Unsteady Two-dimensional Flow Analysis of Nanofluid through a Porous Channel with Expanding or Contracting Walls using Adomian Decomposition Method

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Abstract
In this paper, unsteady two-dimensional flow of nanofluid in a porous channel through expanding or contracting walls with large injection or suction is analyzed using Adomian decomposition method. From the results, it is established that increase in Reynolds number decreases the axial velocity at the center of the channel during the expansion while the axial velocity increases slightly near the surface of the channel when the wall contracts at the same rate. Also, as the wall expansion ratio increases, the velocity at the center decreases and increases near the wall. For every level of injection or suction, in the case of expanding wall, increasing acceleration leads to higher axial velocity near the center and the lower axial velocity near the wall. The approximate analytical solution is verified by numerical solution using shooting method coupled with Runge-Kutta method. The results of the Adomian decomposition method are in good agreements with the results of the numerical method. The present study will provide better physical insights into the flow phenomena such as biological fluids flow through contracting or expanding vessels, regression of the burning surface in the solid rocket motors, binary gas diffusion, transpiration cooling, flow in multichannel filtration systems etc.

Keyword: Nanofluid; Porous Channel; Expanding or Contracting walls; Adomain decomposition method.

1.0 Introduction
The vast biological and engineering applications of laminar flow through a porous channel or pipe with contracting or expanding channel or pipe permeable walls have received considerable amount of research efforts in the past few decades. This is evident in transport of biological fluids through contracting or expanding vessels, filtration in kidneys and lungs, flow inside lymphatics, the synchronous pulsation of porous diaphragms, air circulation in respiratory system, the regression of the burning surface in the solid rocket motors, binary gas diffusion, chromatography, ion exchange, ground water movement, transpiration cooling, the separation of ^{235}U from ^{238}U by gaseous diffusion and flow in multichannel filtration systems such as the wall flow monolith filter used to reduce emissions from diesel engines.(Berman, 1953; Terill, 1964, 1965; Dauenhauer and Majdalani, 2003; Majdalani and Roh, 2000; Majdalani, 2000, 2001; Majdalani and van Moorhem, 1997; Oxarango, *et al*, 2004). The permeable walls have received considerable amount of research efforts in the past few decades. The equations governing such flow are generally nonlinear. In order to predict and determine the actual flow behavior, different analytical, approximate analytical and numerical methods have been employed to solve the nonlinear governing equations. In the past studies of laminar flow through a porous channel, (Majdalani, 2001) and Majdalani and Roh, 2000, adopted asymptotic formulations using Wentzel-Krammers-Brillouin (WKB) and multiple-scale techniques to study the oscillatory channel flow with wall injection. Jankowski and Majdalani (2006) also applied the multiple-scale techniques to analyze oscillatory channel flow with arbitrary suction. Jankowski and Majdalani (2006) developed an analytical solution by means of the Liouville-Green

transformation for laminar flow in a porous channel with large wall suction and a weakly oscillatory pressure while Zhou and Majdalani (2002) used finite difference method and asymptotic technique (variation of parameters and small parameter perturbations) to investigate the meanflow for slab rocket motors with regressing walls. The results from the two methods were compared for different Reynolds numbers *Re* and the wall regression rate *a*, and it was observed that accuracy of the analytical solution deteriorates for small *Re* and large *a*. A good agreement between the solutions was observed for large values of *Re*. A similar analysis was done by Majdalani and Zhou (2003) for moderate-to-large injection and suction driven channel flows with expanding or contracting walls. Multiple solutions associated with this problem have been reported by (Robinson,1976; Zarturska,*et al*. 1988; and Si,*et al*.2011a, 2011b, Majdalani,*et al*. 2002). They applied regular perturbation method to study two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. In a recent study, Dinarvand,*et al*. (2008) adopted homotopy analysis and homotopy perturbation methods to solve Berman's model of two-dimensional viscous flow in porous channels with wall suction or injection. They concluded that the HPM solution is not valid for large Reynolds numbers, a weakness earlier observed in the case of other perturbation techniques. Using the homotopy analysis method, Xu *et al*. (2010) developed highly accurate series approximations for two-dimensional viscous flow between two moving porous walls and obtained multiple solutions associated with this problem. Also, the same method was adopted by Dinarvand and Rashidi, (2010) to analyze two dimensional viscous flow in a rectangular domain bounded by two moving porous walls. Although, the homotopy analysis method is a reliable and efficient semi-analytical technique, it however suffers from a number of limiting assumptions such as the requirements that the solution ought to conform to the so-called rule of solution expression and the rule of coefficient ergodicity. Moreover, the solution comes with large number of terms. In practice, analytical solutions with large number of terms and conditional statements for the solutions are not convenient for use by designers and engineers (Sobamowo, 2016)

Adomian decomposition method (ADM) for solving linear and nonlinear differential equations has fast gained ground as it appears in many engineering and scientific research papers. It is an approximate analytical method that could solve differential equations, difference equation, differential-difference equations, fractional differential equation, pantograph equation and integro-differential equation. It solves nonlinear integral and differential equations without linearization, discretization, closure, restrictive assumptions, perturbation, approximations, round-off error and discretization that could result in massive numerical computations. It reduces complexity of expansion of derivatives and the computational difficulties of the other traditional approximation analytical or perturbation methods. It provides excellent approximations to the solution of non-linear equation with high accuracy. Moreover, the need for small perturbation parameter as required in traditional PMs, the rigour of the derivations of differential transformations or recursive relation as carried out in DTM (Zhou, 1986), the restrictions of HPM to weakly nonlinear problems as established in literatures, and the lack of rigorous theories or proper guidance for choosing initial approximation, auxiliary linear operators, auxiliary functions, auxiliary parameters, and the requirements of conformity of the solution to the rule of coefficient ergodicity as done in HAM have all been overcome by the ADM. The other difficulties that have been overcome by the ADM include the search Langrange multiplier as carried in VIM, and the challenges associated with proper construction of the approximating functions for arbitrary domains or geometry of interest as in Galerkin weighted residual method (GWRM), least square method (LSM) and collocation method (CM).Although, the method presents its own difficulty in determining the Adomian polynomials, A_m, the resulting solutions from the method are more physically realistic. Therefore, in this present work, Adomian decomposition method is applied to analyze unsteady two-dimensional flow of nanofluid through a porous channel with expanding/contracting walls. Also, the developed solutions are used to study the effects of the flow parameters in the expanding or contracting porous channel. From the present analysis, the results obtained by the method for solving the problem under investigation are compared with the numerical solution for the non-linear case and very good agreements are established.

