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BEYOND CALCULATIONS

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By
PROF. ADETOKUNBO BABATUNDE SOFOLUWE

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BEYOND CALCULATIONS

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By

ADETOKUNBO BABATUNDE SOFOLUWE

BSc. (Lagos), MSc., Ph.D (Edinburgh)

Professor of Computer Science

Deputy Vice Chancellor

Management Services

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PREAMBLE

Vice Chancellor, Prof. Oyewusi Ibidapo-Obe, Deputy Vice Chancellor (Academic and Research), Professor Soga Sofola, Registrar and other Principal Officers, Members of Senate and Council of University of Lagos, Distinguished Guests, Ladies and Gentlemen. I am dedicating this lecture to all my teachers of the past, present and future. In particular, I do crave your indulgence to allow me dedicate it especially to my late parents together with my four brothers who in my humble opinion established the foundation that enabled me stand before you today. I must also mention late Archdeacon B. A. Adelaja, the Principal, my other teachers and contemporaries at the C.M.S. Grammar School, Bariga. Finally, Olufunmilayo Oladiti Olagbaiye of the Psychology Department and her siblings do deserve special recognition and mention. You and others not in the audience are included in my long list of teachers. I would like to believe that you have all done your very best for me. The imperfections exhibited by me are entirely not your fault but that of my good self.

On Friday, 25th February 1977, the late Professor Ayodele O. Awojobi, a former Physics Teacher of mine, delivered at this University, an Inaugural Lecture entitled, **"BEYOND RESONANCE"** [1]. Those of us, privileged and fortunate, to have had acquaintance with the late Professor will be glad to borrow from the title of his lecture. This is the reason for my own title, **"BEYOND CALCULATIONS"**. I do hope that you will pardon my impudence, sympathize with me for having a great ambition and accept my effort or attempt by the end of this presentation. It is my intention to start you off very gently with concepts that I suspect you are likely to be familiar with. I shall gradually step things up as I proceed. I hopefully will end by leaving you with issues to ponder over. Following this preamble, the lecture is divided into eight sections. Each section contains some of **our** relevant modest contributions. The emphasis on the word **our** in the last sentence is due to the fact that I cannot claim absolute credit due to formal and informal contributions through

discussions and other means of communication with several people, too numerous to list and/or remember. The first section which concerns the concept of "Mathematical Models" is central to this presentation. Several Mathematical Models subsequently will be presented in the other following sections. The investigations I carried out alone and with others involved Mathematical Models. Computers (analogue and digital) are employed in studying these Models. The usefulness of the obtained results to scientists, engineers, etc. in each case is indicated.

MATHEMATICAL MODELS

Over the years, it has been found very convenient to represent several natural phenomena that arise in the Sciences (Physical, Chemical, Biological, Environmental, Management and Social), Engineering, etc. by what are referred to as Mathematical Models. There are several such models. I shall initially propose those which I hope are elementary.

Let me start off with a quiz:

"What is my age if twice my age is 112?"

To unravel this quiz, I can for convenience denote my age by the letter, x .

Twice my age is 2 times x , which can be written as $2x$.

If this is 112, then I can state this as,

$$2x = 112. \quad (1)$$

If I now divide both sides of the expression (1) by 2, I shall get $x = 56$. This is my age in years.

Next, consider another quiz:

"What number am I thinking of, if its square is equal to four times the number less 3"? Here, I can proceed as follows:

Suppose the number I am thinking of is represented by x . Then, this same quiz can be written as

$$x^2 = 4x - 3. \quad (2)$$

The representations, equation (1) and equation (2) are Mathematical Models. There are several tricks that can be played on these equations to obtain their solutions.

