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A Decision Support Model and Analysis for Aircraft Maintenance Planning

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Abstract

Aircraft maintenance planning and scheduling is a major decision airline operators must make. If properly done, this can result in huge cost savings and high productivity. Maintenance downtime (from daily inspections, component replacements and different types of scheduled checks) is a key operational bottleneck that must be minimized while maintaining aircraft for safety and reliability. This paper provides a planning model which determines the optimal solutions to aircraft maintenance planning and scheduling using a Non-Linear Pure Binary Mathematical Program solved using the Evolutionary Algorithm search engine of the Excel Solver. Results obtained using data from an actual airline for the proposed model and heuristic results in a higher likelihood of meeting operational flight schedule at relatively minimized overall downtime. There is an optimal balance between constraints set by irregular component failures, regulatory maintenance requirements and the anticipatory expectations of meeting projected flight schedule over a given period operation.

1 Introduction

Commercial aircrafts are machines that wear out in the course of their operations. In order to meet up with flight projections and schedules over a time period, an aircraft must remain in an air-worthy state. This condition of air worthiness is met through regular maintenance. Maintenance is any administrative or technical restorative or preventive activity against deterioration from an inherent safety and reliability level of components in the design of the aircraft. Maintenance of aircraft is also a regulatory requirement. It is not a revenue generating activity and should be optimally planned such that the overall aircraft down time due to maintenance is minimized.

Aircraft maintenance planning is the forecasting, scheduling and control of aircraft maintenance in order to keep an aircraft in air-worthy state for profitable service. The series of checks during aircraft maintenance proceeds in increasing detail and scope. Unscheduled fixes can however occur such as when airworthiness check directives come from a regulatory body. The length of flight hours and the number of take-off and landing cycles govern the rate at which checks are performed. According to [1] aircraft maintenance checks can be categorized into four, namely: A Checks - a light check carried out usually overnight at an airport gate every month or every 500 flight hours. This schedule may change depending on the type of aircraft and how frequently it is used. The B Check is also a light overnight check done at an airport gate but at a little extended period say after 12 weeks. The C Check on the other hand is a heavy maintenance check performed after one and half years. It entails a disassembly of critical parts, in a hangar. Finally, the D Check is a heavy overhaul check performed after 4 or 5 years in which the whole aircraft is carried out for comprehensive inspection and maintenance.
The table below shows a typical scheduling chart for aircraft checks.

TABLE 1. A Typical Scheduling Chart for Aircraft Checks

FLIGHT HRS ('00s)	2.5	5	7.5	10	12.5	15	17.5	20	25	27.5	30	32.5	35	37.5	40
1A	1A	1A	1A	1A	1A	1A	1A	1A	1A	1A	1A	1A	1A	1A	1A
2A		2A		2A		2A		2A		2A		2A		2A	
4A				4A				4A				4A			
8A								8A							
1C															1C

From the schedule in Table 1, Type B and Type D checks have been spread through check A and check C respectively.

In the typical schedule in Table 1, above, the maintenance job contents for Check B and check D have been spread into Check A and Check C which has led to complete phasing of the Check A and Check C over 4000 flight hours.

In the table, 1A, 2A, 4A, and 8A represent the different maintenance job packages that should be carried out at periodic intervals under the type A-Check. The comprehensiveness of the maintenance in terms of the extent of detailed maintenance work to be carried out is in increasing order as listed. Check 1A is conducted at 250 flight hours intervals, 2A at 500 flight hours, 4A at 1000 flight hours and 8A at 2000 flight hours intervals. The same applies for which checks 1C, 2C, 3C and 6C are also in increasing order of comprehensiveness and detail. The goal is towards a reliability-centered maintenance [2], which reduces down time and allows for more operational flexibility, in accordance with MSG-3 (Maintenance Steering Group-3) maintenance philosophy. The MSG-3 is a task-oriented, maintenance process that uses a decision tree methodology to effectively separate safety-related items from economic related items and completely define how hidden functional failures are to be treated. The logic in this approach is that activities are assessed at the system level rather than the component level. Meaning that a routine maintenance activity may not be carried out if it can be shown that the functional failure of a system does not affect operational safety or if the economic consequences are insignificant. [3].

However the research focus here is more on maintenance scheduling and how expected component failures based on life expectancy ratings could be managed to reduce aircraft downtime than on the type and content of the maintenance Check. This was what informed the extensive use of Check A only in this work here culminating into only one C-check for a flying hours horizon of 4000. Beyond this is only a repetition of the A/C-check cycle.

Maintenance tasks are developed based on failure effects and their categories. The maintenance program is limited to task that are truly effective which include tasks that prevent deterioration of inherent reliability and operating safety. The maintenance program is mainly generated from the maintenance planning document which is one of the documents given by the aircraft manufacturer to the aircraft operator. This document contains recommendations for aircraft maintenance.

In the maintenance program, aircraft systems and components are classified according to the air transport association format. These systems and components are monitored on hard time, condition and on-condition maintenance basis. Hard time maintenance is a type of preventive maintenance where by components are removed for inspection, repair or overhaul at the end of a specified period (flight hours or flight cycles) regardless of the state the component. On-condition monitoring is a type of maintenance in which the airworthiness status of a component is determined by visual inspection or any other appropriate mode of

evaluation while Condition monitoring is a type of maintenance in which components are monitored at intervals and data obtained are compared with a base line to ascertain if there is any deterioration trend in performance.
In the overall, planning for aircraft maintenance has evolved into a carrier within the airline industries with personnel devoted to learning and mastering the art of aircraft maintenance of scheduling, these planners strive to balance the demands of aircraft maintenance with operational requirements of flight projections made by airline companies, these schedules are generated manually based on the experience of the planner, there seems to be a need to always try and model the scheduling process so as to always take informed decision rather than just rely on experience that might not always be easily available.