2. 0 Problem Formulation

Consider a fully developed unsteady, laminar, isothermal, and incompressible flow in a two-dimensional porous channel bounded by two permeable surfaces or walls that enable the nanofluid to enter or exit during successive expansions or contractions as shown in Fig. 1. One end of the channel is closed by a compliant solid membrane. Both walls are assumed to have equal permeability and to expand uniformly at a time dependent rate, $\dot{a}(t)$. A coordinate system is chosen with the origin at the center of the channel as shown in the figure.

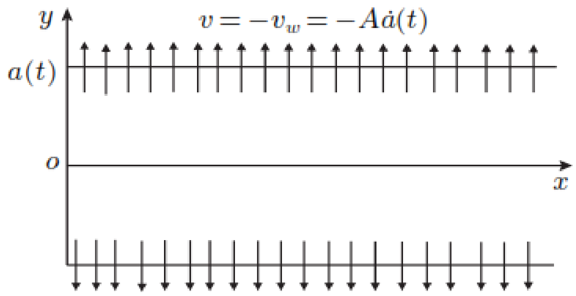

Fig. 1. The model of the porous channel with expanding or contracting walls.

Assuming Following the assumptions, the equations for continuity and motion are

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu_{nf}\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) \tag{2}$$

$$\rho_{nf}\left(\frac{\partial \bar{v}}{\partial t} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + v\frac{\partial \bar{v}}{\partial \bar{y}}\right) = -\frac{\partial \bar{p}}{\partial \bar{y}} + \mu_{nf}\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \tag{3}$$

where

$$\rho_{nf} = \rho_f\left(1-\phi\right) + \rho_s\phi \tag{4}$$

$$\mu_{nf} = \frac{\mu_f}{\left(1-\phi\right)^{2.5}} \tag{5}$$

The above Eqs. (4) and (5) are Brinkman's model of nanofluid.

Assuming no slip condition, the appropriate boundary conditions are given as

$$\bar{y} = a(t), \quad \bar{u} = 0, \quad \bar{v} = -V_w = -\frac{a}{c}$$

$$\bar{y}=0, \quad \frac{\partial \bar{u}}{\partial \bar{y}}=0, \quad \bar{v}=0$$

$$\bar{x}=0, \quad \bar{u}=0 \tag{6}$$

where $c=\frac{\dot{a}}{V_w}$ is the wall presence or injection/suction coefficient i.e. which is the measure of permeability

On introducing the following stream functions and the mean flow vorticity

$$\bar{u}=\frac{\partial \bar{\psi}}{\partial \bar{y}}, \quad \bar{v}=\frac{-\partial \bar{\psi}}{\partial \bar{x}} \tag{7}$$

The pressure term in Eqs. (2) and (3) can be eliminated and the vorticity transport equation is obtained as

$$\rho_{nf}\left(\frac{\partial \bar{\xi}}{\partial t}+\bar{u}\frac{\partial \bar{\xi}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{\xi}}{\partial \bar{y}}\right)=\mu_{nf}\left(\frac{\partial^2 \bar{\xi}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{\xi}}{\partial \bar{y}^2}\right) \tag{8}$$

where

$$\bar{\xi}=\frac{\partial \bar{v}}{\partial \bar{x}}-\frac{\partial \bar{u}}{\partial \bar{y}} \tag{9}$$

Also, the above partial differential equation can be converted to ordinary differential equation using the following similarity variables

$$\bar{u}=\frac{\mu_{nf}}{\rho_{nf}a^2(t)}\bar{f}'(\eta), \quad \bar{v}=\frac{-\mu_{nf}}{\rho_{nf}a(t)}\bar{f}(t) \tag{10}$$

where

$$\eta=\frac{\bar{y}}{H}, \quad \bar{f}'(\eta)=\frac{d\bar{f}(\eta)}{d\eta}$$

Substituting Eq. (9) and (10) into Eq. (8), we have a fourth order ordinary differential equation

$$\bar{f}_{\eta\eta\eta\eta}+\frac{\rho_{nf}v_f}{\mu_{nf}}\left\{\left[\alpha(t)\left(\eta\bar{f}_{\eta\eta\eta}+3f_{\eta\eta}\right)\right]+\bar{f}\bar{f}_{\eta\eta\eta}-\bar{f}_{\eta}\bar{f}_{\eta\eta}\right\}-\frac{\rho_{nf}a^2}{\mu_{nf}}\bar{f}_{\eta\eta t}=0 \tag{11}$$

And the following boundary conditions becomes

$$\bar{f}=0, \quad \bar{f}_{\eta\eta}=0, \quad when \quad \eta=0$$

$$\bar{f}=Re, \quad \bar{f}_{\eta}=0 \quad when \quad \eta=1 \tag{12}$$

where $\alpha\left(t\right)=\dfrac{\rho_{nf}\,a\dot{a}(t)}{\mu_{nf}}$ is the non-dimensional wall dilation rate which is positive for expansion and negative for contraction.