Note that the Mathematical Model, equation (1) belongs to a general class of Models referred to as Linear Algebraic Equations while the Mathematical Model, equation (2) belongs to a general class referred to as Nonlinear Algebraic Equations. Such equations as mentioned earlier, do arise in various real life situations. The solutions of these equations are significant as they assist Scientists, Engineers and others in making meaningful and informed decisions. It is to be noted that obtaining such solutions can be very tasking as unexpected complications can arise. Some of these will be presented in due course. The important message for the audience at this juncture is not to feel uncomfortable or intimidated when confronted with more complicated equations than equation (1) and equation (2) as they are nothing more than Mathematical Models in the simplest sense and there are many ways of dealing with them using a Computer.

NUMERICAL SOLUTION OF NONLINEAR ALGEBRAIC EQUATIONS

In its most general form, the problem associated with Nonlinear Algebraic Equations is to obtain a value or some values or all of the values of the independent variable, say, x , such that the dependent variable, say, y of x , which can be written as $y = f(x)$, vanishes.

Important Economic, Engineering and Scientific models/ equations are represented by these equations. A small class of such equations can easily be solved without resorting to the use of Computers. However, a larger class of these equations can only be solved using Computers. It is instructive and beneficial to investigate some of the numerical methods of solving these equations as they can lead to unexpected surprises.

Equation (2) in Section 1 can easily be solved to obtain the values $x = 1$ or $x = 3$. However, it is not likely that values of x that satisfy the equation

$$1.000001x^{10} - 4.333339x^7 + 5.000347x^3 + 7.88897x - 4.456789 = 0 \quad (3)$$

can easily be obtained.

Equation (3) can be a mathematical model of an Economic or an Engineering situation and its solution can assist in determining whether an important investment should be made or not. Usually, when faced with a task like this, there is a need to resort to the use of Computers. Several numerical methods for doing so on a Computer are in existence [2]. The choice of method will always depend on a number of factors one of which could be the experience of the individual. However, great care must be taken as was discussed in [3] when systems of linear algebraic equations are considered. I now consider it relevant and appropriate to present my observations in [4] to the audience.

Some Remarks Concerning the Solution of the Equation

$$x^x - 10 = 0 \quad [4].$$

In [4], I investigated a very simple looking equation:

$$x^x - 10 = 0 \quad (4)$$

The equation was posed by Bajpai, *et al* in [2]. I subsequently proposed the numerical solution of this equation to a class of mine in a Semester Examination during the 1990/91 session. Surprisingly, as reported in [4], the behaviour of the employed numerical method was unexpected, baffling and quite interesting. The method rather than give the required solution raised more questions than envisaged. By simple calculation, the required x value must lie between 2 and 3 since 2^2 is 4, 3^3 is 27, 10 lies between 4 and 27. Using the numerical method stated in [4], the sequence of approximate values of x initially did not exhibit any meaningful or useful pattern. This was discouraging. After additional iterations, the sequence then settled to an approximate value of x . It is an example of surprises that can come ones way when using the Computer to investigate Mathematical Models. It was possible for an inexperienced person to have halted the computational exercise erroneously concluding that the equation does not have a result or he could arrive at an incorrect result. Next, I present to you another model.

NUMERICAL SOLUTION OF SYSTEMS OF LINEAR ALGEBRAIC EQUATIONS

For a discussion of Numerical Solution of Systems of Linear Algebraic Equations, let us consider another quiz.

"What is the cost of one orange and one banana, if the cost of 5 oranges and 2 bananas is ₦31.00 and the cost of an orange and a banana is ₦8.00"?

To express the statement mathematically, in other words, establishing its equivalent Mathematical Model, I can proceed as follows:

Suppose the cost of an orange is x Naira and the cost of a banana is y Naira. Then, I can write

$$5x + 2y = 31 \quad (5a)$$

$$x + y = 8. \quad (5b)$$

The next step is to solve the above equations (5a) and (5b) for x (cost of an orange) and y (cost of a banana).

The system of equations, (5a) and (5b) is an example of a Mathematical Model, popularly referred to as a System of Linear Algebraic Equations.

