others, even to opposing points of view. Assessing an issue from alternative points of view sometimes is difficult especially if one has egocentric tendencies because they regard oneself and one’s opinions or interest as most important. As we attempt to empathise with the view of others, our own self awareness becomes increasingly important. In fact, it will give us the opportunity to view various alternative that will give us insight on how to take decisions when facing various challenges.

A third component of the model is ASSUMPTIONS. As critical thinkers, we need to be aware of the beliefs we hold to be true that have formed from what we have previously learned and no longer question. There is need to process information based on assumptions about the way the world works that are ingrained in our psyche and typically operate below the level of consciousness. All the assumptions one has and many more will affect one's judgement with respect to possible courses of action for dealing with different problems. The arrows in the model shows that assumptions influence all aspect of the model: our POINT of VIEW, Inferences, whether we decide a problem is worthy of Critical Thinking, and many components of our thought processes. The more in touch an individual is with his assumptions, the more effective he will be a critical thinker.

Another component of the Critical Thinking model that need to be considered is INFERENCES. Critical thinkers need to be skilled at making sound inferences and at identifying when others are making inferences. An inference is a step of the mind or an intellectual leap by which one concludes that something is true in light of something else being true or seeming to be true based on a perception as to how the facts and evidence of a situation fit together.

The next step is EVALUATION OF INFORMATION. One of the characteristics of critical thinkers is decision making and it carries a significant burden for evaluating data and information. Any rational decision making model are rooted in several assumptions. First, the model assumes that the problem or goal is clearly defined. Second, the information that is required to make a decision is available or can be acquired. Third, there is an expectation that all options generated can be adequately considered, compared an evaluated to identify an optimal solution. Fourth, the environment is presumed to be relatively stable and predictable, and finally, there is sufficient time for working through the decision making processes. In evaluating information, there is need to analyse arguments from different points of views because some argument are selfish, and biased which might affect the overall assessment of any decisions taking.

2.2.5 Integration of Critical Thinking to Mathematics Curriculum

The field of education has recognized for decades the need to concentrate on the promotion of Critical Thinking (CT) skills. The Foundation for Critical Thinking seeks to promote essential change in education and society through the cultivation of fair-minded Critical Thinking, thinking predisposed toward intellectual empathy, humility, perseverance, integrity, and responsibility (Paul and Elder, 2008). A rich intellectual environment is possible only with Critical Thinking at the foundation of education. This is because only when students learn to think through the content they are learning in a deep and substantive way, that they can apply what they have learnt to solve real life situation. Moreover, in a world of accelerating change, intensifying complexity, and increasing interdependence, Critical Thinking is now a requirement for economic and social survival (Paul & Elder, 2008).

The question is how this can be best accomplished. Some educators feel that the best path is to design specific courses aimed at teaching CT, which is called the *general skills approach*. By contrast, integrating the teaching of these skills in regular courses in the curriculum is known as the *infusion approach*. The question at the heart of the argument is, whether CT skills are general or depend on content and on the system of concepts specific to that particular content. According to Swartz (1992), the infusion approach aims at teaching specific CT skills along with different study subjects, and instilling CT skills through teaching the set of instructional material.

Swartz also emphasizes that the students should not only employ CT skills in class, but also be able to activate them in real-life situations and to recognize situations when these skills should be used. For this, an appropriate motivation should be fostered; otherwise these skills will remain passive. The integration of CT on the content was conducted according to the infusion approach. The combined mathematical content of an existing learning unit "Probability in Daily Life" (Lieberman & Tversky, 2001) with CT skills according to Ennis' taxonomy, restructured the curriculum. Aizikovitsh, and Amit (2009) tested different learning units and evaluated the participants' CT skills, to examine whether the modified learning unit "Probability in Daily Life," taught in the infusion approach, does indeed develop CT.

Three fundamental components were build on this study and they are: the infusion approach, Ennis' taxonomy and the modified "Probability in Daily Life" learning unit.

The Infusion Approach

In light of the evidence that has accumulated in the field of teaching thinking, the question arises whether thinking skills are general or content-dependent (Perkins & Salomon, 1989). Out of this question, there are developed four major approaches: the general approach, the infusion approach, the immersion approach, and the mixed approach. The general approach teaches thinking skills as a range of general skills detached from other study subjects, as a separate course in the curriculum. In the infusion approach the skills are taught in the framework of a specific study subject, and thinking turns into an integral part of teaching specific materials, while general principles and terminology of thinking are explicitly emphasized. In the immersion approach, the study material is taught in a thought-provoking way and the students are “immersed” in the topic of study, without explicit reference to the principles of thinking. The mixed approach combines the general and the infusion approaches.

Ennis’ Taxonomy

Ennis (1987, 1991) claims that CT is a reflective and practical activity aiming for a moderate action or belief. There are five key concepts and characteristics defining CT: practical, reflective, moderate, belief and action. In accordance with the categories this definition employs, Ennis developed a taxonomy of CT skills that include both an intellectual and a behavioural aspect. In addition to skills, Ennis’ taxonomy also includes dispositions and abilities. Ennis’ definition and taxonomy of CT was chosen because it distinguishes between abilities and dispositions, and because teaching thinking skills according to a taxonomy suits the hierarchical structure of learning unit in probability studies.

The Learning Unit "Probability in Daily Life"

This unit in probability studies is part of the formal high school curriculum of the Israeli Ministry of Education. It was chosen because its rationale is to make the students to "study issues relevant to everyday life, which include elements of Critical Thinking” (Lieberman & Tversky 2001). In this unit, students must analyze problems using statistical instruments, as well as raising questions and thinking critically about the data, its collection, and its results. Students learn to examine data qualitatively as well as quantitatively. They must also use their intuitions to estimate probabilities and examine the logical premises of these intuitions, along with misjudgment of their application.

The unit is unique because it explores probability in relation to everyday problems. This involves CT elements such as tangible examples from everyday life, evaluating reliability of information, accepting and dismissing generalizations, rechecking data, doubting, comparing new knowledge with the existing knowledge. This unit is characterized by questions such as "Define the term 'Critical Thinking'," "Give examples of a problem while using a controlled experiment," "Give examples of failures and misleading commercials," and "Give examples of a scientific truth that was dismissed." While studying the subject, the connection is checked between statistical judgment and intuitive judgment, and intuitive mechanisms that produce wrong judgments are explored (Aizikovitsh & Amit, 2009). While studying the unit, students are expected to acquire the tools for CT. In the beginning, students learn the mathematical tools necessary for performing calculations, and later on use the probability part: causal connection, and mechanisms of intuitive judgment, which are considered more of a psychological projection.

2.2.6 Teaching Critical Thinking and Problem Solving in Mathematics

For teaching and learning, there are many strategies used in Critical Thinking. These are: think/pair/share, jigsaw, one stay other stray, reciprocal teaching, mini-lecture, active lessoning and so on. Among the different strategies there is need to select the appropriate strategy of instruction according to the subject, content and topic of the instruction. Also the main aspects of the selection of strategy is also based on the number of students, level of students, Individual differences, geography of classroom and etc.

Acharya (2008) developed phases of Critical Thinking and Problem solving Approach in Classroom:

Phases of Critical Thinking approach

The main approach of Critical Thinking ABC pattern they are:

- Anticipation
- Building knowledge
- Consolidation

Anticipation

In anticipation phase, the teacher begins with a structured overview. In this case a short talk about the topic-just enough to frame the students thinking about the topic and to raise their curiosity. The anticipation phase serves to:

- call of the knowledge learns /participants already have.
- informally assess what they already have, including misconceptions.
- set purposes for learning.
- provide a context for understanding new ideas.

Building Knowledge Phase

In building knowledge phase, the teacher prepares the student to read the text. The student will use the methods of paired reading/paired summarizing to think about the materials they are reading. Since this method is new to our students, the teacher takes time to thoroughly introduce it. The building knowledge phase serves to:

- compare expectations with what is being learned.
- revise expectations or raise new ones.
- identify the main points.
- monitor personal thinking.
- make inferences about the materials.
- make personal Connections to the lesson.
- questions the lesson.

Consolidation PHASE

Consolidation phase is the parts of the lesson where the students think back over what they learned, apply the ideas, and consider what they already knew before in light of what they have learned. The consolidations phase serves to:

- summarize the main ideas.
- interprets the ideas.
- share opinions
- make personal responses.
- Test out the ideas.
- Assess learning.

- Ask additional questions.

Education system is the brain of any society and also it is called backbone of any system. Mathematics takes an important and major place in school curriculum as well as the curriculum of higher level. Now a day is not only mathematics but also the social sciences are becoming more and more mathematical. Hence, the teaching of mathematics is very important and essential in different discipline and aspects of learning. Also the teaching must be appropriate. Therefore, in teaching mathematics for different level, groups, different situations and other different context, a single strategy is no longer suitable so we need to use different appropriate strategies to teach mathematics from different backgrounds and levels.

2.2.7 Critical Thinking: Teaching Strategies and Classroom Techniques

Critical Thinking teaching strategies is an active process in promoting and enhancing students performance in schools. Listening to lectures in the classroom, to most students is a passive activity because students only listening and will not have the opportunity to asks questions when the lectures is going and this makes the class dull. The intellectual skills of Critical Thinking--analysis, synthesis, reflection, etc.--must be learned by actually performing them. Schafersmen (1991) suggests the following Critical Thinking strategies and classroom techniques to school instructors. Classroom instruction, homework, term papers, and examinations. Therefore, the teacher should emphasize students' active intellectual.

Lectures: Enhancement of Critical Thinking can be accomplished during lecture by the teacher periodically stopping and asking students searching and thoughtful questions about the material he/she has just presented, and then waiting for an appropriate time for them to respond. In achieving this, the teacher should do the following: Do not immediately answer such questions yourself; leave sufficient time for students to think about their answer before they state it. If you constantly answer such questions yourself, students will quickly realize this and not respond. Learn students' names as quickly as possible and ask the questions of specific students that you call upon by name. If an individual cannot answer a question, help them by simplifying the question and leading them through the thought process: ask what data are needed to answer the question, suggest how the data can be used to answer the question, and then have the student use this data in an appropriate way to come up with an answer.

Teachers may ask simple questions by merely asking students to regurgitate factual information they have learnt during their lecture period. Many students have trouble with these factual questions because they are not paying attention in class, they simply have never learned how to listen to a lecture and take mental and written notes, or they don't know how to review their notes and the textbook in preparation for an exam. Perhaps the most basic type of Critical Thinking is knowing how to listen to a lecture actively rather than passively; many students don't know how to do this because they were never taught how to do it and they were able to get through the educational system to their present situation without having to practice it.

It is probably wise to begin asking the factual type of questions so that students will realize that they have to pay attention. However, the goal of Critical Thinking requires that you eventually ask questions that require students to think through a cause and effect or premise and conclusion type of argument. This obliges them to reason from data or information they now possess through the lecture to reach new conclusions or understanding about the topic. For example, in chemistry, after presenting information about chemical reactions, you could ask students to describe chemical reactions that occur to them or near them every day by the combination of commonplace chemical materials. Ask them to explain what type of reaction it is (oxidation, reduction, etc.) using whatever knowledge they possess of the reactant materials and their new knowledge of chemical reactions.

After lecture but before the class ends, the teacher should ask students to write one-minute papers on the most significant thing they have learnt in class today and what single thing they still feel confused about. With this act, teachers will get immediate feedback about what the students are learning and what they still need to understand (technically, this is an application of what is called "classroom research" or "classroom assessment," the deliberate discovery of what and how much students are learning compare with what and how the teacher is teaching). This will improve their Critical Thinking Skills (reading and writing skills).

In the class, teachers should encourage questions from students. Always respond positively to questions; never brush them off or be-little the questioner. Instead, praise the questioner (for example, say "Good question!" or "I bet a lot of you want to know that"). Questions from students mean they are thinking critically about what you are saying; encourage that thinking!

During lecture, bring in historical and philosophical information about Mathematics and science because it enables students to understand that all scientific and mathematical knowledge was gained by someone practising Critical Thinking in the past, and or by act of great courage and tedious painstaking work in the face of seemingly insurmountable difficulties.

Laboratories: Many science courses have laboratories connected with them. Science laboratory exercises are all excellent for teaching Critical Thinking. Student learn the scientific method by actually practising it. In Laboratory, students might be given a task to find out the reaction or effect of mixing two or more chemicals in order to arrive at a reliable and valid result. These involve Critical Thinking because the quantity and conditions of the chemicals must be critically and painstakingly measured. This method of teaching Critical Thinking is so clear and obvious that Critical Thinking is promoted more in secondary education and higher schools.

Homework: Innumerable opportunities exist to promote Critical Thinking by homework assignments. For reading homework, teachers should provide students with the general questions before they begin reading, and insist that they organize their notes around these questions. Make the students to transform the information and make it their own by requesting them to solve, paraphrase, summarize, or outline all reading and writing assignments. This can be done by suggesting that their grading will be based on their written efforts with oral quiz that can be structured to require abstract conceptualization and graded as students speak, for most students will prepare carefully in order to avoid failing repeatedly in public. Thereafter, teacher will collect, grade, and return their written efforts.

As stated above, getting students to write and solve more questions is the best, and perhaps the easiest way to enhance Critical Thinking because it makes them to be creative and independent. Writing forces students to organize their thoughts and think critically about the material. Ask students to write short papers about pertinent topics, review science articles, even paraphrase news articles and textbook chapters. These exercises can be as elaborate as you wish to make them. Students are asked to solve a short word problem questions as take home assignment from the popular media (newspaper, science magazine, etc.), contemplate a list of take-home questions. The teacher prepares the questions and distribute them to all the students to solve biweekly intervals about six or seven times a semester. The ultimate goal of these exercises is to improve students ability to interprete, analyse and evaluate mathematical questions logically.

Quantitative Exercises: Problem solving is Critical Thinking; thus, courses such as Mathematics, chemistry, and physics, that require the solution of various mathematical problems, automatically teach Critical Thinking to some extent just by following the traditional curriculum. When students are required to solve Mathematics problems, they are practising Critical Thinking, whether they know it or not. Mathematics, chemistry, and physics problems belong, of course, to only a limited subset of Critical Thinking, but this subset is an important one. Indeed, all science courses including those that do not traditionally require mathematical problem-solving skills at the introductory level, such as biology, geology, oceanography, astronomy, and environmental science should begin to incorporate some mathematical problems in the curriculum. Asking students to solve mathematics problems in a science gets them thinking about nature and reality in empirical and quantitative terms, which are key components of Critical Thinking.

It appears that some of the mathematical problems and exercises will give the student the facility to manipulate numbers, but will not teach Critical Thinking except when it raised inform of Mathematical word problems. By asking the students to approach the empirical world from a numerical or quantitative viewpoint, would enhance their thinking ability. Indeed, Mathematics students who do not intend to take higher-level Mathematics courses should be educated in the context of word problems to the greatest extent possible. Obviously, students who are given Mathematics problems to solve in the sciences are essentially working on word problems, which is Critical Thinking.

Here are some examples of Mathematical word problems

1. A ship leaves port and travels 21km on a bearing of 032^0 and then 45km on a bearing of 287^o.

a) Calculate its distance from the port. b) Calculate the bearing of the port from the ship.

2. An aircraft flies round a triangular course. The first leg is 200km on a bearing of 115^o and the second leg is 150km on a bearing of 230^o. How long is the third leg course and what bearing must the aircraft fly?

3. A man is 37 years old and his child’s age is 8. How many years ago was the product of their ages 96?

4. A boat can travel 36 miles downstream in 1 hour and 48 minutes, but requires 4 hours for the return trip upstream. Assuming the boat and the stream have constant velocities, find the velocity of the stream and the velocity of the boat in still water.

5. The periods of time required for two painters to paint one square yard of floor differ by one minute. Together, they can paint 27 square yards in one hour. How long does it take each painter to paint one square yard?

Term Papers: Term papers promote Critical Thinking among students by requiring that they acquire, synthesize, and logically analyze information, and that they then present this information and their conclusions in written form. This technique can be used in any Mathematics or science course and is strongly recommended as a way to improve students' Critical Thinking Skills. Perhaps as they research and write it, they will begin to think critically about the benefits of keeping up with lectures and studying for exams.

Examinations: Examinations should require that students write or, at least, think. For written exams, short-and-long-answer essay questions are the obvious solution. A few short-answer essay questions on each exam test the ability of students to analyze information and draw conclusions. This commonly-used technique, by itself, helps to teach Critical Thinking.

2.3 Concept of Peer Assessment

Traditionally, students were seen as passive receivers of information in the classroom who were expected to provide samples of their knowledge in teacher-made tests. Teachers were both the source of information and the judge who evaluated student success. More recently however, alternative ways of assessment are being tried and one of which is Peer Assessment which is defined as a student’s evaluation of his own success (Khadijeh, 2010). Peer Assessment’ is used to describe the process undertaken by students to assess the performance/contribution of themselves and their peer group, in relation to a group task (Loddington, 2008). It could also been described as peer moderated marking of students work based on sets of success criteria from the teacher.

Peer Assessment requires students to provide either feedback or grades (or both) to their peers on a product or a performance, based on the criteria of excellence for that product or event which students may have been involved in determining (Falchikov, 2007). Peer Assessment is an assessment method through which the peers of a candidate or student are requested to provide information about his performance. It is considered by many educators and teachers to be a key technique to get students to take more responsibility for their learning. Reinders and Lazaro (2007) claimed that that if conducted appropriately, Peer Assessment can provide numerous benefits for the learners. Peer-assessment has the advantage of helping students to critically examine the learning in progress. Through this, students understand their own learning better. It also helps the students to foster collaboration skills and improve autonomy.

Peer Assessment can help self-assessment. When students judge their peers' work, they can actually have the opportunity to examine their own work as well. Peer and self-assessment help students develop the ability to make judgements (Brown and Knight 1994). Simply defined, Peer Assessment could be seen as students' evaluating their peers. Topping (1998) defines Peer Assessment as a process in which individuals judge the amount, level, value, quality, or success of the outcomes of their peers. Van Den Berg, Admiraal, and Pilot (2006) define Peer Assessment as a process in which students assess the quality of their fellow students' work and provide each other with feedback.

2.3.1 Advantages and Disadvantages of Peer Assessment

Race (1998) and Bostock (2000) argued about the usefulness of Peer Assessment and listed its advantages as follows:

- Peer Assessment gives students a sense of belonging to the assessment process and fosters their motivation;
- Peer Assessment encourages a sense of ownership of the process in a sense that students feel they are a part of the evaluation process;
- Peer Assessment improves learning;
- Peer Assessment makes assessment a part of the learning process;
- Peer Assessment encourages students' sense of autonomy in learning;
- Peer Assessment helps students identify their weak and strong points;

- Peer Assessment encourages students to analyze each other's work;
- Peer Assessment improves self-assessment capabilities;
- Peer Assessment encourages deep, meaningful learning;
- Peer Assessment helps students to become more involved in the learning process;
- Peer Assessment helps students recognize assessment criteria;
- Peer Assessment reduces the instructor's marking load;
- Peer Assessment provides better quality feedback;
- Peer Assessment gives students a wider variety of feedback;
- Peer Assessment saves time since several groups can be evaluated without teacher's presence; and
- Peer Assessment develops a wide range of transferable skills that can be later transferred to future employment.

Disadvantages of Peer Assessment

In spite of the advantages of Peer Assessment, it can cause potential problems which need to be taken into account. Bostock (2000) and White (2009) argued that there are some potential problems in Peer Assessment. They claimed that, at first sight, the validity and reliability of assessment done by students will be under question. It is not clear whether the feedback from fellow students is accurate and valuable. Indeed, students may not be qualified enough to be able to evaluate each other; students may not take the assessment process seriously. The danger is that students may be influenced by friendships and solidarity among themselves; students may not like peers' marking because of the possibility of being negatively or unfairly evaluated by their peers, or being misunderstood. Another problem that may arise here is that since teachers are not involved in the evaluation process, students may provide each other with false information.

Given the fact that Peer Assessment is not void of problems, Karaca, (2009) have presented some rules for Peer Assessment to be taken into consideration; these rules can considerably decrease the problems of Peer Assessment and hence make it more effective. The rules are listed below:

- Students should be presented with brief information on what they are supposed to do

and what is expected of them;

- Students need to be familiar with the purpose of the evaluation;
- Students need to know what criteria to follow;
- Teachers need to make sure that students are following the criteria clearly and appropriately;
- Students need to practice the process in stress-free environments;
- Teachers should cooperate with colleagues who have already used Peer Assessment; and
- Teachers should not expect Peer Assessment to be perfect at first attempt.

2.3.2 Peer Assessment Principles

In constructivist education, learners construct knowledge and make meaning through social dialogue and interaction with the environment(Vgostky, 1978). This form of collaborative leraning, Dolittle and Camp (1999) posits , is underpinned by a set of theoretical principles. These principles are:

- Learning should take place in authentic and real-world environments;
- Learning should involve social negotiation and mediation;
- Content and skills should be made relevant to the learner;
- Content and skills should be understood within the framework of the learner's prior knowledge;
- Students should be assessed formatively, serving to inform future learning experiences;
- Students should be encouraged to become self-regulatory, self-mediated, and self-aware;
- Teachers serve primarily as guides and facilitators of learning, not instructors;
- Teachers should provide for and encourage multiple perspectives and representations of content (paragraph 29).

In line with Dolittle & Camp's theoretical principles Peer Assessment were the most appropriate learning and assessment methods. In this context, the tutor/facilitator acts more as a guide on the side (Race, 1998; Topping, 1998).

2.3.3 The Peer Assessment Process

The adoption of Peer Assessment in assess learning in the online tutoring course necessitated the development of a framework to ensure that the assessment method is constructively aligned to teaching and learning methods (Juwah, 2003). The framework developed for the Peer Assessment involved a **seven**-stage process.

The seven-stage process

1. **Explicit rationale** : The participants were provided with detailed appropriate information in the course handbook on the rationale of the assessment method. In addition, participants were also provided with a Guide on Peer Assessment containing examples on how to devise assessment criteria, develop an assessment grid/rubric and a brief on how to give and receive feedback after assessment.
2. **Engage learners in an authentic learning context:** In developing the desired skills and capabilities of facilitating, moderating, reviewing, summarizing, assessing and giving feedback on individual and group performance, as well as reflecting one's own personal and professional practice and development, the course was designed and delivered in a way that the learning activities and assessment tasks involved each participant in taking turns to fulfill each of the below listed roles. This ensured that learning was authentic and contextualized, as well as provided the opportunity for the participants to learn by doing. See Table 3.

Table 3. Knowledge, Skills and Capabilities Development in Tutoring course

Skills	
Cognitive	Knowledge of online education; Comprehension of the pedagogy of online education; Application – link theory to practice; Analysis and interpretation of facts and situations; Synthesise new knowledge from available information and evidence; Evaluation of learning and situations.
Transferable	Assessing learning, grading work, giving and receiving feedback Communication – communicate effectively in different situations and audiences using appropriate techniques, media and technology; Team-working.
Competencies/ Capabilities	Decision making and judgement;
Attributes	Trustworthy and honest; Reliable;
Values	Ethics; Accountability – take account of own actions; Fairness – fair in all dealings with others; Respect and value the opinion and belief of others.
Personal Development	Self aware; Self esteem and confident; Reflects on own practice and continually identifying new learning needs for own growth and development.

3. Involve students in setting assessment criteria: As part of their learning and acquiring the desired tutoring/facilitating repertoires, participants through tasks were involved in devising assessment criteria and developing assessment rubric based on a staged learning process. This staged learning involved learning by examples. Participants were given a step-by-step guide to Peer Assessment including devising criteria, allocating weightings for grades/marks (Baume, 2001). Next, the participants were involved in devising relevant assessment criteria to a given task/activity and/or completing partially supplied assessment rubrics (see exemplar below)

Lastly, participants were asked to devise and design from scratch assessment criteria and rubric for given tasks.

4. Assess learning and give feedback: Participants were required to be involved in formative and summative assessment of learning and giving feedback to peers on their performance and development through set activities.

5. Coach for effective performance: Participants were coached to promote the development and acquisition of desired skills and capabilities, as well as to ensure best practice. Coaching involves demonstrating/modeling by example, prompting, questioning, supporting and providing re-assurance and encouragement (Murphy, 2001).

6. Reflect on learning: Participants were encouraged to reflect on their learning, performance and practice. These were done via dialogue with peers and tutor/facilitator, responses to online tutorial questions and reflective journaling. Frameworks for reflection and/or trigger questions were provided as guides to help participants reflect effectively.

7. Teacher check to assure quality: Teacher checked and monitored the assessment process to verify that the stated criteria for the course were fully met and to assure quality and standard of learning.

2.4 Concepts of Instructional Rubrics as an Assessment tools

Wiggins (1998) observes that the word rubrics derives from rubber, the Latin word for red. In medieval times a rubric was a set of instructions or a commentary attached to a law or liturgical service and typically written in red. Thus, rubric came to mean something that authoritatively instructs people. However, in student assessment, a rubric is a set of scoring guidelines for evaluating students' work. Wiggins (1998) further notes that a rubric contains a scale of possible points to be assigned in scoring on a continuum of quality. High numbers are usually assigned to the best performances. Moreover, a rubric provides descriptors for each level of performance to enable more reliable and unbiased scoring.

Rubrics assessment is itself valid and reliable when proper descriptors are formulated because descriptors contain criteria which often refer to standards. The criteria are the conditions that any performance must meet to be successful. For example, the descriptor "effectively listen" may have two criteria as taking apt steps to comprehend and making the speaker feel heard. Arther & McTighe (2000) define a rubric as a set of general criteria used to evaluate a student's performance in a given outcome area. Rubrics consist of a fixed measurement scale and a list of criteria that describe the characteristics of products or performances for each score point. According to Nitko (1996), a scoring rubric is a coherent set of rules that we use to assess the quality of a student's performance. These rubrics may be in the form of a rating scale or a checklist.

Quinlan (2006) contrasts the differences between rating scales and checklists. She states that rubrics are better than checklists or performance lists as the latter items do not usually define clearly the criteria that the evaluator may have in mind. If teachers would like their students to reliably create quality material and thoroughly understand each scoring point, they may want to prefer a rubric than a checklist or a performance list. Rubrics provide students with: expectations about what will be assessed, information on the standards that need to be met, and indications of where they are in relation to goals (Gunawardena**,** 2010). They also increase consistency in teacher ratings of performance, products, or understanding and provide teachers with data to support grades.