And $Re=\dfrac{\rho_{nf}\,aV_w}{\mu_{nf}}$ is the permeation Reynolds number, which is positive for injection and negative for suction.

Using the following variables,

$$\psi=\frac{\bar{\psi}}{a\dot{a}},\quad u=\frac{\bar{u}}{\dot{a}},\quad v=\frac{\bar{v}}{\dot{a}},\quad f=\frac{\bar{f}}{Re}\quad\Rightarrow\quad\psi=\frac{xf}{c},\quad u=\frac{xf'}{c},\quad v=\frac{-f}{c},\quad c=\frac{\alpha}{Re}\tag{13}$$

Eqs. (11) and (12) are normalized as

$$f_{\eta\eta\eta\eta}+\alpha(t)\left(\eta f_{\eta\eta\eta}+3f_{\eta\eta}\right)+Re\left(ff_{\eta\eta\eta}-f_{\eta}f_{\eta\eta}\right)-\frac{\rho_{nf}a^2}{\mu_{nf}}f_{\eta\eta t}=0\tag{14}$$

with the boundary conditions

$$f=0,\quad f_{\eta\eta}=0,\quad when\quad \eta=0$$

$$f=1,\quad f_{\eta}=0,\quad when\quad \eta=1\tag{15}$$

For self-similar solution, we have considered $f=\dfrac{\bar{f}}{Re}$ by transformation introduced by Uchida and Aoki (1977), Dauenhauer and Majdalani (2003) and Majdalani (1997, 2000, 2001, 2002, 2003). This can lead us to consider the case where $\alpha\left(t\right)=\dfrac{\rho_{nf}\,a\dot{a}(t)}{\mu_{nf}}$ is constant during the flow process, and $f=f(\eta)$. Therefore $f_{\eta\eta t}=0$ and Eq. (14) reduces to

$$f_{\eta\eta\eta\eta}+\alpha\left(\eta f_{\eta\eta\eta}+3f_{\eta\eta}\right)+Re\left(ff_{\eta\eta\eta}-f_{\eta}f_{\eta\eta}\right)=0\tag{16}$$

The boundary conditions still remain as Eq. (15).

If $\alpha=0$ in the above Eq. (16), the Berman's model (1953) for channels with stationary walls is recovered.

3. Approximate Analytical Methods of Solution: Adomian Decomposition Method

It is very difficult to generate a closed form solution for Eq. (7). Therefore, in this work, recourse is made to Adomian decomposition method which is an approximation analytical method.

Principle of Adomian decomposition method

The principle of the method is described as follows. The general nonlinear equation is in the form

$$Lu+Ru+Nu=g\tag{17}$$

The linear terms are decomposed into $L + R$, with L taken as the highest order derivative which is easily invertible and R as the remainder of the linear operator of less order than L where g is the system input or the source term and u is the system output, Nu represents the nonlinear terms, which is assumed to be analytic, $L^{-1}$is regarded as the inverse operator of L and is defined by a definite integration from 0 to x, i.e.

$$[\mathrm{L}^{-1} f](x) = \int_0^x f(v)dv \tag{18}$$

If L is a second-order operator, then L^{-1} is a double integral i.e. L^{-1} could be expressed as

$$[\mathrm{L}^{-1} f](x) = \int_1^x \int_0^x f(v)dvdv \tag{19}$$

Applying the inverse operator$L^{-1}$to the both sides of Eq. (17), and using the given conditions, the resulting equation could be written as

$$u = \mu(x) - L^{-1}Ru - L^{-1}Nu \tag{20}$$

Where $\mu(x) = \lambda_x + L^{-1}g$ and λ_x represents the term arising from integrating the source term g(x).

The Adomian methods decomposes the solution u(x) into a series

$$u = \sum_{m=0}^{\infty} u_m \tag{21}$$

and the nonlinear term into a series

$$Nu = \sum_{m=0}^{\infty} A_m \tag{22}$$

where A_m*'s* are Adomian's polynomials of u_0 ,u_1, . . ., u_m and are obtained for the nonlinearity $Nu = f(u)$from the recursive formula

$$A_m = \frac{1}{m!}\left[\frac{d^m}{d\zeta^m}[fu(\zeta)]\right]_{\zeta=0} = \frac{1}{m!}\left[\frac{d^m}{d\zeta^m} f\left(\sum_{i=0}^{\infty}\zeta^i y_i\right)\right]_{\zeta=0} \quad m = 0,1,2,3,... \tag{23}$$

where ζ is a grouping parameter of convenience.
The Adomian decomposition method defines the solution of the function $f(x)$ to be approximated as

$$f(x) = \sum_{m=0}^{\infty} f_m(x) \tag{24}$$

Eq. (16) might be written as

$$Lf = -\alpha\left(\eta f_{\eta\eta\eta} + 3 f_{\eta\eta}\right) - Re\left(ff_{\eta\eta\eta} - f_\eta f_{\eta\eta}\right) \tag{25}$$

Where L is a fourth-order differential operator i.e. $L = \frac{d^4}{d\eta^4}$, and so the fourfold integration operator L^{-1} (the inverse operator which is an integral operator) is defined by

$$L^{-1}(\bullet) = \int_0^\eta \int_0^\eta \int_0^\eta \int_0^\eta (\bullet) d\eta \tag{26}$$

Operating with L^{-1} on both sides of Eq. (25), we have

$$f(\eta) = f(0) + f'(0)\eta + f''(0)\frac{\eta^2}{2} + f'''(0)\frac{\eta^3}{6} - L^{-1}\left[\alpha\left(\eta f_{\eta\eta\eta} + 3 f_{\eta\eta}\right) + Re\left(ff_{\eta\eta\eta} - f_\eta f_{\eta\eta}\right)\right] \tag{27}$$

From the boundary conditions in Eq. (15)

$$f(0) = 0, \quad f''(0) = 0$$

However,

$f'(0)$ and $f'''(0)$ are unknown which we be represented by k_1 and k_2, respectively.