2 Literature Review

According to [4] the scheduling problem organizes and executes a set of tasks with time constraints such as deadlines and priority constraints as well as capability and capacity constraints which are the resources that are required for the tasks. Scheduling is concerned with the optimal allocation of scarce resources to activities over time [5]. [6] in the first comprehensive handbook of scheduling covered quite a number of advances in scheduling. He traced the development of efficient algorithms developed to obtain optimal solutions to the scheduling problem since the 1950s until the problems became more sophisticated and shown to be NP-hard in the 1970s. In the aviation literature, a lot of work has focused on crew scheduling, flight scheduling, fleet scheduling and routing, equipment selection and usage, demand forecasting and passenger-mix optimization. Not as much has been done on optimization models for solving aircraft maintenance scheduling problems. The aircraft maintenance scheduling problem is one that can be approached by solving a mathematical model which takes into consideration, the relevant operational decision constraints and then optimizing the model based on these decisions. For some maintenance tasks, the manufacturer prescribes the need for check flights to be carried out in the aircraft's maintenance manual. [7] considered only maintenance checks of short intervals and crew constraints to formulate a basic the fleet assignment problem, since they mostly interrupt operational schedules of flight projections. Authors in [8] formulated the maintenance scheduling problem for a fleet of aircraft undergoing Type A and Type B checks, and an efficient heuristic approach to solve it. The heuristic was a hybrid of random search and depth first search that reduces the astronomically large solution time for solving large integer-programming models of the problem. The objective values of solutions generated were within 5% of global optimum solutions and in very reasonable amount of time. Other studies on maintenance routing problem are found in [9] and [10].

Most of the previous studies have approached the formulation of maintenance-scheduling problem as a multi-commodity network flow model with a wide variety of solution approach. From the literature it appears reasonable to observe that the key challenge of researchers in aircraft scheduling, routing and maintenance scheduling is not that of problem formulation but that of solving the formulated models to have optimal solutions in reasonably short time. This paper proposes an efficient approach to developing an aircraft maintenance planning model and then optimizing the model using Excel Evolutionary solver.

3 The Aircraft Planning Model Formulation

The model was formulated by first identifying all possible decisions that could be considered when planning for aircraft maintenance. The attendant decision variables were used to form the building blocks of a decision network. An aircraft maintenance planning model was then developed from this decision network. The model was solved using Excel evolutionary solver.

Aircraft maintenance is carried out at specific flight hours depending on the type which varies in work scope and level of task been done. Routine inspections are carried out at more frequent intervals depending on the utilization of the aircraft. At any given point of inspection, various aircraft systems and component are assessed to determine their airworthiness state, at this point a decision of whether a given component had failed or not is made this forms a decision variable. Aircraft checks (type A and C) are done at longer flight hours intervals as compared to those of the routine inspection. During routine inspection the anticipation of a component failure before the next routine inspection time especially if the component is critical affects the extent of component replacement and thus the rescheduling of its next component replacement time (in aircraft flying hours) instead of grounding the aircraft for every such component replacement/maintenance, hence the decision of whether it is time for scheduled component replacement/maintenance or not forms a decision variable. Maintenance significant items are those systems or assemblies whose failure could affect safety both on ground or inflight. For the modeling purpose, components that make up these systems or assemblies are classified as critical components while those that do not constitute maintenance significant items are classified as non-critical components. Hence at the point of inspection a decision of whether the failure involves only non-critical components or at least one critical component becomes a decision variable. In order not to always disrupt flight projections in cases where incident component failure is close to the time for its expected failure, an admissible tolerance range for scheduled maintenance is incorporated such that the decision of whether component failure falls within the tolerance range for its scheduled maintenance or not becomes a decision variable. Hence the decision variables, at any inspection point in time (in aircraft flying hours), for the objective function of the model include:

- The decision of whether a component failure is expected or not.
- The decision of whether it is time for routine maintenance (A or C-check) or not.
- The decision of whether the expected component failures involve non critical components or at least one critical component.
- The decision of whether the expected failures would occur within the tolerance range for scheduled component maintenance/replacement or not.

3.1Model Parameters and Decision Variables

The model is a Mixed Integer Linear Mathematical Program. Maintenance decisions are engrained in four key binary decision variables culminating in a decision tree at whose terminal points are seven key mutually exclusive decision events with corresponding maintenance actions. The seven decision events constitute the seven components of the objective function built to minimize downtime of aircraft.

The first binary decision variable, X_k defined as,

$$X_k = \begin{cases} 1, & if\ failure\ of\ at\ least\ a\ component\ is\ expected\ at\ an\ inspection\ time\ k \\ 0, & otherwise \end{cases} \quad (1)$$

This monitors if one or more component failures is expected at the kth inspection time, S_k (in aircraft flying hours). A decision event occurs when no component is expected to fail at that instance. Two possibilities of actions involving downtime resulting into two separate decision events captured by the first two terms of the objective function occur. If the inspection time, k, does not correspond to the routine maintenance time (A or C check), the cumulative aircraft downtime is only incremented by the inspection duration, t_I. However, if the kth inspection time, S_k, corresponds to a scheduled routine maintenance time (A or C check), the cumulative downtime is incremented by both inspection time, t_I, and the duration, m_H, for routine scheduled maintenance. These decision events are captured by the first two terms of the objective function respectively. The decision as to whether or not kth inspection time, S_k, corresponds or not to a routine scheduled maintenance is captured by the second binary decision variable, Y_k defined as,

$$Y_k = \begin{cases} 1, & \textit{if it is not time for scheduled routine maintenance at inspection time } k \\ 0, & \textit{otherwise} \end{cases} \quad (2)$$