Instructional Rubrics have two general categories: analytic scoring rubrics and holistic scoring rubrics. An analytic scoring rubric requires that the teacher identifies the important aspects of a good solution and then assign points to each aspect. On the other hand with a holistic scoring rubric, teachers must determine the overall quality of the constructed-response. Holistic rubrics place student work into predetermined categories that reflect the quality of the response. According to Nikto (1996), the holistic rubric could be extended to include comments and this creates another rubric type as annotated holistic rubrics. The holistic rubric ratess or score the product or the process as a whole without first scoring parts or components separately (Nikto, 1996).

The analytic rubric rate or score on the other hand, separate parts or characteristics of the product or the process first and then sum these part scores to obtain a total score. In an annotated holistic rubric, raters use a holistic rating first and then they rate or describe a few characteristics that are strengths and weaknesses to support their holistic ratings. Among the three types, using an analytic scoring rubric is a more time-consuming task since the rater has to look for and separately rate each component of a performance. This level of detail is useful when the focus is on diagnosis or helping students to understand the expectations for each part of the performance. This may be especially useful for helping students to learn even though it is time-consuming. According to Arther & McTighe (2000), an analytic trait rubric divides a product or performance into essential traits or dimensions so that they can be judged separately. A separate score is provided for each trait. In general, the above principles apply to any learner in an assessment situation although it is important to look at how concepts in self assessment affect adult learners.

2.4.1 Uses of Instructional Rubrics in Assessing Students learning

The importance of rubrics for Peer assessment is:

- it values the experience of the learners in terms of age, IQ and environment
- rubrics engages in reflection of the learners' experiences
- it empowers the students to excel in the tests and examinations
- rubrics exposes the success criteria to students to enable them understand the concept better.
- it encourages a willingness to make changes based on learning experiences.
- Instructional Rubrics can help teachers analyze and describe students' responses to complex tasks and determine students' levels of proficiency
- In addition, rubrics gives students more specific criteria detailing what is expected and what constitutes a complete response. (Gunawardena**,** 2010).

One important idea from the above discussion is that students bring with them a variety of life experiences to the class. These experiences often include prior grading experiences, both positive and negative. Today, rubrics are widely used in many self /peer assessment situations. Rubrics can be effectively employed to empower students, improve their learning, and assess them individually. Therefore, using rubrics with learners helps to address issues of fairness in grading by assessing each assignment as an individual work.

Arthur (1995) believes that because self-evaluation is linked to self-direction, it is more appropriate than some other evaluation measures for adult learners who are considered to be mature, self-motivated individuals. Theories of learning for adult learners are different from non-adult learners. The theories for adult learning are called "andragogy" while the theories for non-adult learning are called "pedagogy".

Quinlan (2006) cited some learning differences between adult and non-adult learners. According to her, in terms of students' self-concept, adults are independent learners who are able to self-direct their learning. In terms of student prior experiences, they often bring a wealth of life experiences to the learning situation. Adults are often ready to learn the individual needs for their perceived roles. This is cited as student readiness to learn. In terms of application of learning, what adults learn may sometimes need immediate applications in workplace situations. Therefore, students' motivation is intrinsic as their learning has an immediate impact on their lives. Those so-called characteristics should be taken into account when preparing evaluation schemes for adult learners.

2.4.2 Creating Grading Rubrics

Rubrics are becoming increasingly popular with educators moving toward more authentic, performance- based assessments (Brewer, 1996; Marzano et al, 1993). Chances are, however, that you will have to develop a few of your own rubrics to reflect your own curriculum and teaching style. To boost the learning leverage of rubrics, the rubric design process should engage students in the following steps:

1. **Look at models:** Show students examples of good and not-so-good work. Identify the characteristics that make the good ones good and the bad ones bad.
2. **List criteria:** Use the discussion of models to begin a list of what counts in quality work.
3. **Articulate gradations of quality:** Describe the best and worst levels of quality, and then fill in the middle levels based on your knowledge of common problems and the discussion of not-so-good work.
4. **Practice on models:** Have students use the rubrics to evaluate the models you gave them in Step 1.
5. **Use self- and peer-assessment:** Give students their assignment. As they work, stop them occasionally for self- and Peer Assessment.
6. **Revise:** Always give students time to revise their work based on the feedback they get in step 5.
7. **Use teacher assessment:** Use the same rubric students used to assess their work, yourself.

Step 1 may be necessary only when you are asking students to engage in a task with which they are unfamiliar. Steps 3 and 4 are useful but time-consuming; you can do these on your own, especially when you've been using rubrics for a while. A class experienced in rubric-based assessment can streamline the process so that it begins with listing criteria, after which the teacher writes out the gradations of quality, checks them with the students, makes revisions, then uses the rubric for self-, peer-, and teacher assessment.

2.4 .3 Reliability and Validity of Scoring Rubrics

Reliability of Scoring Rubrics

Reliability refers to the consistency of assessment scores. For example, on a reliable test, it is expected that a student would attain the same score, regardless of when the student completed the assessment, when the response was scored, and who scored the response (Moskal & Leydens, 2000). On an unreliable examination, a student's score may vary based on factors that are not related to the purpose of the assessment. Many teachers are probably familiar with the terms " test-retest reliability", "equivalent-forms reliability", "split-half reliability" and "Kuder-Richardson reliability" (Ilogu,2005). Each of these terms refers to statistical methods that are used to establish consistency of students' performances within a given test or across more than one test. These types of reliability are of more concern on standardized or high stakes testing than they are in the classroom assessment. In a classroom, every student's knowledge is repeatedly assessed and this allows the teacher to adjust as new insights are acquired.

The two forms of reliability that are typically considered in classroom assessment and in rubric development involve rater (or scorer) reliability. Rater reliability generally refers to the consistency of scores that are assigned by two independent raters, and at different points in time. The two forms are: Inter-rater and Intra-rater Reliability. Inter-rater reliability refers to the concern that a student's score may vary from rater to rater. For example, one manner in which to analyse an essay exam is to read through the students' responses and make judgement as to the quality of the students' written products based on set criteria to guide the rating process, because two independent raters may not assign the same score to a given response. Although scoring rubrics do not completely eliminate variations between raters they can reduce the occurrence of these discrepancies.

Intra-rater reliability refers to each of the situations in which the scoring process of a given rater changes over time due to some factors, e.g. fatigue, rater's mood, etc. The inconsistencies in the scoring process result from influences that are internal to the rater, rather than true differences in students' performances. Well-designed scoring rubrics respond to the concern of intra-rater reliability by establishing a description of the scoring criteria in advance. Raters are expected to revisit the established criteria in order to ensure that consistency is maintained.

Validity of Scoring Rubrics

The validity of a test is the ability of the test to measure what it is supposed to measure (Ilogu,2005) .There are usually four types of validity namely content validity, concurrent validity, criterion-related validity and construct validity. Content Validity: This is an attempt to ensure that the test covers adequately the contents and the behaviours contained in the behavioural objectives which the students are expected to have acquired as a result of the instruction. These contents and behaviours are expected to have formed the respective two dimensions of the Table of Specification (TOS) from which the test items were developed. The instructional objectives, stipulated content of the lesson, the test items with its percentages relative for the content and behaviours being measured will be sent to panel of experts who are specialist in their field.

The members of the panel will moderate the submitted items and suggests some corrections if necessary. They are usually odd in number. Concurrent Validity :This validity basically refers to the extent to which the test measure the student's ability in an area as a standardized test can measure. In determination of this validity, two different tests are compared or run together at the same time. One of the test is the teacher made test and the standardized test. Both test are administered to the students at the same time. The scores are correlated using Pearson product moment correlation formula.

Criterion-related or Predictive Validity: This is the extent to which the teacher-made test can predict the performance of the candidates in a future test. This is determined by administering the teacher-made test now and the relevant standardized test later on. The two test scores are then correlated using Pearson product moment correlation formula. For example, Mock exams and Senior Secondary Certificate Examination (SSCE).

Construct Validity : This is the ability of the test to measure psychological construct, talent or trait inside the candidate, e.g intelligence or a particular skill such as the verbal and quantitative abilities. This is determined by administering the teacher-made test and the relevant standardized test(e.g Intelligent Test) simultaneously to the candidates. The scores are compared by correlating the two sets of scores, using Pearson product moment correlation formula.

2.5 Critical Thinking and Performance in Mathematics

Mathematics can be described as a combination of calculation skill and competence in mathematical reasoning (Hannula, Maijala & Pehkonen, 2004). Mathematical knowledge answers the question 'What', and one may remember mathematical facts. Mathematical skill answers the question 'How'; which includes, for example, the traditional calculation skill (procedural knowledge). Only Mathematical understanding answers the 'Why' question; it allows one to reason about mathematical statements. The influence of Critical Thinking Skills or meta-cognition on mathematical problem solving has attracted research from Ennis (2005). In contrast to the traditional text book dominated approach, the Mathematics classrooms of the present are encouraged to be a place where discussion and collaboration are valued in building a climate of intellectual challenge. The primary goal of Mathematics teaching being the development of the ability to solve complex Mathematics problems, therefore, mathematical instruction should emphasize the process rather than the product (Kosiak, 2004).

The first research question is to determine if students' reasoning skills, using TER, predicts academic achievement in Mathematics Test, when using TBC. Based on the studies of Yeh and Wu (1992) and Frisby (1992), there is a high correlation between Critical Thinking and academic performance. Yet the findings of the Third International Mathematics and Science (TIMS) study indicate that correlations between Critical Thinking and school achievement were not considered significant. The first research question in the study asks: Do students' abilities to analyze, evaluate, make inferences, and use inductive and deductive reasoning as indicated by the TER predict student achievement as shown by the TAKS 9th Grade Mathematics Test when using a TBC. The study tested the following directional research hypothesis: Students' abilities to analyze, evaluate, make inferences and use inductive and deductive reasoning as indicated by the Test of Everyday Reasoning can predict students' achievement as shown by the TAKS test when using a TBC.(Beth-Bos, 2007).

Promoting Critical Thinking and problem solving in Mathematics education is crucial in the development of successful students. Critical Thinking and problem solving go hand in hand. In order to learn Mathematics through problem solving, the students must also learn how to think critically. Marcut (2005) identified four values of teaching Mathematics through Critical Thinking/problem solving:

1.Critical Thinking and problem solving focus on students' attention, ideas and sense making rather than memorization of fact.
2. Critical Thinking and problem solving develop the student's belief that they are capable of doing Mathematics and that Mathematics makes sense.
3. It provides ongoing assessment data that can be used to make instructional decisions, help students succeed, and inform parents.
4. Teaching through Critical Thinking and problem solving is fun and when learning is fun, students have a better chance of remembering it later.

Mathematics is often held up as the model of a discipline based on rational thought, clear, concise language and attention to the assumption and decision-making techniques that are used to draw conclusions (Marcut, 2005).

Fawcett (1938), introduced the idea that students could learn Mathematics through experiences of Critical Thinking. His goals include¸ the following ways that students could demonstrate that they were, in fact, thinking critically, as they participated in the experiences of the classroom:

1. Selecting the significant words and phrases in any statement that is important, and asking that they be carefully defined.
2. Requiring evidence to support conclusions they are pressed to accept.
3. Analyzing that evidence and distinguishing fact from assumption.
4. Recognizing stated and unstated assumptions essential to the conclusion.
5. Evaluating these assumptions, accepting some and rejecting others.
6. Evaluating the argument, accepting or rejecting the conclusion.

7. Constantly reexamining the assumptions that are behind their beliefs and actions.

Fifty years later, the Critical Thinking is still present in the goals, but it has been subsumed by more holistic notions of what it means to teach, do and understand Mathematics. The students will be able to:

1. organize and consolidate their Mathematical thinking through communication;
2. communicate their Mathematical thinking coherently and clearly to peers, teachers, and others;
3. analyze and evaluate the mathematical thinking and strategies of others;
4. use the language of Mathematics to express mathematical ideas precisely.

These ideas are very similar to those promoted by Fawcett in 1938. Little has changed in the mainstream ways that people tend to define critical thinking in the context of mathematics education. Students are expected to search for the strengths and weaknesses of each and every strategy offered. It is no longer good enough to reach an answer to a problem that was posed.

Now, students are cajoled into communicating their own ideas well, and to demand the same communication from others. A shift has occurred from listing skills to be learned toward attributes of classrooms that promote Critical Thinking as part of the experience of that classroom. Such a class to promote Critical Thinking can be created by providing methods conditions for the students to communicate with one another in order to reflect together on the solution to the problem.

The first condition is for the students to feel free in expressing their ideas. Then, they must be able to listen attentively to their classmates and show interest in their ideas so that they can communicate both for learning Mathematics and in mathematical terms. On the other hand, the students get accustomed to group work which implies mutual help and cooperation for a mutual aim.

In a study undertaken by Barry, Ada and Jenny (2003) in respect to analysing Critical Thinking Skills , they administered CAT Critical Thinking test as pre-test/post-test to two different courses in the social sciences, with the consent of the instructors. Both courses were for junior level classes. One course was specifically designed to improve students' Critical Thinking and problems solving skills, while the other course served as a control. Students in both courses took the pre-test during the first two weeks of the course and then took the post-test during the last week of classes. There were approximately 13 weeks between the pre-test and post-test in each course. The pre-test and the post-test were scored by the same group of faculty, from a broad spectrum of disciplines. 16 students in the control group took both the pre-test and the post-test. No significant change was observed in the performance of students between the pre-test and post-test. The test-retest reliability coefficient was 0.6p<0.01. The overall test performance was remarkably similar on the pre-test and post-test in the control course. 19 students in the Critical Thinking and problem solving class took both the pre-test and post-test. A significant improvement (p<0.05) was observed between scores on the pre-test and scores on the post-test, for students in this course.

M˘arcut (2005) who also carried out a study on Critical Thinking Skills and performance in Mathematics, found out that students who received training in Critical Thinking Skills significantly improved positively in their performance than those who were not trained.
Charles, Renae and Rospond (2004) carried out a research on Critical Thinking instruments to assess Pharmacy students' Critical Thinking Skills and dispositions, and to identify areas for curriculum reforms. The California Critical Thinking instruments were administered to 676 Pharmacy students selected randomly at a private, Mid-west college at various points in their 6- year PharmD PROGRAM. The program is a 2-4 year program; 2 years of pre-pharmacy, followed by 4 years in the professional program (DP1- DP4). In order to identify any changes in the Critical Thinking scores across the curriculum, Pre- and Post- California Critical Thinking Skills Test (CCTST) measurement were used to determine whether students coming into the program with relatively low CCTST scores improved more than those coming in with relatively high scores. When comparing the pre-test and post-test skills scores for 2003 pharmacy graduates, a statistically significant improvement was observed. The mean overall skills score improved nearly 2 points from when the students took the skills test in 1998 as entering students until they took the post-CCTST in 2002 during their 3rd professional year.

In addition, students who scored low on the pre-test showed a significant greater improvement in their post- test scores compared with students who scored high on the pre-test. The scores of students who initially scored low showed a mean improvement of 3.3 points on the post-test, compared with 0.8 point increased in the post-test scores of students who scored high on the pre-test (P=0.0136). Scott and Markert (1994) tested Critical Thinking Skills in medical students using the Watson-Glaser Critical Thinking Appraisal and found that Critical Thinking Skills correlated with students' academic success in the first two years of medical school and with MCAT scores. They concluded that Critical Thinking Skills are one factor involved in a student's success in the first two years of medical school.

Allen and Bond (2001) studied Critical Thinking as a predictor of success in pharmacy school. In addition they administered Pharmacy college admission tests (PCAT) test, interview ratings, and academic performance prior to pharmacy school, they also included the California critical thinking skill tests (CTST). In their population, scores on the CCTST and the PCAT were the strongest predictors of success in practice-related courses and clerkships. Adamcik et al used the Watson-Glaser Critical Thinking Appraisal inventory (WGCTA) to show a correlation between Critical Thinking and both pharmacy GPA and therapeutics course grades.

The WGCTA utilizes reading passages along with 40 questions to assess 5 Critical Thinking Skills : inference, recognition of assumptions, deduction, interpretation, and evaluation of arguments. Unlike Allen and Bond (2001), Adamcik et al (1996) measured Critical Thinking in students towards the end of their pharmacy program. Odedina et al (2001) used the CCTST to explore a relationship between Critical Thinking and performance in a pharmacy administration course. They also found a positive correlation between Critical Thinking and academic performance. Likewise, Kidd and Latif (2002) found a significant relationship between scores on the CCTST and students' didactic Grade Point Average (GPA).

Nursing researchers have produced the most data on CT assessment among healthcare professions. Several researchers have identified changes in Critical Thinking Skills and dispositions based on class year. Both McCarthy et al and Colucciello evaluated the CT skills and dispositions among nursing students. Their research revealed higher scores among nursing students at varying points in the curriculum. However, they were not able to show improvement in scores over the course of a curriculum since both used cross-sectional designs where students at each class level were independent groups.

Leppa (1997) used a pretest-posttest design to evaluate changes in Critical Thinking scores over time. This research found that RN-baccalaureate nursing students did not improve in their CT skills, but did have statistically significant improvements in their dispositions to think critically.

2.6 Peer Assessment and Performance in Mathematics

Onuka (2007) carried out a study on teacher-Initiated and guided students- Peer Assessment programme to improving learning assessment in Mathematics and English Language in large classes using 280 participants. Peer Assessment training was given to the treatment group while the control group were not given treatment. The findings shows that the treatment group achieve better than the control group in both Mathematics and English Language. There were increases in the achievement by all groups after the treatment of teacher-initiated and guided-student- Peer Assessment and traditional instructional methods.

The treatment groups in Mathematics had a mean increase rate of 5.7 in achievement (from 56.4 to 62.1 mean rates in the pretest and posttest scores respectively), while the control group recorded mean achievement rates of 58.2 and 59.9 respectively in Mathematics culminating in a difference (an increase of 1.7) in mean rate of achievement between their achievement in pretest and posttest scores respectively. This findings shows that there was significant difference between the treatment group and the control group at 0.05 level of probability with a t-value of 20.40. It was evident that Peer Assessment training was efficacious in enhancing students in Mathematics because students were able to learn from each other as they grade each other's work.

The result supported the findings of Onuka and Oludipo, (2006) who reported that the performance of students in the experimental group outweighed those from the control group. This shows that feedback, which is an outcome of evaluation, and systematic school based assessment may assist in improving students performance and in cognitive learning objectives respectively.

McLaughlin and Simpson (2004) studied how first year university students felt about peer assessment. Working in a context of a construction management course, their Peer Assessment model asked students to assess their peers' group work. The researchers found that in this peer assessment model, students extremely liked the Peer Assessment process and regarded peer assessment as a very positive and helpful assessment experience. Students' perspectives about Peer Assessment showed that they felt they had learned a lot during the Peer Assessment process, and that they enjoyed assessing their peers' work; a significant number of students preferred Peer Assessment to the assessment merely provided by the teacher. The researchers finally came to the conclusion that the assessment process needs to be a learning tool that helps the learning process considerably.

In a study focused on meta-cognition and learning, Moss (1997) found out that a group of elementary teachers who were exposed to a systematic "self-reflection" process (in this case, using a rubric) outperformed those who attended the same workshop but did not receive the rubric. The systematic self-reflection group tended to set goals, select interventions to match those goals, and exhibit a deeper level of understanding of the content presented. These findings have further implications for intervention practices, which require students to participate by creating assessment criteria and scoring rubrics. This suggests that allowing students to participate in creating criteria for their own assessments may enhance learning.

The study by Schafer et al (2001) was motivated by the speculations made in the literature that a better understanding of rubrics assessment by teachers would lead to the design of effective instructional experiences. Pairs of teachers were selected from different schools based on the course taught (Mathematics, Biology, English, and Government) as well as on the ability level of the students taught. Teams between four and six teachers, and content specialists from different schools brainstormed the criteria for the development of rubrics that would be appropriate for the assessment. Generic analytic scoring rubrics (0- 4 points) in each content area were developed which were to be used for scoring students' Constructed-Response (CR).

One member of each pair of teachers was selected randomly and assigned to receive rubric training for two days. After a period of instruction, all teachers were asked to administer three tests in each of the four content areas. The tests which were blind scored by two independent readers, contained both selected-response and constructed-response items. The results favoured the achievement of students whose teachers had received rubric training in three of the eight (two types in each of the four content areas) comparisons. The other five comparisons were not significant. The significant results were in Mathematics for both item types and in Biology for CR items.

Wen and Tsai (2006) investigated university students' views towards Peer Assessment. Having collected data from 280 university students in Taiwan employing a 20-item instrument, the researchers sought the students' attitudes towards and perceptions of Peer Assessment. The results revealed that students generally liked Peer Assessment since it gave them the chance to compare their work with their classmates; however, students were less appreciative of being criticized by peers and expressed a lack of self-confidence to peer assess their classmates. Students believed that on-line Peer Assessment is not merely a learning aid, but a technical instrument that facilitates the Peer Assessment process.

An interesting outcome of the study was that males had more positive view towards Peer Assessment than females, and that students who had experienced Peer Assessment before had less negative views towards Peer Assessment. Furthermore, the majority of students held the view that Peer Assessment scores should account for merely a small part of the final score. Vu and Alba (2007) investigated Australian university students' experience of Peer Assessment in a professional course. The Peer Assessment component was planned and structured to evaluate and promote student learning in the classroom.

The authors reported that in their case study, Peer Assessment processes were useful for the students' learning. It was found that Peer Assessment had a positive effect on students' learning experiences with most students acknowledging learning from both the process and from their peers. The researchers finally enumerated several conditions for the successful implementation of the Peer Assessment process. These conditions were:

1) providing adequate and appropriate preparation for the successful implementation of Peer Assessment;

2) specifying the objectives of the course as well as the purpose of Peer Assessment;

3) determining the degree of teachers' assistance given during the Peer Assessment process;

4) teachers' handling of fruitful discussion periods following Peer Assessment.

Saito (2008) examined the effects of training on Peer Assessment regarding oral presentations in EFL classrooms. In the first study, both the treatment and control groups were given instruction on skill aspects. However, only the treatment group was given extra 40-minute training on how to assess performances. The results did not show any significant differences between the treatment and control groups. In the second study, only the treatment groups were given longer training. Again, no significant correlation differences were noticed between the treatment and control groups. The final conclusion of the study was that Peer Assessment is a solid technique, which can be enhanced if assessors are trained suitably and effectively.

Karaca (2009) investigated teacher trainees' opinions about the usefulness of Peer Assessment in order to determine whether or not their opinion differs according to such variables as (a) their gender, (b) having taken part in the Peer Assessment process before, and (c) believing in helpfulness of Peer Assessment process. Having gathered data from 175 teacher trainees, the researcher came to the conclusion that the teacher trainees thought positively about Peer Assessment, and that their beliefs were significantly related to the variables of their study.

The results of this research also revealed that the teacher trainees thought of Peer Assessment as a useful assessment method that encouraged students to critically analyze their peer's work, allowed students to take part in the assessment process and fostered interaction among students in a course. Furthermore, the results indicated that teacher trainees believed that Peer Assessment could have some disadvantages. Such disadvantages, to them, included the fact that students might not be capable enough to evaluate each other, and that their evaluation might be affected by their friendly or hostile relationship.

In an attempt to identify secondary school students' perception of Peer Assessment and feedback, Peterson and Irving (2008) carried out an investigation. Using a mixed-method approach including focus groups, semi-structured interviews, questionnaires, and notes, the researchers arrived at feedback on students' perceptions of Peer Assessment. The students had a positive view about Peer Assessment, finding it a useful strategy for both students and teachers. They saw Peer Assessment as fun. With regard to feedback, students believed that feedback motivated them, provided information, and helped them seek new information.

In another piece of research, Bryant and Carless (2009) attempted to investigate how primary school students and their teachers perceive Peer Assessment. After collecting and analyzing data through extensive interviews and classroom observations, the researchers arrived at interesting findings. One outcome of the study was that teachers believed that Peer Assessment was a good technique that strengthened their efforts to improve their students' writing skill. However, some students considered Peer Assessment useful, while some others felt disappointed if they noticed that their peer was not providing good comments.

A noteworthy point that was found in the study was that students' perception of Peer Assessment differed according to their language proficiency level and that of their peer. Students who had their work assessed by a student with higher level of language proficiency expressed dissatisfaction with the work since they could not identify the errors and hence would assume that their peer who was more proficient was right. High proficiency students, on the other hand, complained that their peer could not provide useful comments because he or she was less proficient than them. In this regard, some students did not favor Peer Assessment since they did not receive helpful feedback and comments from their peers. Instead, they preferred to receive feedback from their teacher who was a more reliable source of information in comparison with their peers.

Overall, the results suggested that students had a positive view towards Peer Assessment because they learned from each other and also they had the opportunity to take responsibility for their own work. An important advantage of Peer Assessment pointed out by students who took part in this study was that Peer Assessment helped the students to prepare for examination and transmission to secondary school education. Students argued that through Peer Assessment, they would be able to identify in advance the type of mistakes that they were likely to make in the examination and therefore find techniques to avoid them. Teachers' conception of feedback was similar to that of students in that they, too, saw Peer Assessment useful, and that it would help learners become more successful in their learning. Allowing students to grade tests themselves offers four potential advantages over teacher grading (Sadler and Eddie, (2006):

• *Logistical:* Because an entire classroom of students can be grading simultaneously, tests can be marked in a short amount of time. This saves teacher time (Boud, 1989). Grading can take place immediately following a quiz or during the next meeting of the class. This results in quicker feedback for students (McLeod, 2001). Peers can often spend more time and offer more detailed feedback than the teacher can provide (Weaver & Cotrell, 1986).

• *Pedagogical:* Judging the correctness of answers is an additional opportunity for students to deepen their understanding about a topic. Reading another's answers or simply spending time *pondering* another's view may be enough for students to change their ideas or further develop their skills (Bloom & Krathwohl, 1956; Boud, 1989).