Using the above statement and inserting the boundary conditions of Eq. (15) into Eq. (27), we have

$$f(\eta) = k_1\eta + \frac{k_2\eta^3}{6} - L^{-1}\left[\alpha\left(\eta f_{\eta\eta\eta} + 3 f_{\eta\eta}\right) + Re\left(ff_{\eta\eta\eta} - f_\eta f_{\eta\eta}\right)\right] \tag{28}$$

For the sake of conformity to Adomian analysis, Eq. (28) will be written in form of

$$f(\eta) = k_1\eta + \frac{k_2\eta^3}{6} - L^{-1}\left[\alpha\left(\eta f''' + 3f''\right) + Re\left(ff''' - f'f''\right)\right] \tag{29}$$

The above makes the equation to be easily amendable to Adomian decomposition analysis.
From the Adomian decomposition analysis, the linear operator is defined as

$$Rf = 3\alpha f'' \tag{30}$$

While the nonlinear operator is defined as

$$Nf = \alpha\left(\eta f'''\right) + Re\left(ff''' - f'f''\right) \tag{31}$$

In the Adomian method, the nonlinear term is represented by an infinite series of the so-called Adomian polynomials

$$Nf = \sum_{m=0}^{\infty} A_m \tag{32}$$

Therefore, on substituting Eq. (30) and (31) into Eq. (28), Eq. (28) takes the form

$$f(\eta) = k_1\eta + \frac{k_2\eta^3}{6} - L^{-1}\left[Rf_m + \sum_{m=0}^{\infty} A_n\right] \tag{33}$$

The Adomian's polynomial, *Ai's*, are expressed as

$$A_0 = \alpha\eta f_0'''' + Re\left(f_0 f_0''' - f_0' f_0'' \right)$$

$$A_1 = \alpha\eta f_1'''' + Re\left(f_0 f_1''' + f_1 f_0''' - f_0' f_1'' - f_1' f_0'' \right) \tag{34}$$

$$A_2 = \alpha\eta f_2'''' + Re\left(f_1 f_1''' + f_0 f_2''' + f_2 f_0''' - f_2' f_0'' - f_1' f_1'' - f_0' f_2'' \right)$$

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The other polynomials are generated in similar way.

The linear terms are given by

$$Rf_0 = 3\alpha f_0''$$

$$Rf_1 = 3\alpha f_1'' \tag{35}$$

$$Rf_2 = 3\alpha f_2''$$

From the above analysis, it can easily be shown that the series solution is given as

$$f_0(\eta) = k_1\eta + \frac{k_2\eta^3}{6}$$

$$f_1(\eta) = -\frac{\alpha k_2\eta^5}{30} + \frac{Rek_2^2\eta^7}{2520} \tag{36a-c}$$

$$f_2(\eta) = \frac{\alpha^2 k_2\eta^7}{210} + \frac{\alpha k_2 k_1\eta^7}{630} - \frac{\alpha k_2^2\eta^9}{22680} - \frac{\mathrm{Re}\, k_2^2 k_1\eta^9}{45360} - \frac{\alpha Rek_2^2\eta^9}{22680} - \frac{Re^2 k_2^3\eta^{11}}{2494800}$$

Following the same procedures, $f_3(\eta)$, $f_4(\eta)$, $f_5(\eta)$, $f_6(\eta)$, $f_7(\eta)$, $\ldots f_N(\eta)$ are obtained. Recall that the Adomian decomposition method defines the solution of the function *f(η)* to be approximated as

$$f(\eta) = \sum_{m=0}^{\infty} f_m(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) + \ldots \tag{37}$$

Substituting Eq. (36) into Eq.(37), we therefore have the series solution give by ADM as

$$f(\eta) = k_1\eta + \frac{k_2\eta^3}{6} - \frac{\alpha k_2\eta^5}{30} + \frac{Rek_2^2\eta^7}{2520} + \frac{\alpha^2 k_2\eta^7}{210} + \frac{\alpha k_2 k_1\eta^7}{630} - \frac{\alpha k_2^2\eta^9}{22680} - \frac{\mathrm{Re}\, k_2^2 k_1\eta^9}{45360} - \frac{\alpha Rek_2^2\eta^9}{22680} - \frac{Re^2 k_2^3\eta^{11}}{2494800} + \ldots \tag{38}$$

where the constants k_1 and k_2 are determined using the boundary conditions in Eq. (15) i.e.

$$f(1)=1, \quad f'(1)=0$$

Also, the first-order derivatives of *f(1)* is developed as

$$f'(\eta)=k_1+\frac{k_2\eta^2}{2}-\frac{\alpha k_2\eta^4}{6}+\frac{\alpha^2k_2\eta^6}{30}+\frac{\alpha k_2k_1\eta^6}{90}-\frac{\alpha k_2^2\eta^8}{2520}-\frac{\mathrm{Re}\,k_2^2k_1\eta^8}{5040}-\frac{\alpha Re k_2^2\eta^8}{2520}-\frac{Re^2k_2^3\eta^{10}}{226800}+\ldots \tag{39}$$

5.0 Results and Discussion

The efficiency and accuracy of ADM is demonstrated in Table 1 and 2 for the obtained results of velocity distributions as compared with the numerical procedure as shown Tables 1-8. Also, the validity of ADM compared to the numerical method (NM).