If on the contrary, the failure of one or more components is expected at that kth inspection time, S_k, (X_k=1), and S_k corresponds to a routine maintenance time, (Y_k=0), then the cumulative downtime is incremented by both inspection time, t_I, and the duration, m_H, for routine maintenance constituting the third component term of the objective function regardless of whether all the components expected to fail are not critical (Z_k=0) or at least one is (Z_k =1) where Z_k is defined as,

$$Z_k = \begin{cases} 1, & \textit{if at least one critical component is expected to fail} \\ 0, & \textit{if no critical component is expected to fail} \end{cases} \quad (3)$$

It should be noted that the essence of monitoring Z_k value is also for extra maintenance vigilance. While all components are to be maintained or replaced at their expected component failure times within inspection tolerance range, special attention which may not necessarily result in more increase in downtime necessary to fix the concerned critical components are paid to them especially as may cause chain reaction in the aircraft system.
However, if $X_k = 1$ and the kth inspection time, S_k does not correspond to Routine Maintenance Time ($Y_k = 1$), for either $Z_k = 0$ or 1, the model further verifies whether or not the inspection time falls within a tolerance range specified for fast-tracking maintenance of components so that they do not fail before the next inspection time. Thus, another binary decision variable, V_k, is defined such that,

$$V_k = \begin{cases} 1, & \textit{if expected failure time is within tolerance range for next inspection or routine maintenance} \\ 0, & \textit{otherwise} \end{cases} \quad (4)$$

The combinations of possible decisions from the Z_k and V_k decision points generate four different decision events.

(i) $X_k = 1$, $Y_k = 1$, $Z_k = 0$ and $V_k = 0$ for which both the inspection duration time and maintenance/replacement times of the individual non-critical components expected to fail are added to the cumulative downtime at this instance of decision event. This event is responsible for the fourth decision event and the fourth term of the objective function.

(ii) $X_k = 1$, $Y_k = 1$, $Z_k = 0$ and $V_k = 1$ for which both the inspection duration time and maintenance/replacement times of the individual non-critical components expected to fail as well as those which are expected to fail before next inspection time (within tolerance range) are added to the cumulative downtime at this instance of decision event. This event is responsible for the fifth decision event and the fifth term of the objective function.

(iii) $X_k = 1$, $Y_k = 1$, $Z_k = 1$ and $V_k = 0$ for which both the inspection duration time and maintenance/replacement times of the individual critical and non-critical components expected to fail are added to the cumulative downtime at this instance of decision event. This event is responsible for the sixth decision event and the sixth term of the objective function.

(iv) $X_k = 1$, $Y_k = 1$, $Z_k = 1$ and $V_k = 1$ for which both the inspection duration time and maintenance/replacement times of the individual non-critical components expected to fail as well as those which are expected to fail before next inspection time (within tolerance range) are added to the cumulative downtime at this instance of decision event. This event is responsible for the seventh decision event and the seventh term of the objective function.

The decision tree is illustrated in Figure 1.

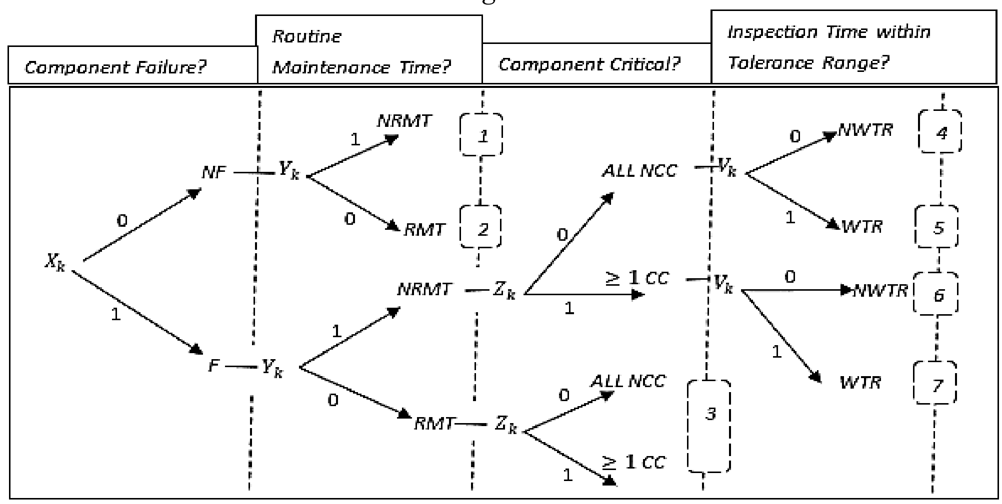
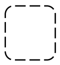


Fig. 1: The Decision network

Notations

	Decision event
NF	No Failure
F	Failure
NRMT	Not routine maintenance time
RMT	Routine maintenance time
ALL NCC	All non-critical component
≥1 CC	At least one critical component
NWTR	Not within tolerance range

There are two types of time measurement units in the model. Aircraft Flying Hour is the unit for determination of the state of the aircraft for component replacement/maintenance and life span as well as regulatory routine maintenance while the Man Hour is the time unit required for inspection and maintenance activities which culminate into the aircraft downtime.
In the model, S_k represents the kth inspection time in aircraft flying hours, k = 1, 2, 3..., K at regular intervals of t_s flying hours, such that $S_k = kt_s$ or $S_k = S_{k-1} + t_s$. The cumulative flying hours P_i (F_i) for each component i, i = 1, 2, . . ., I for non-critical components and P_{I+j} (F_{I+j}), j = 1, 2, . . ., J for critical components represents the cumulative flying hours within the planning horizon that a component is expected to fail (the expected component life in flying hours or mean flying hours to failure) respectively. P_i and P_{I+j} are regularly updated as $S_k + F_i$ and $S_k + F_{I+j}$ respectively as components are scheduled for replacement or maintenance within the planning horizon at an instance of inspection k. In order to update the cumulative component replacement/maintenance flying hour point, binary

variables, f_i and f_{I+j} are employed to determine at any inspection point k whether a component is expected to fail or not.
In this respect,

$$f_i = \begin{cases} 1 & if\ non-critical\ component\ i\ is\ expected\ to\ fail\ at\ inspection\ time \\ 0 & otherwise \end{cases} \quad (5)$$

and

$$f_{I+j} = \begin{cases} 1 & if\ critical\ component\ j\ is\ expected\ to\ fail\ at\ inspection\ time \\ 0 & otherwise \end{cases}$$