• *Metacognitive*: Embedding grading as a part of a student's learning experience can have benefits that go beyond *learning* specific subject-matter content (Brown, 1987). Grading can help to demystify testing. Students become more aware of their own strengths, progress, and gaps (Alexander, Schallert, & Hare, 1991; Black & Atkin, 1996). Pupils develop a capacity to take initiative in evaluating their own work (Darling-Hammond, Ancess, & Faulk, 1995) and use higher order thinking skills to make judgments about others'work (Bloom, 1971; Zoller, 1993; Zoller, Tsaparlis, Fastow, & Lubezky, 1997). Self-evaluation and peer review are an important part of future, adult, professional practice, and test grading is a good way to develop these skills (Boud, 1989).With increasing awareness of the workings of tests, students can also formulate test items that can be used on later exams (Black & Harrison, 2001).

• *Affective*: Affective changes can make classrooms more productive, friendlier, and cooperative, and thus can build a greater sense of shared ownership for the learning process (Baird & Northfield, 1992; McLeod, 2001; Pfeifer, 1981;Weaver & Cotrell, 1986; Zoller, Ben-Chaim, & Kamm, 1997).

The reason for tests is illuminated when students compare and judge the veracity of answers. Students develop a positive attitude toward tests as useful feedback rather than for "low grades as punishment for behavior unrelated to the attainment of instructional objectives" (Reed, 1996, Sadler, 1989). Viewed as a time-saving scheme, students' grading of their own or peers' work can only be considered a satisfactory substitute for teacher grading if the results of these grading practices are comparable to the teacher's. If student feedback or grades are very different from the teacher's judgment, the teacher is obligated to correct the papers herself.

Ideally, student-assigned grades would be indistinguishable from grades assigned by the teacher. Although this may be easy to achieve when questions are of the form of multiple-choice or fill-in-the-blank, such agreement is more difficult in the case of more open-ended responses (Bloom & Krathwohl, 1956). Because subjective judgment is often required for grading, particularly for more open-ended questions, students must learn how to correct tests (Boud, 1989; Neukom, 2000). Considered by many teachers as "guild knowledge," making judgments about students' understanding is a most arcane skill, acquired by apprenticeship and rarely revealed to the uninitiated (Sadler, 1989). A grading rubric or criteria sheet, which lays out different levels of response and equivalent point values, helps to specify a teacher's evaluative criteria and results in a greater agreement with a teacher's potential grade (Baird & Northfield, 1992; Boud, 1989; Weaver & Cotrell, 1986).

This particular analytic approach of using a rubric depends on the teacher's effort in selection and codification of a subset of items from a large number of potential criteria that graders can consider separately but add up overall (Sadler, 1989). Student-grading has several variations but only two major forms are addressed here. Students can correct either their own papers or those of others. Variations in peer-grading include the scoring of the work of students in one class by members of other classes and blind review with names undisclosed. Students can be rewarded for accuracy in grading of their own or another's work, with extra points earned if their judgments are similar to those of their teacher's.

2.7 Instructional Rubrics and Mathematics assessment

Meier (2006) reported a study in which middle school teachers used rubrics on eighth grade students to score non-traditional mathematical tasks. The teachers used analytic scoring rubrics that outlined three categories for evaluation namely mathematical knowledge, strategic knowledge, and explanation. Mathematical knowledge addressed concerns related to mathematical accuracy and correctness of terminology, while strategic knowledge was related to the identification of important parts of the problem and discussion of the methods of the solution. The explanation category dealt with a description of what was done, and required a discussion of why it was done with a written explanation for any diagrams or tables.

Meier (2006) further stated that under mathematical knowledge, students are expected to demonstrate their knowledge of Mathematical concepts, principles, and procedures. This requires an understanding of relationships among problem elements and use of mathematical terminology and notation. It may also require students to recognize when a procedure is appropriate, execute a procedure, verify results of a procedure, and generate or extend familiar procedures. Under strategic knowledge, students are expected to use models, diagrams, and symbols to represent and integrate concepts in addition to being systematic in their application of strategies. For some assessment tasks, students are expected to justify their answers.

This justification requires an appropriate mode of communication (e.g. written, pictorial, graphical or algebraic methods) for expressing the integration of mathematical ideas, conjectures, and arguments. For other assessment tasks, students are required to describe their procedures or strategies. Ultimately, students are also expected to communicate their mathematical ideas in writing: symbolically, or visually, use mathematical vocabulary, notation, and structure to represent ideas, and describe relationships. In a study conducted by Popham (1997), an analytic rubric was used to measure mathematical skills requiring students to complete three subtasks: averaging, graphing, and concluding. Some of the important features of this rubric were that each subtask had teachable evaluative criteria. Those criteria are applicable across a wide range of similar subtasks. Also, this rubric does not delineate the nuances of each evaluation criterion so that different people using the rubric would invariably score students’ responses in an identical manner confirming instrument reliability.

Lane (1993) provided a four component framework that provides guidelines in the construction of assessment tasks. The first component, cognitive processes and task demands, includes: understanding and representing problems, discussing mathematical relations, organizing information, using and discovering strategies and heuristics, using and discovering procedures, formulating conjectures, evaluating the reasonableness of answers, generating results, justifying answers or procedures, and communicating. The second component, mathematical content, includes content categories such as number and operations, measurement, estimation.

Modes of representation is the third component which refers to the internal mental model. This includes problem solving constructs in the forms of written, pictorial, graphical, tabular forms and so on.

Finally, task control is the component in which valid assessment tasks represent the ways in which knowledge and skills are used in “real world” contexts and stresses the need for embodying them. Some of those components are applicable to many types of mathematical tasks. The above sub categories are very lengthy and in my opinion, one has to choose a manageable number of criteria and performance tasks in order to have a handy instrument. A five-point rating scale of 0 to 4 would be a reasonable choice for an analytic scoring rubric. The focus in this study is to construct an analytic scoring rubric to self assess and peer assess mathematical problem solving tasks.

This rubric will serve three main purposes. First, students can use the rubric as a learning tool by identifying their own strengths and weaknesses. Second, the information gathered from this rubric will help the teacher to redesign future teaching and in that sense it will act as a teaching tool. Finally, this rubric can be used for student assessment purposes, providing the teacher a workable tool for assessment. The emphasis was to assess the process of problem solving for better learning rather than evaluating the product.

2.8 Gender and Age in Mathematics

Gender differences in Mathematics performance has been a great controversy issue in educational domain and research documents show great discrepancies among girls and boys performance in school Mathematics (Sprigler & Alsup (2003). Long research history in this area has shows that male advantage in Mathematics achievement is a universal phenomenon (Janson, 1996, Mullis et al., 2004). While early research (Fennema & Sherman, 1977) indicated that males outperformed females in Mathematics achievement at the junior high and high school levels, there were also significant differences in attitudes toward Mathematics between the two groups.

Although, globally, the issue of gender inequality in Science, Technology and Mathematics Education (STME) has produced inconclusive results, one meta-analysis covering the period

1974-1987 on Mathematics and gender led to two conclusions: the average gender gap is very small (statistically insignificant), and the fact that the differences tend to decline with time (Friedman,1989). Conclusively, Smith (2007) asserted that male students are found to prefer numeration, algebra process and construction than their female counterpart.

Knol and Berger (1990) carried out a study to investigate the influence of sex on the performance of students in Mathematics. They administered 72-item multiple choice Mathematics test items from the ACT College Mathematics placement program, to represent a range of items covering content from elementary Mathematics . Two hundred and forty six (246) students who enrolled in three Introductory Statistics classes at the 8^{th} and 9^{th} grade school in Washington were used as participants. Three factor-fixed Analysis of Covariance (ANCOVA) was used. The result showed that the effect of sex, as a variable on performance among the students, was significant.

In a study carried out by Bassey, Joshua and Asim (2008) in secondary schools in Calabar, Cross Rivers State, on gender differences and Mathematics achievement among students in rural schools, 2000 students (50% male and 50% female) were selected using stratified and simple random sampling, and a 30-item four options multiple choice Mathematics achievement test (MAT) was constructed (KR20 of 0.87 and item difficulty $0.40 \leq p \leq 0.82$) and administered. The independent t-test analysis of significance revealed that there is a significant difference between the Mathematics achievement of the rural male and female students (t-cal $5.43 \geq$ t-crit 1.645 at 0.05 level of significance and 98 degree of freedom).

In Nigeria, gender-achievement studies include Abiam and Odok (2006), who found no significant relationship between gender and achievement in number and numeration, algebraic processes, and statistics. They however found the existence of a weak significant relationship in geometry and trigonometry. Also, Adedayo (2006) found no significant difference between the scores of male and female students in Mathematics tests.

Gallagher and kaufman (2006) recognized that the Mathematics achievement and interest of boys are better than the girls. However they explained that they don’t know the main cause of these differences. O’Connor-Petruso, Schiering, Hayes & Serrano(2004) have shown that gender differences in Mathematics achievement become apparent at the secondary level when female students begin to exhibit less confidence in their Mathematics ability and perform lower than males on problem solving and higher level Mathematics tasks.

In spite of research evidences for male's superiority in Mathematics achievement, some research findings do not support the difference between two genders in Mathematics achievement (Ali and Mansoureh (2003). As an example, Sprigler & Alsup (2003) refer to researcher indications that shown no gender difference on the Mathematical reasoning ability at elementary level. Finding from longitudinal study about gender differences in Mathematics show that there is no difference among boys and girls in Mathematics achievement (Ding, Song and Richardson; 2007). This study show that growth trend in Mathematics among two genders was equivalent during the study times.

According to a recent international study conducted by IEA, on average across all countries, there was essentially no difference in achievement between boys and girls at either the eighth or fourth grade (Mullis et al., 2004). Finding of two recent consecutive International studies (TIMSS 1999 &2003) in Iranian educational system (a system that co-education is prohibited and female teachers teach in the girls' schools and male teachers teach in the boys' schools) also confirms that there is no significant differences between boys and girls in Mathematics achievement. Findings from these studies show the significant decrease in the boys' Mathematics achievement score from the time of TIMSS 1999 and the significant improvement in the girls' achievement over the same period. Teacher job satisfaction and the positive perspective of female teachers regarding teaching of Mathematics may be the factors behind the better Mathematics performance of Iranian girls than boys at Grade 8 in Iran (Kiamanesh, 2006).

Coleman, Campbell, Hobson, McParland, Mood, Weinfield, and York (1966) studies showed that as students become older, the correlation between age and school achievement diminishes. Grissom (2004) in his study concluded that the negative relationship between age and achievement remains constant over time. White (1982) said that since schools provide equalizing experiences to both old and young students, there won't be any significant difference between age and achievement in Mathematics. This could be because, as students get older in life, there is tendency for him or her to adjust to challenges or tasks in given to him or her in Mathematics.

Crosser (1991), Kinard and Reinherz (1986), and La Paro and Pianta (2000) were of the view that older children fare better academically than their younger, age appropriate peers. In support of this view, Uphoff & Gilmore (1985) used research evidence about the relationship between age and achievement as well as other evidence to argue that the older and/or more mature students in a class fare better than younger classmates. In contrast DeMeis and Stearns (1992) and Dietz & Wilson (1985) found no significant relationship between age and achievement.

Khata, Krissana, Kungu, Yahya and Mohd (2011) did a study on influence of age and gender on students achievement in Mathematics. The study described the graduating high school students in the U.S. by age, gender and their academic achievement in Mathematics. The study compared the Mathematics achievement between age groups and gender. The comparison revealed that there were statistically significant differences in mathematics GPA scores between age groups and gender; however the effect sizes were small.

Many studies have shown that girls perform better in school than boys in all major subjects (Epstein, Elwood, Hey & Maw, 1998 ; Wong, Lam, & Ho 2002) and that they graduate from high school with higher grade point averages (GPAs) than their male peers (Perkins, Kleiner, Roey, &Brown, 2004). Hillman and Rothman (2003) Praat (1999), Thiessen and Nickerson (1999) and Weaver-Hightower (2003) showed evidence of a growing gender gap in educational achievement in a number of developed countries. According Alton-Lee & Praat (2001); Mullis, Martin, Gonzalez, & Kennedy (2003) the educational statistics in Boston indicated that females are outperforming males at all levels of the school system, attaining more school and post-school qualifications, and attending university in higher numbers

Habibollah, Rohani,. Tengku, Jamaluddin, and Kumar (2009) also did a study on Creativity, age and gender as predictors of academic achievement among undergraduate students. The findings showed that there is interaction effects between creativity, age and gender as low predictors of academic achievement. The findings also show a lower correlation of CGPA and the independent variables of this study. No significant difference between CGPA and gender was observed.

2.9 Gender and Age in Critical Thinking and Mathematics

Mitrevski and Zajkov (2012) examine the effectiveness of physical lab , critical thinking and gender difference among physics students in Macedonia. The results revealed that irrespective of the training on critical thinking, there is no statistical gender differences among group. In support of this findings, Michael (1999) did a study on Mathematics and Science and the findings show no statistically significant gender difference in science achievement among Macedonian eighth grade students. Four years later another generation eighth grade student from the Republic of Macedonia showed statistically significant gender difference in science achievement. Although on average, across most of the countries, boys outperformed girls at the eighth grade, gender difference of Macedonian students favored girls (Micheal.2003). Graybill found evidence of a gender difference in problem-solving tasks, where girls lagged behind boys in the development of logical thinking ability. The differences start to show around the age of 11 years. A moderate correlation has been found between positive attitudes toward science and higher achievement in science (Weinburgh, 1995).

Michael (2012) compared the effectiveness of non traditional versus traditional lecture-based teaching method on students' Critical Thinking, measured with subject specific CT test. In order to have an equal control and experimental group (C and E), classes were selected according to the physics marks, overall achievement and the teacher's suggestions. Lab physics and practical work was used in E during teaching the unit "Electric current". Many practical activities, such as demonstrations, conducting experiments and research activities were performed in E. Students were pretested and post-tested using CT test. The test was developed by the authors. Evaluation process has included checking, revision and modifying the questions using both pilot and focus group methods. This test measures subject specific CT skills in a specific content area-physics. Students were scored on a scale ranging from 0 to 50, with higher scores representing the better achievement. Results show that lab physics and practical work teaching method is not effective in terms of stimulating CT skills, because the data have indicated no statistically significant difference between groups. Also, the findings of the study indicate that the gender difference does not exist in terms of students' achievement on Critical Thinking test.

Myers and Dyers (2006) carried out a study on the influence of students learning style on critical thinking skills in a college of Agriculture and life sciences at University of Florida. The results of their findings showed that no critical thinking skill differences existed between male and female students in this study. Students with deeply embedded Abstract Sequential learning style preferences exhibited significantly higher critical thinking skill scores. No differences in critical thinking ability existed between students of other learning styles.

Ricketts, Rudd and Rick (2004) did a study on discipline-specific critical thinking in Agriculture and leadership in respect to age and gender. According to the descriptive data, female scores were higher than male scores in terms of the critical thinking skill of Analysis, meaning females in the sample may be more adept at "identifying the intended and actual inferential relationships among statements, questions, concepts, descriptions of other forms of representation intended to express beliefs, judgments, experiences, reasons, information, or opinions" (Facione, 1998) They also scored higher than males in their ability to make inferences, meaning females in the sample were more able to "identify and secure elements needed to draw reasonable conclusions; to form conjectures and hypotheses; to consider relevant information and to deduce the consequences flowing from data, statements, principles, evidence, judgments, beliefs, opinions, concepts, descriptions, questions, or other forms of representation" However, Rudd, Baker, & Hoover, (2000) had reported that, an increase in age did not have significant impact on thinking ability of the students, and that the performance of both older and younger students were relatively the same in Mathematics.

2.10 Gender and Age in Peer Assessment and Mathematics

Adediwura (2012) did a research on effect of peer and self assessment on the self-efficacy and students' learner autonomy in the learning of mathematics as well as determining the attitude of male and female students towards the use of peer and self assessment in state public senior secondary schools in Osun state using senior secondary three students . The result of the study showed that, the use of peer and self-assessment in mathematics lessons enhance students' self-efficacy and promote learner autonomy in learning the subject. It was discovered that while there is no significant relationship between sex and enhancement of self-efficacy as a result of students' engagement with the use of peer and self-assessment, the enhanced students' leaner autonomy that was noticed in the sampled students is significantly influenced by their sex. Furthermore, the study revealed that the students have positive attitude towards the use of peer and self-assessment and that their attitude towards the use of these assessment strategies is independent of sex.

Mononen, & Aunio (2013) carried out a study on Early Mathematical Performance in Finnish Kindergarten and Grade One in Helsinki, Finland. The mathematical skills of the pupils were assessed once using researcher-developed paper-pencil tests. The variance analysis (ANOVA) was used to study the effects of age and gender on performance level. Boys and girls performed similarly in both samples, but age effects were found in the kindergarten and first grade; older children performed higher than younger ones. The older children may have had more opportunities to practise and get acquainted with mathematical issues, as the age difference between the youngest and the oldest child in the classroom can be up to one year.

2.11 Conceptual Framework of the Relationship between Critical Thinking Skills, Peer Assessment and Performance in Mathematics.

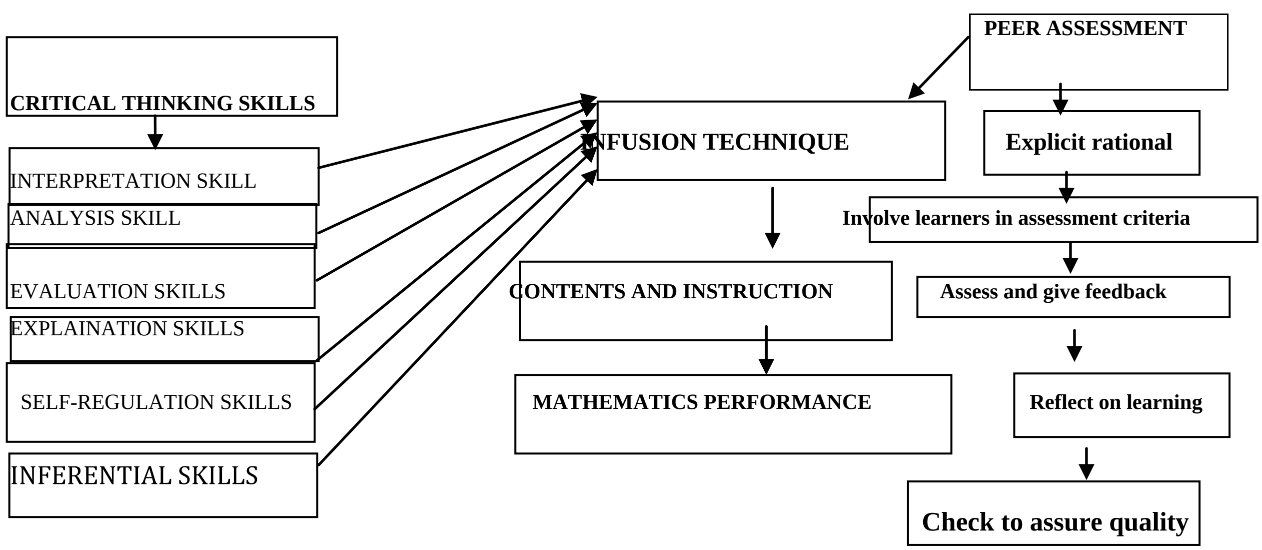


Fig 3 : Researcher's Model showing the relationship between Critical Thinking Skills , Peer Assessment and Performance in Mathematics.

The conceptual framework above illustrated how Critical Thinking Skills and Peer Assessment were used to enhance performance in Mathematics. In the diagram, thinking skills were infused into contents in Mathematics to enable the participants have a deeper understanding of the topics taught to improve performance in Mathematics. Peer Assessment was also introduced to allow participants have a look at each other's Mathematical work so as to enable the weak participants to learn from strong ones. This process is shown to have enhanced performance in Mathematics.

2.12 Appraisal of Literature Review

The development of knowledge base in Mathematics terms and topics are viewed as an integrated whole, rather than an isolated piece of information. This is strongly emphasized as a key goal in the current Mathematics education literature (Herbert and Carpenter, 1992; Ilogu, 2005). Relevant literatures were reviewed on Critical Thinking Skills, Peer Assessment, Instructional Rubrics, Gender, Age and Mathematics Performance. There are mixed results with regards to gender and age difference in science achievement. Some researchers were of the view that there are significant age and gender differences in Mathematics (Clearly & Heng-Kietisak, 1989; DeBaz; 1994; Lee and Burkam, 1996; Hedges and Newell 1995; Wong,, Lam, & Ho, 2002; Perkins, Kleiner, Roey & Brown, 2004; Knol & Berger (1991) while others found no significant difference (Abubaka & Adegboyega, 2012; Ding, Song & Richardson; 2007; Coleman, Campbell, Hobson, McParland, Mood, Weinfield & York 1966). The dichotomy in performance by students in Mathematics has in recent time attracted considerable academic attention by interested and concerned stakeholders. Despite the introduction of different approaches adopted to learning and instruction, this dichotomy has been attributed to success and failure rate of students in schools. Some factors attributed to the above statement include inadequate instructional materials, poor teaching methods, students and teachers attitude towards Mathematics. Thus, effective instruction should enable students to investigate the connections between various concepts and topics in Mathematics.

However, the researcher has developed a new approach that could be used to enhance performance in Mathematics as more effective, long lasting, students-centered and teacher-directed. Critical Thinking Skills and Peer Assessment have been considered the most comprehensive techniques used in education. Critical Thinking has been included in the

National policy of Educations but little emphasis was placed on it by teachers, because some of them lack the required skills to integrate it in the teaching and learning of Mathematics. This could have provided one avenue for teachers and students to emphasize this often neglected teaching and learning objective. More so, it helps students to improve in their academic performance through skills acquisition for understanding information and arguments, evaluating the information and arguments, developing and defending their views with well-supported arguments. This is true of all the literature reviewed.

In the classroom environment, the use of Peer Assessment and Rubric may also improve performance in Mathematics by making teachers' objectives clearer and showing students how to meet these expectations. Again, the use of Peer Assessment can be very effective in improving the performance of the students because it involves activities that engage students in exchanging ideas and grading each other's work which is evident in most research literature. However, insufficient literature on the use of Critical Thinking and Peer Assessment on performance in Mathematics has created a wide gap which may serve as means of instruction to produce a better performance in Mathematics. For this reasons, this study is significant with respect to impact of Critical Thinking Skills and Peer Assessment on performance of students in Mathematics.

CHAPTER THREE

RESEARCH METHODOLOGY

This section focused on the following sub-headings: research design, area of study, population, sample and sampling technique, research instruments and their validation, administration of instruments and statistical methods of data analysis.

3.1 Research Design

The research designs for this study were descriptive survey and quasi-experimental pre-test/ post test control group. The purpose of the survey was to obtain a baseline data and isolate the unique elements in the population for the study while the use of quasi-experimental design was appropriate because it involves human behavior and does not permit complete randomization of subjects and control of all variables (Ilogu, 2005). It was also used to expose participants to training, compare the training and control group on the dependent measures and analyse the effect of the training. The study consisted of two experimental groups---the training and control groups. The training group was exposed to training instructions on Critical Thinking Skills and Peer Assessment while the control group was not exposed to training instructions.

Dependent variable: Performance in Mathematics

Independent variables: Critical Thinking and Peer Assessment

Moderating variables: Gender and Age.

3.2 Study Area

The study was carried out in Delta State. Delta state is a State in South-Southern Nigeria and was created on 27th August, 1991 from the defunct Bendel State. Bendel State was formerly known as the Midwestern Region at the time it gained regional status in August, 1963, from the then Western Region. At the inception of Delta State it was made up of twelve political division called Local Government Areas (L.G.A), which were later increased to 19 in 1996. Presently the State is made up of 25 Local Government Areas and has 12 Educational Zones. The State is divided into three Senatorial Districts, which are Delta North, Delta South and Delta Central. Delta State has 29 constituencies and 29 members in the State House of Assembly. Delta State has 453 public secondary schools and 438 private secondary schools.

The major ethnic groups in the State are Urhobo, Ijaw, Isoko, Ukwuani, Igbo and Itsekiri. Basically, they have identical customs, beliefs and cultures. The cultural identity is manifested in their festivals and traditional marriage ceremonies, while certain words are common to most of the tribes. Delta State has 149 clans or Communities and 100 Traditional Rulers. Their systems of Traditional Administration tend to be identical as well as their folktales, dances, arts and crafts. Farming, Fishing and Hunting are the major occupations of the inhabitants of Delta State, as about 80% of the active labour force are engaged in these occupational activities with the remaining 20% engaged in other occupations. In Delta State there are two Universities, one federal university at Warri-Effurun and one state owned University with three campuses at Abraka, Oleh and Asaba; three Polytechnics at Oghara, Ogwashi-Uku and Ozoro; and three Colleges of Education at Agbor, Warri and Mosogar, one Federal College of Education (Technical) at Asaba. The state is one of the leading producers of crude oil and Natural Gas in Nigeria. Delta State has an International Airport at Osubi and two Domestic Airports at Warri and Asaba. Delta State has the deepest river in Africa; the Ethiope River (100ft in depth) and pure fresh water which is used as a tourist attraction centre and to generate revenue for the State. These attractions make Delta state to be highly populated because people from different ethnic groups in the country go there to either work, school or both

Presently the state covers a landmass of about 18,050 Km^2 of which more than 60% is land. Delta State lies approximately between Longitude 5°00 and 6°.45' East and Latitude 5°00 and 6°.30' North. It is bound in the North by Edo State, on the East by Anambra State, on the South-East by Bayelsa State, and on the Southern flank is the Bight of Benin which covers approximately 160 kilometres of the State's coastline. Delta State is generally low-lying without remarkable hills. The State has a wide coastal belt inter-laced with rivulets and streams, which form part of the Niger-Delta (Delta State Government bulletin, 2012).

3.3 Population of the Study

The target population for this study comprised senior secondary II students in Delta State. The accessible population consisted of all male and female senior secondary II students in Kwale Educational Zone in Delta North, Delta State. SSII was chosen because it is a semi-terminal and important senior class in the secondary school system. SSII is also a stable class since SSIII will be too busy and occupied with preparing for external examinations such as

WAEC/NECO. In the researcher's opinion, the selected SSII students were likely to benefit from the study as they would be taking their final examinations the following year. By participating, their Critical Thinking Skills and Peer Assessment processes would have improved as they would have been exposed to different thinking skills and steps in presenting, interpreting, analyzing and evaluating data and information.