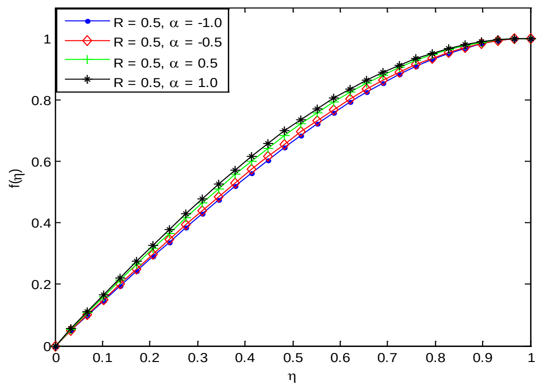
Table 1: Comparison of results of flow for large Reynolds number and suction

$f(\eta)$ when Re = 5, α=0.5			
η	NM	ADM	Diff.
0.0	0.000000	0.000000	0.000000
0.1	0.152874	0.152943	0.000069
0.2	0.301551	0.301686	0.000131
0.3	0.442606	0.442776	0.000170
0.4	0.573196	0.573375	0.000179
0.5	0.690876	0.691038	0.000162
0.6	0.793373	0.793501	0.000128
0.7	0.878373	0.878458	0.000085
0.8	0.943297	0.943340	0.000043
0.9	0.985090	0.985101	0.000011
1.0	1.000000	1.000000	0.000000

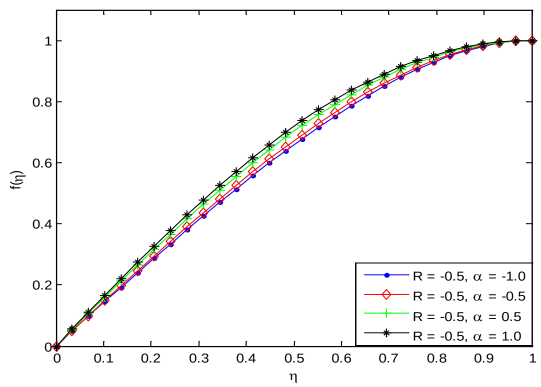
Table 2: Comparison of results of flow for large Reynolds number, injection and suction

$f'(\eta)$ when Re = 5, α=0.5			
η	NM	ADM	Diff.
0.0	1.536154	1.536154	0.000000
0.1	1.151411	1.147136	0.004275
0.2	1.453855	1.420974	0.032881
0.3	1.362554	1.345095	0.017459
0.4	1.245207	1.246060	0.000853
0.5	1.104631	1.123815	0.019184
0.6	0.941489	0.976409	0.034920
0.7	0.754210	0.800039	0.045829
0.8	0.539188	0.591549	0.052361
0.9	0.290458	0.345618	0.055160
1.0	0.000000	0.000000	0.000000

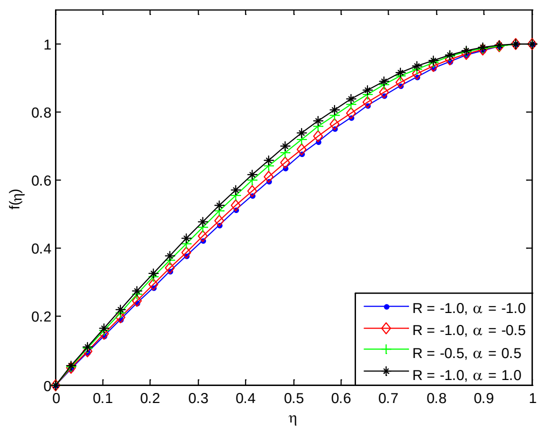
Comparing the solution of ADM with the numerical solutions using shooting method coupled with Runge-Kutta scheme shows that obtained solutions through the ADM demonstrate good accuracy with the numerical solutions. This accuracy gives high confidence about validity of ADM in providing solutions to the problems.



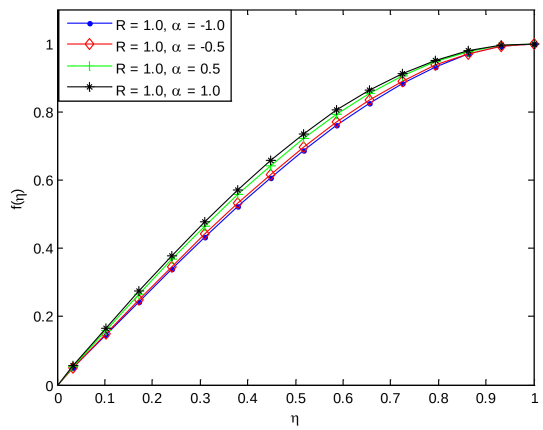
(a)



(b)



(c)



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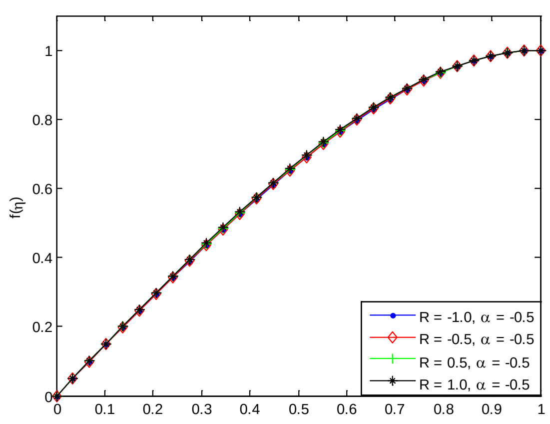