The Mathematical Models referred to as systems of Linear Algebraic Equations actually arise in several real life situations. The given model/system of equations is easy or difficult to solve depending on whether the number of equations given are the same as or less (underdetermined) or more (over determined) than the unknown variables that we are to obtain (determine). The model can be made more interesting depending on whether it is homogeneous or not. A very important point to note is that the computational effort under normal circumstances will increase when the number of equations and unknowns becomes very large. This is usually the case for important and meaningful models such as the one I investigated and discussed in [5]. Then, the displacements (unknowns) in a dam when subjected to some water load are expressed as a system of linear algebraic equations of more than 200 equations involving the same number of unknowns. The obtained results after solving the system on

the Computer indicate the safety or otherwise of the dam. Interestingly and fortunately, the fatigue humans experience with large systems of linear algebraic equations is not experienced by the Computer that can handle such very easily.

It should be remarked here that in numerical parlance, there are two distinct classes of numerical methods, Direct and Indirect Methods for solving systems of linear algebraic equations/models when the conditions are favourable. Occasionally, there are some methods that are sometimes referred to as semi-direct and semi-indirect, some sort of hybrid. I shall not give the description of these methods here as several excellent well written texts exist for this purpose [2]. I shall only report some of my contributions involving the numerical solution of systems of linear algebraic equations.

I shall start off by presenting the investigation carried out in [5]

An Innovative Approach to the Analysis of Arch Dams: The Dokan Arch Dam in Iraq [5]

The safety of a dam is given among other factors by its stress concentrations under various water loadings. In [5], a 3-dimensional rigorous analysis was proposed. The resulting mathematical model was a coupled system of nonlinear partial differential equations. Numerical Solutions of some Partial Differential Equations (to be discussed in a later Section) do lead to numerical solutions of usually large systems of linear algebraic equations. This was the case for this Dam. The system was subsequently solved by some Direct and Indirect Methods. The elegantly obtained numerical results compared favorably with the results of others. They confirmed the safety of the dam. The main thrust in this work is the economic way that the results are obtained. Engineering work usually involve experimentation which are very expensive. Fortunately, solving Mathematical Models on the Computer is much cheaper.

I wish now to present the work reported in [6].

Numerical Solution of Groundwater Flow in a Semi-confined Aquifer [6]

The study of flows in aquifers (underwater storage reservoirs) is very important to hydrologists. The ability or otherwise of an aquifer to transmit, store and yield water is given by such studies. In [6], the results obtained by applying a numerical method to the Mathematical Model, which in this case is again a Partial Differential Equation is more favourable and useful than the non-numerical result which is more difficult to obtain. The point to be noted is that, again, the solution process involved solving large systems of linear algebraic equations. The Numerical approach is again cheaper than the full Engineering one which will involve actual experimentation.

Our attention must be drawn to a class of systems of linear algebraic equations referred to as *Ill Conditioned Systems*. They are systems of linear equations whose solutions are very sensitive to small changes in their coefficients and constants. Examples of such systems abound in literature. I am presenting the one taken from [7] below for illustration.

The system:

$$2x + y = 4 \quad (6a)$$

$$2x + 1.01y = 4.02 \quad (6b)$$

has the exact solution $x = 1$ and $y = 2$.

The system:

$$2x + y = 3.82 \quad (7a)$$

$$2.02x + y = 4.02 \quad (7b)$$

obtained by making small changes in the coefficients and constant of the first has the exact solution $x = 10$ and $y = -16.18$. Ill conditioned systems have to be handled with great care when using the computer. There may not be any reason to suspect that a given large system of equations is ill-conditioned. Any wrong entry of coefficients and/or constants in a system of equations will produce a totally wrong result which may be difficult to detect. One can imagine how this phenomenon could have affected our results in respect of the Dam and Aquifer Problems when misleading results could have given rise to misleading predictions

of safety. Subsequently, this could have resulted in loss of lives and properties.