3.4 Sample and Sampling Procedure

Simple random sampling was used to select Kwale Educational Zone out of the 12 Educational Zones in Delta state. This was done by writing out all the names of the educational zones on separate pieces of paper each of which was folded and put in a box. The box was properly shaken after which the researcher picked one piece of paper from it; on unfolding it, it was found to be Kwale Educational zone. Kwale Educational zone which fell under Delta North senatorial district of Delta State was used for the study. Four public Senior Secondary Schools in Kwale Educational Zone were selected using the same stratified random sampling method. To do this, all the senior secondary schools in Kwale zone were first stratified into three groups- Co-educational, boys' schools and girls' schools. There were 60 senior secondary schools in all (52 co-educational, 3 boys' school and 5 girls' school). 2 schools were randomly chosen among those in the co-educational category because of their large number. In addition, one school each was chosen from the boy's only and the girl's only categories using Hat and Draw method. Among the four schools selected, simple random sampling was used to select two schools for the Training Group, while the other two schools were used for the Control Group. The stratified random sampling technique was used to obtain the initial sample of two hundred and forty six (246) students from the four senior secondary schools which were selected for the study. The selection of the 246 students was stratified based on school type, age and gender. The choice of single sex schools was to find out whether there would be any difference in performance in Mathematics between the coeducational and the single sex schools.

The base-line assessment for the study was done by administering the Watson-Glasser Critical Thinking Appraisal (W-GCTA) on the students. Only participants whose scores fell below 27 were qualified for the main study because they performed below 50% of the total score of 65 (Watson & Glasser, 1980). By this, the 34 students who scored above 50% did not qualify to participate in the study. The final sample for the main study was 212 SSII students consisting of 115 males and 97 females. The selection process is describe numerically as shown in Table 4 below while Table 5 shows the distribution of the participants by gender in the experimental group.

Table 4: Baseline Assessment Results and Distribution of Students by School type, Age, gender and group

Schools	Mean Age	Male	Female	Total	Pre-treatment Assessment results	
					Students who scored 50 and above	Students who scored below 50
School 1	15.62	33	28	61	9	52
School 2	15.49	36	29	65	12	53
School 3	16.90	62	-	62	7	55
School 4	16.63	-	58	58	6	52
Total	16.16	131	115	246	34	212

Table 5: Distribution of Participants by Gender and Experimental Groups

Groups	Male	Female	Total	Total Group
School (1) Training group School (2) Control group	27 31	25 22	52 53	 105
School (3) Training group School (4) Control group	55 	 52	55 52	107
Total	113	99	212	212

The training took place in the various schools selected and the researcher used the Mathematics periods to train the treatment group on how they can apply Thinking Skills and Peer Assessment knowledge to enhance their performance in Mathematics for six weeks while the control group received their regular class lessons.

Research Instruments: Three instruments were used for this study. They are as follows:

Mathematics Performance Test (MPT)**,**

Watson-Glaser Critical Thinking Appraisal

Peer-Assessment Mathematics Scale (PAMS)

1. Mathematics Performance Test (MPT):

Multiple Choice tests and essay were developed by the researcher using past WAEC question papers from 2006 to 2011. The items used were based on the table of specification(TOS) constructed for this instrument. (See Table 6).The MPT consisted of 50 objective-items which attracted 50 marks, each with 4 options and 5 essay questions which attracted 10 marks each. As shown in Table 6, emphasis was laid on four topics namely: Probability, Series and Sequence, Bearing and Distance, and Quadratic Equations. 120 items were originally selected but for an item to be included in the final instrument after item analysis, two criteria were met (a) discrimination index range from 0.4 to 0.6 and (b) difficulty index range from 0.30 to 0.60 (Ilogu, 2005). During the pilot study, this instruments was administered twice within an interval of three weeks and the scores were correlated using Pearson product moment correlation to estimate the stability coefficient. A co-efficient value of 0.87 was obtained, thus, the instrument has high reliability.

Table: 6 Test Blue print for the Mathematics Performance Test

CONTENTS	Weight	BEHAVIOURAL OBJECTIVES			Total (100%)
		Knowledge (40%)	Comprehension (30%)	Application (30%)	
Series and Sequence	20%	4	3	3	10
Probability	20%	4	3	3	10
Quadratic Equations	30%	6	4	5	15
Bearing and Distance	30%	6	5	4	15
Total	100 %	20	15	15	50

Below are some examples of the items:

Instruction: Answer all questions, each question is followed by four options letter A-D. Chose the correct option for each item (question).

1. A coin is tossed and a die is thrown. What is the probability of getting a head and a perfect square?
(a) 1/3 (b) 5/12 (c) 1/6 (d) 5/6

2. Calculate the sum of the first ten term of the Arithmetic progression 3 + 5 + 7 +9 +………..
(a) 120 (b) 122 (c) 12 (d) 126

2. Watson-Glaser's Critical Thinking Appraisal (1980)

Watson-Glaser Critical Thinking Appraisal is a test of Critical Thinking. It measures high level of reasoning and is relevant to problem solving and decision-making. This is a 40-item multiple choice tests. It is designed to measures five sub areas of Critical Thinking which are inference, recognition of assumptions, deduction, interpretation and evaluation of arguments. It provides a composite score for overall assessment of Critical Thinking ability. The possible options range from two to five depending on the questions. The maximum score for W-GCTA is 65. Each item has only one correct answer. According to the author, the concurrent validity of the instrument was 0.61 while reliability coefficient using split-half method was 0.75. The alpha reliability values of the subscales range from 0.31 to 0.83. The researcher adapted this instrument by selecting 26-items for the main study. A test-retest reliability of 0.73 was obtained within three weeks interval.

Exercise 1: In this test, each exercise begins with a statement of fact and you are to chose one out of options provided: True (T), Probably True (PT), Insufficient Data (ID), Probably False (PF) and False (F).

Statement: Two hundred students voluntarily attended a recent weekend students conference in Abuja. At this conference, the topics of tribe relations and means of achieving lasting world peace were discussed, because these were the problems the students selected as being most vital in today's world.

1. The students who attended this conference showed a keener interest in solving broad social problems than most other students in their young age?

(a) T (b) PT (c) ID (d) PF (e) F

2. The majority of the students had not previously discussed the conference topics in their schools. (a) T (b) PT (c) ID (d) PF (e) F
3. **Peer-Assessment Scale Mathematics (PASM)**: This is a 15-item Peer Assessment instrument on 4-point options developed by the researcher. The instrument was designed to assess the level of students' interest and cooperation on how to grade each other without bias in Mathematics exercises using Instructional Rubrics. The responses ranged from Strongly Agree to Strongly Disagree. The content validity was determined by researcher's supervisors and other experts from the Mathematics Education Unit. A draft consisting of twenty five items 4 points lickert scale was developed by the researcher. The draft was given to three Mathematics education lecturers, two English

lecturers and my supervisors to peruse before the final selection of the items. Their assessment covered the following aspects:

- Coverage of content.
- Relevance of items to stated objective.
- Clarity of language, whether suitable or not suitable for the target population.

The outcome of these assessments led to retaining the good items, modification of the fairly good items and removal of bad items. At the end, 15 items were adjudged to be good items which were used for the main study. A test-retest reliability coefficient value of the instruments was obtained at 0.76 from the pilot study. The scoring of the instruments was by assigning 4, 3, 2 and 1 for positive statements where responses are SA, A, D and SD. The points were awarded in the reverse order for the negative statements. The addition of the direct and reverse gave the overall scores. The maximum score is 60 while the minimum score is 15.

Below are some samples of the item in PASM

S/N	ITEMS	1	2	3	4
1	Grading other students' work in Mathematics using scoring guide deepens my understanding of Mathematics.				
2	Grading each other's work in Mathematics using scoring guide makes me learn Mathematics faster.				
3	Scoring my classmates work in Mathematics using marking guide will not improve my Mathematics skills.				

Pilot Study

Validity and Reliability of Instruments:

Copies of the questionnaires were given to some experts in Measurement and Evaluation, and lecturers in the Department of Mathematics Education and English education for content validity. The comments from the lecturers were used to redefine the instruments. A pilot study was carried out outside the main study to have a tryout of the instruments and to determine their psychometric properties. Abraka Grammar School, Abraka, was a public school (co-educational) and was randomly selected from the schools in Delta Central Senatorial District of Delta State through simple random sampling. Thirty students consisting of 15 boys and 15 girls were randomly selected to participate in the exercise. Three instruments were administered to a set of SSII students and after two weeks it was re-administered to the same set of SSII students. The results of the two tests were collated and analyse using Pearson

Product Moment Correlation statistics to estimate the test-retest reliability coefficient. Concurrent validity was also determined by administering the researchers instruments and the standardized instruments at the same time to the participants. The scores of two tests were analysed using Pearson Product Moment Correlation statistics to estimate the validity coefficient of the instruments. The estimated values for the instruments are presented in Table 7.

Table 7: Estimated values for the Research instruments (Test-retest reliability and Concurrent Validity N=30)

Instruments	Variable	No of Items	No of Stds	Test	Mean	Sd	Rtt		Var.	Mean	Sd	rtt
Mathematic Tests		50	**30**	1^{ST} 2^{ND}	25.26 53.40	4.46 7.23	**0.87**		X Y	28.42 27.68	4.23 5.12	**0.74**
Watson-Glaser Thinking Appraisal test	**W-GCTA**	26	**30**	1^{ST} 2^{ND}	45.78 52.93	15.95 16.46	**0.73**		X Y	43.25 40.29	12.40 13.64	**0.69**
	Recognition of Assumption	5	30	1^{ST} 2^{ND}	20.87 25.24	3.42 4.96	0.53		X Y	23.78 24.56	3.88 3.64	0.55
	Logical Interpretation	6	30	1^{ST} 2^{ND}	23.21 28.44	4.15 5.13	0.56		X Y	25.40 23.46	4.18 4.07	0.61
	Deductive Reasoning	5	30	1^{ST} $2ND$	22.68 26.40	3.19 4.82	0.54		X Y	23.89 22.51	4.20 4.35	0.57
	Evaluation of Argument	5	30	1^{ST} 2^{ND}	24.10 27.22	3.49 4.64	0.56		X Y	26.43 26.50	3.61 3.54	0.52
	Inference	5	30	1^{ST} 2^{ND}	21.64 25.53	4.16 5.20	0.61		X Y	23.46 23.21	4.52 4.68	0.58
Peer Assessment	**PAMS**	**15**	**30**	1^{ST} 2^{ND}	58.13 60.26	6.72 6.79	**0.76**		X Y	59.73 58.13	5.92 5.86	**0.68**

* "X" represents researcher's developed instruments test scores while "Y" represents standardized instruments test scores.

Evidence from the table above, shows that the test-retest reliability indices of MPT gives 0.87,W-GCTA gives 0.73 and PAMS gives 0.76. The Concurrent validity indices of MPT gives 0.74, W-GCTA gives 0.69 and PAMS gives 0.68. The values were adjudged to be satisfactory, hence they were suitable, reliable and valid to be used for the study.

Procedures For Data Collection

A letter of introduction was obtained from the Head of Department of Educational Foundations, University of Lagos, Akoka and sent to Delta State Ministry of Education, Asaba and the Principals of the selected Senior Secondary Schools. With this letter, the researcher sought permission to use the selected secondary schools for research and it was granted.

Appointment and Training of Research Assistants

Four Mathematics teachers who were B.Sc (Ed) Mathematics holders and postgraduate students from Delta State University, Abraka were appointed as Research Assistants for the collection and recording of data for the study. The Research Assistants had spent five years in the teaching profession. They were properly trained for a period of two hours, twice a week for two weeks.

Phase 1: Pre-training Assessment:

Introduction of the researcher as the new Mathematics teacher to the participants was done by the Vice Principal of the respective schools. The instrument was administered to the participants with the help of the Research Assistants for pre-test (MAT, W-GCTA and PASM).

Phase 2 Training for Critical Thinking Skills and Peer Assessment:

There were two experimental groups namely: One Training Group and one Control Group.

The researcher prepared lesson notes and infused Critical Thinking Skills and Peer Assessment on each of the topics for the Training group and a normal lesson note for the Control group. Meetings were held with each group twice a week for eight weeks. Each session lasted for 1hr 20 minutes.

Week one :

- Intimating the participants with the purpose of the training.
- Administration of all research instruments. The researcher used Research Assistants for the distribution and collection of the instruments.

Week Two: Critical Thinking Training

The essence of this training was to help participants develop Critical Thinking Skills in order to enhance inquiry and problem solving skills in Mathematics thereby improving their learning outcomes in Mathematics. These skills includes raising questions using (a case study), evaluation of reliable source, identifying variables and search for alternative (Aizokovitsh and Amit, 2008).

Step 1: **Explanation of Concepts to students**

Teacher defined Critical Thinking and explained the processes involved in imparting Critical Thinking Skills to the participants.

Step 2: Teacher Raised Questions by Presenting a short Article or Text (case study).

Your brother woke up in the middle of the night, crying and complaining about a stomach ache. Your parents are not at home. You later decided to give him aspirin, but after an hour he woke up again, suffering from bad nausea and vomiting. The doctor who takes care of your brother regularly is out of town and you consider whether to take your brother to the hospital, which is far from your home. You read from a book about children's diseases and found out that there are children who suffer from a deficiency in a certain type of enzymes and as a result, 25% of them develop a bad reactions to aspirin, which could lead to paralysis or even death. Thus, giving aspirin to these children is forbidden. On the other hand, the general percentage of cases in which bad reactions such as these occur after taking aspirin is 75 %. 3 % of children lack this enzyme. Will you take your brother to the hospital at the middle of the night? or will you repeat aspirin drugs ? or Will you apply another self medication?

Step 3: Open Class Discussion in Small Group about the Article and the Questions.

- Initial suggestion for the resolution of the questions.
- No intervention by the teacher.

Step 4: Further Discussion was directed by the Teacher: Open Class Discussion. During the discussion, the teacher asked the students different questions to foster their thinking skills and curiosity and to encourage them to ask their own questions.

- Various suggestions from students in class were made.
- Interaction between groups of students on the case study.
- Reaching a consensus across the whole class or just across the group.

Step 5. Evaluation of Sources of Information.

The teacher brought out a medical book and showed them different diseases to enable them know why children suffer from deficiency in order to justify his arguments.

Step 6: Identifying Variables (Enzymes Deficiency and Adverse Reactions)

The teacher identified enzymes that may be responsible for vomiting and nausea and its adverse reactions to aspirin.

Step 7: Searching for Alternative and Inference

The teacher and the students suggested different alternative measures to remedy the situation based on verifiable facts before them.

Step 8: Critical Thinking Skills and Mathematical Knowledge (Teaching).

The teacher focused on Critical Thinking and instilling Mathematical knowledge (Bayes formula) statististical connections by referring to students' questions and further discussions. The discussion was focused on Critical Thinking Skills which are: source identification: Medicine book; Source reliability: High; Variable Identification: A-enzyme deficiency D-adverse reaction to aspirin; Mathematical Knowledge: Data:

P (D/A) = 0.25, P(D) =0.75 P(A)=0.03, To Prove: P(A/D)=?

Using Bayes formula (or a two dimensional matrix) the result is:

Only 1% of the children without the enzymes develop an adverse reaction to aspirin.

Week Three :Peer Assessment Training

- Teacher defined Peer Assessment and explained the processes involved in assessing work in Mathematics.
- Distribution of sample work in Mathematics and the success criteria.
- Teacher discussed how participants can learn from sample responses in Mathematics.
- Teacher taught participants how they can assess each other's work in Mathematics.
- Participants worked on the problem on their own.
- Participants worked together to improve their work or sample work.
- Participants exchanged their work to see how they can help each other.
- Participants were graded as: 1 for poor, 2 for fair, 3 for good, and 4 for excellent.

Week 4: TOPIC: PROBABILITY THEORY

Critical Thinking Skills	OBJECTIVES	TEACHERS ACTIVITIES	STUDENTS ACTIVITIES
Interpretation Skills	To develop skills in comprehending and expressing the meaning of data, rules, procedures, event and information. E.g. 1. Determine the probability of mutually exclusive events and independent event. 2. Identify all possible outcome 3. Use outcome table to determine the probability of independent event. 4. Determine the probability of obtaining an odd number when a die is thrown once.	Teacher helped the students to identify skills in interpreting questions, experience and beliefs through i. categorization ii. decoding information iii. clarifying meaning e.g. which is the probability of obtaining an odd number when a die is thrown once. This question is interpreted by listing out favourable outcomes	Students were able to identify all the possible outcomes for a die i.e. 1,2,3,4,5,6. Students also were able to identify odd numbers from the possible outcomes i.e. 1,3,5. Also were able to state the probability formular.

		(sample) and possible likely outcome (population) i.e. 1,3,5 and 1,2,3,4,5,6. Teacher later state the formular Pr. Of obtaining odd no = no. of favourable outcome / no. of possible likely outcome no. of favorably outcome = 3 / no. of possible likely outcome = 6	
Analysis Skills	1. To develop skills for detailed examination of information, questions, statement and concepts 2. To develop skills in identifying intended and actual inferential relationship among statements, questions and concepts.	Teacher analysed the information by substituting the figure into the formular. Pr (odd) = $^3/_6$ = <u>½</u>	Students were able to show workings and steps in arriving it the answer e.g. Prob.(odd) = no. of Fav. Outcome / no. of Possible outcome = $^3/_6$ = <u>½</u>
Explanation Skills	To develop skills that help students state the results of one's reasoning, justify the reasons in terms of evidential, conceptual and methods use in arriving at conclusion.	Teacher helped the students to interpret and analyse questions, statement and justify methods of arriving at a conclusion through: i. stating the result ii. justifying procedures iii. presenting argument	The students were able to: i. state the results ii. justified the procedures in getting the result. iii.Present the argument in getting the result iv.determine the probability of mutually exclusive event in the population.
Evaluation Skill	To develop skills in assessing or judging the credibility or value of statements or questions and methods for given purpose in relation to rules, principles and procedures.	Teacher helped the student to evaluate information, statements and questions through: i. assessing claims ii. assessing arguments. The teacher assessed the results of the question asked by critically looking at the methods and procedures used and check if it logically follows the rules guiding probability through e.g. state the formular calculating mutually exclusive and independent event. Ensure that the steps applied for the rules of probability theory.	Students were able to evaluate the question by answering the question raised by the teacher through logical presentation of data's. .

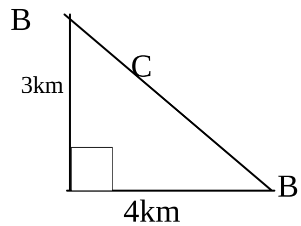
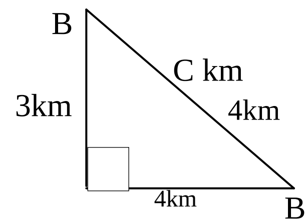
Inferential Skills	To develop skills in drawing reasonable conclusion from data, statement, judgment, questions relating to probability theory.	The teacher assisted the students to take relevant decisions using evidence and conjecturing alternatives. This was done by ensuring that all the questions were solved through systematic approach and by application of almighty formular to justified the result obtained.	Students were able to draw relevant conclusion by following the procedure to arrive at the answers.
Self regulation	To develop skills that will help students monitors one's cognitive activities with a view of correcting oneself.	Teacher assisted the students to apply skills in reassessing one's judgment through: i. self examination ii. self correction	The students were able to re-assessed their work by cross-checking the procedures used in arriving at the answers.

Peer Assessment Training

- Assessment: Teacher distributed rubrics on probability to participants for grading.
- Participants exchanged their solved work on probability for grading using rubrics as a guide. 1 for poor, 2 for fair, 3 for good, and 4 for excellent.
- Teacher collected the scripts and explained further on how to answer difficult areas in the questions. This was done to cross-check for quality grading among the participants.
- Teacher gave out assignments to the participants on probability.

Week 5: TOPIC: BEARING AND DISTANCE

Critical Thinking Skills	OBJECTIVES	TEACHERS ACTIVITIES	STUDENTS ACTIVITIES
Interpretation Skills	To develop skills in comprehending and expressing the meaning of data, rules, procedures, event and information. e.g. 1. Calculate the bearing and distance of the port and from the port. 2. Expressed the bearing as a 3-figure bearing or compass bearing.	Teacher helped the students to identify the skills in interpreting question through: i. categorization ii. decoding and ii. clarifying meaning e.g A boy walked 3km due south and then 4km due east. How far is the boy from his starting position. This question was interpreted by establishing the starting point, the direction walked and the distance covered and putting them on a diagram or	Students were able to note starting point and sketch the distance walked including the change in direction and their respective bearing i..e

	3. How far is the boy from his starting position if he walked 3km due south and 4km due east.	sketch i.e.  Note that the bearing on angle substended. Also state that the formular for calculating distance i.e $=C^2=\alpha^2$ +2ab CosC identify each of the figure variables.	 They were able to state the formular as. $C^2= \alpha^2 + b^2 - 2ab$ Cos C
Analysis Skill	1. To develop skills for detailed examination of information, questions, statement and concepts 2. To develop skills in identifying intended and actual inferential relationship among statements, questions and concepts.	Teacher analysed the information by substituting the figure of each variable into the equation. $C^2 = 3^2 + 4^2 - 2 \times 3 \times 4 \times \text{Cos } 9^0$ $C^2 = 9 + 16 - 24 \times 0$ $C = \sqrt{25} = 5$km	Students were able to show working and steps by introducing each figures to their respective variables. To look for unknown (C) distance from his starting point to where he is. $C^2 = 3^2 + 4^2 - 2 \times 3 \times 4 \times \text{Cos } 9^0$ $C^2 = 9 + 16 - 24 \times 0$ C2 = 25 - 0 $C = \sqrt{25}$ C = 5km
Explanation Skill	To develop skills that help students state the results of one's reasoning, justify the reasoning in terms of evidential, conceptual and methods used in arriving at conclusion.	Teacher helped the students to interprete and analyze the questions, statement and justify method, of arriving at a conclusion through: 1. Starting 2. Justifying procedures 3. Presenting argument.	1. Students were able to state the results 2. Justified the procedures 3. Present the argument in getting the results. 4. Calculate the distance and bearing of walked made by the boy.
Evaluation Skill	To develop skills in assessing or judging the credibility or value of statement or questions and methods for a given purpose in relation to rules, principles and procedures.	Teacher helped the students to evaluate information, statement and question through: i. assessing claims ii. assessing argument. The teacher assessed the results of the question by critical	Student were able to evaluate the question by answering the question raised by the teacher through logical presentation of data's.

		looking at the methods and procedures used and check if the results logically follows the rules guiding bearing and distance e.g. state the formular for calculating distance and bearing.	
Inferential Skill	To develop skills in drawing reasonable conclusion from data, statement, judgment, questions relating to bearing and distance.	The teacher assisted the students to take relevant decision using evidence and conjecturing alternatives. This was done by ensuring that all the questions were solved through systematic approach and by application of almighty formular to justified the result obtained..	Students were able to draw relevant conclusion by following the procedures in arriving at the answer.
Self Regulating Skill	To develop skills that will help students monitors one's cognitive activities with a view of correcting oneself.	Teacher assisted students to apply skills in reassessing one's judgment through i. self examination ii. self correction	The students were able to re-assessed their work by cross-checking the procedures, used in arriving at the answers.

Peer Assessment Training

- Assessment: Teacher distributed Instructional rubrics on Bearing and Distance to participants for grading.
- Participants exchanged their solved work on Bearing and Distance for grading using Instructional rubrics as a guide.
- Teacher collected the scripts and explained further on how to answer difficult areas in the questions. This was done to cross-check for quality grading among the participants.
- Teacher gave out assignments to the participants on Bearing and Distance.

Week 6: TOPIC: QUADRATIC EQUATION

Critical Thinking Skills	OBJECTIVES	TEACHERS ACTIVITIES	STUDENTS ACTIVITIES
Interpretation Skills	To develop skills in comprehending and expressing the meaning of data, rules, procedures, event and	Teacher helped the students to identify skills in interpreting questions, experience and beliefs through: i. categorization ii. decoding information and iii. clarifying meaning e.g. Find the value of χ if $\chi^2 - 5x + 6 =$	Students were able to identify equation. Also they were able to state the almighty formular with ease.

	information. E.g. 1. state the almighty formular 2. solve quadratic equation with irrational roots 3. use the method of completing the square to solve quadratic equations. 4. solve word problem leading to quadratic equation 5. find the value of χ if $\chi^2 - 5x + 6 = 0$	0. This question is interpreted by using the reference equation $ax^2+bx- C =0$. Teacher later state the almighty formular as $\chi = b\pm\sqrt{\frac{b2-4ac}{2a}}$	
Analysis Skills	1. To develop skills for detailed examination of information, questions, statement and concepts. 2. To develop skills in identify intended and actual inferential relationship among statements questions and concepts	Teacher analysed the information by introducing each figure of the element into the formular $\chi = -(-5) \pm \sqrt{\frac{(-5)^2\ (4\ x\ 1\ x\ 6)}{2\ x\ 1}}$ $\frac{5\ \pm\ \sqrt{1}}{2}$ $= \frac{4}{2}\ \ or\ \ \frac{6}{2}$ = 2 or 3	Students were able to show working and steps in arriving at the answer. $\chi = -(-5) \pm \sqrt{\frac{(-5)^2\ (4\ x\ 1\ x\ 6)}{2\ x\ 1}}$ $= 5 \pm \sqrt{\frac{25\ -\ 24}{2}}$ $= \frac{5+1}{2}$ or $\frac{5-1}{2}$ $= {}^{6}/_{2}$ or ${}^{4}/_{2}$ = <u>3</u> or <u>2</u>
Explanation Skills	To develop skills that help students state the results of one's reasoning, justify the reasons in terms of evidential, conceptual and methods use in arriving at conclusion.	Teacher helped the students to interprete and analyse questions, statement and justify methods of arriving at a conclusion through: i. stating the result ii. justifying procedures iii. presenting arguments.	The students were able to expanciate on the topic taught well as procedures adopted for each of the topic. Solve quadratic equation by completing the square.
Evaluation Skill	To develop skills in assessing or judging the credibility or value of	Teacher helped the students to evaluate information, statements and questions through: i. assessing claims	Students were able to evaluate the question by answering the questions

	statements and method for given purpose in relation to rules, principles and procedures	ii. assessing arguments. The teacher assessed the results of the question asked by critically looking at the method and procedures use and check if it is logic ally follows the rules guiding quadratic equation e.g. state the formular for calculating quadratic equation and ensure that the steps applied for the rules of quadratic equation.	raised by the teacher through logical presentation of data's.
Inferential Skills	To develop skills in drawing reasonable conclusion, from data, statement, judgment, question and concept relating to quadratic equation.	The teacher assisted the students to take relevant decisions using evidence and conjecturing alternatives. This was done by ensuring that all the questions were solved through systematic approach and by application of almighty formular to justified the result obtained.	The students were able to draw relevant conclusion by following the procedure to arrive at the answer.
Self-regulation	To develop skills that will help students monitors one's cognitive activities with a view of correcting oneself	Teacher encouraged the students to apply skills of reassessing their work through: i. Self-examination ii. Self correction	The students were able to re-assessed their work by cross-checking the procedures used in arriving at the answers.