Similarly t_r , $0 < t_r < t_s$ is the tolerance aircraft flying hours such that P_i or P_{I+j}, can be fast-tracked to inspection time S_k whenever $S_{k+1} - t_r \leq P_i < S_{k+1}$ or $S_{k+1} - t_r \leq P_{I+j} < S_{k+1}$. Since components are expected not to fail for safety, despite any factor of safety built into component expected failure time specifications, in the worst scenario, $t_s = t_s - 1$. This fast-tracks component replacement and is a major trust of this work.
Total number of components under maintenance is thus, $N = I + J$, made up of I non-critical and J critical components.
In man hour time unit, t_I, is the inspection man hour for a single inspection time. m_H is the routine maintenance man hour while $T_{ni}(T_{cj})$ represents the sum man hour required to fix non-critical component ,i, (critical component, j) components at an instance of inspection. These four parameters cumulatively determine the aircraft downtime (in man hours) at any instance and are thus engrained in the objective function.

3.2 The Model

The objective of the model is to reduce the overall downtime for aircraft maintenance measured in Man Hours. The maintenance downtime is accumulated from inspections, component replacement/maintenance times and routine checks like A-checks over a projected period of flight operations measured in Flying Hours.

Based on the identified parameters, decision variables and decision events discussed in Section 3.1, the Pure Binary Non-Linear Mathematical Programming model for the problem under discussion is given by;

$$\begin{aligned}\text{Minimize} \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\{&[Y_k(1-X_k)(f_i + f_{I+j})t_I] \\ &+ [(1-X_k)(1-Y_k)(f_i + f_{I+j})(t_I + m_H)] \\ &+ [X_k(1-Y_k)(f_i + f_{I+j})(t_I + m_H)] \\ &+ [X_kY_k(1-Z_k)(1-V_k)(t_I + T_{ni})f_i] \\ &+ [X_kY_k(1-Z_k)V_k(t_I + T_{ni} + m_H)f_i] \\ &+ [X_kY_kZ_k(1-V_k)(f_i + f_{I+j})(t_I + T_{cj})] \\ &+ [X_kY_kZ_kV_k(f_i + f_{I+j})(t_I + T_{cj} + m_H)]\}\end{aligned}$$

Subject to:

$$X_k\left(\sum_i f_i + \sum_j f_{I+j}\right) + (1-X_k)\left(1 - \sum_i f_i - \sum_j f_{I+j}\right) \geq 1$$

for $i = 1, 2, \dots I$
and $I + j = I + 1, I + 2, \dots\dots, I + J$ (7)

$$Y_k(S_k - T_H) + (1 - Y_k)(1 - S_k + T_H) < 0$$

for $k = 1, 2, \ldots, K$ (8)

$$Z_k[(\sum_i f_i + 1)\sum_j f_{I+j}] + (1 - Z_k)[\sum_i f_i(1 - \sum_j f_{I+j}] \geq 1$$

for $k = 1, 2, \ldots, K;\ i = 1, 2, .., I$ *and* $I + j = I + 1, I + 2, \ldots\ldots, I + J$ (9)

$$(1 - V_k)(S_k - P_i + t_r + 1) + V_k(P_i - S_k - t_r) \leq 0$$

for $i = 1, 2, \ldots, I$ and $k = 1, 2, \ldots, K$ (10)

$$(1 - V_k)(S_k - P_{I+j} + t_r + 1) + V_k(P_{I+j} - S_k - t_r) \leq 0$$

for $j = 1, 2, \ldots, J$ and $k = 1, 2, \ldots, K$ (11)

$$f_i(P_i - S_k)(S_{k+1} - P_i - 1) - (1 - f_i)(P_i - S_k)(S_{k+1} - P_i - 1) \geq$$

for $k = 1, 2, \ldots, K$ *and* $i = 1, 2, \ldots.., I$ (12)

$$f_{I+j}(P_{I+j} - S_k)(S_{k+1} - P_{I+j} - 1)(1 - f_{I+j})(P_{I+j} - S_k)(S_{k+1} - P_{I+j} - 1) \geq 0$$

for $k = 1, 2, \ldots, K$ *and* $I + j = I + 1, I + 2 \ldots, I + J$ (13)

$$Pi = S_k + fi\,F$$

for $i = 1, 2, \ldots.., I;\ k = 1, 2, \ldots, K$ (14)

$$P_{I+j} = P_{I+J} + f_{I+j}\,F_{I+j}$$

for $j = 1,2, \ldots.., J$ or $I + j = I + 1, I + 2, \ldots\ldots, I + J$ (15)

$$T_{ni} = f_i t_{n,i}$$

for $i = 1,2, \ldots.., I$ (16)

$$T_{cj} = f_{I+j} t_{c,I+j}$$

for $j = 1,2, \ldots.., J$ or $I + j = I + 1, I + 2, \ldots\ldots, I + J$ (17)

The model objective comprises of seven terms corresponding respectively to the seven mutually exclusive decision events labelled in Figure 1. Table 2 shows that the seven Binary Decision terms corresponding to the decision events are mutually exclusive terms for all combinations of X_k, Y_k, Z_k and V_k required in the model.