In the next section we shall encounter another important class of Mathematical Models.

NUMERICAL SOLUTION OF ALGEBRAIC EIGENVALUE PROBLEMS

The class of problems referred to as Algebraic Eigenvalue Problems do arise in the mathematical modeling of vibrations of dynamical and structural systems, nonlinear optimization, mathematical economics and information system design. In its simplest form, the statement of the problem is:

If A is a given $n \times n$ real square matrix, obtain a real or complex number λ and vector $\bar{x} \neq \bar{0}$ such that $A\bar{x} = \lambda \bar{x}$. (8).

From the literature, several analytical (non-numerical) and numerical methods exist for solving this class of problems. Interested listeners are referred to the Internet and/or some standard texts such as [8] for this. Some of the interesting and relevant investigations [9, 10, 11, and 12] carried out together with some others are now summarized.

Oscillatory Behaviour of the Simple Iterative Method for Eigenvalue Problems [9]

The Simple Iterative or Power Method as its name suggests is a simple numerical method that can be used to obtain dominant eigenvalue and corresponding eigenvector of a given $n \times n$ real square matrix. The method iteratively generates a sequence of vectors

$$\bar{x}_{n+1} = A\bar{x}_n \quad (n = 0, 1, 2, 3, \dots) \quad (9)$$

with $\bar{x}_0 \neq \bar{0}$ judiciously guessed.

Several numerical experiments based on the simple iterative method were carried out in [9]. A remark concerning the unexpected oscillatory behaviour of some of the obtained results was made.

Following this, I collaborated with a researcher to investigate the Simple Iterative or Power Method with 1-Norm in [10]

The Behaviour of the Simple Iterative Method with 1-norm for Algebraic Eigenvalue Problems [10]

In this work, some numerical experiments based on the simple iterative or power method using the 1-norm were carried out. A remark concerning the performance and efficiency of the method was subsequently made.

Next, I continued to collaborate with the same researcher to investigate the Simple Iterative method with fractional norms in [11].

Simple Iterative Method with Fractional Norms for Solving Algebraic Eigenvalue Problems [11]

Numerical experiments based on the simple iterative or power method using fractional norms were carried out. A remark regarding the performance and consequently, the efficiency of this method was made with attention drawn to the nature of the considered matrices.

It is interesting also to report the work I carried out with some others in [12]. Before I do so, I would like to draw our attention to the fact that apart from The Simple Iterative or Power Method and its variants, other Methods generally referred to as Transformation Methods, do exist for solving Algebraic Eigenvalue Problems. A combination of such methods gave rise to the work reported in the publication [12].

An Householder/QL Algorithm for the Functional Eigenproblem [12]

An algorithm for the computation of the eigenvalues $E(\mathcal{E})$ and eigenfunction $Y(\mathcal{E})$ of a smooth matrix function $A(\mathcal{E})$ in an interval $0 < \mathcal{E} < 1$ was developed. It was based on a sequential use of Taylor's series expansion, Householder's tridiagonalisation and QL decomposition.

The next section introduces us to yet another class of models.

NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

An Ordinary Differential Equation (ODE) is an equation that involves an independent variable, say, x , a dependent variable, say, y , and at least one differential coefficient. An example of an ODE is

$$y + x + y' = \sin(x - y) \quad (10)$$

where $'$ denotes the derivative of y with respect to x .

Ordinary Differential Equations (ODEs) are employed as Mathematical Models in numerous Engineering and Scientific investigations. Some of those that I have spent considerable time investigating are reported in [13,14,15,16,17,18,19,20,21,22,23 and 24].

At the most elementary level, interest is in obtaining what is referred to as analytical or closed form solution to a given ODE. Unfortunately, for most important equations, which are models of real life situations, such solutions may not exist or may be very difficult to obtain if they exist. The alternative to this is to obtain Numerical Solutions to ODEs.

I now present some of my contributions.