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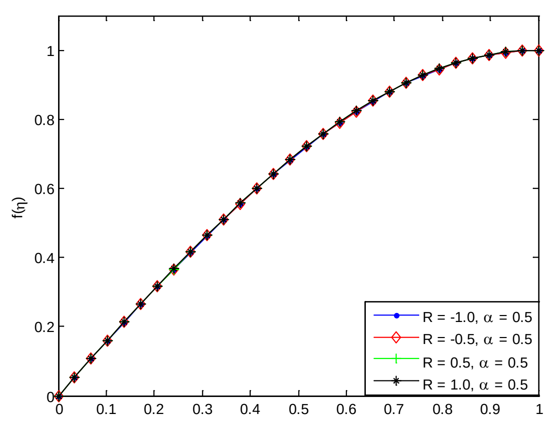
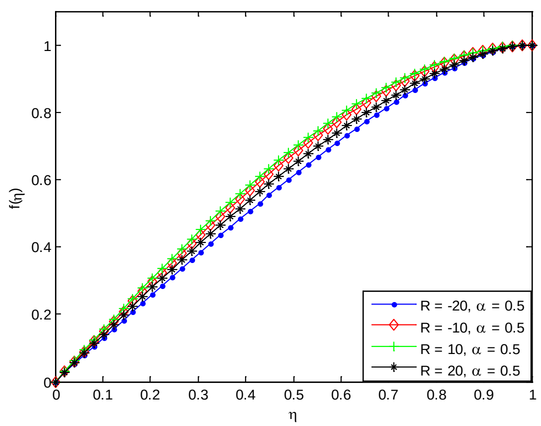
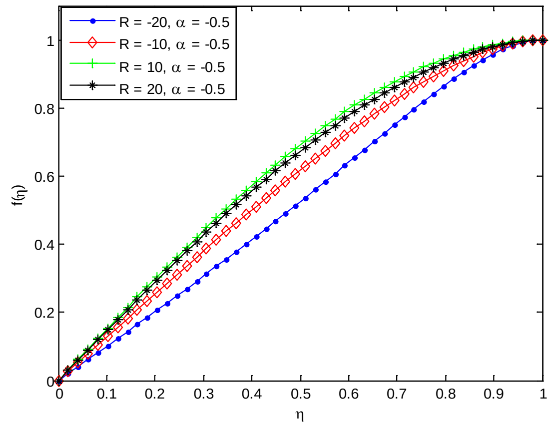


(f)



(g)



(h)

Fig. 1 Variation of f (η) for different expansion and contraction ratio, α and different small values of Re

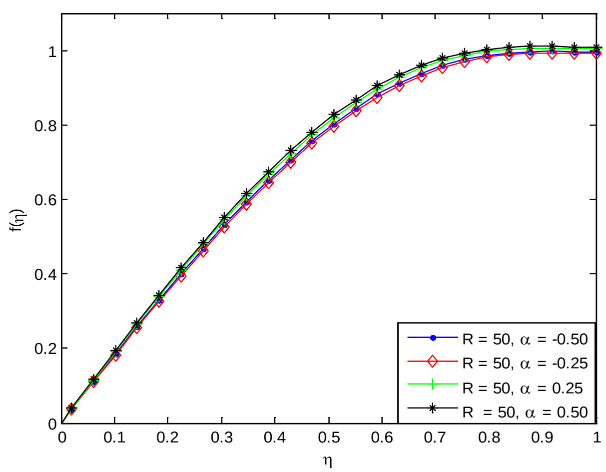
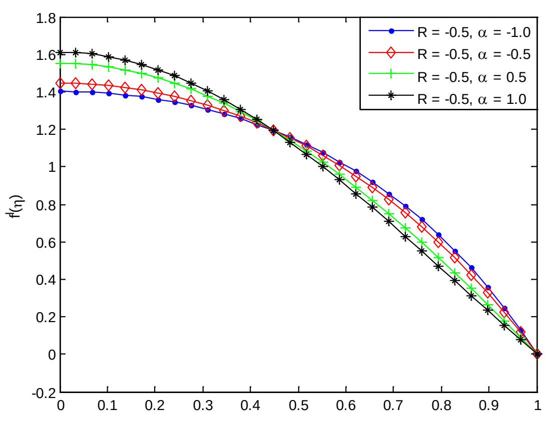
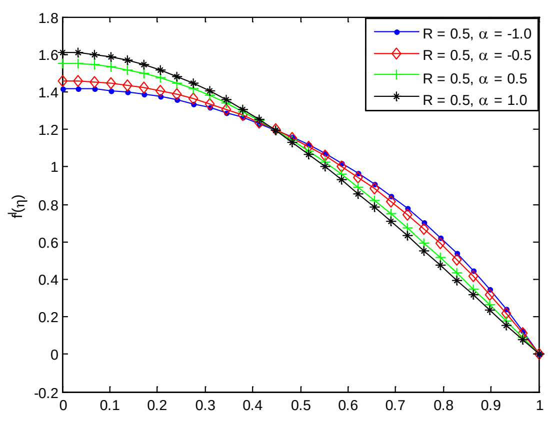


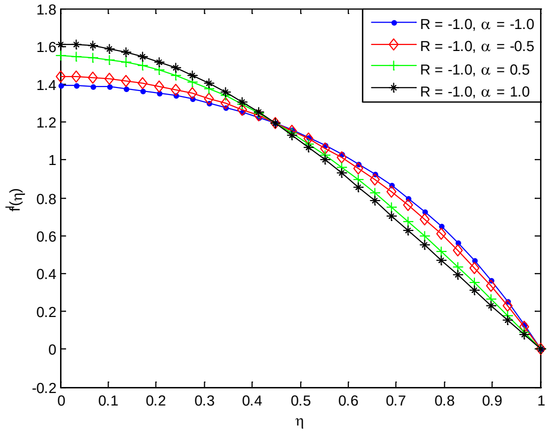
Fig. 2 Variation of f (η) for different expansion and contraction ratio, α and large value of Re

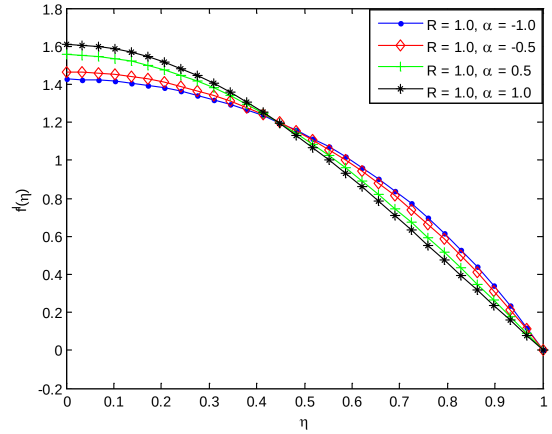


(a)