Table 2: Mutual Exclusivity of the Binary Decision Terms of the Objective Function

Event	Variable Term	Decision Level 1		Decision Level 2		Decision Level 3			
		(X_k, Y_k)		(X_k, Y_k, Z_k)		(X_k, Y_k, Z_k, V_k)			
		(0, 1)	(0, 0)	(1, 0, 0)	(1, 0, 1)	(1,1,0,0)	(1,1,0,1)	(1,1,1,0)	(1,1,1,1)
1	$Y_k(1-X_k)$	1	0	0	0	0	0	0	0
2	$(1-X_k)(1-Y_k)$	0	1	0	0	0	0	0	0
3	$X_k(1-Y_k)$	0	0	1	1	0	0	0	0
4	$X_kY_k(1-Z_k)(1-V_k)$	0	0	0	0	1	0	0	0
5	$X_kY_k(1-Z_k)V_k$	0	0	0	0	0	1	0	0
6	$X_kY_kZ_k(1-V_k)$	0	0	0	0	0	0	1	0
7	$X_kY_kZ_kV_k$	0	0	0	0	0	0	0	1

3.2.1 The Constraints

Constraint set (7) with Constraint sets (12) and (13) fixes whether there is at least one component failure or not for which $X_k = 1$ and $X_k = 0$ respectively.

Constraint set (7) is exclusively valid for either of the cases at any inspection instance, k, for which the values of f_i and f_{I+j} are determined relative to the prevailing values of the updated Expected Failure time of component i, P_i and component j, P_{I+j} in flying hours and the current Cumulative Inspection time, S_k. For this constraint, the inequality (7) ensures that $X_k = 0$ (no component is expected to fail) as $\sum_i f_i + \sum_j f_{I+j} \le 0$ and by implication the definition of the variable, $\sum_i f_i + \sum_j f_{I+j} = 0$ and also that $X_k = 1$ (at least one component is expected to fail) as $\sum_i f_i + \sum_j f_{I+j} \ge 1$.

Constraints (7) in conjunction with Constraint set (12) fixes condition for which f_i (as 0 or 1) takes its value and Constraint set (13) the condition for f_{I+j} (as 0 or 1) takes its value.

Constraint set (8) determines values of Y_k (0 or 1) at an instance of inspection, k, subject to prevalent value of S_k relative to the updated Routine Maintenance time, T_H.

The inequality (8) returns $Y_k = 1$ (Not time for Routine Maintenance) as $S_k > T_H$ and $Y_k = 0$ (Time for Routine Maintenance) as $S_k > 1 + T_H$ and by implication$S_k \ge T_H$.

Constraint set (9) determines the values of Z_k at an instance of inspection, k, relative to the prevailing values of f_i and f_{I+j} for all components. The inequality (9) establishes $Z_k = 0$ (All components expected to fail are non-critical) as $\sum_i f_i(1 - \sum_j f_{I+j}] \ge 1$ can only be valid if the sum (critical components) $\sum_j f_{I+j} = 0$ and thus $\sum_i f_i \ge 1$ while $Z_k = 1$ (At least one component expected to fail is a critical one) as $(\sum_i f_i + 1) \sum_j f_{I+j} \ge 1$, the worst scenario being that $\sum_i f_i = 0$ for which the sum (for critical components) $\sum_j f_{I+j} \ge 1$.

In Constraint set (10), the values of V_k (1 or 0) are obtained to determine whether not inspection time is within tolerance range for routine maintenance given the Cumulated Component Maintenance/Replacement P_i, for non-critical components, P_{I+j} for critical omponents, prevalent inspection time, S_k, and tolerance range of time t_r. The inequalities (10) and (11) thus ensure $V_k = 0$ (Inspection Time not within tolerance range) as $S_k \le P_i - t_r - 1$ and $S_k \le P_{I+j} - t_r - 1$ while $V_k = 1$

(Inspection Time within tolerance range) as $S_k \geq P_i - t_r$ and $S_k \geq P_{I+j} - t_r$ for each $i = 1, 2, ..., I$, $j = 1, 2, . . . , J$ and $k = 1, 2, ..., K$.
Constraint sets (14) and (15) update Expected Failure time, P_i, of non-critical component i, and P_{I+j}, of critical component j given the expected component life in flying hours or mean flying hours to failure, F_i and F_{I+j} respectively.
Finally, Constraint sets (16) and (17) indicate non-critical component maintenance/replacement time, $t_{n,i}$ and critical component maintenance/replacement time, $t_{c,I+j}$.

4. Computational analysis

4.1 Test Problem

This work uses data from an airline in Nigeria whose flight projection from previous utilization for one its aircrafts has been estimated to be 4320 flight hours for the next 24 months. If its routine inspection is carried out at every 25 flight hours, its A-check is carried out at every 250 flight hours and C-check every 4000 flight hours. It takes an average of 0.75 hours to complete a routine inspection, 10 hours to complete an A-check, it takes 400 hours to complete a C-check. The aircraft had already flown 875 flight hours as at the time this flight projection is made, the table below presents the data on aircraft component.
Table 3: Component data table

Systems/Components	Category	Maintenance/Replacement Man hours ($\bar{t}_{n,i} / \bar{t}_{c,i}$)	Average Flight hours to failure
Direct Reading Tire Pressure Gauge	Non Critical (32,D)	0.20	1160
APU Cockpit Hour meter	Non Critical (49,D)	0.10	800
Automatic Cargo Loading Systems	Non Critical (45,D)	0.4	2000
Starter Valve Arming System	Non Critical (80,D)	0.15	2500
Lavatory Smoke Detection System	Non Critical (26,D)	0.75	3000
Left and Right Engine Fuel Filter	Critical (22,A,)	0.20	550
Voice Recorder Unit	Critical (23,A,C)	0.15	280
Emergency Exit Light	Critical (33,B,C)	0.25	245