Determination of the Meridian and Stresses of the Drop-Shaped Tank [15]

The drop-shaped tank is a shell of constant strength used on land for storing drinking water or liquefied petroleum gas (LPG). In offshore oil exploration, it is sometimes necessary to store crude oil in containers due to bad weather that may prevent the transportation of the same by tankers onshore for refining. I shall like to state here that I was introduced to this shape/tank by my Ph.D supervisor, Dr. Rodney Royles of Edinburgh University. I do acknowledge with gratitude his patience, guidance, love and understanding during the period of our interaction.

When deriving the equations of the meridian of the drop-shaped tank, membrane shell theory is employed. The derived mathematical model is a nonlinear ordinary differential equation

which does not have a closed form of solution as far as I know. The graphical and numerical methods suggested so far are doubtful. This assertion is confirmed in the publications entitled, "ECHINODOME: SOME APPROACHES TO THE ANALYSIS OF THE DROP SHAPED TANK" [13] and "AN IMPROVED NUMERICAL APPROACH TO THE ANALYSIS OF THE ECHINODOME" [14].

The name ECHINODOME was given to the drop-shaped tank in these and some other publications due to the similarity of the shape of the drop-shaped tank with that of the Sea Urchin, *Echinus-Echinus Esculentis*. In [13], the variable coefficients nonlinear ordinary differential equations describing the meridian of the drop-shaped tank were investigated. The second order equivalent form of the system was investigated analytically and numerically. Results obtained indicated that the latter approach is the most feasible.

The investigation carried out in [14] was an extension of the one carried out in [13]. In practice when the drop-shaped tank is to be constructed, it is necessary to have an efficient scheme of shape generation. This was the spirit of the investigations in [14] where some numerical methods were used in solving the differential equations of the drop-shaped tank. One of the considered methods was finally proposed as the most reliable and efficient based on certain criteria.

In pursuance of the shape generation, the system of nonlinear equations describing the meridian of the drop-shaped tank was transformed into another equivalent form in [15]. An algorithm based on this new formulation of problem is used in generating the co-ordinates of the meridian of the drop-shaped tank.

After obtaining a reliable scheme for shape generation, a Finite Element Simulation was used to obtain the stresses and moments of the drop-shape, due to the varying hydrostatic pressure head. The usefulness of this approach was assessed.

Membrane Approximation of the Behaviour of the Drop-Shaped Tank Under Symmetrical Loading [16]

The work reported here involved the theoretical investigation of the response of the drop-shaped tank under symmetrical loading. An algorithm based on membrane shell theory was developed and implemented on the digital computer. The obtained results were quite encouraging and satisfactory when compared with experimental ones.

The Behaviour of the Drop-Shaped Tank Under Unsymmetrical Loading [17]

A more generalized problem than the one reported in [16] was investigated and reported in [17]. An algorithm for predicting the response of the drop-shaped tank under unsymmetrical loading was developed and implemented on the digital computer. The listener should observe that the previous investigation was a special case of the one carried out here.

Form for Underwater Storage Vessels [18]

An optimum form of design for large tanks suitable for underwater storage was described in [18]. Installation and operating procedures were outlined together with the relative merits of founding such structures on the seabed or anchoring them as submerged floating vessels. The general features of loading were discussed in addition to the specific design criteria. Several computer simulations which were carried out were presented in [18].

Treatment of the Singular Initial Conditions of the Drop-Shaped Tank [19]

The mathematical model which describes the meridian of the drop-shaped tank is an initial value problem of a nonlinear ordinary differential equation. Unfortunately, its initial conditions are given at a singular point.

A method for incorporating the initial conditions of the differential equations of the drop-shaped tank given at a singular point was developed and presented in [19]. The approach showed how to avoid

singularities in initial conditions when employing numerical methods for the drop shaped tank. This approach may be employed in integrating ordinary differential equations having similar features.