Peer Assessment Training

- Assessment: Teacher distributed Instructional rubrics on Equation with Irrational roots to participants for grading.
- Participants exchanged their solved work on Equation on Irrational roots for grading using Instructional rubrics as a guide. 1 for poor, 2 for fair, 3 for good, and 4 for excellent.
- Teacher collected the scripts and explained further on how to answer difficult areas in the questions. This was done to cross-check for quality grading among the participants.
- Teacher gave out assignments to the participants on Equation on Irrational Roots.

Week 7: TOPIC: SERIES AND SEQUENCE (ARITHMETIC PROGRESS)

Critical Thinking Skills	OBJECTIVES	TEACHERS ACTIVITIES	STUDENTS ACTIVITIES
Interpretation Skills	To develop skills in comprehending and expressing the meaning of data, rules, procedures, event and information. E.g. Sequence and series. 1. Determine the pattern of a sequence 2. Determine any particular terms of a given sequence. 3. Find the nth term of AP 4. Find the common difference and the number n of the terms of an AP Find the 20thterm of an Arithmetic progression 3, 0, -3-6…	Teacher helped the students to identify skills in interpreting questions, data, rules, procedures, situations, experience and beliefs through: i. categorization (ii.) decoding information and (iii.)clarifying meaning e.g. Find the 20th term of an Arithmetic progression 3, 0, -3-6…This questions will be interpreted by looking for the difference of the 1st, 2nd, 3rd figure in that order, Then if the difference is the same all through, it means the sequence is AP. Teacher later stated the formular as: Tn =a+ (n-1)d. Where a is first term = 3, d is the difference = -3 and nth is the number we are looking for	Students were able to interpret data, questions and statement on sequence and series (AP) through categorizing information, decode information and clarifying meaning. Students were able to subtract 1st, 2nd, 3rd figures are arrange in either ascending or descending order. E.g 3-0=3. Again they were able to state the formula of A.P
Analysis Skills	To develop skills for detailed examination of information, questions, statement and concepts. To develop skills in identify intended and actual inferential relationship among statements, questions and concepts.	Teacher assisted the students in breaking down materials, statements, questions and concepts into component elements or parts such that its organizational structure would clearly be understood through (i) examing ideas and (ii) detecting arguments. Teacher analyse the information by introducing the figures into the formular T20 = a + (20-1)d = 19d. T20 = 3 + 19 x -3. T20= 3-57= -54	Students were able to examining ideas and detecting argumants by showing workings and steps in arriving at the answer. E.g. T20 = a + (20-1) = 19d. T20 = 3 + 19 x 3. T20 = 3-57 = -54.
Evaluation Skills	To develop skills in assessing or judging the credibility or value of statements and methods for give purpose in relation to rules principles and procedure.	Teacher helped the students to evaluate information, statements and questions through (i) assessing claims and (ii) assessing arguments. The teacher assessed the results of the question asked by critically looking at the method, procedures use and whether it logically follows the rules guiding arithmetic progression e.g. stating the A.P formula and ensure that the steps applied followed the rules of A.P.	Students were able to evaluate the question by answering the questions raised through logical presentation of data's.
Inferential	To develop skills in drawing	The teacher assisted the students to take	The students were able to draw

Skills	reasonable conclusions from data, statement, judgment, questions and concepts.	relevant decision using evidence and conjecturing alternatives. This was done by ensuring that all the questions were solved through systematic approach and by application of almighty formular to justified the result obtained. T20 = 3-57=-54.	relevant conclusion by following the procedure to arrive at the answer.
Explanation Skills	To develop skills that help students state the result of one's reasoning, justify the reasons in terms of evidential, conceptual and methods use in arriving at conclusions.	Teacher helped the student to interpret and analyze questions, statement and justify the reasons and methods of arriving at a conclusion through (i) stating results (ii)justifying procedures and (iii) presenting arguments.	The students were able to expanciate on the topic taught as well as procedures adopted for each of the topics.
Self-regulation Skills	To develop skills that will make students monitors one's cognitive activities with a view of correcting one's self.	Teacher encouraged the students to apply skills of reassessing their work through (i) Self-examination and (ii) Self-correction.	The students were able to reassessed their work by cross checking the procedures and the answers they got.

Peer Assessment training

- Assessment: Teacher distributed Instructional Rubrics on Arithmetic Progression to participants for grading.
- Participants exchanged their solved work on Arithmetic Progression for grading using Instructional Rubrics as a guide. 1 for poor, 2 for fair, 3 for good, and 4 for excellent.
- Teacher collected the scripts and explained further on how to answer difficult areas in the questions. This was done to cross-check for quality grading among the participants.

Teacher gave out assignments to the participants on Arithmetic Progression.

Training for the Control Group

The Control Group was chosen from a different location that was far from the experimental schools to avoid experimental contamination. During the first session, Mathematics Performance Test (MPT), Watson-Glaser Critical Thinking Appraisal Test (WGCTAT) and Peer Assessment Scale in Mathematics (PASM) were administered to the participants. The participants were taught the same topics for the same duration as the experimental group by the researcher but they did not receive the Critical Thinking and Peer Assessment Training Module. In addition, normal weekly class tests were conducted and in the eighth week, post tests were administered. The control group was later exposed to Critical Thinking Skills and Peer Assessment training because they worked.

Probability (Theoretical and experimental probability)

- Introducing the topic probability of mutually exclusive events and independent event.
- Teaching of mutually exclusive and independent event to the participants using prepared lesson notes with Instructional materials.
- Teacher used sets of cards and letter words to teach participants how to find probability.
- Teacher gave participants room for questions in areas not clear and later responded.
- Evaluation: Teacher asked participants to solve questions on mutually exclusive events.
- Teacher collected the scripts and explained further on how to answer difficult areas in the questions.

Teacher gave out assignments to the participants on theoretical and experimental probability.

Bearing and Distance

- Collection of assignment given for grading.
- Teaching of the next topic Bearing and Distance to the participants using prepared lesson notes with Instructional Materials.
- Introducing the topic (1) Calculation of bearing and distance of the port and from the port. (2) Expressed the bearing as a 3-figure bearing or compass bearing.
- Teacher used Mathematical sets to teach participants how to construct, measure and calculate the distance from a point to another point. These can be expressed either as 3-figure bearing or a compass bearing.
- Teachers calculated the distance and bearing in terms of kilometers [km] and degrees [^{o}]
- Teacher gave participants room for questions in areas not clear and later responded.
- Evaluation: Teacher asked participants to solve questions relating to distance and bearing and ask the participants to put their answers in 3 significant figures, degrees and Kilometers.
- Teacher collected the scripts and explained further on how to answer difficult areas in the questions.

Teacher gave out assignments to the participants on bearing and distance

Quadratic Equation (Equations with Irrational Roots)

- Collection of assignment given for grading.
- Teaching of the next topic Quadratic Equation (Equation with Irrational roots) to the participants using prepared lesson notes with Instructional materials.
- Introducing the topic (1). State the almighty formular (2). Solve quadratic equation with irrational roots
- Teacher gave participants room for questions in areas not clear and later responded.
- Evaluation: Teacher asked participants to solve questions relating to Equation with Irrational roots. Teacher collected the scripts and explained further on how to answer difficult areas in the questions.

Teacher gave out assignments to the participants on Equation with irrational roots

Sequence and Series (Arithmetic Progression)

- Teaching of the new topic; Sequence and Series (Arithmetic Progression) to the participants using prepared lesson notes with Instructional Materials.
- Introducing the topic (1) Determine the pattern of a sequence (2) Determine any particular terms of a given sequence. (3). Find the nth term of AP (4). Find the common difference and the number n of the terms of an AP
- Teacher gave participants room for questions in areas not clear and later responded.
- Evaluation: Teacher asked participants to solve questions relating to Arithmetic progression. E.g. Find the 5^{th} and 8^{th} terms of the sequence whose nth terms is (a) $2n + 1$ (b) $3-5n$.
- Teacher collects the scripts and explained further on how to answer difficult areas in the questions.

Teacher gave out assignments to the participants on Arithmetic Progression**.**

Phase 3: Post-test

Week 8: The researcher revised the training process of Critical Thinking, Peer Assessment using Instructional Rubrics with the participants and later administered all the instruments on them for post-test.

Data Analysis

The data that were collected from the participants using the various instruments were coded and subjected to both descriptive and inferential statistics. All the hypotheses were tested at 0.05 level of significance. The means and standard deviations for pre and post tests assessment measures were computed. Hypotheses 1, 2, 3, 4, 5 ,6 and 7 were tested using Analysis of Covariance while Hypothesis 8 was tested using Multiple Regression Analysis.

Analysis of Covariance (ANCOVA). ANOVA is particularly applicable in this study because there was need to adjust the criterion scores Y, for an initial variable X. More so, it was used to determine the effect of Critical Thinking Skills and Peer Assessment training on Mathematics performance using pre and post test scores of the independent variables while partialling out or removing the effect of other variables. The covariance corrects for effect of pretesting. The use of ANCOVA is justified because of its ability to statistically correct the initial differences in the pre-test scores of the assessment instruments among the participants.

Multiple Regression Analysis: This was used to predict scores of sets of independent variable such as Critical Thinking Skills, Peer Assessment and Gender on Mathematics performance . More so, it was used to established which of the sets of the observed variables gave rise to the best prediction on the criterion variable.

CHAPTER FOUR

DATA ANALYSIS AND PRESENTATION OF RESULTS

The data that were collected from the participants using the various instruments were coded and subjected to both descriptive and inferential statistics. All the hypotheses were tested at 0.05 level of significance. The means and standard deviations for pre and post tests assessment measures were computed. Hypotheses 1,2, 3,4,5,6,7 and 8 were tested using Analysis of Covariance (ANCOVA), while Hypothesis 8 was tested using Multiple Regression Analysis.

4.1 Testing of Hypotheses

Hypothesis One: There is no significant main effect of treatment on students' performance in Mathematics.

Table 8: Descriptive Data on Pre and Post test scores of the Participants across the Experimental conditions.

Groups		Pre-test		Post-test		
	N	Mean	SD	Mean	SD	Mean Difference
Training Group	108	29.62	5.16	54.82	8.93	25.20
Control Group	104	29.79	5.68	38.54	9.21	8.75
Total	212	29.71	5.41	46.54	9.07	16.83

Evidence from Table 8 shows that participants exposed to training instructions had the highest mean difference of 25.20, whereas the Control Group had 8.75. To determine whether significant difference exists in Mathematics performance scores among participants, one-way ANCOVA was used and the results are presented in Table 9

Table 9: ANCOVA Test of Difference in Post-test Mathematics Performance between Training and Control Groups.

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	14721.22	2	7360.61	92.63	*
Covariates	686.09	1	686.09	8.63	*
Experimental Group	14128.68	1	14128.68	177.80	*
Error	16607.31	209	79.46		
Corrected Total	31328.54	211			

*Significant at 0.05; df=1 & 209; F-cal = 177.80, F-critical=3.89

The ANCOVA results presented in Table 9 shows that for the Experimental condition, the F–value of 177.80 was greater than the F-critical value of 3.89, given 1 and 209 degrees of freedom at the .05 level of significance. Since the calculated F-value was greater than the F-critical value, hypothesis 1 was rejected. This therefore suggests that training on Critical Thinking Skills and Peer Assessments was significant in improving the Mathematics performance of the students. This also showed that Critical Thinking Skills and Peer Assessment Training had impact on the participants as different from those in the control group who were not trained.

Hypothesis Two: There is no significant main effect of gender on students' performance in Mathematics.

Table 10: Descriptive Data on Pre and Post test scores of participants in Mathematics test due to gender.

Groups		Pre-test Scores		Post-test Scores		
Gender	**N**	**Mean**	**SD**	**Mean**	**SD**	**Mean Difference**
Male	113	29.92	5.50	47.85	12.88	17.93
Female	99	29.46	5.31	46.67	11.28	17.21
Total	212	29.69	5.40	47.26	12.08	17.57

Evidence from Table 10 shows that the male students had mean difference of 17.93 while the female students had a mean difference of 17.21 this shows that male participants did better than their female counterparts. To determine whether significant difference exists due to gender, a one way ANCOVA was utilised and results are presented in Table 11

Table 11: ANCOVA Post-test Difference in Mathematics Performance due to gender

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	812.06	2	406.03	2.78	n.s
Covariate	560.91	1	560.91	3.84	*
Gender	219.52	1	219.52	1.50	n.s
Error	30516.47	209	146.01		
Corrected Total	31328.54	211			

n.s=Not significant at 0.05; df= 1 & 209; F-cal= 1.50; F-critical=3.89

Table 11 shows that a calculated F-value of 1.50 was less than critical F-value of 3.89 for gender at 0.05 level of significant, given 1 and 209 degrees of freedom. Since the F-calculated value was less than F-critical value, hypothesis two was accepted. Therefore there is no significant main effect of gender on student's performance in Mathematics.

Hypothesis Three: There is no significant effect of age on students' performance in Mathematics.

Table 12: Descriptive Data on Pre and Post test scores of participants in Mathematics tests due to Age.

Age Groups		Pre-test Scores		Post-test Scores		
Age	**N**	**Mean**	**SD**	**Mean**	**SD**	**Mean Difference**
14-15 years	113	29.84	5.65	47.22	12.43	17.38
16 years & above	99	29.55	5.14	46.50	12.09	16.95
Total	212	29.69	5.39	46.86	12.26	17.17

Evidence from Table 12 shows that participants whose age falls between 14 and 15 years old had a higher mean difference of 17.38 whereas those that falls between 16 years and above had a mean difference of 16.95. This further shows that participants from 14 to 15 years did better than those from 16 years and above in mathematics. To determine whether significant difference exists due to age, a one way ANCOVA was utilised and results are presented in Table 13.

Table 13: ANCOVA Post-test Difference in Mathematics Performance due to Age

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	627.09	2	313.54	2.13	n.s
Covariates	599.90	1	599.90	4.08	*
Age	34.54	1	34.54	.235	n.s
Error	30701.45	209	146.89		
Corrected Total	31328.54	211			

n.s=Not significant at 0.05; df= 1 & 209; F-cal= 0.235; F-critical=3.89

The ANCOVA results presented in Table 13 shows that for age, the calculated F–value of 0.235 was less than F-critical value of 3.89, given 1 and 209 degrees of freedom at the .05 level of significance. This therefore suggests that age does not have significant effect on students performance in Mathematics. Therefore hypothesis 3 was accepted.

Hypothesis Four: There is no significant interaction effect of treatment and gender on students' performance in Mathematics.

Table 14: Descriptive Data on Pre and Post test scores of participants between Experimental Condition and Gender in Mathematics.

Groups		Pre-test Scores			Post-test Scores		
	Gender	**N**	**Mean**	**SD**	**Mean**	**SD**	**Mean Difference**
Training Group	**Male**	58	29.86	5.43	56.20	9.59	26.34
	Female	50	29.36	4.86	53.22	7.89	23.86
Control Group	**Male**	55	29.52	5.95	39.05	9.65	9.53
	Female	49	29.42	5.87	37.97	8.76	8.55
Total		212	29.54	5.52	46.61	8.97	17.07

Evidence from Table 14 shows that the male and female participants exposed to Training Instructions had mean differences of 26.34 and 23.86, whereas the Control Group had 9.53 and 8.55 for male and female participants. This shows that male students did better than their female counterparts. To determine whether significant difference exists due to gender and experimental conditions, a one way ANCOVA was utilised and results are presented in Table 15

Table 15: 2 x 2 ANCOVA Tests of the Effects of Experimental Condition and Gender on Post-test Mathematics Performance of Students

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	14957.59	4	3739.39	47.28	.000
Covariates	652.95	1	652.95	8.25	*
Gender	185.98	1	185.98	2.35	n.s
Experimental Group	13930.83	1	13930.83	176.14	*
Experimental Group * Gender	47.00	1	47.00	.59	n.s
Error	16370.95	207	79.08		
Corrected Total	31328.54	211			

n.s=Not significant at 0.05; df= 1 & 207; F-cal= 2.35; F-critical=3.89
n.s=Not significant at 0.05; df= 1 & 207; F-cal= 0.59; F-critical=3.89
*Significant at 0.05; df=1 & 207; F-cal= 176.14; F-crtical= 3.89

Table 15 shows that a calculated F-value of 2.35 for gender was not significant at 0.05 level, given 1 and 207 degrees of freedom because it was less than the F-critical value of 3.89. The F-value of 178.61 for experimental condition was significant since it was greater than the F-critical value of 3.89 at 0.05, given 1 and 207 degrees of freedom while the calculated F-value of 0.593 for interaction between experimental conditions and gender was not significant since it was less than F-critical value of 3.89 at 0.05 level of significance, given 1 and 207 degrees of freedom. Therefore, hypothesis four was accepted. It was concluded that there is no significant interaction effect of experimental conditions and gender on student's performance in Mathematics.

Hypothesis Five: There is no significant interaction effect of treatment and age on students performance in Mathematics.

Table 16: Descriptive Data on Pre and Post test scores of participants between Experimental Condition and Age in Mathematics.

Groups	Age	Pre-test Scores N	Mean	SD	Post-test Scores Mean	SD	Mean Difference
Training Group	**14-15yrs**	57	29.98	5.24	55.27	9.08	26.34
	16yrs & above	51	29.23	5.08	54.42	8.85	23.86
Control Group	**14-15 yrs**	56	29.79	6.08	38.66	9.45	8.95
	16yrs & above	48	29.84	5.23	38.46	9.09	8.67
Total		212	29.63	5.41	46.70	12.18	7.10

Evidence from Table 16 shows that participants whose age falls between 14 years and 15 and 16 years and above and were exposed to Training Instructions had mean difference of 26.34 and 23.86, whereas those in Control Group had a mean difference of 8.95 and 8.67. This shows that those in the experimental group did better than their control group counterpart. To determine whether significant difference exists due to age and experimental conditions, a two way ANCOVA was utilised and results are presented in Table 17.

Table 17: 2 x 2 ANCOVA Tests of Interaction Effects of Experimental Condition and Age on Mathematics Performance of Students

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	14754.68	4	3688.67	46.07	*
Covariates	698.69	1	698.69	8.72	*
Age	21.05	1	21.05	0.263	n.s
Experimental Group	14106.83	1	14106.83	176.18	*
Experimental Group * Age	11.77	1	11.77	.147	n.s
Error	16573.86	207	80.06		
Corrected Total	31328.54	211			

n.s=Not significant at 0.05; df= 1 & 207; F-cal= 0.263; F-critical=3.89
n.s=Not significant at 0.05; df= 1 & 207; F-cal= .147; F-critical=3.89
*Significant at 0.05; df=1 & 207; F-cal= 176.18; F-critical= 3.89

Table 17 shows that a calculated F-value of 0.263 for age was not significant since it was less than F-critical value of 3.89 at 0.05 level of significant given 1 and 207 degree of freedom. The calculated F-value of 176.18 for experimental condition was significant since it was greater than the F-critical value of 3.89 at 0.05, given 1 and 207 degree of freedom. However, the calculated F-value of .147 for interaction between experimental conditions and age was not significant since it was less than F-critical value of 3.89 at 0.05 level of significance given 1 and 207 degree of freedom. Therefore, hypothesis five was accepted. It was concluded that there is no significant interaction effect between experimental conditions and gender in mathematics performance test.

Hypothesis Six: There is no significant interaction effect of Age and Gender on students performance in Mathematics.

Table 18: Descriptive Data on Pre and Post test scores of participants between Age and Gender in Mathematics.

Groups		Pre-test Scores			Post-test Scores		
Age	**Gender**	**N**	**Mean**	**SD**	**Mean**	**SD**	**Mean Difference**
14-15yrs	**Male**	57	30.26	6.00	48.22	12.18	17.96
	Female	56	29.84	5.29	44.75	11.67	14.91
16yrs & above	**Male**	56	29.58	4.97	47.48	13.65	17.90
	Female	43	29.51	5.40	46.88	10.78	17.37
Total		212	29.79	5.41	46.83	12.07	17.04

Evidence from table 18 shows that participants whose age falls between 14 and 15 years (male and female) had a mean difference of 17.96 and 14.91, whereas those from 16 years and above (male and female) had a mean difference of 17.90 and 17.37. This shows that male students whose age falls between 14 and 15 years did better than male participants whose age falls between 16 years and above. However, female students whose age falls between 16 years and above did better than female participants whose age falls between 14 and 15 years . To determine whether significant difference exists due to gender and age, a two way ANCOVA was utilised and results are presented in Table 19.

Table 19: 2 x 2 ANCOVA Tests of Interaction Effects of Age and Gender on Mathematics Performance of Students.

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	928.57	4	232.14	1.58	n.s
Covariates	550.96	1	550.96	3.75	n.s
Gender	188.73	1	188.73	1.28	n.s
Age	31.97	1	31.97	.218	n.s
Gender * Age	91.88	1	91.88	.626	n.s
Error	30399.97	207	146.86		
Corrected Total	31328.54	211			

n.s=Not significant at 0.05; df= 1 & 207; F-cal= .218; F-critical=3.89
n.s=Not significant at 0.05; df= 1 & 207; F-cal= 1.28; F-critical=3.89
n.s=Not significant at 0.05; df=1 & 207; F-cal= .626; F-crtical= 3.89

Table 19 shows that a calculated F-value of 0.218 for age was not significant because it was less than critical F-value of 3.89 at 0.05 level of significance, given 1 and 207 degree of freedom. The calculated F-value of 1.28 for gender was not significant since it was less than F-critical value of 3.89 at 0.05 level of significance given 1 and 207 degrees of freedom. Also F-value of .626 for interaction between age and gender was also not significant because it was less than F-critical value of 3.89 at 0.05 given 1 and 207 degrees of freedom. Hypothesis six was therefore accepted. It was concluded that there is no significant interaction effect between age and gender in Mathematics performance test.

Hypothesis Seven: There is no significant interaction effect of treatment, age and gender on students performance in Mathematics.

Table 20: Descriptive data of Pre and Post Tests Scores of Participants between experimental conditions, Age and Gender in Mathematics

				Pre-test Scores		Post-test Scores		
Experimental group	**Age**	**Gender**	**N**	**Mean**	**Std. Dev**	**Mean**	**Std. Dev**	**Mean Difference**
Training Group	14-15 years	Male	30	30.40	5.91	55.80	9.17	25.40
		Female	27	29.51	4.45	52.88	8.40	23.37
	16 years and above	Male	28	29.28	4.91	56.64	10.17	27.36
		Female	23	29.17	5.39	53.60	7.43	24.43
Control Group	14-15years	Male	27	30.11	6.21	39.81	9.29	9.71
		Female	29	29.34	6.04	37.17	8.80	7.83
	16 years and above	Male	28	29.89	5.10	38.32	10.11	8.43
		Female	20	29.90	5.54	39.15	8.67	9.25
Total			212	29.70	5.44	46.69	9.00	16.97

Evidence from table 20 shows that participants whose age falls between 14 and 15 years (male and female) who were exposed to training instructions had a mean difference of 25.40 and 23.37, whereas those from 16 years and above (male and female) who were exposed to training instruction also had a mean difference of 27.36 and 24.43. However, the participants from control group whose age falls between 14-15 years (male and female) had a mean difference of 9.71 and 7.83 whereas those from 16 years and above (male and female) had a mean difference of 8.43 and 9.25. This shows that male participants in the treatment group whose age falls between 14 and 15 years did better than their female counterparts. Also male participants in the treatment group whose age falls between 16 years and above did better than their female counterparts. The male participants in the control group whose age falls between 14-15 years did better than their female counterparts. However, female participants in the control group whose age falls between 16 years and above did better than male participants. To determine whether significant difference exist due to treatment, gender and age, a two way ANCOVA was utilised and results are presented in Table 21.

Table 21: 2 x 2 ANCOVA Tests of Interaction Effects of experimental conditions, Age and Gender on Mathematics Performance of Students

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Corrected Model	15053.27	8	1881.65	23.47	*
Covariates	654.96	1	654.96	8.16	*
Gender	168.07	1	168.07	2.09	n.s
Age	18.97	1	18.97	0.237	n.s
Experimental Group	13666.90	1	13666.90	170.46	*
Gender * Age * Experimental Group	128.21	4	32.05	0.400	n.s
Error	16275.27	203	80.17		
Corrected Total	31328.54	211			

Not significant at 0.05; df= 1 & 203; F-cal= 0.237; F-critical=3.89

Not significant at 0.05; df= 1 & 203; F-cal= 2.09; F-critical=3.89

* significant at 0.05; df=1 & 203; F-cal= 170.46; F-critical= 3.89

Not Significant at 0.05; df= 4 & 203; F-cal= 0.400; F-critical= 2.41

Table 21 shows that a calculated F-value of 0.237 for age was not significant because it was less than F-critical value of 3.89 at 0.05 level of significance given 1 and 203 degrees of freedom. Also the calculated F-value of 2.09 for gender was not significant since it was less pothan F-critical value of 3.89 at 0.05 level of significance given 1 and 203 degrees of freedom while calculated F-value of 170.46 for experimental was significant since it was greater than F –critical value of 3.89 at 0.05 level of significance given 1 and 203 degree of freedom. However, the F-value of 0.400 for interaction between, treatment, age and gender was not significant since it was less than F-critical value of 3.89 at 0.05 level of significance given 4 and 203 degrees of freedom. Hypothesis seven was therefore accepted. It was concluded that there is no significant interaction effect between treatment, age and gender in Mathematics performance test.

Hypothesis Eight:

There is no significant linear relationship between Mathematics Test scores and a set of independent variables (Critical Thinking Skills , Peer Assessment, and Gender). The hypothesis was tested using Multiple Regression analysis and the results are presented in Table 22, 23, 24 and 25.