Low Oil Pressure Warning Switch	Critical (79,B,C)	0.15	1200
Engine Oil Quantity Indicator	Critical (79,B,C)	0.20	1200
Autopilot Actuators (Aileron) (Elevator)	Critical (22,B,C)	1.40	4000
DFCS Mode Control Panel	Critical (22,B,C)	0.20	4000
Passenger Address Amplifier	Critical (23,B,C)	0.10	1500
Panel, Audio Selector	Critical (23,A,C)	0.05	4000
Handsets, Headsets	Critical (23,A,C)	0.05	4000
Engine-Driven AC Generators	Critical (24,B,H)	0.50	4000
Transformer Rectifier	Critical (24,B,C)	0.10	4000
Window & Pitot Heat Module	Critical (30,B,C)	0.10	4000
Flight Recorder	Critical (31,A,C)	0.25	4000
Main Gear Actuator	Critical (32,A,O)	0.75	4000
Main Gear Lock Actuators	Critical (32,B,O)	1.00	4000
Airspeed Indicator	Critical (34,A,C)	0.25	4000
Standby Artificial Horizon Indicator	Critical (34,B,C)	0.15	4000
Air Speed Warning System	Critical (34,B,C)	0.25	4000

Table 4: Test Problem Parameters

Parameters	Values
Inspection duration (t_I)	0.75 Man Hours
Routine Maintenance Duration (m_H)	10 Man Hours
Aircraft grounding Time	0.5 Man Hours
Inspection Interval (t_s)	25 Flight Hours
Routine Maintenance Interval (t_f)	250 Flight Hours
Routine Maintenance Tolerance Range (t_r)	24 Flight Hours
Number of Inspection Phases ($K/5$)	26 (Five Inspections per phase); K = 130
Initial Aircraft Flight Hours	875 Flight Hours

4.2 Solution Methodology and Analysis

4.2.1 Solution Methodology

The model is solved using Excel Evolutionary Solver. The evolutionary algorithm is a nature inspired optimization method with a stochastic search tailored after Darwin's theory of evolutionary survival of the fittest. The fitter a solution the better the solution's chance of making it into the next generation. This continues until an optimal or near optimal solution is reached, as determined by a termination criterion.

In the Excel Solver, three options are available: The GRC Non-linear engine is for smooth non-linear problems; the LP Simplex engine is for solving linear programmes; the Evolutionary engine solves non-smooth problems (although can also be used to solve any of the earlier identified types of problems). The current problem fits into the ambient of solution domain of the Evolutionary Solver being non-linear, non-smooth and NP-hard as most combinatory problems of its sort. The parameters of the Evolutionary solver are: Mutation rate of 0.0075, population Sze of 100, Convergence tolerance of 0.0001 and Maximum time without improvement of 300 seconds as termination criterion.

The regular (normal) maintenance plan vis a vis maintenance/replacement of components as at when components are expected to fail was simulated on Excel worksheet using the same aircraft and maintenance planning parameters as used in the model with regulatory frequency of replacement dictated by expected component failure time in order to find a basis of comparing the efficiency of the model developed here.

4.2.2 Results and Analysis

The results obtained are analyzed using tables and charts. The maintenance inspections are segmented into 26 phases with each phase consisting of 5 inspections of 25 flying hours each, covering 13 A-Checks (at intervals of 250 flying hours) and one C-Check (at 4000 aircraft flying hours).

The summary of results and deductions from them as in the succeeding sub-sections.

4.2.2.1 Aircraft Downtime

Table 5 presents the summary of the downtime obtained for Regular and the formulated Model planning.

Table 5: Phase by Phase Downtime Results

Phase	Flying Hours	Check Type	Downtime (Man hrs.)	Phase	Flying Hours	Check Type	Downtime (Man Hrs.)		
			Regular	Model				Regular	Model
1	1000	NRM	7.75	6.5	14	2625	A-Check	17.65	16.25
2	1125	A-Check	17.85	16.6	15	2750	NRM	7.95	6.7
3	1250	NRM	8.3	7.05	16	2875	A-Check	16.4	16.4
4	1375	A-Check	16.25	16.25	17	3000	NRM	8.6	7.35
5	1500	NRM	8	6.75	18	3125	A-Check	17.65	16.4

6	1625	A-Check	16.35	16.35	19	3250	NRM	7.85	6.6
7	1750	NRM	9.35	6.85	20	3375	A-Check	17.85	16.6
8	1875	A-Check	16.25	16.25	21	3500	NRM	9.2	6.7
9	2000	NRM	8.3	7.05	22	3625	A-Check	16.6	16.6
10	2125	A-Check	16.25	16.25	23	3750	NRM	7.9	6.5
11	2250	NRM	9.35	6.85	24	3875	A-Check	16.45	16.45
12	2375	A-Check	17.7	16.45	25	4000	C-Check	408.4	407.15
13	2500	NRM	5.85	7.25	26	4125	A-Check	16.25	16.25

Using the Excel Statistical Analysis tool-pack, the two streams of result (Regular and Model) has very high correlation, of 0.9995. The analysis of variance (ANOVA) scheme through the same medium for a Single Factor treatment (degree of freedom of 51 and sum of squares of 301814) has an F-critical value of 4.03431 compared with a meagre model f-value of 0.00812 showing that the difference between the means of the two are statistically insignificant, but with a p-value of 0.96615. Figure 2 exhibits the downtime distributions at phases culminating into A-Checks, while Figure 3 exhibits the distribution culminating into In-Between A-Check phases (the Non-Regulatory Maintenance (NRM) inspection phases)