Local Truncation Error of Explicit Euler Method and the Equations of the Drop-Shaped Tank [20]

Evaluation of the truncation error incurred by using the explicit Euler method in the numerical integration of the equations of the drop-shaped tank is carried out in [20]. The work established that the errors were insignificant.

Implicit Euler Method and the Equations of the Drop-Shaped Tank [21]

I shall like to draw your attention to the fact that broadly speaking, there are two classes, **Explicit** and **Implicit** Numerical Methods for solving Initial Value Problems of Ordinary Differential Equations. The Explicit Methods are usually easier to apply to an equation than Implicit Methods. However, the Implicit Methods usually give more accurate numerical solutions than the Explicit Methods. The choice of which Method to use then depends on what is desired by the person.

An algorithm and subsequently a digital computer program that evolved from the application of the Implicit Euler Method to the equations of the drop-shaped tank were reported in [21]. The developed program was employed in generating the coordinates of the meridian of such tank. As expected, the work required more computational effort. However, more accurate results were obtained.

A Collocation Method for Nonlinear Ordinary Differential Equations [22]

A collocation method that was previously applied to linear ordinary differential equations was applied to the nonlinear ordinary differential equation describing the meridian of the drop-shaped tank as a case study in [22]. The obtained numerical results compared favourably with previously obtained ones by other methods, such as in [15] and [21].

Runge-kutta Schemes for Solving Electrical Network Problems [23]

In [23], the Implicit, Semi-Implicit and Explicit Runge-Kutta methods for solving Ordinary Differential Equations were presented. Numerical Experiments involving electrical network problems were carried out giving very interesting results.

I consider it necessary at this point to make a brief comment on different types of computers before discussing the next Mathematical Model.

To date, broadly speaking, there are three classes of Computers: Analogue, Digital and Hybrid. This classification is based on the manner data is represented by the particular Computer. An Analogue Computer represents data in the form of continuous variables. The Digital Computer, on the other hand, represents data in discretized manner. The Hybrid Computer is a combination of both Analogue and Digital. The most popular of these classes, which most of us are familiar with, is the Digital Computer. With some Mathematical Models such as the work presented in [24], the Analogue Computer is the most suitable.

A Note on Stability of Some Parametrically Excited Structural Elastic Systems [24]

In this work, the stability of parametrically excited systems governed by the nonlinear ordinary differential equation:

$$\ddot{x} + \mathcal{E}(1 - x^2 + \mu x^4)\dot{x} + x = 0 \quad (11)$$

where \mathcal{E} and μ are specified continuously varying material and loading parameters were investigated using an analogue computer. By using the analogue computer, qualitative properties of the given systems were easily obtained.

The attention of the listener is now drawn to the important class of ODEs that are referred to as Stiff Equations in the Literature before ending this section.

Numerical solutions of ODEs are generally obtained step by step using appropriate fixed or varying step lengths. There is a

general feeling that the smaller the step length of a particular Numerical Method for a particular Equation is, the better or greater the accuracy of the obtained values. This will not always be true as there are roundoff errors that may affect the results as I reported in [14]. With some problems and appropriate methods, very small step lengths may have to be used to enable us obtain reliable and accurate results. This is the case with the class of Stiff Equations. In fact, special methods, usually, Implicit Schemes have to be deployed when solving such systems on the Computer.

I now go on to another class of Mathematical Models (in the next Section).

NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

A Partial Differential Equation (PDE) is an equation that contains an unknown function, say, u , dependent on two or more independent variables, say x and y , and at least one of its partial derivatives with respect to these variables. An example of a PDE is

$$u_{xx} + u_{yy} = 0 \quad (12)$$

where u_{xx} is the second partial derivative of u with respect to x and u_{yy} is the second partial derivative of u with respect to y .

Again, Partial Differential Equations (PDEs) are employed as Mathematical Models in numerous Engineering and Scientific investigations. Some of those that I have spent considerable time investigating are reported in [5, 6, 26 and 27].