Table 22: Inter-correlation Matrix of the Critical Thinking Skills, Peer Assessment, Gender and Mathematics Performance Test (n=212)

Variables	**Mean**	**Std. Dev**
Critical Thinking	50.81	4.80
Peer Assessment	61.11	4.73
Gender	1.47	0.50
Mathematic Performance	46.84	12.18

Variable	**CTS**	**PAMS**	**Gender**	**MAT**
CTS	1	0.085	0.257*	0.395*
PAMS	0.085	1	-0.096	0.300*
Gender	0.257*	-0.096	1	-0.090
MAT	0.395*	0.300*	-0.090	1

*Correlation is significant at 0.05 level

From the results of the table above, Mathematics post-test scores correlated positively with (r-cal=0.395), Peer Assessment (r-cal=0.300) and correlated negatively with gender -0.090. All the correlation values were statistically significant at 0.05 level of significance. Entered Multiple Regression Analysis was employed to determine the predictor variables to the explained variance. The results were presented in Tables 23, 24 and 25

Table 23 Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.505(a)	.255	.244	10.593

The Independent variables entered accounted for 24.4% of the total variation in the Mathematics Performance Test (Adjusted R^2 = 0.244). The set of independent variable entered in the regression equation correlated 0.225 with Mathematics performance test scores and controlling for the cofounding effects of other factors with a view to evaluate the specific contributions of each variable in the regression equation, to predicting Mathematics tests.

Table 24 : Model summary of influence of Critical Thinking Skills, Peer Assessment and Gender on Mathematics Performance .

Model		Sum of Squares	Df	Mean Square	F	Sig.
1	Regression	7990.669	3	2663.556	23.73	*
	Residual	23337.879	208	112.201		
	Total	31328.547	211			

*Significant $P < 0.05$,F-Cal = 23.73; F-critical at 0.05 (3 , 208) = 2.65

Table 24 indicated that the sets of independent variable in the regression equation correlated with the Mathematics performance test scores because the F- value of 23.73 was greater than F- critical of 2.65 at 0.05 level of significant, given 3 and 208 degrees of freedom.

Table 25: Variable in the Multiple Regression Equation

Model	Variable	Unstandardized Coefficients		Standardized Coefficients	T	Sig.
		B	Std. Error	Beta	B	Std. Error
1	(Constant)	38.89	11.78		3.385	.000
	Critical Thinking skill	1.060	.158	.418	6.710	.000
	Peer Assessment	.639	.156	.248	4.101	.000
	Gender	-4.218	1.520	-.173	-2.775	.006

Dependent Variable: Post-test scores of Mathematics Achievement Test

Evidence from table 24 indicated that Critical Thinking Skills and Peer Assessment were found to have had significant linear effects on Mathematics test scores. The beta weight for Critical Thinking Skills (0.418) was significant at 0.05 probability level (t-cal=6.710), Peer Assessment had beta weight of (0.248) which was also significant at 0.05 probability level (t-cal=4.101), and gender had beta weight of (-.173) which was significant at 0.05 probability level (t-cal =-2.775). The bêta weight was used to ascertain the relative contribution of a particular independent variable in the equation after partialling out the effects of a particular independent variable. Further examination of the data in table 25 shows that gender had made the least contribution in the regression equation in terms of predicting performance in the Mathematics tests. Based on the significance of the their beta weights, it is evidence that Critical Thinking and Peer Assessment are the most important variables for predicting Mathematics performance test of SSS II students.

CHAPTER FIVE

DISCUSSION, SUMMARY OF FINDINGS, CONCLUSIONS, CONTRIBUTIONS TO KNOWLEDGE, IMPLICATIONS FOR FINDINGS, RECOMMENDATIONS AND SUGGESTIONS FOR FURTHER STUDIES

5.1 Discussion Of Findings

The finding in hypothesis one showed that there was a significant difference in Mathematics scores between the Training and Control Group. The reason for the difference could be attributed to acquisition of knowledge in Critical Thinking Skills and Peer Assessment which were infused into the teaching of Mathematics. This finding was supported by M˘arcut (2005) who had shown that students who received training in Critical Thinking Skills significantly improved positively in their performance than those who were not trained. Barry, Ada and Jenny (2003) were also in conformity with the impact of Critical Thinking on students academic performance. They did a study with respect to analysing Critical Thinking Skills and administered Critical Thinking Test (CAT) as pre-test/post-test to two different courses in the social sciences, with the consent of the instructors. A significant improvement ($p < 0.05$) was observed between scores on the pre-test and post-test, for students in this course. They were of the view that the training programme on Critical Thinking Skills organized for the training group had a significant impact on them than their control group counterpart who did not receive any training programme.

Charles, Renae and Rospond (2004) also carried out a study using Critical Thinking instruments to assess Pharmacy students' Critical Thinking Skills and dispositions, and to identify areas for curriculum reforms. They found out that those in the training group that received training on Critical Thinking Skills did better in their post test scores than their control group counterparts. The reason for the improvement was that they had training on Critical Thinking Skills and were exposed to the process of Critical Thinking skills; therefore they were able to solve some problems or challenges that came their ways. The result also supported the findings of Onuka and Oludipe, (2006) who reported that the performance of students in the treatment group outweighed those from the control group. The results of this study also agreed with the findings of Onuka, (2007) that feedback, which is an outcome of evaluation, and systematic school based assessment do assist in remediating students' poor performance and in achieving cognitive learning objectives. The reason for the improvement on the part of the training group was that they were exposed to the process of assessing each other's work from the beginning and that has created confidence among them because they were able to know each other's weaknesses and strength and then built on it. This was done by looking at good works from other students and using it to correct one's own mistakes.

The finding in hypothesis two revealed that there was no significant difference in post-test Mathematics Performance of students due to gender. This could be attributed to the awareness of the importance of the subject by both sexes in the society and that one can hardly survive without it. In support of this finding, Sprigler & Alsup (2003) who carried out a study on gender achievement, found out that there was no gender difference on Mathematical reasoning ability at elementary level. The study of Ding, Song and Richardson, (2007) was also in support of this finding. They were of the view that there was no significant difference between male and female students' performance in Mathematics. They carried out a longitudinal study on gender differences in Mathematics and one of their findings showed that there is no significant between boys and girls in Mathematics achievement. This study showed that growth trend in Mathematics was not gender sensitive. In addition, the findings of (Howes, 2002; Barton, 1998; & Sines, 2006) showed that the effect of gender on performance among students was not significant.

Abiam and Odok (2006) who also carried out a study on gender and Mathematics achievement found no significant relationship between gender and achievement in number and numeration, algebraic processes, and statistics. They however found the existence of a weak significant relationship in geometry and trigonometry. Also, Adedayo (2006) found no significant difference between the scores of male and female students in Mathematics tests. The gender balance in Mathematics could be as a result of their interest and attitude as well as the Critical Thinking Skills and Peer Assessment training received by those in the training group.

However, in spite of research evidences of no significant difference in Mathematics achievement between male and female students, some research findings do support the gender difference in Mathematics achievement. The result of the findings of Knol & Berger (1991) revealed that the effect of sex on performance variability among the students was significant on global scores of the students. More so, Bassey, Joshua and Asim (2008) carried out a study on gender and Mathematics Achievement in secondary schools in Calabar, Cross Rivers State. The result of their findings revealed that there is a significant difference between the Mathematics achievement of the male and female students (t-cal $5.43 \geq$ t-crit 1.645 at 0.05 level of significance and 198 degree of freedom). This was attributed to environmental factor and that the male students showed greater interest in calculation oriented subjects like Mathematics, Physics, Further Mathematics, Chemistry, Accounting or Economics while their female counterparts showed little or no interest in subjects that are calculation oriented but showed greater interest in subjects that are literary oriented such as English Language, Christian/Islamic religion knowledge, History and Literature in English.

The finding in hypothesis three showed that there is no significant main effect of age on students performance in Mathematics. This could be due to the fact that both the younger and older students attended the same classes, listening to their teachers while teaching as well as contributed to the lessons together. In support of this finding, DeMeis and Stearns (1992) found no significant relationship between age and achievement. White (1982) said that since schools provide equalizing experiences to both old and young students, there won't be any significant difference between age and achievement in Mathematics. Grissom (2004) in his study concluded that the negative relationship between age and achievement remains constant over time. However, Crosser (1991), Kinard & Reinherz (1986), and La Paro and Pianta (2000) were of the view that older children fare better academically than the younger children in the same class. In support of this view, Uphoff and Gilmore (1985) used research evidence about the relationship between age and achievement as well as other evidence to argue that the older and/or more mature students in a class fared better than their younger classmates.

The result on hypothesis four revealed that there is no significant interaction effect of treatment and gender. The reason for no interaction could be attributed to acquisition of knowledge by both sexes in Critical Thinking Skills and Peer Assessment which aided the training group. In support of this finding, Mitrevski and Zajkov (2012) examine the effectiveness of physical laboratory, critical thinking and gender difference among physics students in Macedonia. The results revealed that irrespective of the training on critical thinking, there is no statistical gender differences among the groups. Moreover, the findings of Michael (2009) showed no statistically significant gender difference in science achievement among Macedonian eighth grade students. In support of this finding, Adediwura, (2012) who researched on effect of peer and self assessments on the self-efficacy and students' learner autonomy in the learning of mathematics as well as determining the attitude of male and female students towards the use of peer and self assessments in State public senior secondary schools in Osun State using senior secondary three students. The result of the study showed no significant relationship between sex and enhancement of self-efficacy as a result of students' engagement with the use of peer and self-assessments.

However, the findings of Michael (2003) negates the findings of this study. It showed that significant difference exists between male and female students in the eighth grade class in the Republic of Macedonia. Graybill (1975) also found evidence of a gender difference in problem-solving, where girls lagged behind boys in the development of logical thinking ability.

The results on hypothesis five revealed that there was no significant main effect of treatment and age on students performance in Mathematics. The reason for the no significant difference could be as result of exposure of students to Critical thinking skills and Peer assessment which were infused into the teaching of Mathematics in the classroom consisting of students of different ages. This finding was supported by Fisher, (2003) who highlighted that the Critical Thinking Skills training had helped in stimulating students' intellectual capability irrespective of their ages and made to become more engaged in classroom activities. In support of this finding, Rudd, Baker, and Hoover, (2000) who carried out a study on relationship between age and critical thinking among Mathematics students and found out that age did not have significant impact on thinking ability of the students, and that the performance of both older and younger students were relatively the same in Mathematics. More so, DeMeis & Stearns (1992) also found no significant relationship between age and achievement.

However, the findings of Mononen and Aunio, (2013) did not in corroborate this finding. They carried out a study on early Mathematical performance in Finnish Kindergarten and Grade One in Helsinki, Finland. The result showed that age was a significant predictor to success in Mathematics in the kindergarten and first grade; older children performed higher than younger ones. The older children may have had more opportunities to practise and get acquainted with mathematical issues, as the age difference between the youngest and the oldest child in the classroom can be up to one year.

Crosser’s (1991), Kinard and Reinherz’s (1986), and La Paro and Pianta’s (2000) also negated the finding of this study. They supported the finding that age has contributed significantly in predicting performance in Mathematics. They presented evidence that older children fared better academically than their younger ones. Similarly, Uphoff & Gilmore (1985) used research evidence about the relationship between age and achievement as well as other evidence to argue that the older or more mature students in a class fare better than younger classmates.

The finding in hypothesis six revealed that there was no significant interaction effect of age and gender on students performance in Mathematics. The reason for this finding could be as a result of the same knowledge acquisition received by both sexes irrespective of age. In support of this finding, DeMeis and Stearns (1992) who did a research on age and gender as yardstick for predicting achievement in Mathematics, found no significant relationship among age, gender and achievement. However, the findings of Khata, Krissana, Kungu, Yahya and Mohd (2011) was not in corroboration with the findings of this study. Their study revealed that there is a significant difference in achievement in Mathematics between gender and age. Langer, Kalk, & Searls (1984) also found significantly higher achievement of the oldest as compared to the youngest students at age nine.

The result for hypothesis seven revealed that there was no significant interaction effect of treatment, age and gender on students’ performance in Mathematics. The reason for no difference could be attributed to acquisition of knowledge in Critical Thinking Skills and Peer Assessment which were infused into the teaching of Mathematics. The finding of Saito (2008) confirmed the result. He examined the effects of training on Peer Assessment regarding oral presentations in English and Foreign Language (EFL) classrooms. The results did not show any significant difference between the treatment and control groups. The findings of DeMeis and Stearns (1992) was also in consonance with the result above, They researched into age and gender as yardstick for predicting achievement in Mathematics, they found no significant relationship between age and achievement and gender and achievement.

However, M˘arcut (2005) carried out a study on Critical Thinking Skills and performance in Mathematics and found out that students who received training in Critical Thinking Skills significantly improved in their performance than those who were not trained. Again, Peterson and Irving (2008) carried out an investigation on perception of students towards Peer Assessment in secondary schools. The researchers found out that the students had a positive view of Peer Assessment because it is a useful strategy for both students and teachers to assess themselves. They saw Peer Assessment as fun. With regard to feedback, students believed that feedback motivated them, provided information, and helped them seek for new information.

However, the findings of Khata, Krissana, Kungu, Yahya and Mohd (2011) negate this result. They studied the influence of age and gender on students achievement in Mathematics. The results revealed that there were statistically significant differences in mathematics GPA scores between age groups and gender. Habibollah, Rohani,. Tengku, Jamaluddin, and Kumar (2009) also studied Creativity, age and gender as predictors of academic achievement among undergraduate students. Their finding showed that creativity, age and gender are predictors of academic achievement.

Finally, the test of hypothesis eight revealed that there was significant relationship between Mathematics performance and a sets of variables such as Critical Thinking Skills, Peer Assessment, and gender. The results was in consonance with M˘arcut (2005) who believes that Critical Thinking has positive relationship with performance in Mathematics. Again, Onuka (2007) also believed that there was positive relationship between Peer Assessment and performance in Mathematics in secondary schools because the more students assessed themselves the better they are exposed to procedures and rules expected from them. This of course improves academic performance in Mathematics. Abiam and Odok (2006) view was not in line with the findings above as their study found out a significant negative relationship between gender and academic performance in Mathematics.

5.2 Summary of Findings

The study was conducted primarily to determine the impact of Critical Thinking Skills and Peer Assessment on performance in Mathematics. Based on the research hypotheses formulated for this study, the following are the highlights of the findings:

1. Students who were exposed to training strategies (Critical Thinking Skills, Peer Assessment) combined with good Instructional Rubrics procedures generally improved significantly in their performance in Mathematics than their counterparts that were not exposed to the training instructions.
2. There was no significant difference in students' performance in Mathematics due to gender.
3. There was no significant main effect in students' performance in Mathematics due to age.
4. The study found out that there was no significant interaction effect of treatment and gender;
5. The findings showed that there was no significant interaction effect of treatment and age
6. There was no significant interaction effect in students' performance in Mathematics due to age and gender.
7. The study found out that there was no significant interaction effect of treatment, age and gender.
8. Finally the study showed that Critical Thinking Skills and Peer Assessment had significant linear effects on Mathematics test scores.

5.3 Conclusion

In the light of the preceding discussions and summary of findings, the following conclusions can be drawn:

1. There was a significant difference in Mathematics performance post-test scores among the experimental groups. The study found out that both Critical Thinking Skills and Peer Assessment are all significantly effective in teaching and learning Mathematics.
2. There was no significant difference due to gender in post-test scores of Mathematics performance test among the participants.

3. There was no significant difference due to age in post-test scores of Mathematics performance test among the participants.
4. There was no significant interaction effect of treatment and gender on students performance in Mathematics.
5. There was no significant effect of treatment and age on students performance in Mathematics.
6. There was no significant interaction effect of age and gender on students performance in Mathematics.
7. There was no significant interaction effect of treatment, age and gender on students performance in Mathematics.
8. There was a significant linear relationship between Mathematics performance post-test scores and a set of independent variables Critical Thinking Skills, Peer assessment and Gender.
9. Lastly, the study revealed that Critical Thinking Skills and Peer Assessment were predictors of Mathematics performance post-test.

5.4 Contributions to Knowledge

1. The study has produced training packages consisting of Critical Thinking Skills , Peer Assessment and Instructional Rubrics which have been used to teach Mathematics in school and confirmed to enhance academic performance in Mathematics because students were able to solve Mathematics and other problems using interpretation, analyses, explanation, self-regulation, evaluation and inferential skills to arrive at valid and reliable conclusions.
2. The study also confirmed that the infusion of Critical Thinking Skills and Peer Assessment into the Mathematics curriculum improved the students' learning and performance in Mathematics. The students understood the contents better and easily.
3. The study has demonstrated that gender has no effect on Mathematics performance in spite of Critical Thinking Skills and Peer Assessment training instructions infused in the teaching of Mathematics.
4. The study has demonstrated that age has no interaction effects on Mathematics performance in spite of the Critical Thinking Skills and Peer Assessment training instructions infused into the teaching of Mathematics.

5. The study has demonstrated that age and gender have no interaction effects on Mathematics performance in spite of Critical Thinking Skills and Peer Assessment training instructions infused into the teaching of Mathematics.

5.5 Implication of Findings

- This study points out that Critical Thinking Skills and Peer Assessment were effective in the teaching and learning process because both concepts were used to enhance academic performance in Mathematics. The importance of these concepts has made the students to be critically minded and independent in taking decisions when facing Mathematics and other problems in school and outside school.
- The study also indicates that Peer Assessment was very effective in enhancing students performance in Mathematics because it made students to be active learners. Again students had the opportunity to assess their strengths and weaknesses so that they could do better when next they are confronted with similar problems or challenges.
- This study also proves that Critical Thinking Skills was very effective in improving academic performance in Mathematics because students who were exposed to the training did better than students who were not exposed to the training. This further indicated that Critical Thinking Skills should take its rightful place in the school curriculum as it has helped in building confidence among students in terms of interpreting, analysing, and evaluating data and information in Mathematics.
- Instructional Rubrics have also helped the students to be independent and creative in developing their success criteria they were used to judge their performance in class activities. The students were able to develop criteria for themselves based on teacher's guidelines. As the students got involved in this activities, they were gradually learning the procedures and steps in presenting solutions to the answers systematically and logically which later improved their performance in Mathematics.
- The study affirms that gender has no effect on Mathematics achievement using Critical Thinking Skills and Peer Assessment. This further proved that irrespective of the intervention programme infused into the teaching of Mathematics male students do not perform better than their female counterparts.

- The study confirmed that age has no effect on Mathematics achievement using Critical Thinking Skills and Peer Assessment. This further proved that irrespective of the intervention programme infused into the teaching of Mathematics younger students do not perform better than the older students.
- The study confirmed that age and gender has no effect on Mathematics achievement using Critical Thinking Skills and Peer Assessment. This further proved that irrespective of the intervention programme infused in the teaching of Mathematics, younger student do not perform better than the older students. Also male students do not perform better than their female counterparts.
- Infusion technique has been very effective in the teaching and learning of Mathematics in schools because students learnt the contents easily, better and their performance improved in the post-test administration of the Mathematics performance test.

5.6 Recommendations

Based on the findings of this study, the following specific recommendations are put forward for consideration:

1. Training in Critical Thinking Skills and Peer Assessment as confirmed by this study are practicable means to enhance students' achievement in Mathematics. This therefore, suggests that Critical Thinking Skills and Peer Assessment serve as a viable means of improving low academic performance in Mathematics. Based on this, effort should be made to formally train Mathematics teachers on the rudiments of Critical Thinking Skills and Peer Assessment and how to integrate the skills into the school curriculum for learning.
2. Training on Peer Assessment as confirmed by this study is an effective means of improving on low academic performance in Mathematics. Peer Assessment serves as tool, which places students at the center of the learning process. Therefore, secondary school teachers should integrate Peer Assessment when assessing the students.
3. The infusion techniques introduce while teaching in the classroom should be emphasized because it made the students understand the concept better. Therefore, teachers should consistently and explicitly emphasize specific Critical Thinking skills and peer assessment process that aid learning.

4. Since the national policy on Education includes Critical Thinking as one of the objectives of Nigerian education. The curriculum developers, implementors and educational evaluators should ensure teachers incorporate Critical Thinking Skills into subject curriculum and classroom experiences. This is one of the approaches that can produce citizens that will be prepared to solve the myriad of problems encountered daily at individual and corporate levels. The dream of ensuring that Nigerians are able to find meaningful solutions to scientific, technological, economic, social and political problems confronting the nation can be realized.

5. The Ministry of Education and heads of schools should ensure that teachers use instructional rubrics to assess their students in schools because it involves self-evaluation which is linked to self-direction. Ultimately, teachers will teach the students how to develop grading rubric which will enhance their academic performance in Mathematics. This will also help the students to communicate their mathematical ideas in writing: symbolically, visually, use mathematical vocabulary, notation, and structure to represent ideas, and describe their relationships.

6. Finally, Critical Thinking, Peer Assessment and Instructional Rubrics should be integrated in the secondary school curriculum, scheme of work, lesson note, lesson plan and in the classroom when teaching and learning is taking place because the three concepts serve as learning and teaching aids.

Suggestion for further studies

In view of the experience gained by the researcher in this study, the following suggestions are put forward for further study.

1. It is recommended that future researchers should investigate further on the impact of these two training interventions on other core subjects other than Mathematics, covering more schools and for longer period of time in other locations.

2. There is need to determine which of these two training instructions is easier to apply in large classes as it applies to public schools.
3. Moreover, there is need to study the perception of Peer Assessment in Mathematics among secondary school students.
4. There is also the need to investigate the role and implementation of Instructional Rubrics in other subjects taught in public secondary schools.

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APPENDICES

Students Questionnaire

Introduction

Dear Students,

This questionnaire is for research purpose only. It is not a test as there is no wrong or right answers. please kindly respond to the statement below honestly All information provided will be treated as confidential.

Section A **(BlO-DATA)**

Gender: (Male). (Female)

Age: (a) 14-15 years (b) 16 years and above

Class: SS11

Class type: (a) Science (b) Art (c) Commercial

Mother's highest educational qualification (a) First school leaving Cert

(b) SSCE/WAEC (C)OND/NCE (d). Bed / B.sc (e) Med I M.sc (f) P-hd (h) Others

Father's highest educational qualification (a) First school leaving Cert (b) SSCEIWAEC (C) OND / NCE

(d) Bed / B.sc (e) Med / M.sc (f) P-hd (h) Others.

Mother's Occupation (a) Farmers (b) teachers (c) Nurses (d) Accountant

(e) Artisans (1) Drivers

(g) journalist (h) Journalists (i) Engineers

(j) Medians (k) Military & Para military (I) Others

Father's Occupation (a) Farmers (b) teachers (c) Nurses (d) Accountant (e) Artisans (f) Drivers (g) Doctors (h) Journalists (i) Engineers (j) Mechanics (k) Military & Para-military (i) Others:

To answer each item, simply tick one of the choices SA, A, U, D, SD below:

SA means Strongly Agree
A means Agree
U means Undecided
D means Disagree
SA means Strongly Disagree

PEER ASSESSMENT SCALE IN MATHEMATICS

For each statement, tick ( ) the response that applies to you.

1	Grading other students' work in Mathematics using scoring guide deepens my understanding of Mathematics.	SA	A	D	SD
2	Grading other students' work in Mathematics using scoring guide help me write my own solution clearer.				
3	Solving Mathematics problems together is good for learning Mathematics.				
4	Working Mathematics in groups is good for communicating Mathematics ideas,				
5	Working Mathematics in groups is more inspiring				
6	Grading each others work in Mathematics using scoring guide make me learn Mathematics faster.				
7	Scoring students' work in Mathematics using scoring guide Will enable me know my weaknesses				
8	Assessing students' work in Mathematics using scoring guide wilt enable me know my strengths and weaknesses.				
9	Assessing students work in Mathematics using scoring guide will not make students solve problem on their own.				
10	Since other students will see my work in Mathematic, I will work harder.				
11	1 often compared my Mathematics solutions homework with other students' before submitting mine to my teacher.				
12	1 worked on homework problems in Mathematics with my classmates before submitting to my teacher.				
13	Studying step by step procedures in Mathematics with my classmates before submitting to my teacher.				
14	Solving Mathematics questions together using the same scoring guide will make students do better in Mathematics.				
15	Working in group is good for teaching Mathematics ideas.				

PEER-ASSESSMENTS MATHEMATICS INVENTORY

Please check one box that best describes your opinion on Mathematics.

1= Definitely not
2=No
3=May be not applicable
4=Yes
5=Definitely.

S/N	**DO YOU AGREE WITH THE FOLLOWING STATEMENTS?**	**1**	**2**	**3**	**4**	**5**
1	I enjoy working with mathematical problems.					
2	I am among the strongest mathematically in the tutorial group.					
3	I enjoy explaining mathematical ideas to others.					
4	I spend a lot of time on mathematics homework.					
5	I would spend more time on homework, if the benefit was greater.					
6	Since other students see my work, I will work harder.					
7	I understand the motivation for this type of tutorial.					
8	It is a good idea to have a tutorial like this.					
9	There should be more tutorials like this one.					
10	Tutorials like this make it more difficult to have both fast and slow students in the same group.					
11	Working in groups is good for learning mathematics.					
12	Working in groups is more fun and inspiring.					
13	Working in groups is good for communicating and teaching mathematical ideas.					
14	Assessing other students' solutions deepens my understanding of the material.					
15	Assessing other students' solutions helps me write my own solutions clearer.					

Kari and Hasto (2004)

CRITICAL THINKING APPRAISAL (WATSON-GLASER, 1994)

For each statement, tick ( ) the response that applies to you.

Exercise 1: Inference

In this test, each exercise begins with a statement of facts that you are to regard as true. For each inference, you will find spaces sheet on the answer sheet labeled T, PT, ID, PF and F. For each inference, make a mark on the answer sheet under the appropriate heading as follows:

T =if you feel that the inference is definitely **TRUE**; that it properly follows beyond a reasonable doubt from the statement of facts given.

PT= if, in the light of facts given, you think the inference is **PROBABLY TRUE;** that is more likely to be true than false.

ID= if you decide that there are **INSUFFICIENT DATA**; that you cannot tell from the facts given whether the inference is likely to be true or false; if the facts provide no basis for judging one way or the other.