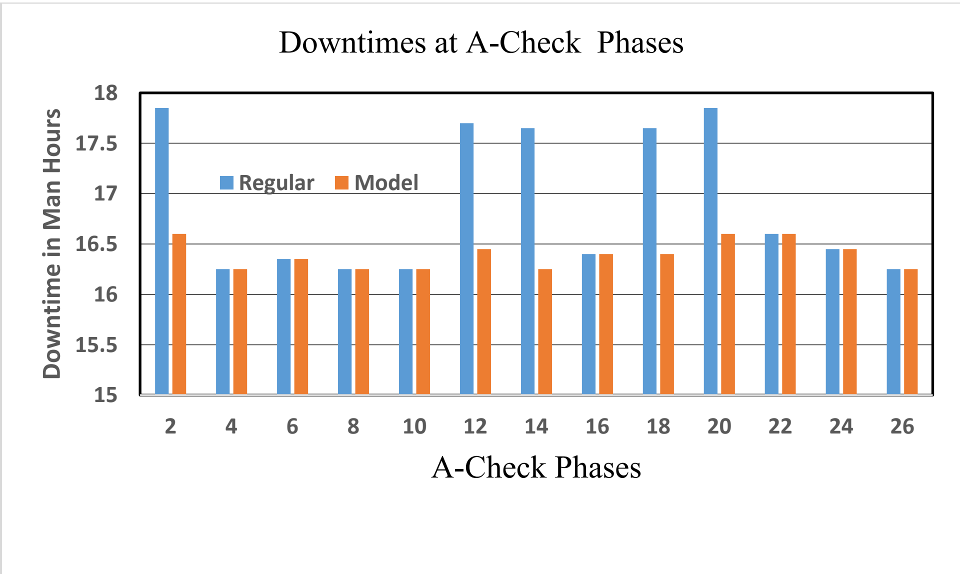


Figure 2: Downtime Distribution at A-Check Phases

However, a cursory look at the two streams of results shows that the objective of this work (minimizing downtime) is accomplished at all phases, the Model solution being superior at each phase (as graphically evident in Figures 2 and 3). In addition, a total of 331.7 Man hours overall will be saved using this model for just 3300 flying hours for a single aircraft. The savings is enormous considering the huge revenue that can accrue to the airline for that period of time.

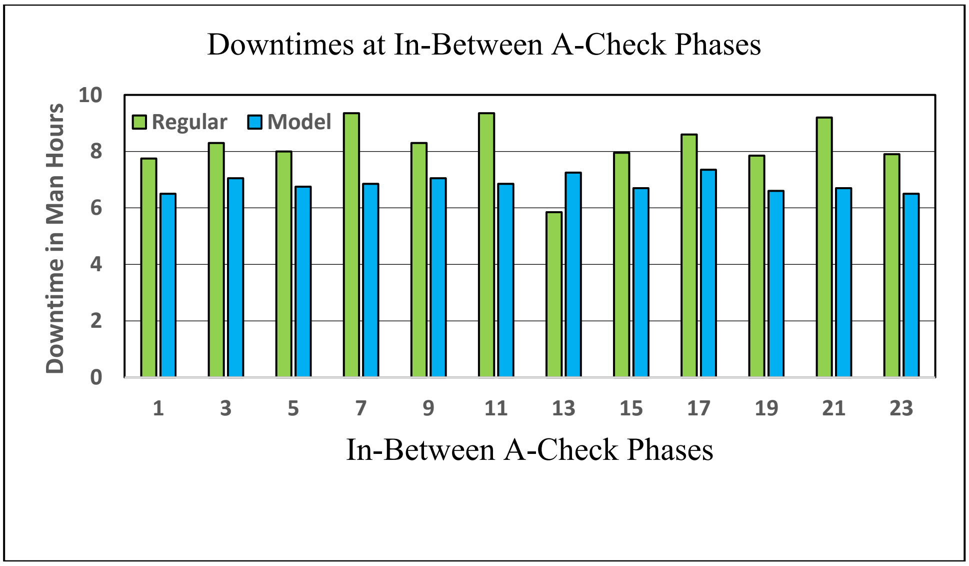


Figure 3: Downtime Distribution at In-Between A-Check Phases

4.2.2.2 Component Maintenance/Replacement Flying Hour Analysis

One other key contribution of this model is in component maintenance/replacement fast-tracking so as not to cause regular grounding to replace components as at when due. How this is enhanced is illustrated by Table 6 using components 6, 7 and 8 by virtue of their comparatively low expected failure flying hours (of 550, 280 and 245 respectively).

Table 6: Selected Component Times

Round	Maintenance/Replacement Time (Flying Hours)					
	Component 6		Component 7		Component 8	
	Regular	Model	Regular	Model	Regular	Model
1	1100	1100	1120	1120	980	975
2	1650	1650	1400	1400	1225	1225
3	2200	2200	1680	1675	1470	1450
4	2750	2750	1960	1950	1715	1700
5	3300	3300	2240	2225	1960	1950
6	3850	3850	2520	2500	2205	2200
7			2800	2800	2450	2450
8			3080	3075	2695	2675
9			3360	3350	2940	2925
10			3640	3625	3185	3175
11			3920	3900	3430	3425
12					3675	3675
13					3920	3900

Table 6 above shows discrepancies in flying hours of component maintenance/replacement for the Regular and Model cases. It also demonstrates the capability of the Model to restore parity of flying hours of maintenance/replacement where fast-tracking is not feasible (at highlighted pairings). So, the fast-tracking mechanism may not necessarily lead to higher frequency of maintenance/replacement and hence higher cost outlay for such components. Obviously, component maintenance/replacement flying hours for both cases may exhibit no discrepancies (as exhibited for component 6) and may exhibit cycles of discrepancies and restoration of parity (as exhibited for components 7 and 8 in Table 6).