At the most elementary level, interest is in obtaining analytical or closed form solution to a given PDE. Unfortunately, for most important equations, which are meaningful models of real life situations, such solutions may not exist or may be very difficult to obtain. The alternative to this is to obtain numerical solutions to PDEs. To obtain the desired goal, some transformation is made using some well known techniques [25]. The employed technique would transform the given PDE into a system of linear algebraic equations, which is then solved as described in an earlier Section.

I now return to the two earlier presented models. These are,

An Innovative Approach to the Analysis of Arch Dams: The Dokan Arch Dam in Iraq [5]

and

Numerical Solution of Groundwater Flow in a Semi-confined Aquifer [6]

The mathematical models of the important engineering problems investigated in [5] and [6] were PDEs. To be amenable to numerical methods of solutions, these (continuous) equations were replaced by their equivalent Finite Difference Schemes. After applying the schemes to the respective Model in the domain of interest, large systems of Linear Algebraic Equations were obtained. The resulting systems were then solved as reported in an earlier Section. The more economically obtained results provided very useful information and engineering insight into these practical problems. Two additional Models will now be presented.

On Finite Difference Solution of Linear Second Order Parabolic Partial Differential Equations in Two Space Variables [26]

The linear second-order parabolic partial differential equation in two space variables is investigated using explicit and implicit finite difference schemes. The algorithm established that there is a considerable increment in computational time and memory requirements when compared with the case of one space variable.

Some Remarks Concerning Finite Difference Solution of Linear Second Order Parabolic Partial Differential Equations in Three Space Variables [27]

In this work, the linear second-order parabolic partial differential equation in three space variables was investigated using explicit and implicit finite difference schemes. The computer implementation of this approach indicated that there is a considerable increment in computational time and memory requirements when compared with the cases of one and two space variables.

ARTIFICIAL INTELLIGENCE

Computer Scientists and others working in several diverse fields of human endeavours, such as Psychology, Philosophy, Linguistics, etc., have shown a lot of interest in an area popularly referred to as Artificial Intelligence (AI). The motivation is to explore and adopt new paradigms in order to overcome the limitations that have been identified in the manner that computations are carried out. The branch of Computer Science dealing with AI is focusing on intelligent behaviour, learning and adaptation. The interest is in designing and building machines that exhibit intelligent behaviour. Some of my modest contributions to this are reported below in [28, 29,30,31,32, 33, 34 and 35].

Pattern Recognition Using Single Layer Perceptron [28]

In this paper, the Single layer Perceptron is simulated and taught the first five letters of the English alphabets. The network was later employed to recognize the patterns it was taught and also used in recognizing distorted versions of the original patterns.

Determining Salient Input Features for Feedforward Nets [29]

A technique for improving the predictive accuracy, robustness and training time of neural nets is presented in this paper.

A Survey of Biocomputing Paradigms: Biologically Inspired Approaches for the Intelligent Industry [30]

A general survey of biocomputing paradigms is presented in this paper. Three major approaches for building biocomputing systems with their representative paradigms were discussed.

Some Biologically Inspired Paradigms for Solving NP Hard Problems, Case Study: The Travelling Salesman Problem [31]

The Traveling Salesman Problem involves finding the shortest route a traveling salesman has to take to visit a set of cities, with the condition that each city is visited once before returning to his starting point (city). This innocently posed problem is not quite easy to solve, especially as the number of cities grows. Several

approaches including ours reported here and in [33] have been proposed. It is quite interesting to remark that our own approaches are based on AI.

Here the work carried out showed that genetic algorithms and neural networks can be employed in solving intractable problems. A genetic algorithm was applied to solving 6 and 10 instances of the Traveling Salesman Problem.

On Fuzzy Logic and Neural Networks [32]

This paper critically examines fuzzy systems and their applications in robust neural net modeling. Empirically, we show how neural network may be useful in modeling adaptive fuzzy systems. Problems and limitations associated with the application of each of these computing models are presented.