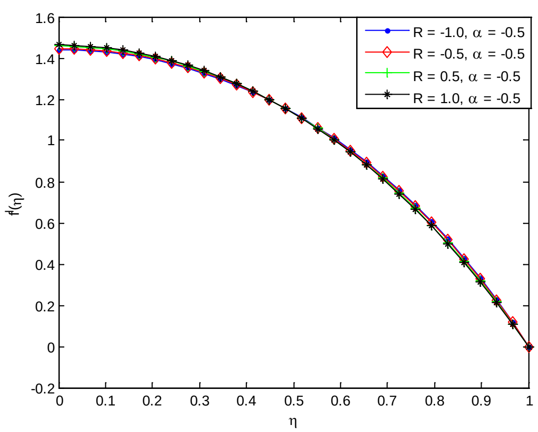


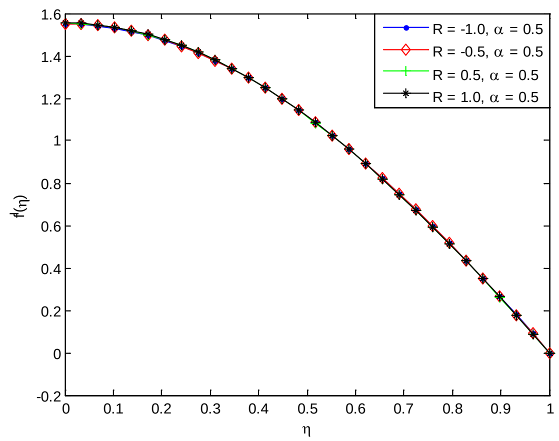
(b)





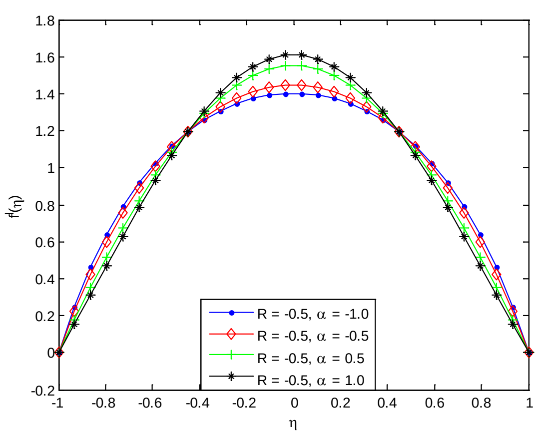
(c) (d)

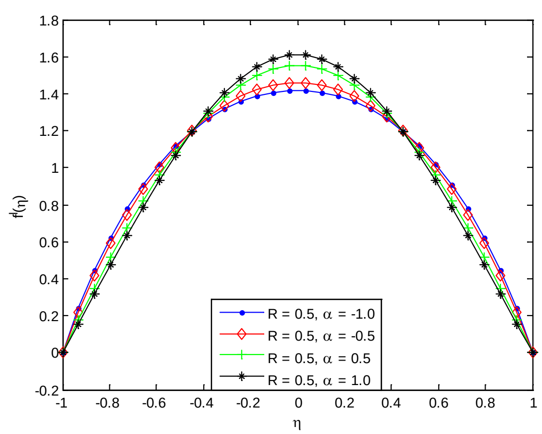




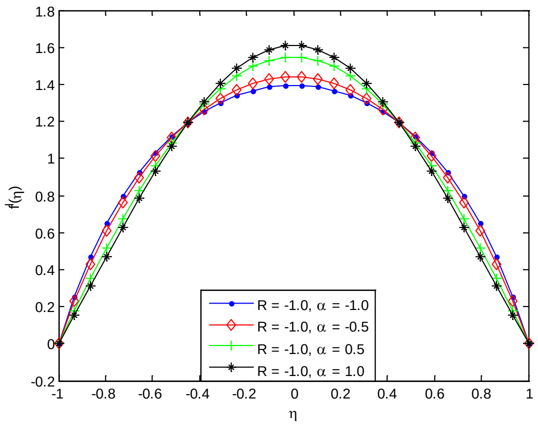
(e) (f)

Fig. 3 Variation of f' (η) for different expansion and contraction ratio, α and different small values of Re

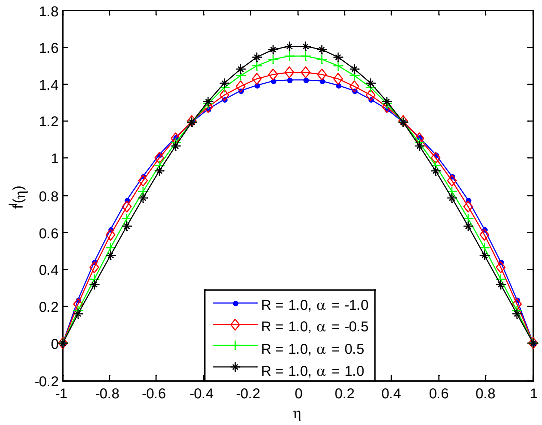




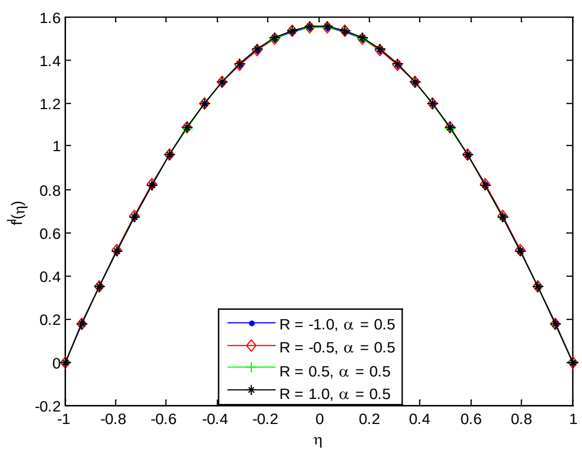
(a) (b)