4.2.2.3 Component Types Expected to Fail By Phases

Table 7 below further give the combinations of component types expected to fail for both the Regular and our Model Maintenance Plan.

Table 7: Combinations of Components Expected to Fail by Phases

Phase	Components Expected to Fail (NC – Non-Critical, C – Critical)				
	Regular	Model	Phase	Regular	Model
1	0NC,1C	0NC,1C	14	0NC,1C	None
2	0NC,2C	0NC,2C	15	0NC,2C	0NC,2C
3	1NC,3C	1NC,3C	16	0NC,1C	0NC,1C
4	None	None	17	1NC,2C	1NC,2C
5	0N,2C	0N,2C	18	0N,1C	0N,1C
6	1N,0C	1N,0C	19	1N,1C	1N,1C
7	0N,3C	0N,3C	20	0N,2C	0N,2C
8	None	None	21	1N,1C	1N,1C
9	1NC,2C	0NC,2C	22	0N,2C	0N,3C
10	None	1N,0C	23	0N,2C	0N,1C
11	0NC,3C	0NC,3C	24	0N,1C	0N,1C
12	1N,0C	1N,0C	25	2N,2C	1N,2C
13	2NC,3C	2NC,4C	26	0N,2C	0N,0C

A number of observations can be made from Table 7 which further buttress the efficacy of the Model Plan. There are only 8 out of 26 (highlighted), representing about 30% of the number of phases in which there are slight discrepancies in the combinations of types of components expected to fail. The implication of this is that the fast-tracking scheme for component maintenance/replacement does not completely distort component maintenance/replacement substantially despite its earlier proven advantage and at the same time it maintains the regulatory checks. Furthermore, there are expected to be 10 instances (2 per component and 0.384 per phase) of Non-Critical Component maintenance/replacements for both the Regular and the Model plan and 39 instances (6.5 per component and 1.5 per phase) of Critical component maintenance/replacement for the Regular plan as against 37 instances (6.167 per component and 1.423 per phase) for the Model plan. This further reinforces the advantage of the Model plan especially for fast-tracking component maintenance/replacement which reduces downtime and the same time comparing favorably (and even slightly better) than the Regular plan in terms of instances of component maintenance/replacement.

4.2.2.4 The C-Check

From Table 5, the phase culminating in Regular plan C-Check returned a downtime of is 408.4 as against a slightly less value of 407.15 flying hours for the Model plan. It should however be noted that the downtimes accruable to components expected to fail at exactly 4000 flying hour (the regulatory C-Check time) as given in Table 3 are not included in the analysis as the addition of such is trivial and will not influence any variation between the Regular and Model plans under discuss.

5. Conclusion

The focus of this research study is the formulation of a model for aircraft maintenance planning process in commercial airline operation, given that the current process is time consuming and inefficient especially in cases where experienced hands are not easily available. The developed model has the potential to significantly improve the aircraft maintenance planning process especially in the areas of real time decision making, taking into cognizance the daily challenges faced by airline operators in trying to meet operational flight schedules while being limited by regulatory and maintenance constraints.

The model was developed by first identifying key decision variables that interplay during maintenance planning process. These decision variables were used to form a connecting frame work, from which the maintenance planning model was formulated. The maintenance planning model was optimized around the decision variables using Excel Evolutionary Solver in phases of five inspections per phase and results were obtained for twenty six phases.

From the results obtained, it is clearly seen that downtime of aircraft was substantially reduced using the model compared to the Regular maintenance strategy. The tolerance range attached to component maintenance/replacement time positively influenced the timely maintenance/replacement of the components at specific inspection time during which an aircraft may be grounded. Operationally, the implication is a higher possibility of meeting flights scheduled over a projected period at relatively minimized overall downtime. Hence there is an optimal balance between constraints imposed by irregular component failures, regulatory maintenance requirements and the anticipatory expectations of meeting projected flight schedules over a given period operation.

There are however, significant opportunities for improvement in the area of aircraft maintenance and its planning process which are areas of further research. The fundamental issue of how maintenance/replacement logistics with respect to availability of personnel, equipment and in and out of aircraft space constraints can further reduce the gross downtime used in this work is a subject matter for future research. The availability of spares in cases of component failures need to be incorporated into the model. Intelligent agents can be also be incorporated into the maintenance process to help in real time decision making during operation.

References

[1] Kinnison, H., Aviation Maintenance Management. 2004. 2004, McGraw-Hill, New York.
[2] Smith, A.M., Reliability-centered maintenance. 1993: McGraw-Hill New York.
[3] Ackert, M.S.P., Basics of Aircraft Maintenance Programs for Financiers. 2010.
[4] Lopez, P. and F. Roubellat, Production scheduling. 2013: John Wiley & Sons.
[5] Brucker, P., Scheduling and constraint propagation. Discrete Applied Mathematics, 2002. **123**(1): p. 227-256.
[6] Leung, J.Y., Handbook of scheduling: algorithms, models, and performance analysis. 2004: CRC Press.
[7] Hane, C.A., et al., The fleet assignment problem: Solving a large-scale integer program. Mathematical Programming, 1995. **70**(1-3): p. 211-232.
[8] Sriram, C. and A. Haghani, An optimization model for aircraft maintenance scheduling and re-assignment. Transportation Research Part A: Policy and Practice, 2003. **37**(1): p. 29-48.
[9] Clarke, L., et al., The aircraft rotation problem. Annals of Operations Research, 1997. **69**: p. 33-46.
[10] Vance, P.H., et al., Airline crew scheduling: A new formulation and decomposition algorithm. Operations Research, 1997. **45**(2): p. 188-200.