A More Efficient Monte Carlo Algorithm for Solving Travelling Salesman Problem [33]

This paper presented a more efficient Monte Carlo sampling strategy for obtaining optimal solutions to some n -city instances of the Traveling Salesman Problem in $O(n^2)$ time. The application of this new scheme in the generation of fitter starting population for Genetic Algorithms based solutions is finally discussed.

A Backprop Variant and Some Practical Applications [34]

A variant of the well-known Backpropagation (backprop) of Rumelhart, Hinton and William was developed and presented in [34]. This model was applied to some practical problems which include character recognition, bacteria identification and the exclusive OR (XOR). The obtained results were quite encouraging.

User Modeling Systems [35]

In [35], User Modeling and User Modeling Systems were examined. The philosophy, the architecture and current applications of such systems were discussed. Effort that was

being made to improve the performance of some components of the user modeling systems was presented.

BEYOND CALCULATIONS

In the preceding sections, I have presented to you some effort that I have made over the years to qualify me as a Professor of Computer Science in the University of Lagos. I must state here that it has not been my effort alone. Several others have assisted as you can observe from my list of references. In my opinion, it has been very exciting and quite interesting. Starting from Mathematical Models, I have tried to obtain results using the computer. In the process, I have learnt about possible pitfalls and I have tried to pass the experience on to others, especially to my loving students. The students have made my time in the University quite thrilling. A number of the Students, both in this country and outside it have climbed the ladder of Computer Science to the highest point. The University of Lagos which I have associated with since 1969 after I left C. M. S. Grammar School, Bariga and Old Swinford Hospital School, Stourbridge, England, is continuing to grow in stature and strength, with the support and guidance of the present Governing Council led by its very energetic Chairman, vibrant Senate under the leadership of the present Vice Chancellor, Principal Officers of the University and other members of the University community. The University is amongst the best in Nigeria. However, greater assistance is required by the Institution from all stake holders, which include Foreign, Federal and State Governments, Foreign and Local Agencies, and more importantly, the Alumni and others. There is a need to have a very strong and generous Alumni Association as is the case in many foreign universities. It is expected that the Institution to a very large extent is the "Mother Teacher". As for some of the reported investigations, with time, they will become trivial, obsolete and irrelevant. Some of the methods that were considered very relevant several years ago have naturally gone into extinction. For example, there are very few researchers left still grappling with Finite Difference Schemes as more

sophisticated methods are now available. More researchers are using AI approaches to investigate Traveling Salesman and other Combinatorics Problems now than before. These (approaches) are proving more successful than the classical ones. Calculations that were done by the old classical approaches are now being done by simply using some packages, such as, MATHEMATICA. Such is the nature of knowledge and more importantly, life. "Everything must change", recalling the title of a popular song. Living must not be a struggle for anyone. Effort should be made towards improving the quality of life of all. After all, the unfortunate, exploited and downtrodden fellow that is in your neighbourhood could have been you. How then will you feel if this was to be the case?

Incidentally, I recollect the similarity in the title of this lecture and the book, "*BEYOND CALCULATION: The Next Fifty Years of Computing*", edited by Peter J. Denning and Robert M. Metcalfe [36]. Please observe that the title of this book is slightly different from mine. It has Calculation while I have Calculations. The book contains 20 articles written by some great minds. The writers in their various contributions attempted to outline and indicate the possible directions of Computing in the next fifty years. Many of their submissions are superb and brilliant. It is instructive that you try and acquaint yourself with their thoughts. However, in my humble opinion, what is "Beyond Calculations", that which makes them meaningful, useful and worthwhile is the greatest force on earth, and that is, GOD.

The Vice Chancellor, Ladies and Gentlemen, Thank you for listening and for your patience. God keep you all safe and well.

Adetokunbo Babatunde Sofoluwe

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