(c)

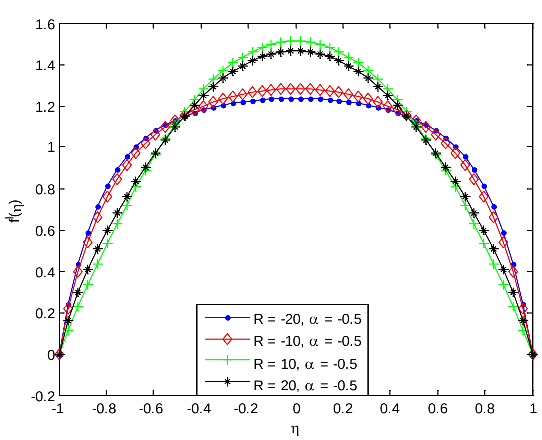


(d)

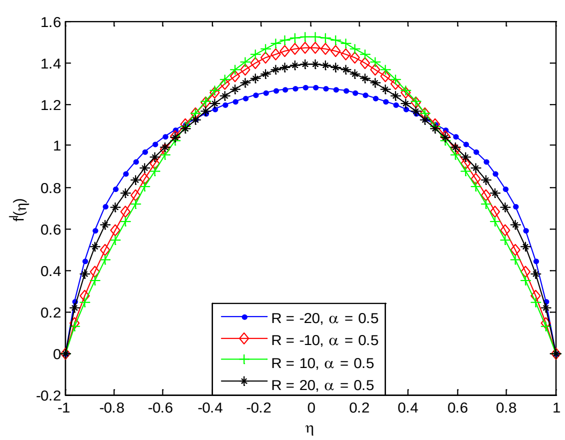


(e)

Fig. 4 Variation of f' (η) for different expansion and contraction ratio, α and different small values of Re



(a)



(b)

Fig. 5 Variation of f' (η) for different expansion and contraction ratio, α and different values of Re

Figs. 1 show the effects of the permeation Reynolds number and non-dimensional wall dilation rate on the dimensionless flow velocities. Fig. 1e, 1f, 1g, 1h, 4e, 5a and 5b show the effects of Reynolds number, Re, on the velocity at constant non-dimensional wall dilation rate on the dimensionless axial velocity. Increase in the Reynolds number decreases the axial velocity at the center of the channel during the expansion while the axial velocity increases slightly near the surface of the channel when the wall contracts at the same rate. The behavior of axial velocity for different permeation Reynolds number, over a range of non-dimensional wall dilation rate were plotted in Figs. 1a-1d, 2, 3a-f, 4a-4d. The figures depict that, for every level of injection or suction, the velocity is maximum at the center of the channel and near the point, the velocity is increased when the channel is expanding and decrease when the channel contracts. That is because the flow toward the center becomes greater to make up for the space caused by the expansion of the wall and as a result the axial velocity also becomes greater near the center. As the wall expansion ratio increases, the velocity at the center decreases and increases near the wall. Similarly, for the case of contracting wall as shown in Fig. 4a-d and 5a, increasing contraction ratio leads to lower axial velocity near the center and the higher near the wall because the flow toward the wall becomes greater and as a result the axial velocity near the wall becomes greater. So, both the expansion and suction through the wall reinforce the flow through the channel and similarly does the wall contraction and injection through the surface. The results of the present study show that for every level of injection or suction, in the case of expanding wall, increasing $a(t)$ leads to higher axial velocity near the center and the lower axial velocity near the wall.

5.0 Conclusion

In this work, unsteady two-dimensional unsteady flow of nanofluid in a porous channel through expanding/contracting walls with large injection or suction has been analyzed using Adomian decomposition method. From the results, it was established that increase in the Reynolds number decreases the axial velocity at the center of the channel during the expansion while the axial velocity increases slightly near the surface of the channel when the wall contracts at the same rate. Also, as the wall expansion ratio increases, the velocity at the center decreases and increases near the wall. For every level of injection or suction, in the case of expanding wall, increasing wall dilation rate leads to higher axial velocity near the center and the lower axial velocity near the wall. The comparison of the results of Adomian decomposition method with results of numeral method using Runge-Kutta with shooting method showed remarkable accuracy. This work will enhance the understanding biological fluids flow through contracting or expanding vessels, filtration in kidneys and lungs, flow inside lymphatics, the synchronous pulsation of porous diaphragms, air circulation in respiratory system, the regression of the burning surface in the solid rocket motors, binary gas diffusion, chromatography, ion exchange, ground water movement, transpiration cooling, the separation of ^{235}U from ^{238}U by gaseous diffusion and flow in multichannel filtration systems such as the wall flow monolith filter used to reduce emissions from diesel engines.

Nomenclature

$\dot{a}$ time-dependent rate

B_0 electromagnetic induction

Ha Hartmann number

$\bar{p}$ pressure

Re permeation Reynolds number

t time

$\bar{u}$ velocity component in x-direction

$\bar{v}$ velocity component in y-direction

V_w fluid inflow velocity at the wall

$\bar{x}$ coordinate axis parallel to the channel walls

$\bar{y}$ coordinate axis perpendicular to the channel walls

ρ_{nf} density of the nanofluid

ρ_f density of the fluid

μ_{nf} dynamic viscosity of the nanofluid

ρ_s density of the nanoparticles

ϕ fraction of nanoparticles in the nanofluid

σ electrical conductivity

α dimensionaless wall dilation rate

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