PF= if in the light of the facts given, you think inference is **PROBABALY FALSE**; that it is more likely to be false than true. -

F= if you think the inference is definitely FALSE; that it is wrong, either, because it misinterprets the facts given, or because it contradicts the facts or necessary inferences from those facts.

Statement:

Two hundred students in their young age voluntarily attended a recent weekend students conference in Lagos . At this conference, the topics of tribe relations and means of achieving lasting world peace were discussed, because these were the problem the students selected as being most vital in today's world.

1. As a group, the students who attended this conference showed a keener interest in solving broad social problems than most other students in their young age.

(a) T (b) PT (c) ID (d) PF (e) F

2. The majority of the students had not previously discussed the conference topics in their schools.

(a) T (b) PT (c) ID (d) PT (e) F

3. The students came from all sections of the country.

(a) T (h) PT (c) ID (d) PF (e) F

4. The students discussed mainly labour relations problems.

(a) T (b) PT (c) ID (d) PF (e) F

5. Some young students felt it worthwhile to discuss problems of tribe relations and ways of achieving world peace.

(a) T (b) PT (c) ID (d) PF (e) F

Exercice 2 : Recognition of Assumptions

Below are several statements followed by several proposed assumptions? You are to decide for each assumption whether a person, in making the given statement, is really making that assumption. If you think that the given assumption is taken for granted in statement, chose **ASSUMPTION MADE**. If you think the assumption is not necessarily taken for granted in the statement, chose **ASSUMPTION NOT MADE.**

Statement: “We need to save time in getting there so we’d better go by plane.” Proposed assumptions:

6. Going by plane will take less time than by going by some other means of transportation.

(a) Assumption Made

(b) Assumption Not Made.

7. These plane service available to us for at least part of distance to the destination.

(a) Assumption Made

(b) Assumption Not Made.

8. Travel by air piano is more convenient than travel by train.

(a) Assumption Made

(b) Assumption Not Made

Exercise 3: Deduction

In this section, each expertise consists of several statements followed by several suggested conclusions. For the purpose of this study, consider the statements in each exercise as true without exception. After reading the conclusion beneath the statement, please mark whether you think it **FOLLOWS OR DOES NOT FOLLOW** from the statement given, regardless of whether you believe the statement to be true or not from your own experience or knowledge.

Statement:

Some holidays are raining. All raining days are boring. Therefore

9. No clear days are boring.

(a) Follows

(b) Does not follow.

10. Some holidays are boring.

(a) Follows

(b) Does not follow.

11. Some holidays are not boring

(a) Follows

(b) Does not follow.

Exercise 4: Interpretation

The next exercise consist of brief paragraphs follows by several Conclusions. For these questions, please assume that everything in the paragraph is true. The problem is to judge whether or not each of the proposed conclusions logically follows beyond a reasonable doubt from the information given. Please mark - Conclusion follows or Conclusion does not follow after the conclusion.

Statement:

A study of vocabulary growth in children from eight months to six years old shows that the size of spoken vocabulary increases from zero words at age eight months to 2,562 words at age six years.

12. None of the children in this study had learned to talk by the age of six months

(a) Conclusion follows

(b) Conclusion does not follow,

13. Vocabulary growth is slowest during the period when children are learning to walk.

(a) Conclusion follows

(b) Conclusion does not follow.

Exercise 5: Evaluation of Argument

Below are several questions followed by several arguments. For the purpose of this study, please regard each argument as true. The problem Then is to decide whether it is a strong argument that related to the questions or week argument related only to trivial aspects.

Questions:

Should all young adults in Nigeria go to Secondary School?

14. Yes; secondary school provides an opportunity for them to learn songs and cheers,

(a) Strong argument

(b) Weak argument.

15. No; a large percent of young adults do not have enough ability or interest to derive any benefit from secondary school training.

(a) Strong argument (b) Weak argument.

16 . No: excessive studying permanently warps an individuals personality,

(a) Strong argument.

(b) Weak argument.

Watson – Glaser, (1994)

Analysis of Watson Glaser's Critical Thinking Test

TEST 1: Inference

Statement:

Two hundred school students in their early early young age voluntarily attended a recent weekend student conference in Lagos. At this conference, the topics of tribe relations and means of achieving lasting world peace were discussed, because these were problems that the students selected as being most vital in today's world.

1. As a group, the students who attended this conference showed a keener interest in broad social problems than do most other people in their early teens. (PT), because, as is common knowledge, most people in their early teens do not show so much serious concern with broad social problems. It cannot be considered definitely true from the facts given because these facts do not tell how much concern other young teenagers may have. It is also possible that some of the students volunteered to attend mainly because they wanted a weekend outing.

2. The majority of the students had not previously discussed the conference topics in their schools. (PF), because the students' growing awareness of these topics probably stemmed at least in part from discussions with teachers and classmates.

3. The students came from all parts of the country. (ID), because there is no evidence for this inference.

4. The students discussed mainly industrial relations problems. (F), because it is given in the statement of facts that the topics of tribe relations and means of achieving world peace were the problems chosen for discussion.

5. Some young students felt it worthwhile to discuss problems of tribe relations and ways of achieving world peace. (T), because this inference follows from the given facts; therefore it is true.

Test 2: Recognition of Assumptions

‘We need to save time in getting there so we’d better go by plane.’
Proposed assumptions:

6. Going by plane will take less time than going by some other means of transportation. (Assumption made), it is assumed in the statement that the greater speed of a plane over the speeds of other means of transportation will enable the group to reach its destination in less time.

7. There is a plane service available to us for at least part of the distance to the destination. (Assumption made), this is necessarily assumed in the statement as, in order to save time by plane, it must be possible to go by plane.

8. Travel by plane is more convenient than travel by train. (Assumption Not made), this assumption is not made in the statement – the statement has to do with saving time, and says nothing about convenience or about any other specific mode of travel.

Test 3: Deduction

Statement:
Some holidays are rainy. All rainy days are boring. Therefore:
Proposed Conclusions:

9. No clear days are boring. (NO), the conclusion does not follow. You cannot tell from the statements whether or not clear days are boring. Some may be.

10. Some holidays are boring. (YES), the conclusion necessarily follows from the statements as, according to them, the rainy holidays must be boring.

11. Some holidays are not boring. (NO), the conclusion does not follow, even though you may know that some holidays are very pleasant.

Test 4: Interpretation

Statement:

A study of vocabulary growth in children from eight months to six years old shows that the size of spoken vocabulary increases from 0 words at age eight months to 2,562 words at age six years.

Proposed Conclusions:

12. None of the children in this study had learned to talk by the age of six months. (YES), the conclusion follows beyond a reasonable doubt since, according to the statement, the size of the spoken vocabulary at eight months was 0 words.

13. Vocabulary growth is slowest during the period when children are learning to walk. (NO), the conclusion does not follow as there is no information given that relates growth of vocabulary to walking.

Test 5: Evaluation of Arguments

Statement:

Should all young adults in Nigeria go to secondary school?

Proposed Arguments:

14. Yes; secondary school provides an opportunity for them to learn school songs and cheers. (WEAK), this would be a silly reason for spending years in college.

15. No; a large percentage of young adult do not have enough ability or interest to derive any benefit from secondary school training. (STRONG). If it is true, as the directions require us to assume, it is a weighty argument against all young people going to college.

16. No; excessive studying permanently warps an individual's personality. (WEAK), this argument, although of great general importance when accepted as true, is not directly related to the question, because attendance at secondary schools does not necessarily require excessive studying.

CRITICAL THINKING SKILLS INVENTORY

Please check one box that best describes your opinion on Mathematics.

4= Strongly Agree

3=Agree

2=Disagree

1=Strongly Disagree

Please check one box that best describes your opinion

S/N	STATEMENTS	1	2	3	4
1	Criticizing the weaknesses of an experts is my hobby.				
2	Am always compose before I take any decision.				
3	I think deep before answering any questions.				
4	I find easy to understand an argument.				
5	I always break down information before taking a decision.				
6	I can demonstrate what happened without missing words.				
7	Am good at breaking data for better understanding.				
8	I love people that present arguments with facts.				
9	I know how to evaluate source material.				
10	I can spot inconsistencies in an argument easily.				
11	I can present my argument clearly without bias.				
12	If am not sure of any statement, I will find out to research more.				
13	I pay serious attention to people’s point of argument.				
14	Am good at separating true statement from false statement.				
15	I enjoyed arguing logically.				
16	I respect people that make intelligent statement.				
17	I do cross-checked my ideas before taking final decision.				
18	I find it easily to evaluate the evidence to support a point or view.				
19	Am good at reading between lines.				
20	I understand how to structure an argument.				

Mathematics Performance Test

Section A (Objective Test)

1. Which of the following is an Arithmetic progression?
   (a) 11, 9, 5, 2……...
   (b) 2, 4, 8, 16……..
   (c) 11, 12, 13, 14………
   (d) 1, 6, 36, 72…..
2. The nth term of Arithmetic progression is?
   (a) a + (n -1)d
   (b) a + (1 – n)d
   (c) a – (n -1)d
   (d) a +d(n +1)
3. Find the 20^{th} term of an A.P 3, 0, -3, -6………..
   (a) 54
   (b) -54
   (c) 45
   (d) -45

4. If the 5^{th} term of an A.P. is 17 and the 12^{th} term is 31. What is the first term?
   (a) 6
   (b) 7
   (c) 8
   (d) 9

5. Calculate the sum of the first ten term of the Arithmetic progression 3 + 5+7+9+….
   (a) 120
   (b) 122
   (c) 124
   (d) 126

6. Which of these is an example of Geometric progression?

(a) -1, 2, -4, 8, -14…….

(b) 1, 2, 4, 16………..

(c) -1, -2, -4, -8,-16…..

(d) 1, 2, 4, 6, 8……….

7. The nth term of a Geometric progression is?

(a) ar^{n-1}

(b) ar^{n}

(c) arn^{-1}

(d) ar^{1-n}

8 . The second term of Geometric is 18 and the fourth is 162. Find the common ration.

(a) 3

(b) 4

(c) 5

(d) 6

9. Calculate the sixth term of the Arithmetic progression 6, 9, 12, 15…………..

(a) 20

(b) 21

(c) 22

(d) 23

10 The 6^{th} term of a G.P is 2000. Find its first term if its common ratio is 10.

(a) 0.01

(b) 0.04

(c) 0.03

(d) 0.02

11. The 6^{th} term of a G.P is 2000. Find its 10^{th} term if its common ratio is 10.

(a) 10×10^6

(b) 40×10^6

(c) 20×10^6

(d) 30×10^6

12. The 6^{th} term of a G.P is 2000. Find its 20^{th} term if its common ratio is 10.

(a) 10×10^{16}

(b) 40×10^{16}

(c) 30×10^{16}

(d) 20×10^{16}

13 A coin is tossed and a die is thrown. What is the probability of getting a head and a perfect square?

(a) 1/3

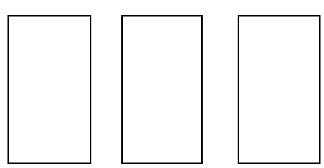
(b) 5/12

(c) 1/6

(d) 5/6

14. Five cards are lettered A, B, C, D, E. Three cards are chosen at random, one after the other, without replacement and are placed in the order shown below:

1_{st} 2_{nd} 3rd



What is the probability that the cards spell the word BED?

(a) 1/125

(b) 1/5

(c) 1/20

(d) 1/60

15. When two dice are thrown, what is the probability of the total score being a prime number?

(a) 11/12

(b) 5/12

(c) 7/12

(d) 1/2

16. When three dice are thrown together what is the probability of getting a total score of 10?

(a) 1/3

(b) 1/6

(c) 1/8

(d) none of the above

17. A statistical survey shows that 28% of all men take size 9 shoes. What is the probability that your friends father takes size 9 shoes?

(a) 18/25

(b) 9/28

(c) none of the above

(d) 7/25

18. A school contains 357 boys and 323 girls. If a student is chosen at random, what is the probability that a girl is chosen?

(a) 19/40

(b) 21/40

(c) 29/40

(d) none of the above

19. A State Lottery sells $1^{1/2}$ millions tickets of which 300 are prizewinners. What is the probability of getting a prize by buying just one ticket?

(a) 1/500

(b) 1/5000

(c) 1/50

(d) 1/5

20. Statistics show that 92 out of every 100 adults are at least 150cm tall. What is the probability that a person chosen at random from a large crowd is less than 150cm tall?

(a) 23/25

(b) 21/25

(c) 2/25

(d) none of the above

Give all distances correct to 3 significant figure and all angles and bearings correct to 0.1^0 in question 21- 34.

21. From a point on the edge of the sea, one ship is 5km away on a bearing S 50^0 E and another is 2km away on a bearing S 60^0 W. How far apart are the ships?

(a) 5.99km

(b) 6.99km

(c) 7.99km

(d) 8.99km

22. A student walks 50m on a bearing 025^0 and then 200m due east. How far is she from her starting point?

(a) 326m

(b) 226m

(c) 526m

(d) 426m

23. Two goal posts are 8m apart. A footballer is 34m from one post and 38m from the other. Within what angle must he kicks the ball if he is to score a goal?

(a) 41^o

(b) 31^o

(c) 21^o

(d) 11^0

24. City A is 300km due east of city B. City C is 200km on a bearing of 123^0 from city B. How far is it from C to A?

(a) 161km

(b) 171km

(c) 151km

(d) 141km

25. A triangular field has two sides 50m and 60m long, and the angle between these sides is 96^0. How long is the third side?

(a) 92.0m

(b) 62.0m

(c) 82.0m

(d) 72.0m

26. Two boats A and B left a port C at the same time on different routes. B travelled on a bearing of 150^0 and A travelled on the north side of B. When A had travelled 8km and B had travelled 10km, the distance between the two boats was found to be 12km. Calculate the bearing of A's route from C.

(a) 157.2^o

(b) 57.2^o

(c) 22.8^o

(d) 67.2^0

27. A surveyor leaves her base camp and drives 42km on a bearing of 032^0. She then drives 28km on a bearing of 154^0. How far is she then from her base camp and what is her bearing from it?

(a) 36.1km, 73.2^0

(b) 46.1km, 73.2^o

(c) 36.1km, 83.2^o

(d) 46.1km, 83.2^o

28. Two ships leave port at the same time. One travels at 5km/h on a bearing of 046^0. The other travel at 5km/h on a bearing of 127^0. How far apart are the ships after 2 hours?

(a) 19.2m

(b) 13m

(c) 20m

(d) 6.5m

29. A boat sails 4km on a bearing of 038^0 and then 5km on a bearing of 067^0.

a) How far is the boat from its starting point?

b) Calculate the bearing of the boat from its starting point.

(a) 9.72km, 54.1°

(b) 8.72km, 64.1°

(c) 8.72km, 054.1^0

(d) 9.72km, 64.1°

30. A photographer is 350m away from a lion and wants to get closer before he takes a photograph. There is a water-hole in the direct line between the lion and himself, so he moves at an angle of 8^0 to this line to a better position 200m further on. Calculate his distance from the lion.

(a) 154m

(b) 164m

(c) 174m

(d) 184m

31. Three towns, A, B and C are situated so that so that |AB|= 60km and |AC|= 100km. The bearing of B from A is 060^0 and the bearing of C from A is 290^0. Calculate the distance |BC|, the bearing of B from C

(a) 156km, 91°

(b) 146km, 81.7°

(c) 146km, 91.7°

(d) 156km, 81.7°

32. A ship leaves port and travels 21km on a bearing of 032^0 and then 45km on a bearing of 287^o. (a) Calculate its distance from the port.

(b) Calculate the bearing of the port from the ship.

(a) 44.5km, 144.1^o

(b) 44.5km, 134.1^o

(c) 55.5km, 144.1^o

(d) 55.5km, 134.1^o

33. An aircraft flies round a triangular course. The first leg is 200km on a bearing of 115^o and the second leg is 150km on a bearing of 230^o. How long is the third leg course and what bearing must the aircraft fly?

(a) 196km, 329.9^o
(b) 196km, 339.9^o
(c) 193km, 329.9^o
(d) 193km, 339.9^o

34. Villages A, B, B, C, D, are such that B is 4km due east of A, C is 3km due south of B and D is 4km S 50 W from C. Calculate the distance and bearing of A from D.

(a) 5.65km, N 9.5^0 W (or 350.5^0)

(b) 6.65km, N 9.5^o W (or 350.5^o)

(c) 5.65km, N10. 5^o W (or 349.5^o)

(d) 6.65km, N10.5^o W (or 349.5^o)

35. A girl moves from a point on a bearing of 60^o to a point Q, 40m away. She then moves from the point Q on a bearing of 120^o to a point R. The bearing of P from R is 255^o . Calculate correct to three significant figures, the distance between P and R.

(a) 29.9km

(b) 39.9km

(c) 49.0km

(d) 47.9km

36. Which of the following is a quadratic expression?

(a) $ax^2 +bx +c = 0$

(b) $ax + c$

(c) $ax^2 + bx + c$

(d) $9x +y = z$

Use quadratic formula to solve question 37-40. Give the roots correct to 2 decimal places when necessary.

37. $x^2 + 5x + 6= 0$

a) 2, 3

(b) -2, -3

(c) -2, 3

(d) 2, -3

38. $X^2 – 5x + 4 = 0$

(a) -1, -4

(b) -1, 4

(c) 1, 4

(d) 1, -4

39. $2x^2 + 5x + 3 = 0$

(a) $-1, 1^{1/2}$

(b) $-1, -1^{1/2}$

(c) $1, -1^{1/2}$

(d) $1, 1^{1/2}$

40. $3x^2 – 4x + 1 = 0$

(a) -1, -⅓

(b) -1, ⅓

(c) 1, -⅓

(d) 1, ⅓

41. Find two numbers which differ by 4 and whose product is 45.

(a) 9, 5 or -5, -9

(b) 9, -5 or -5,-9

(c) -9, 5 or 5,9

(d) none of the above

42. Find the number which, when added to its square, mark 90.

(a) -10 or 90

(b) 10 or 29

(c) -9 or 19

(d) 9 or -10

43. Twice the square of a certain whole number added to 3 times the number makes 90. Find the number.

(a) 8

(b) 7

(c) 6

(d) 9

44. A man is 37 years old and his child's age is 8. How many years ago was the product of their ages 96?

(a) 8years

(b) 7years

(c) 6years

(d) 5 years

45. A certain number is subtracted from 18 and from 13. The product of the two numbers obtained is 66 Find the first number.

(a) 7 or 24

(b) 7 or 21

(c) 8 or 24

(d) 8 or 21

46. Find two consecutive even numbers whose product is 224.

(a) 12, 13 or -13, -12

(b) 14, 16 or -16, -14

(c) 12, 14 or -12, -14

(d) 16, 18 or -16, -18

47. The base of a triangle is 3cm longer than its corresponding height. If the area is $44cm^2$, find the length of its base.

(a) 5cm

(b) 3cm

(c) 8cm

(d) 11cm

Using irrational root method solve the equation in question 48-50

48. $(x-2)^2 = 9$

(a) 5, - 1
(b) -5, -1
(c) 5, 1
(d) -5, 1

49. $(x-7)^2 = 4$

(a) -9, -5

(b) -9,5

(c) 9, 5

(d) 9, -5

50. $(x+3)^2 = 4$

(a) 1, 5

(b) - 1, -5

(c) -1, 5

(d) 1, -5

KEYS

1	C
2	A
3	B
4	D
5	A
6	C
7	A
8	A
9	B
10	D
11	C
12	D
13	C
14	D
15	B
16	C
17	D
18	A
19	B
20	C
21	A
22	B
23	D
24	B
25	C
26	D
27	A
28	B
29	C
30	A
31	C
32	B
33	D
34	A
35	C
36	A
37	B
38	C
39	B
40	D
41	A
42	D
43	C
44	D
45	A
46	B
47	D
48	A
49	C
50	B

ESSAY

Q1. The first term of a G.P is 4 and the last term is 972. Find the sum of the G.P. if the Common ratio is 3.

Q2. Two fair dice are thrown together once. Find the probability of obtaining

(a) a total of 6

(b) a total of 11

(c) a "2" on 1^{st} die and a "5" on the 2nds die

Q3. The fifth term of a G.P. is 8 and the ninth term is 40 ½ calculate the first term and the sum of the first seven terms.

Q4. Find two consecutive even numbers whose product is 224.

Q5. A student leaves home and walks 6km on a bewaring of 032^{o} and then 8km on a bearing of 287^{o}. calculate his distance from home and the bearing of the home from the boy.

SOLUTIONS TO THE MATHEMATICS TEST (THEORY)

Q1. The first term of a G.P is 4 and the last term is 972. Find the sum of the G.P. if the Common ratio is 3.
The sum of a G.P Series is usually found by the use of the formulae.

Sn = $\frac{a(r^n - 1)}{r - 1}$ Where r > 1 or Sn = $\frac{a(1 - r^n)}{1 - r}$

Sn = Sum of the 1st n-terms

a = first term; r = common ratio

Solution

a = 4

r = 3

T_n = 972

$T_n = ar^{n-1}$

972 = 4 x 3^{n-1}

$\frac{972}{4} = 3^{n-1}$

$243 = 3^{n-1}$

$3^5 = 3^{n-1}$

5 = n-1

6 = n

Sn = $\frac{a(r^n - 1)}{r - 1}$

Sn = $\frac{4(3^6 - 1)}{3 - 1}$

Sn = $\frac{4}{2}x(729 - 1)$

Sn = 2×728

Sn = **1456.**

Q2. Two fair dice are thrown together once. Find the probability of obtaining

(a) a total of 6

(b) a total of 11

(c) a "2" on 1^{st} die and a "5" on the 2nd die

Solution: It is convenient to display all the possible outcome

Number on the 2^{nd} die

Number on the 1^{st} die	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

These are 36 possible outcomes, shown as, (1,1), (1,2), (1,3),…., (6, 6)

(a) There are five ways of obtaining a total of 6 on the two dice
(5, 1), (4, 2), (3, 3), (2, 4), (1, 5)

$$P\,(\text{Sum of } 6) = \frac{5}{36}$$

(b) There are two ways of attaining a total of 11, (6, 5, 5, 6)

$$P\,(\text{sum of } 11) = \frac{2}{36} = \frac{1}{18}$$

(c) There are two ways of attaining a 'two' on the 1^{st} die and a 'five' on the 2^{nd} die.

$$P\,(2 \text{ on } 1^{st} \text{ die and on } 2^{nd} \text{ die}) = \frac{1}{36}\ .$$

Q3. Find the sum of 8 +9+10+….+25

The sum of an AP is usually found using:

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right) \quad \text{or} \quad S_n = \frac{n}{2}\left(a + L\right)$$

Where S =Sum of the first n terms

n= number of terms

a = 1st term

d = common difference

L = The last term

Solution

a = 8

d = 1

T_n = 25

Recall,

T_n = a+ (n-1)d

25 = 8 + (n – 1)

25 = 8 + n – 1

25 – 8 + 1 = n

18 = n

Using $S_n = \frac{n}{2}\left(a + L\right)$

$$S_n = \frac{18}{2}\left(8 + 25\right)$$

= 9 x 33

= 297

Q4. Find two consecutive even numbers whose product is 224.

Let the 1st number = x

Then the 2nd number = x + 2

$\therefore \quad x \times (x+2) = 224$

Expanding

$x^2 + 2x = 224$

$x^2 + 2x = 224 = 0$

Factor giving

$x^2 + 16 - 14x - 224 = 0$

$x(+16) - 14x(x+16) = 0$

$(x-14)(x+16) = 0$

$x - 14 = 0 \ or \ x + 16 = 0$

$x = 0 + 14 \ or -16 = 14 or -16$

2nd number = x +2

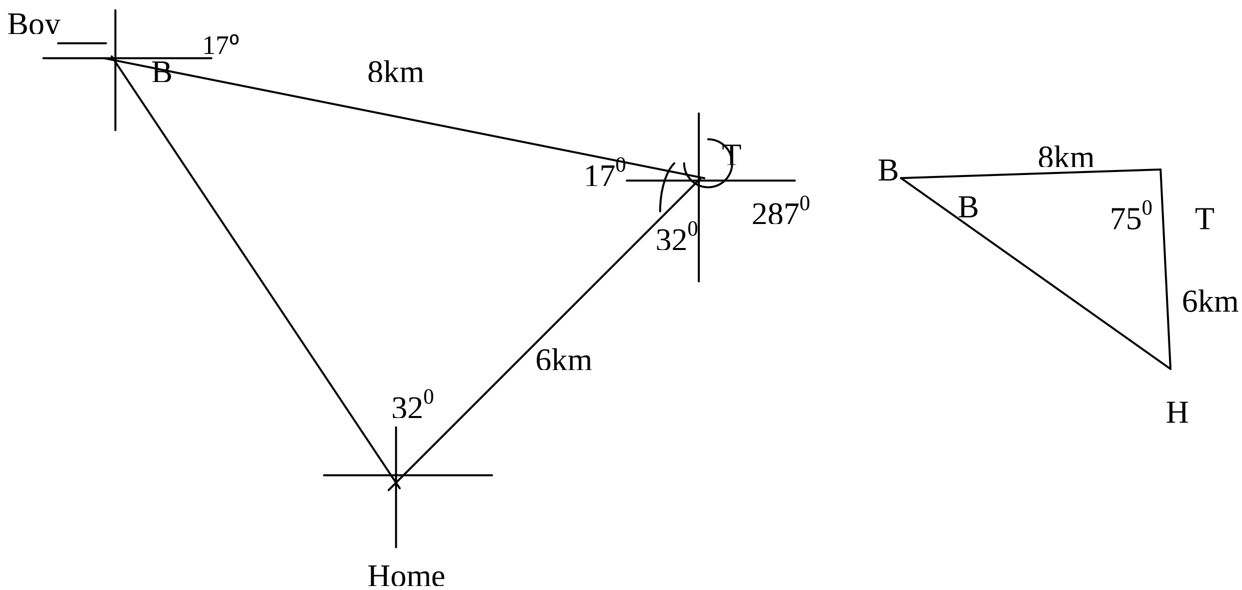
= 14 +2 or -16 +2

= 16 or -14

$\therefore$ The Numbers are

14, 16 or -14, - 16

Q5. A student leaves home and walks 6km on a bewaring of 032ᴼ and then 8km on a bearing of 287ᴼ. calculate his distance from home and the bearing of the home from the boy.



$T = 287^0 - (180^0 + 32^0)$
$t^2 = b^2 + h^2 - 2hb\ Cos\ T$
$t^2 = 6^2 + 8^2 - (2 \times 6 \times 8 \cos 75)^0$
$t^2 = 36 + 64 - (24.847)$
$t^2 = \sqrt{75.153}$
$t = 8.67km$

$$\frac{Sin\ B}{6} = \frac{Sin\ 75^0}{8.67}$$

$$Sin\ B = \frac{6 \times Sin\ 75^0}{8.67}$$

$Sin\ B = \ 0.6685$

B = Sin^{-1} 0.6685

B = 41.9^0

Let the bearing of the home from the Boy = x^0

X = 90ᴼ + 17ᴼ + 41.9ᴼ

= 148.9ᴼ
Ω 149ᴼ

SUMMARY OF THE FOUR LESSONS TAUGHT

The researcher taught the following four topics within the context of Critical Thinking skills, peer assessment and instructional rubrics conditions created. This four topics are Sequences and series, Quadratic equations, Bearing and distance and Probability.

Subject: Mathematics

Class: SSII

Subject: Mathematics

Class: SSII

Topic: Probability

Number of Periods: 4

Duration: 80 Minutes

Previous knowledge: The students are able to select objects by random to represent sample or population of the study.

Behavioural Objectives: At the end of the lesson, the students should be able to :

1. define Probability

2. associate the probability of an event with an exact measure by theory.

3 associate the probability of an event with an exact measure by experimentation.

4. determine the probability of mutually exclusive events in the same population.

Instructional Material: A set of cards, dies, and coins with numbers written on them.

Reference Books: New General Mathematics for Senior Secondary Schools Book 2 (Macrae, Kalejaiye, Garba, Chima, Ademosu, Channon, McleishSmith and Head,2008).

Outline of Content:

- Definition of a Probability: Probability is a game of chance. It could also means a measure of likelihood of the occurrence of a required outcome. To find the probability of an event happening we use the formulae below:
- Probability = $\frac{\text{Number of required outcome}}{\text{Number of possible oucomes}}$

  N.T Maximum Probability =1

  Minimum Probability =0

A probability is a way of assigning every event a value between zero and one, with the requirement that the event made up of all possible results (in our example {1,2,3,4,5,6}) is assigned a value of one. To qualify as a probability, the assignment of values must satisfy the requirement that if you look at a collection of mutually exclusive events (events with no common results, e.g., the events {1,6}, {3}, and {2,4} are all mutually exclusive), the probability that at least one of the events will occur is given by the sum of the probabilities of all the individual events.

The probability of an event A is written as P(A), p(A) or Pr(A). This mathematical definition of probability can extend to infinite sample spaces, and even uncountable sample spaces, using the concept of a measure. The opposite or complement of an event A is the event [not A] (that is, the event of A not occurring); its probability is given by P (not A) = 1 - P(A). As example, the chance of not rolling a six on a six-sided die is 1 – (chance of rolling a six) $= 1 - \frac{1}{6} = \frac{5}{6}$.
If both events A and B occur on a single performance of an experiment, this is called the intersection or joint probability of A and B, denoted as P (A ∩ B)

- **Independent probability**

If two events, *A* and *B* are independent then the joint probability is

$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B),$$

for example, if two coins are flipped the chance of both being heads is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Mutually exclusive

If either event *A* or event *B* or both events occur on a single performance of an experiment this is called the union of the events *A* and *B* denoted as $P(A \cup B)$. If two events are mutually exclusive then the probability of either occurring is

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B).$$

For example, the chance of rolling a 1 or 2 on a six-sided die is
$P(1 \text{ or } 2) = P(1) + P(2) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}.$

Not mutually exclusive

If the events are not mutually exclusive then

$$\mathrm{P}(A \text{ or } B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \text{ and } B).$$

For example, when drawing a single card at random from a regular deck of cards, the chance of getting a heart or a face card (J,Q,K) (or one that is both) is $\frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{11}{26}$, because of the 52 cards of a deck 13 are hearts, 12 are face cards, and 3 are both: here the possibilities included in the "3 that are both" are included in each of the "13 hearts" and the "12 face cards" but should only be counted once.

- **Conditional probability**

Conditional probability is the probability of some event *A*, given the occurrence of some other event *B*. Conditional probability is written P (A $|$ B), and is read "the probability of *A*, given *B*". It is defined by

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}.$$

If $\mathrm{P}(B) = 0$ then $\mathrm{P}(A \mid B)$ is formally undefined by this expression. However, it is possible to define a conditional probability for some zero-probability events using a σ-algebra of such events (such as those arising from a continuous random variable). For example, in a bag of 2 red balls and 2 blue balls (4 balls in total), the probability of taking a red ball is ½ ; however, when taking a second ball, the probability of it being either a red ball or a blue ball depends on the ball previously taken, such as, if a red ball was taken, the probability of picking a red ball again would be $1/3$ since only 1 red and 2 blue balls would have been remaining.

Procedures : Step 1:

The teacher gave a brief overview of last lesson and inform the students that the topic involve addition, subtraction, division and multiplication. He later entertained and asked questions based on last lesson.

Step 2: The teacher rolled dies over on the table to know the number the appears repeatedly.

Step 3: The teacher asked students to write down the numbers that appears as the dies is being rolled over several times.

Step 4: The teacher guided the students on how to determine probability of mutually exclusive events in the same population.

Step 5: The teacher wrote and solved examples on Probability. Example 1: A bag contains 2 white balls, 3 red balls and a black ball. If a ball is selected at random, what is the probability that it is:

(a) Not white
(b) Red
(c) Black

Students' response: The students were given opportunity to ask questions on any area not clear which was answered by the teacher.

Summary: The teacher wrote out all he taught on the chalkboard such as what is Probability, different conditions of probability as well as there formulae.

Evaluation: The teacher asked the students the following questions:

1. Define probability.
2. A card is drawn from a pack of 52 playing cards. Find the probability of the following results:
   (a) The card is an ace
   (b) The card is the ace of the club
   (c) The card is a spade
   (d) The card is a picture.

The teacher gave out Instructional Rubrics or scoring guide to all the students and asked them to exchange their solved work to enable them assessed themselves.

Assignment: The teacher asked the students the following questions:

(a) A bag containing three red balls, four blue balls, five white balls and six black balls . A ball is picked at random. What is the probability that it is either

(a) red or blue

(b) red or white

(c) blue or white

(d) blue or black

(e) red, white, or blue

(f) blue white and black?

Subject: Mathematics

Class: SSII

Topic: Bearing and Distance

Number of Period: 2

Duration: 80 Minutes

Previous knowledge: The students have been taught how to solve trigonometry questions on triangle using cosine /sine formula.

Behavioural Objectives: At the end of the lesson, the students should be able to :

1. Define Bearing and Distance and their uses
2. Mention 3 types of Bearing
3. Express bearing as a 3-figure bearing or a compass bearing.
4. Construct an angle showing distance and bearing from three points
5. Use cosine rule to calculate the distance and bearing from three points
6. Use sine rule to calculate the distance and bearing from three points
7. Solve word problems leading to bearing and distance

Instructional Material: Use of compass, ruler, diagram and Chalkboard.

Reference Books: New General Mathematics for Senior Secondary Schools Book 2 (Macrae, Kalejaiye, Garba, Chima, Ademosu, Channon, McleishSmith and Head,2008).

Outline of Content:

- Introduction and definition of the concept bearing and distance
- Types of bearing
- Problems involving bearing.
- Expressing bearing as a 3-figure or as a compass bearing
- Construct an angle showing distance and bearing from three points
- Use cosine rule to calculate the distance and bearing from three points
- Use sine rule to calculate the distance and bearing from three points
- Solve word problems leading to bearing and distance

Procedures

Step 1: The teacher gave a brief overview of the topic and also define the concept. Thereafter he flashed back the mind of the students on the trigonometry . He also inform the students that the topic involves addition, subtraction, division and multiplication. He later entertained and asked questions.

A bearing is the direction one object is from another object, usually a *bearing* is the compass direction or angle between a line connecting two points and a north-south line, or meridian. Bearings can be measured in two systems, Mils and Degrees.

Step 2. Teacher listed out the types of bearing **:** compass , grid, magnetic and true

Step 3. Teacher wrote out and solves problems involving bearing and distance. Example:

(i). From a point on the edge of the sea, one ship is 5km away on a bearing S 50^o E, and another is 2km away on a bearing S 60^o W. How far apart are the ship?

(ii) A student walks 50m on a bearing 025^o and then 200m due east. How far is she from her starting point?

Students response: The students were given opportunity to ask questions on any area not clear which was answered by the teacher.

Summary: The teacher summarizes by re-echoed what he has taught such as meaning of bearing and distance, type of bearing, solving problems by applying sine rule and expression of bearing as a 3-figure or as a compass bearing.

Evaluation: The teacher evaluates the students by asking the following questions:

(i) Define the concept of bearing and distance.

(ii) Mention 2 types of bearing and explain them with examples

(iii)City A is 300km due east of city B. City C is 200km on a bearing of 123^o from city B. How far is it from C to A?

(iv)Two boats A and B left a port C at the same time on different routes. B travelled on a bearing of 150^o and A travelled on the north side of B. When A had travelled 8km and B had travelled 10km, the distance between the two boats was found to be 12km. Calculate the bearing of A's route from C. Give all distances correct to 3 s.f. and all angles and bearings correct to 0.1^o

The teacher gave out instructional rubrics or scoring guide to all the students and asked them to exchange their solved work to enable them assessed themselves.

Assignment: The teacher asked the students to solve the following questions at home.

(i) Villages A, B, C and D are such that B is 4km due east of A, C is 3km due south of B and D is 4km S50^oW from C. Calculate the distance and bearing of A from D.

(ii) A surveyor leaves her base camp and drives 42km on a bearing of 032^o. She then drives 28km on a bearing of 154^o. How far is she then from her base camp and what is her bearing from it. Give all distances correct to 3 s.f. and all angles and bearings correct to 0.1^o

Subject: Mathematics

Class: SSII

Topic: Quadratic Equation

Number of Periods: 4

Duration: 80 Minutes

Previous knowledge: The students have been taught algebraic expression involving quadratic and they were able to arrange numbers and letters..

Behavioural Objectives: At the end of the lesson, the students should be able to :

1. Define Quadratic Equation
2. Solve quadratic equation with formular method.
3. Derivation of the general form of a quadratic equation
4. Use method of completing square to solve Quadratic Equation (QE)
5. Solve word problems leading to Quadratic Equation (QE)

Instructional Material: Chalk and Chalkboard.

Reference Books: New General Mathematics for Senior Secondary Schools Book 2 (Macrae, Kalejaiye, Garba, Chima, Ademosu, Channon, McleishSmith and Head, 2008).

Outline of Content:

- Definition of Quadratic equation: A quadratic equation is a univariate polynomial equation of the second degree and it is written in the form of $ax^2 + bx + c = 0$, where *x* represents a variable or an unknown, and *a*, *b*, and *c* are constants with $a \neq 0$. (If $a = 0$, the equation is a linear equation.) in other words polynomial equation takes the form of P = Q where P and Q are polynomials with coefficients in field of rational numbers. Quadratic equations can be solved by factorizing, completing the square, graphing and using the quadratic formula (given above).
- Solving a quadratic equation by the formula method

- Derivation of the general form of a quadratic equation
- Use method of completing square to solve Quadratic Equation (QE)
- Solve word problems leading to Quadratic Equation (QE)

Proceedures

Step 1: The teacher gave a brief overview of the topic and also define the concept. Thereafter he flashed back the mind of the students on the topic during SSI. He also inform the students that the topic involves addition, subtraction, division, multiplication and factorization. He later entertained and asked questions.

Step 2: The teacher wrote out the general formula and also generate questions to be solved. The formula for solving quadratic equation is $ax^2 + bx + c = 0$. He later writes out examples and solve the questions on the board: Use the formula to solve these equations. Give the roots correct to 2 decimal places where necessary. Use factorization to check the results. E.g.(i) $X^2+5x+6=0$, (ii) $x^2-5x+4=0$ and (iii) $x^2-4x-5=0$.

Step 3: The teacher used the general form to derive the general equation of the quadratic equation. $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Step 4: The teacher illustrated with example on the board.

Students' response: The students were given opportunity to ask questions on any area not clear which was answered by the teacher.

Summary: The teacher summarized by re-echoed what he has taught such as meaning of quadratic equation, using quadratic equation formula to solve equation and also derive general quadratic equation from quadratic formular.

Evaluation: The teacher evaluated the students by asking the following questions:

1. Define quadraic equation.

2. Solve the following equation using the formular method (i) 3x2-5x-3=0 (ii) 5x2+8x-2=0 (iii) 5x2+3x-3=0

The teacher gave out instructional rubrics or scoring guide to all the students and asked them to exchange their solved work to enable them assessed themselves.

Assignment: The teacher asked the students to solve the following questions at home.

$6x^2+13x+6=0$, (ii) $x^2-2x-4=0$ (iii) $4x^2+7x-3=0$ on page 42 of New General Mathematics for Senior Secondary Schools Book 2 (Macrae, Kalejaiye, Garba, Chima, Ademosu, Channon, McleishSmith and Head,2008).

Subject: Mathematics

Class: SSII

Topic: Sequences and Series

Number of Periods: 4

Duration: 80 Minutes

Previous knowledge: The students are able to round number to a specified decimal place or significant figures. They are also able to determine the round off error of approximation.

Behavioural Objectives: At the end of the lesson, the students should be able to :

1. Define a Sequence and Series
2. Determine the pattern of a sequence

1. Find any particular term of a given sequence
2. Find the nth term of Arithmetic Progression (AP)
3. Find the nth term of Geometric Progression (GP)
4. Calculate the sum of an AP up to the nth term
5. Calculate the sum of an GP up to the nth term

Instructional Material: A set of cards with numbers written on them and arranged in order of sequence.

Reference Books: New General Mathematics for Senior Secondary Schools Book 2 (Macrae, Kalejaiye, Garba, Chima, Ademosu, Channon, McleishSmith and Head,2008).

Outline of Content:

- Definition of a sequence: A sequence is a set of terms arranged according to a rule. The sequence depends on the rule of combination. E.g, the sequence 5,11,17, 23….is generated by the rule 6n-1 while the sequence 1, 10, 39, 116, 166….. is generated bt Tn = n (2^{n+1})-3n. The rule can be given and the sequence required to be drawn from the rule. For example, the nth term of a sequence is given by $3x2^{n-2}$. Write the first three terms of the sequence. We have Arithmetic and Geometric sequence.

- Definition of Series: A series is obtained by adding together the terms of a sequence. A series may be infinite (carrying on without end) or finite (containing a definite numbers of terms).
- Arithmetic progression (AP): Is a sequence of terms that increase or decrease consecutively by a constant amount, the common difference. For an AP with first term a and a common difference d, the nth term is given by U, where, $U_n = a + (n-1)d$ E.g, 24, 15, 16, -3, -12……….. common difference= -9

Procedures : Step 1:

The teacher gave a brief overview of last lesson and inform the students that the topic involve addition, subtraction, division and multiplication. He later entertained and asked questions based on last lesson.

Step 2: The teacher displayed the number cards and used the arrangement to introduce the sequence number.

Step 3: The teacher asked students to give another name for such arrangement of object in order pattern to enable them to think critically.

Step 4: The teacher guided students to generate their own sequence of numbers which form an AP and find the common difference.

Step 5: The teacher solved examples on sequence and AP.

Example 1: The 3^{rd} term of an AP is 9 while the 11^{th} term is 7. Find the first five terms of the AP.

Example 2: Generate the next three terms of the sequence 5,9 18, 34….and hence generate its sequence relation.

Students' response: The students were given opportunity to ask questions on any area not clear which was answered by the teacher.

Summary: The teacher wrote out all he taught on the chalkboard such as what is sequence, series, arithmetic progression, the nth term of an AP is given by $U_n = a + (n-1)d$. d may be a positive or negative integer or fraction.

Evaluation: The teacher asked the students the following questions:

1. Define Sequence and series.
2. Find the 5^{th} and 8^{th} terms of the sequence whose nth terms is (a) 2n + 1 (b) 3-5n.

The teacher gave out instructional rubrics or scoring guide to all the students and asked them to exchange their solved work to enable them assessed themselves.

Assignment: The teacher asked the students to: find the nth of the following APs: (a) 3,7,11….(b) 6, 4, 2 …. (c)$^{1}/2$, $^{3}/4$, 1….(d) 100, 91, 82… on page 179 of New General Mathematics for Senior Secondary Schools Book 2 (Macrae, Kalejaiye, Garba, Chima, Ademosu, Channon, McleishSmith and Head,2008).

Name:
Class: **SSS II**
Subject: **Mathematics**
Topic: **Sequence and Series**
Date: ____________________

Sequence and Series: Arithmetic Progression (AP	**1**	**2**	**3**	**4**
Determine the pattern of a sequence in relation to Arithmetic progression(AP)	Is able to determine the pattern of sequence in relation to Arithmetic progression a little	Is able to determine the pattern of sequence in relation to Arithmetic progression partially.	Is able to determine the pattern of sequence in relation to Arithmetic progression adequately.	Is able to determine the pattern of sequence in relation to meaning of Arithmetic progression adequately and consistently.
Find the nth term of a Arithmetic progression	Is able to find nth term of AP a little.	Is able to find nth term of AP partially.	Is able to find nth term of AP adequately.	Is able to find the nth term of AP adequately and consistently.
Find the nth term of a Geometric progression	Find little or no nth term of GP.	Is able to find partial nth term of GP.	Is able to find adequately the nth term of GP.	Is able to find the nth term of GP adequately and consistently.
Calculate the sum of an AP up to the nth term	Is able to calculate little sum of AP up to nth term.	Is able to calculate partially the sum of AP up to nth term.	Is able to calculate adequately the sum of AP up to nth term.	Is able to calculate the sum of AP adequately and consistently.
Calculate the sum of a GP up to the nth.	Is able to calculate little sum of GP up to nth term.	Is able to calculate partially the sum of GP up to nth term.	Is able to calculate adequately the sum of GP up to nth term.	Is able to calculate the sum of GP adequately and consistently.

Key
1 = rarely or unacceptable
2 = sometimes or below standard
3 = regularly or satisfactory
4 = consistently or proficient

Name:
Class: SSS II
Subject: Mathematics
Topic: Quadratic Equation
Date: ___________________

Quadratic Equation:	**1**	**2**	**3**	**4**
State Quadratic Equation (QE)formulae	Is able to state the formulae of Quadratic Equation (QE) a little.	Partially state the formulae of Quadratic Equation (QE).	Is able to state the formulae of Quadratic Equation (QE) adequately.	Is able to state the formulae of Quadratic Equation (QE) adequately and consistently.
Solve quadratic equation with irrational roots	Is able to solve quadratic equation with irrational root a little.	Partially solve quadratic equation with irrational root.	Is able to solve adequately the quadratic equation with irrational root.	Is able to solve adequately and consistently the quadratic equation with irrational root.
Derive Quadratic Equation formula	Is able to derive QE formula a little.	Partially derive QE formula.	Is able to derive QE formula adequately.	Is able to derive QE formula adequately and consistently.
Use method of completing square to solve QE	Use little or no method of completing square to solve QE.	Partially used the method of completing square to solve QE.	Is able to use the method of completing square to solve QE adequately.	Is able to use the method of completing the square to solve QE adequately and consistently.
Solve word problems leading to QE	Is able to solve word problems in QE a little.	Partially solve word problems in QE.	Is able to solve word problems in QE regularly.	Is able to solve word problems in QE adequately and consistently.

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Name:
Class: **SSS II**
Subject: **Mathematics**
Topic: **Probability**
Date: ___________________

Probability	**1**	**2**	**3**	**4**
Define and state the formula of probability	Is able to define and state the formula with mistakes.	Partially define and state its formula	Is able to define and state its formula adequately..	Is able to define and state its formula adequately and consistently.
Display chart with label	Is able to display little chart with label .	Partially display chart with label..	Is able to display chart with label adequately	Is able to display all chart with label adequately and consistently.
Solve experimental probability	Is able to solve few experimental probability problems .	Partially solve some experimental probability problems.	Is able to solve most experimental probability problems adequately.	Is able to solve all experimental probability problems adequately and consistently.
Solve theoretical probability	Is able to solve few theoretical probability problems.	Partially solve some theoretical probability problems.	Is able to solve most theoretical probability adequately.	Is able to solve all theoretical probability adequately and consistently.
Determine probability by mutually exclusive events	Is able to determine probability by mutually exclusive events little.	Partially determine probability by mutually exclusive events.	Is able to determine probability by mutually exclusive events regularly.	Is able to determine probability by mutually exclusive events adequately and consistently.

Key
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Name:
Class: SSS II
Subject: Mathematics Topic: Bearing and Distance
Date: ____________________

Bearing and Distance:	**1**	**2**	**3**	**4**
Express bearing as 3-figure or as compass bearing	Is able to express bearing as 3-figure bearing or compass a little.	Partially expressing bearing as 3-figure bearing or compass bearing.	Is able to express bearing as 3-figure bearing or compass regularly.	Is able to express bearing as 3-figure bearing or compass regularly and consistently.
Construct angle showing distance and bearing from three points	Is able to construct angle showing the distance and bearing from three points little.	Partially construct angle showing the distance and bearing from three points.	Is able to construct angle showing the distance and bearing from three points regularly.	Is able to construct angle showing the distance and bearing from three points regularly and consistently.
Use cosine rule to calculate the distance and bearing from three points.	Use cosine rule to calculate little distance and bearing from three points.	Sometimes use cosine rule to calculate the distance and bearing from three points.	Is able to use cosine rule to calculate the distance and bearing from three points regularly.	Is able to use cosine rule to calculate the distance and bearing from three points regularly and consistently.
Use sine rule to calculate the distance and from three points.	Use sine rule to calculate little distance and bearing from three points.	Sometimes use sine rule to calculate the distance and bearing from three points.	Is able to use sine rule to calculate the distance and bearing from three points regularly.	Is able to use sine rule to calculate the distance and bearing from three points regularly and consistently.

Key
1 = rarely or unacceptable
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DEPARTMENT OF EDUCATIONAL FOUNDATIONS
(WITH EDUCATIONAL PSYCHOLOGY)
FACULTY OF EDUCATION
UNIVERSITY OF LAGOS, NIGERIA

Head of Department
Prof. (Mrs.) Ayoka Mopelola Olusakin
B. Ed., M. Ed; Ph. D (Ibadan) FCASSON

Tel: 234-1 4932660 - 1
Ext. 2260; 1948

10th August, 2011.

TO WHOM IT MAY CONCERN

LETTER OF INTRODUCTION

This is to confirm that **ASUAI, NELSON CHUKWUYENUM** with Matriculation No. **069034059** is a Ph. D student of Measurement and Evaluation in the Department of Educational Foundations. He is conducting a research on his project ***"Impact of Instructional Rubrics and Critical Thinking on Performance in Mathematics among Senior Secondary School Students in Delta State".***

It shall be greatly appreciated if you could give him the necessary assistance based on the information above.

Thank you.

HEAD
DEPT. OF EDUCATIONAL FOUNDATION
UNIVERSITY OF LAGOS

Prof. (Mrs.) A.M. Olusakin

Department of Educational Foundations

University Of Lagos,

Akoka, Lagos

13th September, 2011

Mr. Adhi' Elijah Obi
Umuaja Mixed Sec. School
Umuaja
Delta State

Dear Sir/ Madam,

PART-TIME RESEARCH ASSISTANT

You are hereby appointed as Research Assistant in the research study that would be carried out in your school, Umuaja mixed Sec. Sch. Umuaja Delta State

The objective of the study is to investigate the Impact of Critical Thinking Skills and Peer Assessment on Senior Secondary School Students' Performance in Mathematics in Delta State, Nigeria.

Indicate your willingness to partake in the study on the duplicate copy of this letter.

Yours faithfully,

12/09/2011

Asuai Nelson Chukwuyenum

(Researcher)

Department of Educational Foundations
University Of Lagos,
Akoka, Lagos

13th September, 2011

Mr. Chiburor Onyebuchi
B. S. S. Obiaruku
Delta state
....

Dear Sir/ Madam,

PART-TIME RESEARCH ASSISTANT

You are hereby appointed as Research Assistant in the research study that would be carried out in your school, Boys Sec. School Obiaruku, Delta State
The objective of the study is to investigate the Impact of Critical Thinking Skills and Peer Assessment on Senior Secondary School Students' Performance in Mathematics in Delta State, Nigeria.
Indicate your willingness to partake in the study on the duplicate copy of this letter.

Yours faithfully,
Asuai 12-09-2011
Asuai Nelson Chukwuyenum
(Researcher)

Department of Educational Foundations
University Of Lagos,
Akoka, Lagos

13th September, 2011

Mrs Ndubusi Charity
Akashiede Girls Sec. school
Obiaruku
Delta state

Dear Sir/ Madam,

PART-TIME RESEARCH ASSISTANT

You are hereby appointed as Research Assistant in the research study that would be carried out in your school, Akashiede Girls' Secondary school, Obiaruku, Delta St.

The objective of the study is to investigate the Impact of Critical Thinking Skills and Peer Assessment on Senior Secondary School Students' Performance in Mathematics in Delta State, Nigeria.

Indicate your willingness to partake in the study on the duplicate copy of this letter.

Yours faithfully,

12th Sept. 2011

Asuai Nelson Chukwuyenum

(Researcher)

Department of Educational Foundations
University Of Lagos,
Akoka, Lagos

13th September, 2011

Mr. Ebokaiwe John
Umutu Mixed Sec. School, Umutu
Delta state

....

Dear Sir/ Madam,

PART-TIME RESEARCH ASSISTANT

You are hereby appointed as Research Assistant in the research study that would be carried out in your school, Umutu Mixed Sec. School, Umutu, Delta state.
The objective of the study is to investigate the Impact of Critical Thinking Skills and Peer Assessment on Senior Secondary School Students' Performance in Mathematics in Delta State, Nigeria.
Indicate your willingness to partake in the study on the duplicate copy of this letter.

Yours faithfully,

Asuai 12/9/2011
Asuai Nelson Chukwuyenum
(Researcher)